JPNT 6(2), 71-78 (2017)
https://doi.org/10.11003/JPNT.2017.6.2.71

JPNT Journal of Positioning, Navigation, and Timing

A Periodic Analysis of Sidereal Shifts for GPS Satellites and the Solar Wind Stream

Changhyun Cho†, Byung-Kyu Choi

Space Geodesy Group, Korea Astronomy and Space Science Institute, Daejeon 34055, Korea

ABSTRACT

The sidereal day of a Global Positioning System (GPS) satellite was intended to equal one half of a sidereal day of the Earth. However, the sidereal day of GPS satellites has become unequal to one half of a sidereal day of the Earth. This is fundamentally caused by the non-sphericity of the Earth and the gravity of the Moon. The difference between sidereal days of GPS satellites and the Earth is known as a sidereal shift. The details surrounding sidereal shifts and their origins have yet to be fully understood. We calculated the periodicity of sidereal shifts for GPS satellites using broadcast ephemeris data. To conduct a periodic analysis of the sidereal shift, we employ the Lomb-Scargle periodogram method. It shows that the orbit periods of GPS satellites have small-amplitude perturbations with a 13.6-day period. In addition, we compare the GPS satellite orbit periods with the periodicity of geomagnetic indices and the solar wind parameters to identify the cause of the perturbations. Our results suggest that the solar wind stream might also affect the 13.6-day period of the sidereal shifts.

Keywords: sidereal shift, period, GPS, orbit, solar wind stream

1. INTRODUCTION

The sidereal day of a Global Positioning System (GPS) satellite was designed to equal half of a sidereal day of the Earth. Therefore, a GPS satellite should be located in the same position at every sidereal day (Agnew & Larson 2007). This repeatability could potentially be utilized to improve GPS positioning accuracy by mitigating multipath noise, which is highly sensitive to the geometry of the satellite constellation and the receiver. This method is known as sidereal filtering (Bock 1991, Genrich & Bock 1992, Ragheb et al. 2007, Zhong et al. 2010, Atkins & Ziebart 2016).

However, it was discovered that the sidereal day of a GPS satellite is not exactly one half of a sidereal day of the Earth and instead appears to demonstrate a variation with time (Seeber et al. 1998, Choi et al. 2004, Axelrad et al. 2005, Agnew & Larson 2007). This time discrepancy is known as a sidereal shift (Lee et al. 2001, Weiss 2007). The sidereal shift of a GPS satellite is defined as the difference between twice the orbital period of the GPS satellite and the sidereal day of the Earth. Previous studies have shown that the sidereal shift for GPS satellites is not zero and that the variation in the sidereal shift with respect to time is different for each satellite (Seeber et al. 1998, Choi et al. 2004, Axelrad et al. 2005, Agnew & Larson 2007). In addition, a recent study showed that the average sidereal shift for GPS satellites is approximately 9 seconds (Choi et al. 2004, Axelrad et al. 2005, Agnew & Larson 2007). Based on this discrepancy, Choi et al. (2004) proposed a modified sidereal filtering method, and it has since been proven that the multipath error could be more effectively eliminated by employing this approach (Schwahn & Söhne 2009, Gang et al. 2012, Lou et al. 2014).

There are some known factors that affect the sidereal shifts of GPS satellites. The oblateness of the Earth is the fundamental factor that induces a drift in the sidereal shift, and the gravity of the Moon contributes to oscillations of the shifts (Choi et al. 2004, Axelrad et al. 2005, Agnew & Larson 2007). Other potential factors, however, including the solar

Received Jan 25, 2017 **Revised** Mar 07, 2017 **Accepted** Apr 12, 2017

†**Corresponding Author**
E-mail: chcho@kasi.re.kr
Tel: +82-42-865-3298 Fax: +82-42-865-3358

http://www.ipnt.or.kr Print ISSN: 2288-8187 Online ISSN: 2289-0866

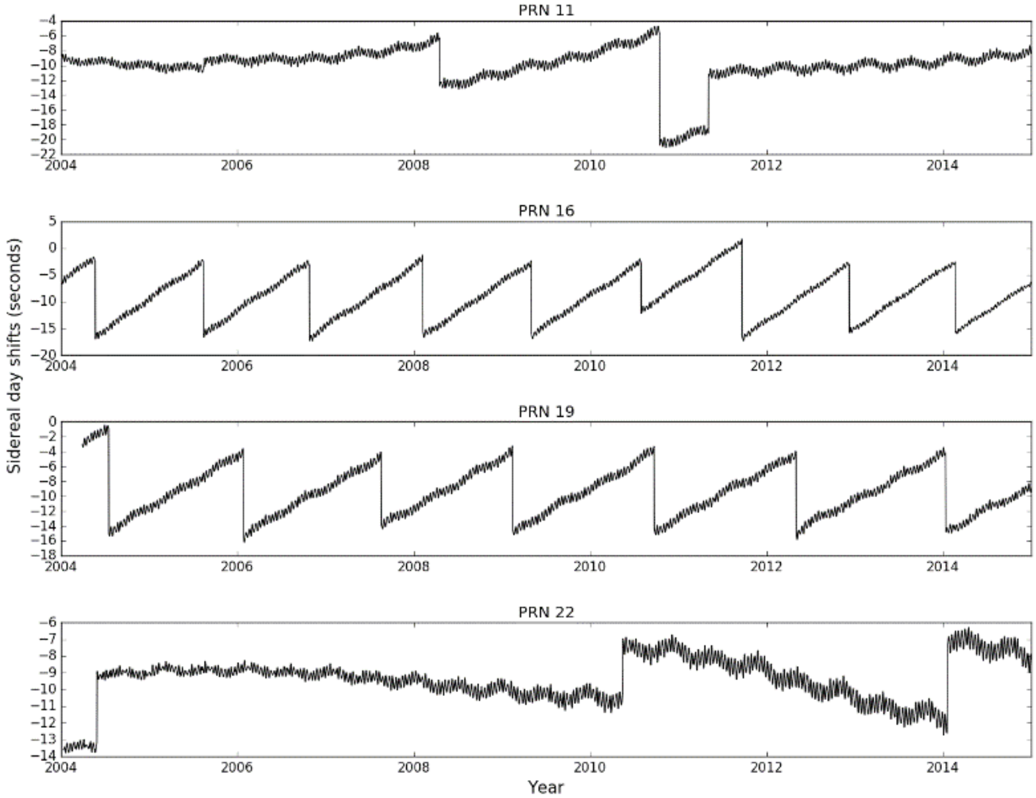

Fig. 1. Sidereal shifts for GPS PRN 11, 16, 19, and 22 from 2004 to 2014.

wind stream, have not been properly investigated.

In this study, we established the reasoning as to why sidereal shifts of GPS satellites are not zero and why they are variable with respect to time. Accurate sidereal shifts were obtained from the broadcast ephemeris within the time span of 2004 to 2014. After calculating the sidereal shifts, we carried out a detailed analysis to find the periodic properties of sidereal shifts. We also investigated the possibility that the Earth's magnetic field and the solar wind stream might affect the orbit of GPS satellites.

2. CALCULATION OF SIDEREAL SHIFTS

To obtain the sidereal shift, it is essential to calculate the exact orbital period of the GPS satellite. There are three methods with which to calculate the repeat time of a GPS constellation. The first method is conducted by using the semi-major axis provided from the broadcast ephemeris data. The second approach is applied by computing, through an interpolation of the precise GPS satellite orbit, the timing with which the satellite crosses the equator. The last method is utilized by observing the correct repeat geometry for a given location and thereafter estimating the time.

The first method is used to obtain the period of GPS satellites in this study. This approach is based on Kepler's third law, which describes the relationship between the semi-major axis and the orbital period. The period of the GPS satellites is computed as

$$P = \frac{2\pi}{n} \tag{1}$$

where n is the mean motion, which is the angular speed required for the satellite to complete one orbit. The mean motion n is written as

$$n = \sqrt{\frac{GM_E}{a^3}} + \Delta n \tag{2}$$

where a and Δn are the semi-major axis of the satellite and the correction of the mean motion, respectively. These two parameters can be obtained from broadcast ephemeris data. GM_E is the gravitational constant of the Earth, the value of which is 3.986005×10^{14} m^3/s^2.

The sidereal shift is defined as twice the orbital period of the GPS satellite minus the mean sidereal day of the Earth. It can be calculated as follows:

$$T = 2P - 86164 \tag{3}$$

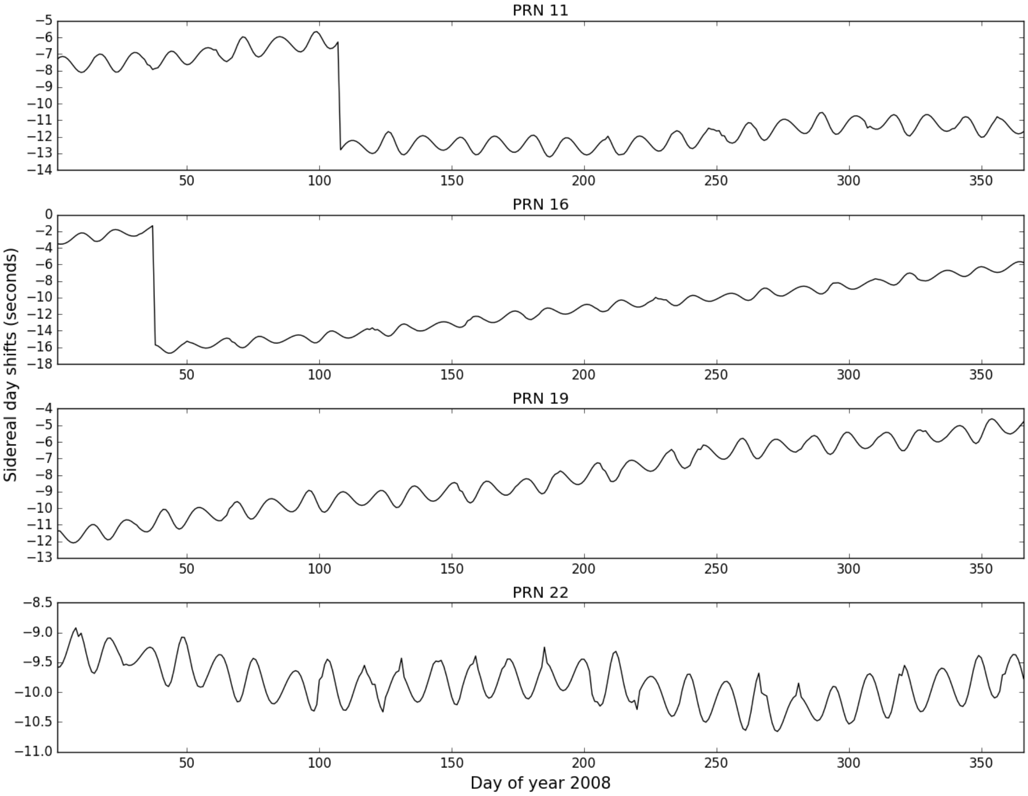

Fig. 2. Sidereal shifts for GPS PRN 11, 16, 19, and 22 in 2008.

where T and P are the sidereal shift and the period of the GPS satellite, respectively. The second term in Eq. (3) represents a mean sidereal day of the Earth, the unit for which is represented in seconds.

Fig. 1 illustrates the sidereal shifts of GPS satellites during the years from 2004 to 2014. It clearly shows that the mean value of the sidereal shifts is not zero. That is, the orbital period of the GPS satellites is not precisely one half of a sidereal day of the Earth. Furthermore, the evolution of the sidereal shifts is clearly different for each satellite. For example, all of the satellites are characterized by linear drifts and a sharply dropped pattern, but the slope of the drifts as well as the period of dropped pattern is different for each satellite. The secular drifts that are observable in Fig. 1 occurred because of the resonance effect with tesseral harmonics in the Earth's gravitational field (Axelrad et al. 2005). The sharply dropped patterns, also shown in Fig. 1, are caused by GPS satellite maintenance maneuvers, which are adjusted by the control segment. This procedure sustains the ascending node of the GPS satellite within ±2 degrees of a nominal value. Other perturbations are apparent in Fig. 1, but they are difficult to recognize since their period is relatively small compared to the 11-year timescale.

Table 1. Properties of sidereal shifts.

PRN	Plane slot	Block	Average of T	Range of T
11	D	IIR	-10.0	-21.1 ~ -4.7
16	B	IIR	-9.0	-17.4 ~ 1.6
19	C	IIR	-9.5	-16.2 ~ -0.4
22	E	IIR	-9.7	-13.9 ~ -6.3

Table 1 indicates the properties of the sidereal shifts. The plane slots of the GPS PRN 11, 16, 19, and 22 satellites are D, B, C, and E, respectively. In addition, all of the satellites' block types are IIR. These facts confirm previously obtained results that the sidereal shifts are independent of the plane slot (Axelrad et al. 2005). The average values of the sidereal shifts are between -10.0 and -9.0, which is similar to findings in previous research (Choi et al. 2004). It is also remarkable that the range of the sidereal shifts is different among each satellite (Seeber et al. 1998, Choi et al. 2004, Axelrad et al. 2005, Agnew & Larson 2007).

Fig. 2 shows the sidereal shifts of the GPS satellites in 2008, within which all of the small-amplitude perturbations, as well as linear drifts and dropping patterns, are observable. These perturbations seem to have roughly1-second amplitudes and a period of approximately 13 days. The small-amplitude oscillations, as was established from previous research, were

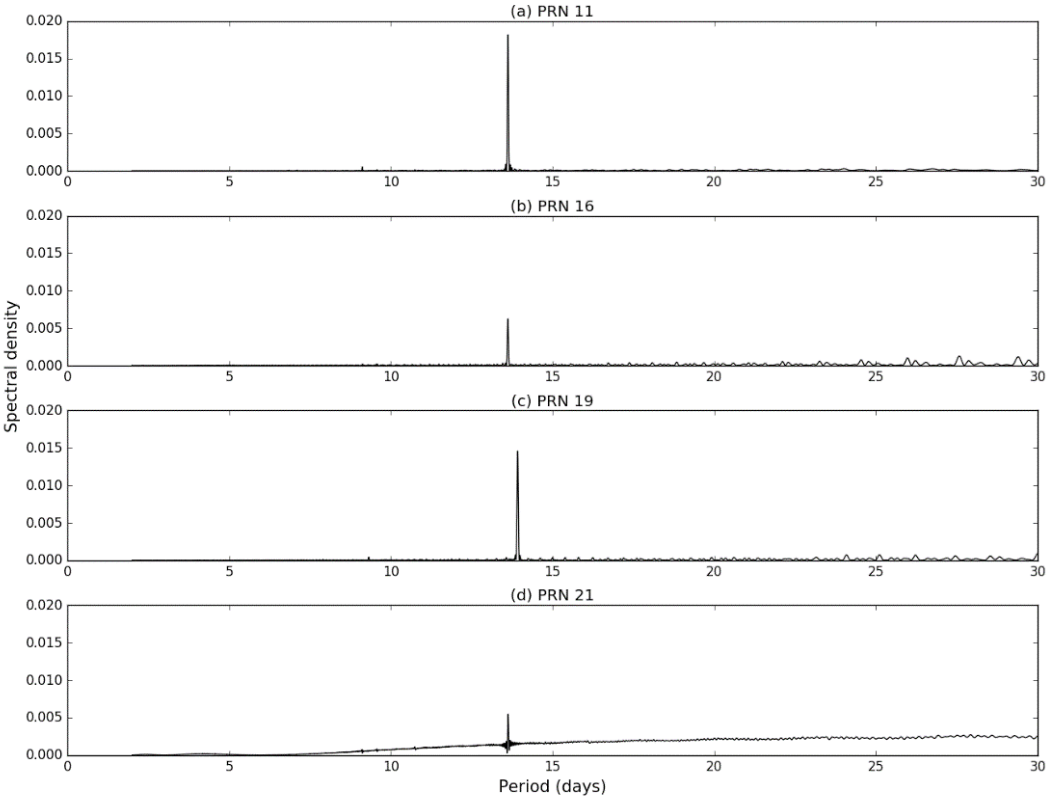

Fig. 3. Lomb-Scargle periodograms of sidereal shifts for (a) GPS PRN 11, (b) PRN 16, (c) PRN 19, and (d) PRN 21 from 2004 to 2014.

caused mainly by the gravitational effect of the Moon (Choi et al. 2004, Axelrad et al. 2005). In this study, we conducted a spectral analysis to detect the periodicities of sidereal shifts to validate the causes of the perturbation of the GPS satellites.

3. PERIODIC ANALYSIS USING LOMB-SCARGLE PERIODOGRAM

The Lomb-Scargle method was applied to identify the periodicities of the GPS satellites. This method is employed to estimate a frequency spectrum based on the least-squares fit of sinusoids to data samples. Therefore, it is also known as a least-squares spectral analysis. It is, in many ways, similar to a Fourier analysis, which is often used to detect periodicity within regularly sampled data. The Lomb-Scargle periodogram, however, can account for unevenly sampled data and produce less long-period noise.

The Lomb-Scargle periodogram at a frequency ω is computed using the following formula:

$$P_x(\omega) = \frac{1}{2}\left(\frac{[X_j cos\omega(t_j-\tau)]^2}{\sum cos^2\omega(t_j-\tau)} + \frac{[X_j sin\omega(t_j-\tau)]^2}{\sum sin^2\omega(t_j-\tau)}\right) \tag{4}$$

where X_j and t_j are the sidereal shifts and time, respectively. The time delay τ is defined by the following equation:

$$tan2\omega\tau = \frac{\sum_j sin2\omega t_j}{\sum_j cos2\omega t_j} \tag{5}$$

where ω is the frequency.

Fig. 3 displays the Lomb-Scargle periodograms of sidereal shifts for GPS satellites. The peaks in Fig. 3 indicate the periodicities of the sidereal shifts. The figure shows that a 13.6-day periodicity is dominant among all of the GPS satellites. In addition, Fig. 4 reveals the periodicities of sidereal shifts during 2009. This peak corresponds to the small perturbations in Fig. 2, indicating that the period of the small oscillations is 13.6 days. Other peaks that appear in the periodograms for GPS PRN 16, 19, and 21 might be induced by noise. In summary, the 13.6-day period in GPS satellite orbit perturbations is a universal phenomenon since they can be observed among all of the satellites and are independent of the satellite plane slot.

To identify what causes these oscillations, we investigated the periodicities of the Earth's geomagnetic activity and solar wind parameters. Fig. 5 illustrates the Lomb-Scargle periodograms of the Kp and Ap indices. The Kp-index measures disturbances in the horizontal component of the magnetic field of Earth with an integer in the range

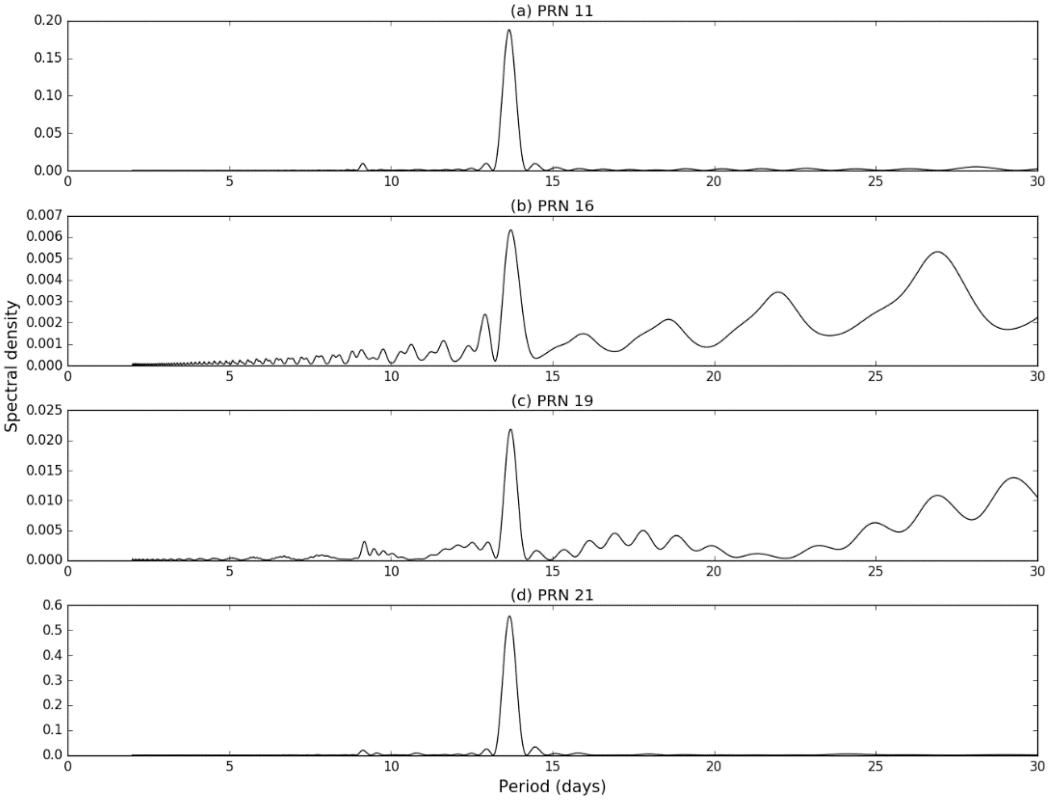

Fig. 4. Lomb-Scargle periodograms of sidereal shifts for (a) GPS PRN 11, (b) PRN 16, (c) PRN 19, and (d) PRN 21 in 2009.

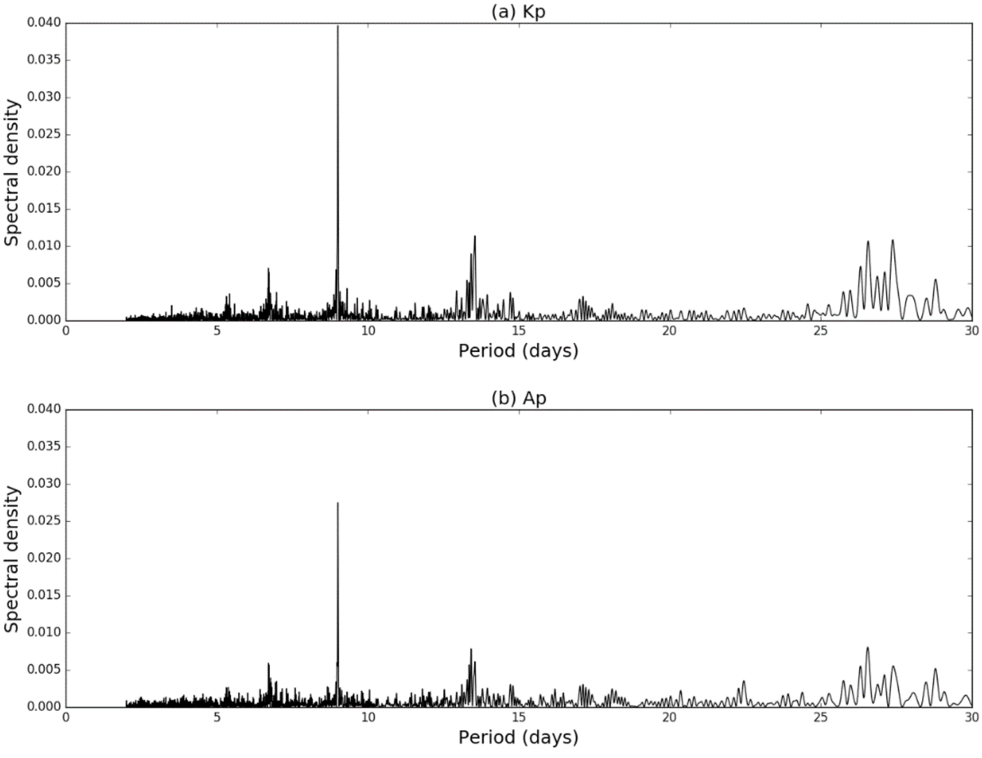

Fig. 5. Lomb-Scargle periodograms of the geomagnetic activity (a) Kp and (b) Ap indices during 2004-2014.

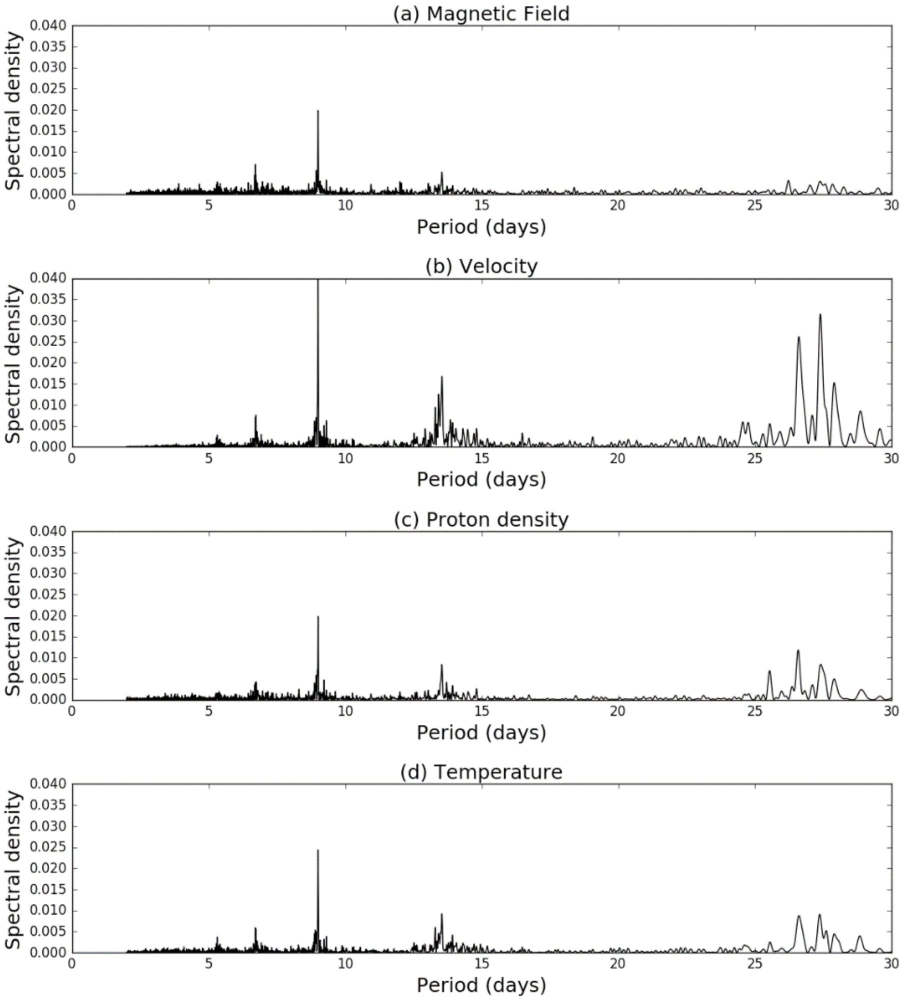

Fig. 6. Lomb-Scargle periodograms of the (a) magnetic field strength, (b) velocity, (c) proton density, and (d) temperature of the solar wind stream from 2004 to 2014.

from 0-9; the Ap-index provides a daily average of the geomagnetic activity over the globe for a given day. These indices are widely used to measure geomagnetic activity. The periodograms of the Kp and Ap indices both exhibit their highest power at 9.0 days. Additionally, 13.5-, 26.6-, and 27.4-day peaks are clearly observed with a similar spectral density.

Fig. 6 indicates the periodicities of the solar wind parameters, namely, the magnetic field strength, velocity, proton density and temperature of the solar wind. Fig. 6 demonstrates a dominant power at 9.0 days and remarkable peaks at 13.5, 26.6, and 27.4 days. All of the periodograms in Fig. 6 show strong resemblances with those of the geomagnetic indices in Fig. 5. These results are consistent with earlier studies, which reported that the solar wind and the geomagnetic parameters have a 9.0-day period in addition to a 13.5-, 26.6- and 27.4-day period (Mursula & Zieger 1996, Pedatella & Larson 2010, Emery et al. 2011).

In summary, sidereal shifts of GPS satellites are characterized by a 13.6-day periodicity. This period can be observed among all of the GPS satellites, regardless of the satellite's plane slot. Meanwhile, the geomagnetic and solar wind parameters also have a 13.5-day period, as well as a 9.0-, 26.6- and 27.5-day period. Based on these similarities of the periodicities, the 13.6-day period of sidereal shifts might be affected by the solar wind stream and the gravitational effect of the Moon simultaneously.

4. CONCLUSIONS

In the present study, we conducted a detailed investigation of the sidereal shifts of the GPS satellites. Accurate sidereal days for each GPS satellite were estimated by broadcast ephemeris data. The results showed that the value of twice the sidereal day of the GPS satellites was not precisely equal to one sidereal day of the Earth. This difference is known as the sidereal shift, which has an average value of 9 seconds. An investigation of 11 years' worth of sidereal shifts revealed that sidereal shifts are characterized by linear drifts, sharply dropped patterns, and small-amplitude oscillations. We further analyzed more detailed periodic behaviors of the sidereal shifts using Lomb-Scargle periodograms and discovered that sidereal shifts of GPS satellites primarily have a 13.6-day period.

To identify what causes this periodicity, we considered that the solar wind stream emitted from the Sun could affect the GPS satellites' perturbations. We discovered that the geomagnetic data and solar wind stream have periods of 9.0, 13.5, 26.6, and 27.4 days. The 13.5-day period is in good agreement with the period of sidereal shifts of GPS satellites. In particular, the sidereal shift of GPS satellites has the largest spectral power at 13.6 days, whereas the solar activity has the largest spectral power at a periodicity of 9.0 days. These corresponding periods do not coincide with the largest spectral powers observed for sidereal shifts and the solar wind stream. Choi et al. (2004) reported that the gravitational effect of the Moon has an approximately 13-day period, and their results are similar to ours. In conclusion, we suggest that the 13.6-day period of the sidereal shifts could be caused by the solar wind stream from the Sun.

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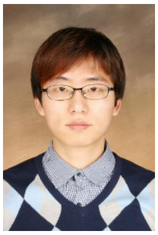

Changhyun Cho received his Master's degree, in astronomy and atmospheric sciences from Kyungpook National University in 2014. His research interests include GNSS positioning algorithm and operation of Global Data Center.

Byung-Kyu Choi received the Doctor's degree in Electronics from Chungnam National University in 2009. He has been working at the Korea Astronomy and Space Science Institute since 2004. His research interests include GNSS PPP, PPP-RTK, and Ionospheric TEC estimation.

https://doi.org/10.11003/JPNT.2017.6.2.71