

# An Offline Electronic Payment System Based on an Untraceable Blind Signature Scheme

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## Abstract

This paper proposes a new offline electronic payment (e-payment) system that satisfies the major security requirements of e-payment, i.e. anonymity, unlinkability, unforgeability, double spending control, conditional traceability, and fraud prevention. The central idea is the use of Hwang et al.'s RSA-based untraceable blind signature (BS), which disables the link between the e-coin and its owner and ensures the anonymity of both the customer and the merchant. It attaches an expiration, a deposit and the transaction dates to each e-coin in order to manage the database of the bank effectively, to correctly calculate the interest on the e-coin and to aid arbitration if a dishonest customer attempts to double-spend the coin. It also ensures the anonymity of the customer as long as the coin is spent legitimately. Only when a fraudulent e-coin transaction is detected can the bank, with the help of the central authority (a trusted entity), determine the identity of the dishonest customer. The system is referred to as offline since the bank does not need to be concurrently involved in transactions between a customer and a merchant. Finally, analyses of the performance of the prototype and the primary security requirements of the proposed system are also presented.

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**Keywords:** Electronic payment system, RSA, Untraceable blind signature, Digital signature

## 1. Introduction

Unlike conventional payment systems, an electronic payment (e-payment) system uses cryptographic protocols and computer networks to exchange an electronic coin (e-coin) and goods between a customer and a merchant with the assistance of a bank. Here, the customer and the merchant involved in this system do not need to interact with each other physically; this is therefore a convenient system with respect to time and conveyance. On the other hand, an e-payment system is more vulnerable than conventional payment systems, since it is easy to copy an e-coin for misuse or double spending. E-payment systems can be categorized as online and offline. In an online system [2-4], the bank is generally able to check each payment between the customer and the merchant. Thus, if double spending occurs, the bank can terminate the transaction instantly. However, since the bank remains online during each payment, this causes transaction congestion; this also sacrifices the anonymity of the customer. In contrast, an offline system [5-7, 10] allows a customer to pay an e-coin to a merchant without the involvement of a bank or any other central authority (CA). Finally, the merchant can deposit the e-coin obtained from the customer to a bank account at a convenient time. Here, double spending is checked only after connecting with the bank at the time of depositing the coin. Thus, an offline system gives rise to less congestion; it also provides anonymity and privacy to the customer from the bank, the merchant and any other third party during the payment. An offline system therefore has specific advantages.

For offline e-payment systems, a number of blind signature (BS)-based systems have been proposed [5, 7, 8, 10, 11, 15]. However, the majority of the existing BS-based systems are not completely untraceable. Unlinkability is an essential requirement for an e-payment system in order to maintain the anonymity of the customer, and relies on untraceable BS. To ensure anonymity in an offline e-payment system, the e-coin usually contains the identity of its owner in the blind form. When using this information, the merchant, the bank or any attackers are unable to identify the owner of the e-coin; only the CA knows how to link the e-coin and its owner to determine the owner's identity if she is dishonest. Thus, anonymity depends on unlinkability, and these are complementary requirements.

In any offline e-payment system, there is a controversial relationship between untraceability and the detection of double spending. Untraceability disables the link between the e-coin and its owner, and thereby ensures the privacy of the honest customer. On the other hand, the detection and prevention of double spending are also important, so that a dishonest customer cannot double-spend an e-coin and a merchant cannot deposit the same e-coin more than once. An ideal e-payment system must therefore satisfy the following requirements [8, 10].

1. *Anonymity*: The eligibility of the customer is proven without publicly disclosing her identity or information. Thus, no one but the CA can determine the identity of the customer from the contents of the e-coin.
2. *Unlinkability*: No one but the CA can determine the link between the e-coin and its owner. This helps to ensure the anonymity of the customer.
3. *Unforgeability*: Only the Bank can produce legal e-coins.
4. *Double Spending Control*: No e-coin is allowed to be spent more than once.
5. *Conditional Traceability*: No one but the CA should be able to determine the identity of a dishonest customer when double spending occurs.
6. *Theft prevention*: No one but the real owner of a valid e-coin can spend it successfully.
7. *Offline Payments*: When transactions between the customer and the merchant occur, the bank does not need to be concurrently involved.

8. *Date Attachability*: Expiration, transaction and deposit dates should be attached to each e-coin.

In order to satisfy the above requirements, this paper proposes a new offline e-payment system. To achieve anonymity and unlinkability, the proposed system exploits Hwang et al.'s [1] RSA-based untraceable BS, in which only the owner of the e-coin and the CA can determine the link between the e-coin and its owner. It should be mentioned that although a number of BS schemes are available in the domain of cryptography, the majority of these cannot meet the requirement for untraceability. However, Hwang et al.'s RSA-based BS fully satisfies all the requirements of an ideal BS scheme [1]. The bank's signature on each e-coin, which uses an RSA digital signature, ensures authenticated and confidential communication. The proposed system also embeds three types of dates with each e-coin: the expiration date, the transaction date and the deposit date [8, 10]. The expiration date allows the information of expired e-coins to be erased from the bank's database. This prevents the database from becoming too large through the constant increase in e-coin information. The transaction date and the deposit date aid in the detection of dishonest customers, who may try to duplicate the e-coin and perform double spending. These dates also help in the detection of dishonest merchants who may try to deposit the same e-coin more than once. In order to prevent the forgery of an e-coin when the bank issues it to a customer, the bank attaches the customer's public verification key to it.

The rest of this paper is organized as follows. Section 2 summarizes some related works. Section 3 briefly explains the cryptographic tools required to develop the proposed system. Section 4 explains the configuration of the system, and Section 5 describes the individual stages of the system. Sections 6 and 7 discuss the experimental analysis and the security analysis of the system respectively. Finally, Section 8 concludes this paper.

## 2. Related Works

Extensive research on e-payment systems or schemes has been carried out previously. The scheme proposed in [14] first introduced offline e-payment systems. In order to ensure the anonymity of the customer, this work introduced the concept of BS, although this system was unable to achieve untraceability. These authors used cut-and-choose technology to address the double spending problem. However, cut-and-choose technology is highly inefficient in terms of the data exchange between customers and merchants during the payment process. To overcome the limitations of this approach, several schemes were later proposed which did not rely on cut-and-choose technology to address the double spending problem [16, 17]. They also aimed to meet the requirements of an ideal offline e-payment system. Unfortunately, none of these were globally acceptable. Most of the recently proposed schemes have been shown to be susceptible to serious security flaws, and some are inefficient to implement in practice. Schemes which address the divisibility and transferability of e-coins are generally inefficient, since all of the transaction history of a payment is recorded within an e-coin for the purposes of detection of double spending. The size of the e-coin therefore gradually increases with every transaction, and this limits the maximum number of transactions.

The scheme proposed in [5] failed to achieve anonymity and unlinkability, which are essential requirements of an offline e-payment system [7]. The offline payment system proposed in [10] also has three security flaws: an attack on the detection of double spending, possible forging of the expiration date of a valid e-coin, and cheating on the exchange protocol, as shown by the cryptanalysis presented in [8]. Again, the cryptanalysis presented in [19]

showed that the offline payment scheme proposed in [8] has security flaws in its verifiability, double spending detection and unforgeability. In this system, a malicious spender can withdraw coins without injecting their actual identity, and can forge a valid coin using the homomorphic property of modular operation. Although the Mondex system developed by the National Westminster Bank in the UK successfully ensures the anonymity of the customer, it is vulnerable to illegal fund transfers which cannot be traced [20]. Bitcoin is a globally successful cryptocurrency following a distributed mode of transaction in which a bank has no role. Its primary advantage is that it reduces the high processing costs to the bank; however, its major drawback is that transactions are public, and sensitive transaction data can be traced [18].

### 3. Cryptographic Building Blocks

This section discusses the required cryptographic tools that are used in developing the proposed e-payment system. These are the RSA digital signature, the RSA cryptosystem, and Hwang et al.'s RSA-based untraceable BS [1]. RSA-based schemes are chosen here since they are by far the easiest to understand and implement of all the public-key algorithms proposed over the years [12].

#### 3.1 The RSA Digital Signature

1. Any entity  $X$  (e.g. the customer, the merchant, the bank or the central authority) chooses two large primes  $p_x$  and  $q_x$ .
2.  $X$  then computes  $n_x = p_x \cdot q_x$  and  $S_x V_x \equiv 1 \pmod{(p_x - 1)(q_x - 1)}$ .
3.  $X$  then retains  $(S_x, p_x, q_x)$  as a private signing key and publishes  $(V_x, n_x)$  as a public verification key.
4.  $X$  chooses a message  $m$  for signature verification.
5.  $X$  carries out the signing operation using  $X$ 's private signing key  $S_x$ , i.e.,  $m_s = m^{S_x} \pmod{n_x}$ .
6. Later on, a verification operation is carried out using  $X$ 's public verification key  $V_x$ , i.e.  $m = m_s^{V_x} \pmod{n_x}$ .

#### 3.2 The RSA Cryptosystem

1. Any entity  $X$  (e.g. the customer, the merchant, the bank or the central authority) chooses two large primes  $p_{x'}$  and  $q_{x'}$ .
2.  $X$  then computes  $n_{x'} = p_{x'} \cdot q_{x'}$  and  $E_{x'} D_{x'} \equiv 1 \pmod{(p_{x'} - 1)(q_{x'} - 1)}$ .
3.  $X$  then retains  $(D_{x'}, p_{x'}, q_{x'})$  as a private decryption key and publishes  $(E_{x'}, n_{x'})$  as a public encryption key.
4.  $X$  chooses a message  $m$  for encryption-decryption.
5.  $X$  carries out the encryption operation using  $X$ 's public encryption key  $E_{x'}$ , i.e.  $m_d = m^{E_{x'}} \pmod{n_{x'}}$ .
6. Later on, a decryption operation is carried out by  $X$  using  $X$ 's private decryption key  $D_{x'}$ , i.e.  $m = m_d^{D_{x'}} \pmod{n_{x'}}$ .

#### 3.3 Hwang et al.'s Blind Signature

Hwang et al.'s blind signature [1] is based on the RSA cryptosystem and consists of five phases, which are described as follows.

1. *Initialization phase*: This phase is the same as the RSA digital signature. The signer keeps  $(S, p, q)$  as the secret signing key and publishes  $(V, n)$  as the public verification key.
2. *Blinding phase*: The customer or the merchant,  $Y$ , requires a message  $m$  (i.e. her identity) to be signed by the bank. First, she randomly chooses two distinct integers  $r_1$  and  $r_2$  as the blinding factors. Following this, she randomly chooses two distinct primes  $a_1$  and  $a_2$  such that  $a_1 \neq a_2$  and the greatest common divisor (GCD)  $(a_1, a_2)$ , is 1. Then,  $Y$  computes the blinded messages  $\alpha_1 = r_1^V \cdot m^{a_1} \bmod n$  and  $\alpha_2 = r_2^V \cdot m^{a_2} \bmod n$ , and sends  $(\alpha_1, \alpha_2)$  to the bank.
3. *Signing phase*: After receiving  $(\alpha_1, \alpha_2)$  from  $Y$ , the bank randomly chooses two distinct primes  $b_1$  and  $b_2$  such that  $b_1 \neq b_2$  and  $GCD(b_1, b_2) = 1$ , and signs the blinded message by computing  $t_1 = \alpha_1^{b_1 S} \bmod n$  and  $t_2 = \alpha_2^{b_2 S} \bmod n$ . Now  $(t_1, t_2)$  is a signed, blinded message. The bank sends  $(t_1, t_2, b_1, b_2)$  to  $Y$ .
4. *Unblinding phase*: After receiving  $(t_1, t_2, b_1, b_2)$  from the bank,  $Y$  computes  $a_1 b_1$  and  $a_2 b_2$ . Due to the use of four distinct primes  $(a_1, a_2, b_1, b_2)$  where  $GCD(a_1, a_2) = 1$  and  $GCD(b_1, b_2) = 1$ ,  $GCD(a_1 b_1, a_2 b_2)$  is also equal to 1. Since  $GCD(a_1 b_1, a_2 b_2) = 1$ , there must be exactly two integers  $w$  and  $t$  that satisfy the equation  $a_1 b_1 w + a_2 b_2 t = 1$ . This is known as the extended Euclidean algorithm [12]. The four parameters  $(a_1, a_2, w, t)$  are kept secret by  $Y$ . Now  $Y$  computes  $s_1 = t_1 \cdot r_1^{-b_1} = m^{a_1 b_1 S} \bmod n$  and  $s_2 = t_2 \cdot r_2^{-b_2} = m^{a_2 b_2 S} \bmod n$ . Following this,  $Y$  derives the signature  $S$  by computing  $S = s_1^w \cdot s_2^t \bmod n$  and then publishes  $(m, S)$ .
5. *Verification phase*: As a result,  $S$  is the signature on the message  $m$ . Now anyone can verify the legitimacy of the signature by checking whether  $S^V = m \bmod n$ .

#### 4. Configuration of the Payment System

The entities involved in the proposed system are a trusted central authority (CA), the bank, the customer and the merchant. The roles of these entities are described below.

##### The Central Authority (CA)

1. The CA is the trusted judge in the system, and conducts the registration of the other entities involved in the system, signing the bank account number of every customer and the merchant. When registering, the bank, the customer and the merchant interact with the CA using their identity; the CA therefore knows their identities. Moreover, the CA manages a database that stores the information of the customer and the merchant involved with the same Bank, and this is shown in **Table 1**.
2. The CA possesses a separate private key and public key (using the RSA cryptosystem/digital signature) for both encryption-decryption and signing-verification. The CA's private signing key is  $(S_{ca}, p_{cas}, q_{cas})$ , the public verification key is  $(V_{ca}, N_{cas})$ , the public encryption key is  $(E_{ca}, N_{cae})$  and the private decryption key is  $(D_{ca}, p_{cae}, q_{cae})$ .
3. In addition, the CA has another secret encryption-decryption key pair, i.e.  $\{(E_s, N_s), (D_s, N_s)\}$ , for encryption and decryption respectively of the identity of the customer and the merchant; these are stored in the database as shown in the first column of **Table 1**.

### The Bank

1. The bank manages bank accounts for both the customer and the merchant, and authenticates these with a bank account number signed by the CA. The bank also manages a database that contains the information of the customer and the merchant, as shown in **Table 2**.
2. Prior to signing the e-coin, the bank attaches an expiration date to it. The bank also stores the withdrawn e-coin in the withdrawal database, as shown in **Table 3**.
3. When a merchant asks to deposit an e-coin, the bank verifies the validity of this e-coin and deposits it in the deposit database, as shown in **Table 4**. The bank also detects and prevents double spending, and discloses the identity of any double spender with the help of the CA.
4. Again, the bank renews a customer's unused but outdated e-coin with a new expiration date, and modifies the withdrawal information of the e-coin, as shown in **Table 3**.
5. The bank possesses a separate private key and public key (using the RSA cryptosystem/digital signature) for both encryption-decryption and signing-verification. The bank's private signing key is  $(S_{bn}, p_{bns}, q_{bns})$ , the public verification key is  $(V_{bn}, N_{bns})$ , the private decryption key is  $(D_{bn}, p_{bne}, q_{bne})$  and the public encryption key is  $(E_{bn}, N_{bne})$ . The bank also possesses an identity  $ID_{bn}$  and two primes  $(p_{bn1}, p_{bn2})$  such that  $p_{bn1} \neq p_{bn2}$  and  $GCD(p_{bn1}, p_{bn2}) = 1$ .

### The Customer

1. The customer withdraws e-coins from her bank account  $AN_{cn}$ . Before withdrawal, she must be authenticated by the bank as a valid bank account holder. The customer also blinds her identity  $ID_{cn}$  (using Hwang et al.'s BS) to keep herself anonymous with respect to the other entities involved in the system and to all other third parties except the CA. She therefore uses her blinded identity  $(\alpha_{cn1}, \alpha_{cn2})$  when withdrawing the e-coin from the bank and paying it to the merchant. She also renews her unused but outdated e-coin from the bank with a new expiration date.
2. She possesses a separate private key and public key (using the RSA cryptosystem/digital signature) for both encryption-decryption and for signing-verification. Her private signing key is  $(S_{cn}, p_{cns}, q_{cns})$ , her public verification key is  $(V_{cn}, N_{cns})$ , her private decryption key is  $(D_{cn}, p_{cne}, q_{cne})$ , her public encryption key is  $(e_{cn}, N_{cne})$  and her identity number is  $ID_{cn}$ . She randomly chooses two distinct blinding factors  $(b_{cn1}, b_{cn2})$  and two primes  $(p_{cn1}, p_{cn2})$  such that  $p_{cn1} \neq p_{cn2}$  and  $GCD(p_{cn1}, p_{cn2}) = 1$ , and another two secret integers  $w$  and  $t$ .

### The Merchant

1. The merchant sells goods to the customer in exchange for the e-coin. He also blinds his identity  $ID_{mn}$  (using Hwang et al.'s BS) to keep himself anonymous from all other entities except the CA. He verifies the e-coin received from the customer. He also attaches a transaction date  $D_m$  and a deposit date  $D_{dn}$  to the valid e-coin, and deposits it with the bank.
2. He possesses a separate private key and public key (using the RSA cryptosystem/digital signature) for both encryption-decryption and signing-verification. His private signing key is  $(S_{mn}, p_{mns}, q_{mns})$ , his public verification key is  $(V_{mn}, N_{mns})$ , his private decryption key is  $(D_{mn}, p_{mne}, q_{mne})$ , his public encryption

key is  $(E_{mn}, N_{mne})$  and his identity number is  $ID_{mn}$ . He randomly chooses two distinct blinding factors  $(b_{mn1}, b_{mn2})$  and two primes  $(p_{mn1}, p_{mn2})$  such that  $p_{mn1} \neq p_{mn2}$  and  $GCD(p_{mn1}, p_{mn2}) = 1$ .

**Table 1.** Database of the central authority

Encrypted identity of customer/merchant	Blinded identity of customer/merchant	The bank's identity
$E(ID_{c1})$	$(\alpha_{c11}, \alpha_{c12})$	$ID_{b1}$
$E(ID_{m1})$	$(\alpha_{m11}, \alpha_{m12})$	$ID_{b1}$
$E(ID_{c2})$	$(\alpha_{c21}, \alpha_{c22})$	$ID_{b2}$
$E(ID_{m2})$	$(\alpha_{m21}, \alpha_{m22})$	$ID_{b2}$
.	.	.
.	.	.
$E(ID_{cn})$	$(\alpha_{cn1}, \alpha_{cn2})$	$ID_{bn}$
$E(ID_{mn})$	$(\alpha_{mn1}, \alpha_{mn2})$	$ID_{bn}$

**Table 2.** Database of the bank, containing the information of the customer and merchant

Blinded identity of customer/merchant	Account number	Account balance (\$)
$(\alpha_{c11}, \alpha_{c12})$	$AN_{c1}$	2000
$(\alpha_{m11}, \alpha_{m12})$	$AN_{m1}$	5000
$(\alpha_{c21}, \alpha_{c22})$	$AN_{c2}$	4000
$(\alpha_{m21}, \alpha_{m22})$	$AN_{m2}$	3000
.	.	.
.	.	.
$(\alpha_{cn1}, \alpha_{cn2})$	$AN_{cn}$	2000
$(\alpha_{mn1}, \alpha_{mn2})$	$AN_{mn}$	1000

**Table 3.** Withdrawal information database of the bank

E-coin	Customer's blinded identity
$(t_{c11}, t_{c12}, V_{c1}, A_{ec1}, D_{ec1}, A_{c1}, D_{c1}, e-coin_{c1})$	$(\alpha_{c11}, \alpha_{c12})$
$(t_{c21}, t_{c22}, V_{c2}, A_{ec2}, D_{ec2}, A_{c2}, D_{c2}, e-coin_{c2})$	$(\alpha_{c21}, \alpha_{c22})$
$(t_{c31}, t_{c32}, V_{c3}, A_{ec3}, D_{new3}, A_{c3}, D_{c3}, e-coin_{c3})$	$(\alpha_{c31}, \alpha_{c32})$
.	.
$(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, e-coin_{cn})$	$(\alpha_{cn1}, \alpha_{cn2})$

**Table 4.** Deposit database of the bank

E-coin	Deposit Date	Transaction Date
$(D_{d1}, t_{c11}, t_{c12}, V_{c1}, A_{ec1}, D_{ec1}, A_{c1}, D_{c1}, e-coin_{c1})$	$D_{d1}$	$D_{t1}$
$(D_{d2}, t_{c21}, t_{c22}, V_{c2}, A_{ec2}, D_{ec2}, A_{c2}, D_{c2}, e-coin_{c2})$	$D_{d2}$	$D_{t2}$
.	.	.
$(D_{dn}, t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, e-coin_{cn})$	$D_{dn}$	$D_{tn}$



## 5. Individual Stages of the System

The proposed system consists of six stages: registration, withdrawal, payment, deposit, renewal, and tracing. Each of these stages is described below.

### 5.1 Registration Stage

The objective of this stage is to register the bank, the customer, and the merchant with the CA, so that the CA can later certify their identities. For this purpose, the bank, the customer and the merchant must provide the necessary information to the CA when they register. This stage is described in detail below and is illustrated in Fig. 1.

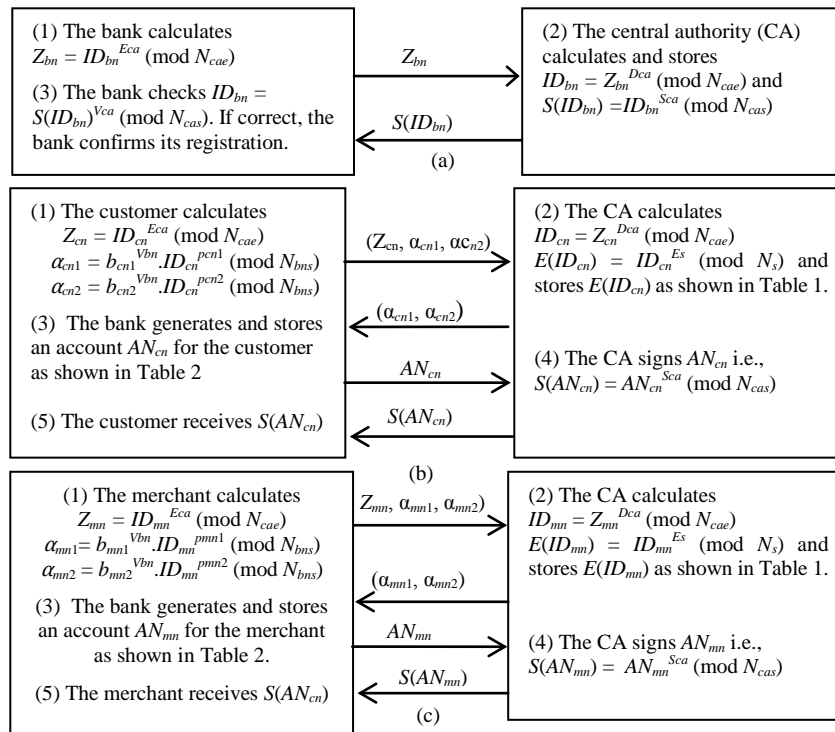


Fig. 1. Data flows at the registration stage for (a) the bank, (b) the customer, and (c) the merchant

#### Step1: Registration of the bank

1. The bank encrypts its identity  $ID_{bn}$ , i.e. the bank calculates  $Z_{bn} = ID_{bn}^{E_{ca}} \pmod{N_{cae}}$  where  $(E_{ca}$  and  $N_{cae})$  form the CA's public encryption key. The bank then sends  $Z_{bn}$  to the CA.
2. The CA decrypts  $Z_{bn}$  with its private decryption key  $D_{ca}$  to obtain and store  $ID_{bn}$  in its database, as shown in Table 1.
3. Now the CA signs  $ID_{bn}$  using its private signing key  $(S_{ca}, N_{cas})$ , i.e. the CA calculates  $S(ID_{bn}) = ID_{bn}^{S_{ca}} \pmod{N_{cas}}$  and sends  $S(ID_{bn})$  to the bank to confirm its registration.
4. The bank verifies  $S(ID_{bn})$  using the CA's public verification key,  $(V_{ca}$  and  $N_{cas})$ , i.e. the bank checks  $ID_{bn} = S(ID_{bn})^{V_{ca}} \pmod{N_{cas}}$ . If this is correct, the bank is convinced of its registration.



### Step2: Registration of the customer

1. First the customer encrypts her identity  $ID_{cn}$ , i.e., the customer calculates  $Z_{cn} = ID_{cn}^{Eca} \pmod{N_{cae}}$  and submits this to the CA, where  $(E_{ca}, N_{cae})$  is the CA's public encryption key. Then, using Hwang et al.'s BS, the customer blinds her identity  $ID_{cn}$  as  $(\alpha_{cn1}, \alpha_{cn2})$ , i.e. the customer calculates  $\alpha_{cn1} = b_{cn1}^{Vbn} \cdot ID_{cn}^{pcn1} \pmod{N_{bns}}$  and  $\alpha_{cn2} = b_{cn2}^{Vbn} \cdot ID_{cn}^{pcn2} \pmod{N_{bns}}$  where  $(p_{cn1}, p_{cn2})$  are two distinct primes and  $(b_{cn1}, b_{cn2})$  are two distinct blinding factors chosen by the customer. Now the customer sends  $Z_{cn}, \alpha_{cn1}, \alpha_{cn2}$  to the CA.
2. After receiving this from the customer, the CA decrypts  $Z_{cn}$  with its private decryption key  $D_{ca}$  to retrieve the customer's identity  $ID_{cn}$ . To keep the customer's  $ID_{cn}$  secret, the CA again encrypts  $ID_{cn}$  with the CA's personal secret encryption key  $(E_s)$  and computes  $E(ID_{cn}) = ID_{cn}^{Es} \pmod{N_s}$ .
3. Following this, the CA stores  $(\alpha_{cn1}, \alpha_{cn2})$  which corresponds to the customer's encrypted identity  $E(ID_{cn})$ , as shown in **Table 1**, and sends  $(\alpha_{cn1}, \alpha_{cn2})$  to the bank.
4. The bank generates and stores a bank account number  $AN_{cn}$  for the customer, which corresponds to  $(\alpha_{cn1}, \alpha_{cn2})$ , as shown in **Table 2**, and sends  $AN_{cn}$  to the CA.
5. The CA signs  $AN_{cn}$ , i.e. the CA calculates  $S(AN_{cn}) = AN_{cn}^{Sca} \pmod{N_{cas}}$  and sends it to the customer. Now the customer can withdraw the e-coin from her account while authenticating herself to the bank using  $S(AN_{cn})$ .

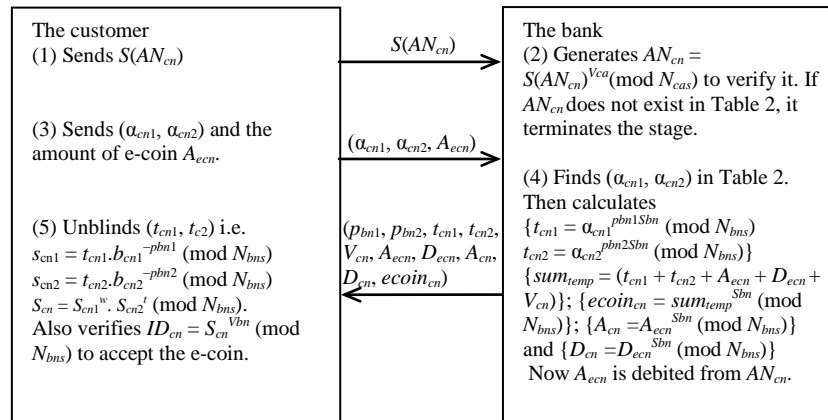
### Step 3: Registration of the merchant

1. First the merchant encrypts his identity  $ID_{mn}$ , i.e., the merchant calculates  $Z_{mn} = ID_{mn}^{Eca} \pmod{N_{cae}}$  and submits this to the CA, where  $(E_{ca}, N_{cae})$  is the CA's public encryption key. Then, using Hwang et al.'s BS, the merchant blinds his identity  $ID_{mn}$  as  $(\alpha_{mn1}, \alpha_{mn2})$ , i.e. the merchant calculates  $\alpha_{mn1} = b_{mn1}^{Vbn} \cdot ID_{mn}^{pmn1} \pmod{N_{bns}}$  and  $\alpha_{mn2} = b_{mn2}^{Vbn} \cdot ID_{mn}^{pmn2} \pmod{N_{bns}}$ , where  $(p_{mn1}, p_{mn2})$  are two distinct primes and  $(b_{mn1}, b_{mn2})$  are two distinct blinding factors chosen by the merchant. Now the merchant sends  $Z_{mn}, \alpha_{mn1}, \alpha_{mn2}$  to the CA.
2. After receiving this from the merchant, the CA decrypts  $Z_{mn}$  with its private decryption key  $D_{ca}$  to retrieve the merchant's identity  $ID_{mn}$ . To keep the merchant's  $ID_{mn}$  secret, the CA again encrypts  $ID_{mn}$  with the CA's secret encryption key  $(E_s)$ , and computes  $E(ID_{mn}) = ID_{mn}^{Es} \pmod{N_s}$ .
3. Following this, the CA stores  $(\alpha_{mn1}, \alpha_{mn2})$ , which corresponds to the merchant's encrypted identity  $E(ID_{mn})$ , as shown in **Table 1**, and sends  $(\alpha_{mn1}, \alpha_{mn2})$  to the bank.
4. The bank generates and stores a bank account number  $AN_{mn}$  for the merchant corresponding to  $(\alpha_{mn1}, \alpha_{mn2})$ , as shown in **Table 2**, and sends  $AN_{mn}$  to the CA.
5. The CA signs  $AN_{mn}$ , i.e. the CA calculates  $S(AN_{mn}) = AN_{mn}^{Sca} \pmod{N_{cas}}$  and sends it to the merchant. Now the merchant can deposit the e-coin to his account while authenticating himself to the Bank using  $S(AN_{mn})$ .

## 5.2 Withdrawal Stage

The objective of this stage is to enable a legitimate customer to withdraw the e-coin deposited in her bank account. For this purpose, the customer first needs to be authenticated by the bank by her bank account number, i.e.  $S(AN_{cn})$ , which has already been signed by the CA. This stage is described in detail below and is illustrated in **Fig. 2**.

1. The customer sends  $S(AN_{cn})$  to the bank. The bank then verifies  $S(AN_{cn})$  with the CA's public verification key, i.e. the bank generates  $AN_{cn} = S(AN_{cn})^{V_{ca}} \pmod{N_{cas}}$  where  $(V_{ca}, N_{cas})$  is the CA's public verification key. If  $AN_{cn}$  does not exist in the bank's database, as shown in **Table 2**, the bank terminates the process at this stage.
2. Otherwise, the bank asks the customer to send her identity in its blinded form, i.e.  $(\alpha_{cn1}, \alpha_{cn2})$  and stores it in the bank's database, as shown in **Table 2**.
3. Following this, the customer requests an e-coin of amount  $A_{ecn}$  from the bank by submitting  $(\alpha_{cn1}, \alpha_{cn2})$ .
4. If  $(\alpha_{cn1}, \alpha_{cn2})$  does not exist in the bank's database, it terminates the process at this stage.
5. Otherwise the bank signs  $(\alpha_{cn1}, \alpha_{cn2})$  using Hwang et al.'s BS, i.e. the bank calculates  $t_{cn1} = \alpha_{cn1}^{p_{bn1}S_{bn}} \pmod{N_{bns}}$  and  $t_{cn2} = \alpha_{cn2}^{p_{bn2}S_{bn}} \pmod{N_{bns}}$ , where  $p_{bn1}$  and  $p_{bn2}$  are the two distinct primes and  $S_{bn}$  is the bank's private signing key.
6. The bank then calculates the withdrawn e-coin as  $sum_{temp} = (t_{cn1} + t_{cn2} + A_{ecn} + D_{ecn} + V_{cn})$ , where  $A_{ecn}$  is the e-coin amount requested by the customer from the bank,  $D_{ecn}$  is the expiration date of the e-coin, and  $V_{cn}$  is the public verification key of the customer. Then the Bank signs  $sum_{temp}$  with its private signing key  $S_{bn}$ , i.e. the bank calculates  $ecoin_{cn} = sum_{temp}^{S_{bn}} \pmod{N_{bns}}$ ,  $A_{cn} = A_{ecn}^{S_{bn}} \pmod{N_{bns}}$ , and  $D_{cn} = D_{ecn}^{S_{bn}} \pmod{N_{bns}}$ . A valid e-coin involves an eight-tuple, i.e.  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$ .
7. The bank then stores this eight-tuple e-coin  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$  corresponding to the customer's blinded identity  $(\alpha_{cn1}, \alpha_{cn2})$  in the bank's withdrawal database, as shown in **Table 3**. The bank also debits  $A_{ecn}$  from the customer's bank account and sends this e-coin to the customer along with its distinct primes  $(p_{bn1}, p_{bn2})$ .
8. After receiving  $(p_{bn1}, p_{bn2}, t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$  from the bank, the customer unblinds  $(t_{cn1}, t_{cn2})$ , i.e. the customer calculates  $\{s_{cn1} = t_{cn1} \cdot b_{cn1}^{-p_{bn1}} \pmod{N_{bns}}, s_{cn2} = t_{cn2} \cdot b_{cn2}^{-p_{bn2}} \pmod{N_{bns}}\}$  and  $S_{cn} = S_{cn1}^w \cdot S_{cn2}^t \pmod{N_{bns}}$ , where  $w$  and  $t$  are the two secret integers held by the customer. Finally the customer verifies  $S_{cn}$  using the bank's public verification key  $(V_{cn}, N_{bns})$ , i.e. the customer calculates  $ID_{cn} = S_{cn}^{V_{bn}} \pmod{N_{bns}}$ .
9. If the correct  $ID_{cn}$  is calculated, the customer accepts the e-coin.



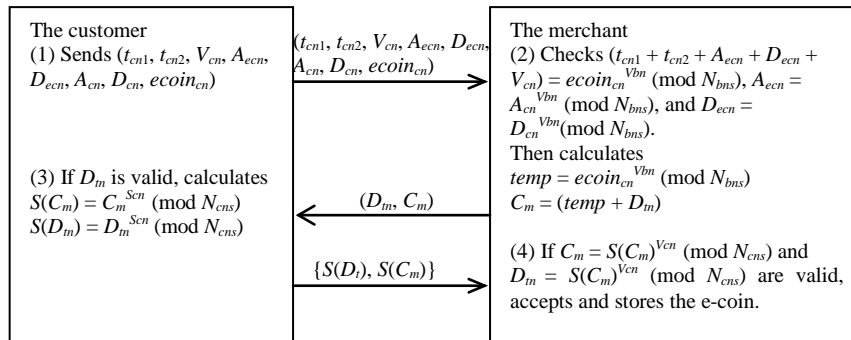
**Fig. 2.** Data flows at the withdrawal stage

### 5.3 Payment Stage

The objective of this stage is to pay the merchant the e-coin for the goods purchased by the customer. After withdrawing the e-coin from the bank, the customer can make this payment. Here, the merchant does not need to communicate with the bank while the customer pays him.

Furthermore, it is easy for the merchant to verify the e-coin received from the customer. This stage is described in detail below and is illustrated in **Fig. 3**.

1. The customer firstly sends the eight-tuple e-coin  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$  to the merchant.
2. After receiving this from the customer, the merchant checks whether  $(t_{cn1} + t_{cn2} + A_{ecn} + D_{ecn} + V_{cn}) = ecoin_{cn}^{Vbn} \pmod{N_{bns}}$ ,  $A_{ecn} = A_{cn}^{Vbn} \pmod{N_{bns}}$  and  $D_{ecn} = D_{cn}^{Vbn} \pmod{N_{bns}}$ . If this is found to be incorrect, the merchant terminates the process at this stage.
3. Otherwise, the merchant decides that it is a valid e-coin. Then he checks the expiration date  $D_{ecn}$  to confirm whether the e-coin is outdated. If it is outdated, the merchant terminates the process at this stage.
4. Otherwise, the merchant calculates  $temp = ecoin_{cn}^{Vbn} \pmod{N_{bns}}$ , from which he again calculates  $C_m = (temp + D_m)$ , where  $D_m$  is the transaction date. Now the merchant sends  $(D_m, C_m)$  to the customer to sign both of these individually.
5. After receiving  $(D_m, C_m)$  from the merchant, the customer checks  $D_m$  to confirm the validity of the transaction's date and time with respect to the current time.
6. If it is valid, the customer individually signs both  $D_m$  and  $C_m$  with her private signing key, i.e. the customer calculates  $S(C_m) = C_m^{Scn} \pmod{N_{cns}}$  and  $S(D_m) = D_m^{Scn} \pmod{N_{cns}}$ , and sends these to the merchant.
7. After receiving  $S(C_m)$  and  $S(D_m)$  from the customer, the merchant decrypts both of them using the customer's public key. If the original  $C_m$  and  $D_m$  sent to the customer are retrieved, he accepts the e-coin and sells the goods to the customer. The merchant stores  $S(C_m)$  and  $S(D_m)$  until he successfully deposits the e-coin. If double spending is detected,  $S(C_m)$  and  $S(D_m)$  can be used to prove that the transaction of the e-coin was valid. Since the e-payment system is offline, the merchant does not need to deposit the e-coin to the bank immediately; it can be deposited at any convenient time.



**Fig. 3.** Data flows at the payment stage

#### 5.4 Deposit Stage

The objective of this stage is to enable the merchant to deposit the e-coin obtained from the customer to his bank account. For this purpose, the merchant attaches the deposit date to the e-coin received from the customer. When the merchant deposits the e-coin, the bank checks for double spending. If the e-coin has already been deposited, the bank simply ignores the deposit of the e-coin, alerting the merchant that he is trying to deposit the same e-coin more than once. Moreover, if the e-coin has already been spent, the bank discloses the identity of the e-coin owner with the help of the CA, as discussed below in Section 5.6 (the tracing stage). This stage is described in detail below and is illustrated in **Fig. 4**.

1. Firstly, for authentication, the merchant submits his bank account number  $S(AN_{mn})$ , which has already been signed by the CA, to the bank. If authenticated, the merchant attaches the signed transaction date  $S(D_m)$  and deposit date  $D_{dn}$  with the e-coin received from the customer and sends  $(S(D_m), D_{dn}, t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$  along with his blinded identity, i.e.  $(\alpha_{mn1}, \alpha_{mn2})$ , to the bank.
2. The bank then checks the correctness of the expiration date  $D_{ecn}$ , the signed transaction date  $S(D_m)$ , and the deposit date  $D_{dn}$ . To verify  $S(D_m)$ , the bank calculates  $D_m = S(D_m)^{V_{cn}} \pmod{N_{cns}}$ .
3. If the Bank finds that the e-coin exists in the withdrawal database, as shown in [Table 3](#), and does not exist in the deposit database, as shown in [Table 4](#), it proceeds to Step 4. Otherwise, the bank rejects the e-coin and sends a rejection message to the merchant.
4. Finally, the bank stores the e-coin with the deposit date  $D_{dn}$  and transaction date-time  $D_m$ , as shown in [Table 4](#), in the merchant's bank account.

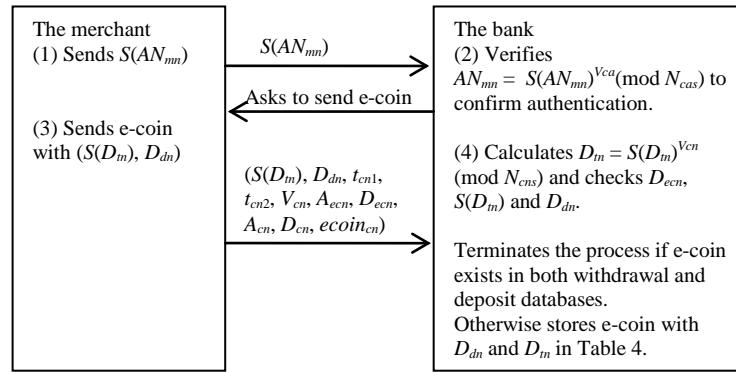


Fig. 4. Data flows at the deposit stage

### 5.5 Renewal Stage

The objective of this stage is to enable the customer to renew any unused but outdated e-coin. This stage protects the database of the bank from becoming prohibitively large due to the continuous operations of withdrawing and depositing e-coin. When an unused e-coin expires, the customer renews it at the bank with a new expiration date. This stage is described in detail below and is illustrated in [Fig. 5](#).

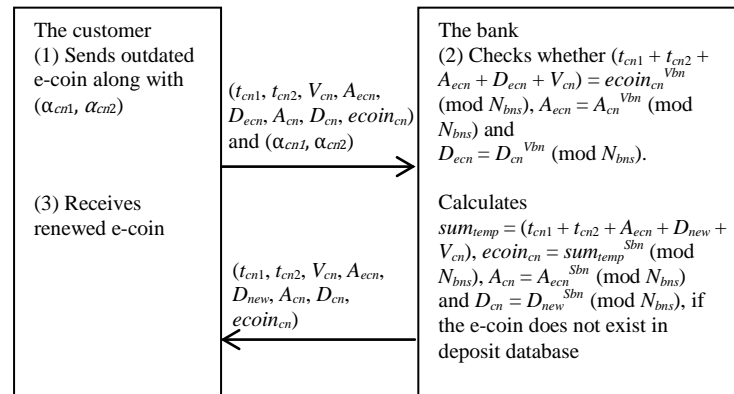
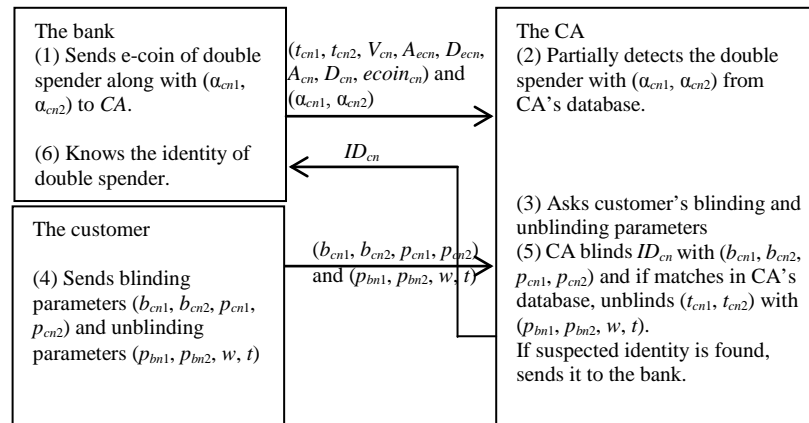


Fig. 5. Data flows at the renewal stage

1. The customer sends an unused but outdated e-coin  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$  along with her blinded identity  $(\alpha_{cn1}, \alpha_{cn2})$ .
2. The bank checks whether  $(t_{cn1} + t_{cn2} + A_{ecn} + D_{ecn} + V_{cn}) = ecoin_{cn}^{vbn} \pmod{N_{bns}}$ ,  $A_{ecn} = A_{cn}^{vbn} \pmod{N_{bns}}$  and  $D_{ecn} = D_{cn}^{vbn} \pmod{N_{bns}}$ . If this is not correct, the Bank terminates the process at this stage.
3. Otherwise, the bank checks whether the e-coin exists in the deposit database, as shown in [Table 4](#).
4. If the e-coin does not exist in the deposit database, the bank attaches a new expiration date  $D_{new}$  to renew the e-coin and calculates  $sum_{temp} = (t_{cn1} + t_{cn2} + A_{ecn} + D_{new} + V_{cn})$ . The bank then signs on  $sum_{temp}$  with its private signing key  $S_{bn}$ , i.e. the bank calculates  $ecoin_{cn} = sum_{temp}^{Sbn} \pmod{N_{bns}}$ , and  $D_{cn} = D_{new}^{Sbn} \pmod{N_{bns}}$ . Now  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{new}, A_{cn}, D_{cn}, ecoin_{cn})$  is a valid e-coin, and the bank sends it to the customer so that she can use it to pay to the merchant.
5. The bank then replaces the previous eight-tuple e-coin in the bank's withdrawal database  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$  with the renewed one, i.e.  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{new}, A_{cn}, D_{cn}, ecoin_{cn})$  corresponding to the customer's blinded identity  $(\alpha_{cn1}, \alpha_{cn2})$ , as shown in [Table 3](#).

## 5.6 Tracing Stage



**Fig. 6.** Data flows at the tracing stage

The objective of this stage is to detect double spending by a customer. When an e-coin is double-spent, this implies the existence of two e-coins with the same record. The bank checks for double spending when it receives these e-coins. If double spending occurs, the bank discloses the identity of the e-coin's owner with the help of the CA. This stage is described in detail below and is illustrated in [Fig. 6](#).

1. When the bank receives the e-coin from the merchant for depositing, the bank checks whether the e-coin already exists in the deposit database and/or the withdrawal database. If the e-coin exists in the withdrawal database but not in the deposit database, the bank terminates the process at this stage.
2. If the e-coin exists in both databases, the Bank sends the eight-tuple e-coin  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$  along with the customer's blinded identity, i.e.  $(\alpha_{cn1}, \alpha_{cn2})$  to the CA.

3. The CA then searches  $(\alpha_{cn1}, \alpha_{cn2})$  in the second column of the CA's database, as shown in **Table 1**, to determine the identity of the dishonest customer. The CA retrieves the identity  $ID_{cn}$  by decrypting the contents of the first column of **Table 1** using its secret decryption key  $(D_s, N_s)$ . Thus, the CA partially detects the double spending.
4. In order to detect any dishonesty of the bank towards the customer, the CA asks the suspended customer to send her blinding parameters, i.e.  $(b_{cn1}, b_{cn2}, p_{cn1}, p_{cn2})$ , and unblinding parameters, i.e.  $(p_{bn1}, p_{bn2}, w, t)$ , as described in Sections 4 and 5.2.
5. Using the blinding parameters of the suspended customer, the CA then blinds the customer's identity  $ID_{cn}$  and checks whether this matches with her previously stored blinded identity, i.e.  $(\alpha_{cn1}, \alpha_{cn2})$ , as shown in **Table 1**. If it matches, the CA confirms that the customer has sent the authentic blinding parameters to the CA.
6. The CA then unblinds  $(t_{cn1}, t_{cn2})$  using  $(p_{bn1}, p_{bn2}, w, t)$  as described in Section 5.2. If the unblinded identity matches that of the suspended customer, the CA confirms double spending and sends the customer's identity to the bank.

## 6. Experimental Analysis

This section evaluates the performance of the proposed system.

### 6.1 Experimental Setup

To measure the computational time requirements for the withdrawal, payment and deposit stages, a prototype of the proposed system was developed using an Intel Core i3-2.20 GHz processor with 2 GB of RAM running on a Windows 7 operating system. For encryption, GMP [13] with a 1024-bit modulus was used. All computation times exclude communication times. Moreover, the various operations of this system that are not related to cryptography are not considered here.

### 6.2 Experimental Results

The operational stages of the proposed system consist of authentication, encryption, decryption, blinding, signing, unblinding and verification. The registration stage involves the highest number of operations; that is, 18 exponent operations and one summation. The withdrawal stage consists of 10 exponent operations and one summation; the payment stage consists of eight exponent operations and two summations; the deposit stage consists of five exponent operations and one summation; the renewal stage consists of six exponent operations and two summations; and the tracing stage consists of 12 exponent operations and two summations. **Table 5** shows the required exponent operations and the computation time for the withdrawal, payment and deposit stages.

**Table 5.** Major cryptographic operations required at each stage of the proposed system and computation time required for the basic three stages

Stage	Major operations	Time requirement (ms)
Registration	18 E	-
Withdrawal	10 E	29
Payment	8 E	24
Deposit	5 E	22
Renewal	6E	-
Tracing	12 E	-

### 6.3 Comparisons

This section compares the proposed system with the systems in [8-10], in terms of operations of modular multiplication ( $M$ ) and modular exponentials ( $E$ ). The operations considered for comparison are the computational cost of withdrawing and paying an e-coin by a customer ( $c_1$ ); the computational cost to the bank at the withdrawal stage ( $c_2$ ); the computational cost to the merchant of verifying the e-coin ( $c_3$ ); the transaction mode ( $c_4$ ); and the fundamental hard problem of the secure e-payment system ( $c_5$ ). These are presented in Table 6 below.

Table 6. Performance comparisons

Case	Eslami and Talebi [10]	Juang [9]	Baseri et al. [8]	Proposed system
c1	5E+9M	3E+6M	6E+8M	7E+11M
c2	1E+2M	1E+2M	2E+1M	6E+6M
c3	6E+3M	2E+2M	6E+2M	3E+3M
c4	Offline	Offline	Offline	Offline
c5	Factoring, DLP	DLP	Factoring, DLP	Factoring

### 6.4 Comparison of Security Requirements

This section compares the proposed system with those in [8-10] in terms of major security requirements. This comparison is presented in Table 7 below.

Table 7. Comparison of requirements

Requirements	[10]	[9]	[8]	Proposed system
Anonymity	√	√	√	√
Unforgeability	√	√	×	√
Double spender detection	×	√	×	√
Date attachment	√	×	√	√

## 7. Security Analysis

The proposed e-payment system satisfies the requirements listed below.

**Anonymity and unlinkability:** In this system, even if an adversary knows all the information of a particular customer except her secret parameters, i.e.  $(p_{cn1}, p_{cn2}, w, t)$ , the e-coin cannot be linked to its owner. Here, a valid e-coin is the eight-tuple  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$ , where  $(t_{cn1}, t_{cn2})$  is the blinded signed identity of the customer who owns this e-coin. An adversary who does not know  $p_{cn1}, p_{cn2}, w$  and  $t$  cannot unblind  $(t_{cn1}, t_{cn2})$ , and therefore cannot identify the owner of the e-coin. Again, if the customer wants to obtain a further e-coin by choosing secret parameters such as  $(pcn1', pcn2', w', t')$ , her second e-coin is then  $(tcn1', tcn2', Vcn, Aecn', Decn', Acn', Dcn', ecoin_{cn}')$ . Thus, no one can detect that these two e-coins originate from the same customer; only the CA can detect this when these e-coins are used dishonestly.

**Unforgeability:** In order to forge a valid e-coin, an adversary needs to alter the content of  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn})$  from the eight-tuple of a valid e-coin, since  $A_{cn}$  is the signature of the bank on the amount of e-coin  $A_{ecn}$ ;  $D_{cn}$  is the bank's signature on the expiration date  $D_{ecn}$ ; and  $ecoin_{cn}$  is the signature of the bank on  $(t_{cn1} + t_{cn2} + A_{ecn} + D_{ecn} + V_{cn})$ . If an adversary alters the content of either  $A_{cn}$  or  $D_{cn}$  or  $ecoin_{cn}$ , it cannot re-generate the bank's signatures on these. Therefore an adversary cannot forge the e-coin.



**Theft prevention:** When a customer pays her e-coin  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$  to a merchant, the merchant verifies the customer as a valid e-coin owner or not through  $V_{cn}$ , the customer's public verification key. In order to do this, the merchant sends  $D_m$  to the customer to get her signature, i.e. the customer calculates  $S(D_m) = D_m^{S_{cn}} \pmod{N_{cns}}$ . The merchant then verifies  $S(D_m)$  using  $V_{cn}$ . If the original  $D_m$  is found, he accepts the e-coin. Since, the customer's private signing key  $S_{cn}$  is secret, no one except the customer can generate the signature on  $D_m$ . Thus, the merchant cannot re-spend or double-spend the e-coin accepted from the customer, and only a valid e-coin owner can spend it successfully.

**Detection of double spending:** No one can alter the customer's public verification key  $V_{cn}$  within the eight-tuple of a valid e-coin  $(t_{cn1}, t_{cn2}, V_{cn}, A_{ecn}, D_{ecn}, A_{cn}, D_{cn}, ecoin_{cn})$  because it is bundled in  $ecoin_{cn}$  and is generated by the signature of the bank on  $(t_{cn1} + t_{cn2} + A_{ecn} + D_{ecn} + V_{cn})$ . Thus, only the valid e-coin owner can double-spend a valid e-coin, as she has the private signing key corresponding to her public verification key  $V_{cn}$ . However, in order to detect double spending, the bank separately maintains withdrawal and deposit databases as shown in [Tables 3](#) and [4](#) respectively. If an e-coin already exists in the deposit database, the bank rejects the deposit of that e-coin. If an already deposited e-coin comes from a different merchant, this shows that the customer has paid the same e-coin to two different merchants. The bank then discloses the identity of the e-coin owner, as discussed in Section 5.6 (the tracing stage).

## 8. Conclusions

In this paper, an offline e-payment system is proposed which fully meets the primary security requirements of e-payment systems. Here, the use of Hwang et al.'s RSA-based untraceable BS ensures the anonymity of any customer, the unforgeability of e-coins and the untraceability of an e-coin to its owner. In this system, the customer is anonymous to all entities (although partially known to the CA) and is prevented from double-spending since her blinded signed identity is attached to the e-coin. With the help of the bank and the attempted spender, only the CA is able to determine the identity of the owner of the e-coin from the contents of the e-coin. Thus the system maintains the anonymity of the customer more securely compared with existing systems. The system ensures the prevention of theft through the attachment of the public verification key to the e-coin. Moreover, because the dates of transactions are attached to the e-coin at various stages in the system, the bank can easily control the size of its database and contribute to arbitration between the double spender and the merchant. Since the system also offers the renewal of e-coins, the customer can renew any unused but outdated e-coins with the bank. A comparison of computational costs at the various stages of the system with several existing systems shows the proposed system is scalable.

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