General linearly constrained adaptive arrays

일반 선형제약 적응배열

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ABSTRACT: A general linearly constrained adaptive array is proposed to improve the nulling performance. The nulling performance is examined in the array weight vector space. It is shown that the constraint plane is shifted to the origin perpendicularly by the gain factor such that the increase of the gain factor results in the decrease of the distance from the constraint plane to the origin. Thus the variation of the gain factor has an effect on the extent of orthogonality between the weight vector and the steering vectors for the interferences such that the nulling performance of the general linearly constrained adaptive array is improved by the gain factor. It is observed that the proposed adaptive array with an optimum value of the gain factor yields a better nulling performance in coherent signal environment and a similar nulling performance in noncoherent signal environment compared to the conventional linearly constrained adaptive array.

Keywords: Adaptive array, Optimum, Gain factor, Linear constraint, Nulling performance

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초   록: 잡음신호제거 성능을 향상시키기 위하여 일반선형제약적응배열이 제안되었다. 잡음신호제거 성능이 배열계수 벡터공간에서 검토되었다. 제약면이 이득요소에 의하여 원점에 대하여 수직 방향으로 이동하는데 이득요소가 커지면 원점과의 거리는 감소하게 됨이 밝혀졌다. 따라서 이득요소의 변화가 계수벡터와 잡음신호 방향벡터와의 직교성의 정도에 영향을 주어서 일반선형제약적응배열의 잡음신호제거 성능이 이득요소에 의하여 향상되게 된다. 제안된 적응배열은 최적의 이득요소 수치에서 간섭적인 신호환경에서는 기존의 선형제약적응배열에 비하여 더 나은 잡음신호제거 성능을 나타내고 비간섭적인 신호환경에서는 비슷한 잡음신호제거 성능을 나타냄이 관찰되었다.

핵심용어: 적응배열, 최적, 이득요소, 선형제약, 제거성능

I. Introduction

The linearly constrained adaptive arrays[1,2] depends on the extent of the correlatedness of the desired signal and the interference signals. If the desired signal is partially or totally correlated with the interference signals, the desired signal is partially or totally cancelled in the array output. A variety of algorithms were proposed to reduce the signal cancellation phenomena.[2-8]

In this paper, a general linearly constrained adaptive array is proposed to improve the nulling performance. The nulling performance is examined in the array weight vector space. It is assumed that the direction of the desired signal is known a priori. The error output is generated by the array output subtracted by the desired response which is formed as the output of the multichannel uniform all-pass filter weighted by a gain factor.

The linearly constrained broadband adaptive array is implemented in coherent and noncoherent signal environments. It is shown that the value of the gain factor affects the nulling performance such that there exists a value of the

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gain factor which yields a best nulling performance.

Adaptive Array processing techniques have been applied in many areas which include radar,[9] sonar,[10] and seismology.[11]

II. General linearly constrained broadband adaptive array

In the conventional linearly constrained adaptive array[1] proposed by Frost, it is assumed that the desired signal is uncorrelated with the interference signals. If the desired signal is correlated with the interferences, it is demonstrated that the desired signal is cancelled in the array output.[2]

A general linearly constrained adaptive array is proposed to reduce the signal cancellation phenomena in coherent and noncoherent signal environments. The general linearly constrained broadband adaptive array with N sensor elements followed by L taps per element is shown in Fig. 1.

The desired signals at each channel are delayed after they pass through the steering time delay elements right after the each sensor such that the desired signal becomes in phase after the steering time delay elements. The desired response is generated by multiplying the output of the multichannel uniform allpass filter (i.e., all weights zero except for the first column of uniform weights) by a gain factor.

The optimum weight vector which yields a minimum mean square error output with a unit gain constraint at the look direction (i.e., the direction of the desired signal) can be found by solving the following constrained optimization problem.

$$\min \ (\boldsymbol{w} - g\boldsymbol{s})^T \boldsymbol{R} (\boldsymbol{w} - g\boldsymbol{s})$$
$$\text{subject to } \boldsymbol{C}^T \boldsymbol{w} = \boldsymbol{f}, \quad\quad\quad (1)$$

where an $NL \times 1$ weight vector $\boldsymbol{w} = [w_1 w_2 \cdots w_{NL}]^T$, the $NL \times 1$ weight vector $\boldsymbol{s}$ of the multichannel allpass filter is given by $\boldsymbol{s} = [\frac{1}{N} \frac{1}{N} \cdots \frac{1}{N} 00 \cdots 0]^T$ in the figure, $c_i = \frac{1}{N}$, , $1 < i < N$. $\boldsymbol{R}$ is an $NL \times NL$ input signal correlation matrix, which is given by $\boldsymbol{R} = \boldsymbol{E}[\boldsymbol{x}\boldsymbol{x}^T]$ and the input signal vector $\boldsymbol{x} = [x_1 x_2 \cdots x_{NL}]^T$. The lth column vector of the $NL \times L$ constraint matrix $\boldsymbol{C}$ consists of elements of 0 except of the lth group of N elements of 1, and the $L \times 1$ constraint vector is given by
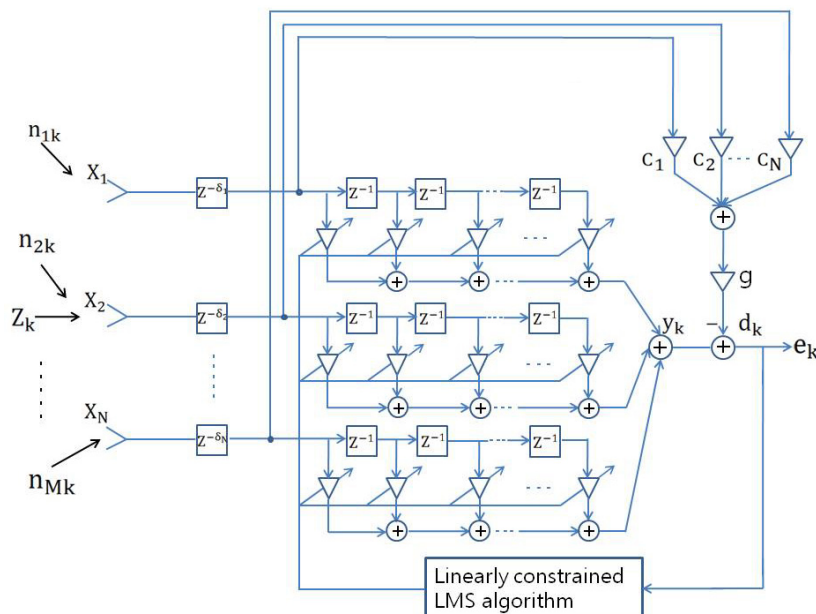
$$\boldsymbol{f} = [100 \dots 0]^T.$$

Fig. 1. General linearly constrained broadband adaptive array.

The optimum weight vector can be found by the method of Lagrange multipliers solving the unconstrained optimization problem with the following objective function.

$$O(w) = (w - gs)^T R(w - gs) + \lambda^T (C^T w - f), \quad (2)$$

where λ is a $L \times 1$ Lagrange multiplier vector. The gradient of the objective function is given by

$$\nabla(w) = \frac{\partial O(w)}{\partial w} = 2Rw - 2gRs + C\lambda. \quad (3)$$

By setting the gradient equal to zero, we have the optimum weight vector as

$$w_o = gs - R^{-1} C\lambda_h, \quad (4)$$

where λ_h is $1/2\lambda$.

The optimum weight vector is obtained by finding λ_h using the linear constraint in (1), substituting the resulting λ_h for that in (4). Then the optimum weight vector is given by

$$w_o = g[I - R^{-1} C(C^T R^{-1} C)^{-1} C^T]s + R^{-1} C(C^T R^{-1} C)^{-1} f. \quad (5)$$

The optimum weight vector in (5) could be interpreted geometrically in the translated weight vector space. If we denote the translated weight vector $(w - gS)$ as v, the optimization problem in the translated weight vector space can be formulated as

$$\min v^T Rv$$
$$\text{subject to } C^T v = (1 - g)f. \quad (6)$$

The objective function with the Lagrange multiplier vector is represented as

$$O(v) = 1/2 v^T Rv + \lambda^T (C^T v - (1 - g)f). \quad (7)$$

The optimum weight vector using the gradient of $O(v)$ is

expressed as

$$v_o = (1 - g)R^{-1} C(C^T R^{-1} C)^{-1} f. \quad (8)$$

From (8), it is observed in the translated weight vector space that the constraint plane is shifted to the origin perpendicularly by the gain factor g such that the increase of the gain factor results in the decrease of the distance from the constraint plane to the origin. Thus the variation of the gain factor has an effect on the extent of orthogonality between the weight vector and the steering vectors for the interferences such that the nulling performance of the general linearly constrained adaptive array may be improved by the gain factor compared to the conventional linearly constrained adaptive array.

III. General adaptive algorithm

The general linearly constrained adaptive algorithm is derived by minimizing the mean square error using the steepest descent method.[12]

$$w_{k+1} = w_k + \mu(-\nabla(w)_k), \quad (9)$$

where μ is a convergence parameter and k is a iteration index. Substituting the gradient in (3) for that in (9), we have the following iterative equation.

$$w_{k+1} = w_k + \mu Rw_k + \mu g + \mu g Rs - \mu C\lambda_h. \quad (10)$$

We find the Lagrange multiplier vector λ_h by applying the $(k+1)$ th weight vector w_{k+1} to the linear constraint in (1) to find the λ_h and substituting the λ_h for that in (10), we have the following general linearly constrained adaptive algorithm.

$$w_{k+1} = P[w_k - \mu R(w_k - gs)] + F, \quad (11)$$

where the $NL \times NL$ projection matrix P is given by

$$P = I - C(C^T C)^{-1} C^T. \tag{12}$$

which projects a vector onto the constraint subspace which is the orthogonal complement of the column space of C and the $NL \times 1$ vector F is given by

$$F = C(C^T C)^{-1} f. \tag{13}$$

which is in the column space of C and normal to the constraint subspace.

A general linearly constrained LMS (Least Mean Square) algorithm can be obtained by substituting a instantaneous correlation matrix $x_k x_k^T$ for R in (11) and rearranging the resulting equation. Then the general linearly constrained LMS algorithm is expressed as

$$w_{k+1} = P[w_k - \mu e_k x_k] + F, \tag{14}$$

where e_k is the output error signal.

The array weights are updated iteratively by the general linearly constrained LMS algorithm in the computer simulation.

IV. Simulation results

The linearly constrained broadband adaptive array with 5 sensor elements and 3 weights per element is employed to demonstrate the nulling performance of the general linearly constrained adaptive array. It is assumed that the incoming signals are plain waves. The incoming signals are generated by passing a white Gaussian random signal through the 4 th-order Butterworth filter such that the bandwidth is 3 Hz with the lower and upper cutoff frequencies 8 Hz and 11 Hz respectively. The sampling frequency is 608 Hz. The convergence parameter is assumed to be 0.0001.

The gain factor is varied to improve the nulling performance in coherent and nocoherent signal environments. The simulation results in[6] are redisplayed to demonstrate the nulling performance.

A) Case for one coherent interference

It is assummed that a coherent interference is incident at 30° with respect to the array normal. The variation of the error power between the array output and the desired signal is displayed in Fig. 2. The optimum value of is shown to be 0.33. The comparison of the array performance for $g = 0.33$, the conventional linearly constrained adaptive array proposed by Frost and the case for $g = 2.0$ are shown in Figs. 3 and 4 with respect to the array output and the desired signal for $k = 1 \sim 10000$ and $28001 \sim 29000$.

It is shown for $28001 \le k \le 29000$ that the case for $g = 0.33$ performs best while the Frost's performs better than the case for $g = 2.0$. The beam patterns are shown in Fig. 5, in which the case for $g = 0.33$ forms a deepest null.
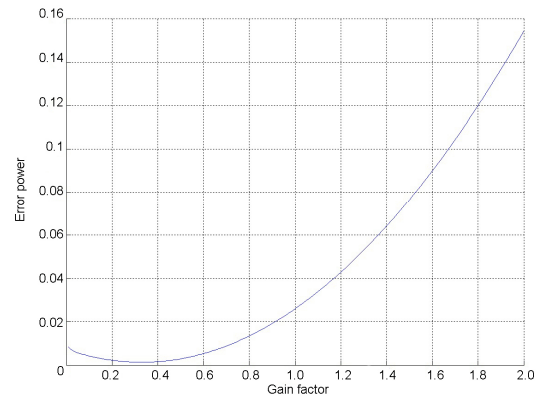

Fig. 2. Variation of the error power in terms of gain factor for one coherent interference case.
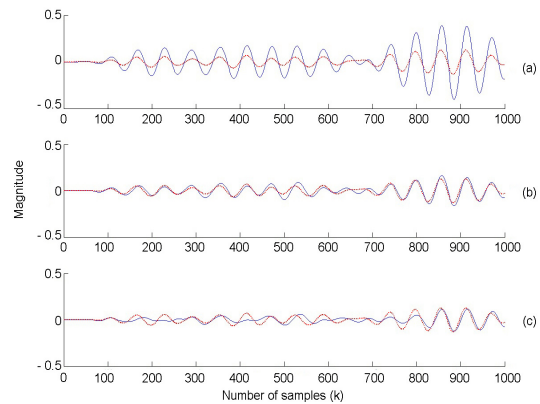

Fig. 3. Comparison of array output (solid line) and desired signal (dotted line) for one coherent interference case: (a) g = 0.33, (b) Frost's, (c) g = 2, for $1 \le k \le 1000$.
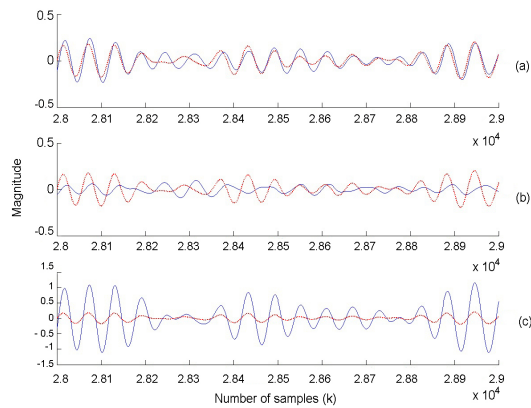
Fig. 4. Comparison of array output (solid line) and desired signal (dotted line) for one coherent interference case: (a) g = 0.33, (b) Frost's, (c) g = 2, for $28001 \leq k \leq 29000$.

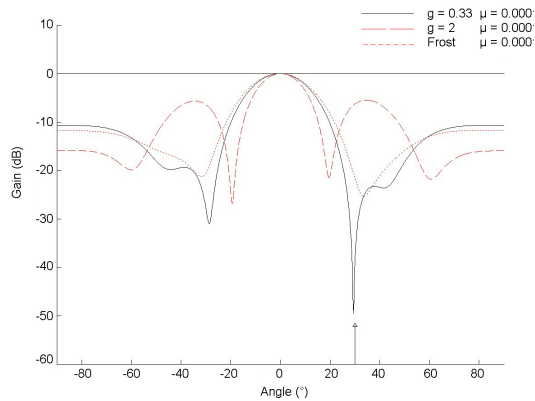Fig. 5. Comparison of beam patterns for one-coherent interference case at 30°.

B) Case for two coherent interferences

It is assumed that two coherent interferences are incident at -54.3° and 57.5°. The variation of the error power between the array output and the desired signal is displayed in Fig. 6. The optimum value of g is shown to be 0.29. The comparison of the array performance for $g = 0.29$. the conventional linearly constrained adaptive array proposed by Frost, and the case for $g = 2.0$ are shown in Figs. 7 and 8 with respect to the array output and the desired signal for $k = 1 \sim 1000$ and $28001 \sim 29000$.

It is shown for $k = 28001 \sim 29000$ that the case for $g = 0.29$ performs best while Frost's performs better than the case for $g = 2.0$ The beam patterns are shown in Fig. 9, in which the case for $g = 0.29$ forms two deepest nulls
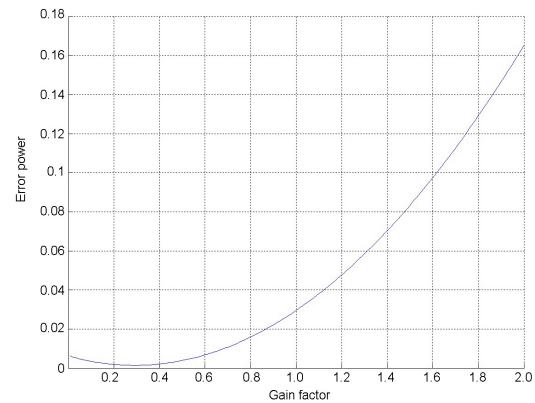

Fig. 6. Variation of the error power in terms of gain factor for two-coherent interference case.
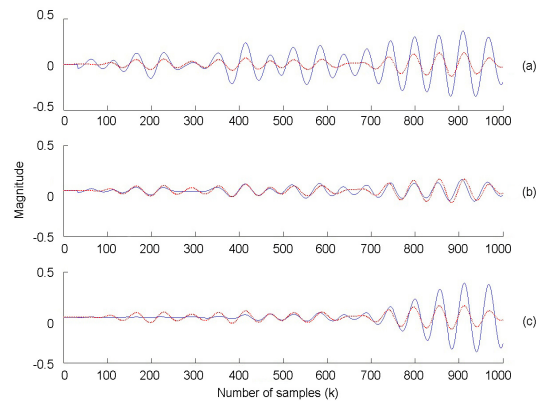

Fig. 7. Comparison of array output (solid line) and desired signal (dotted line) for two-coherent interference case: (a) g = 0.29, (b) Frost's, (c) g = 2.0, for $1 \leq k \leq 1000$.
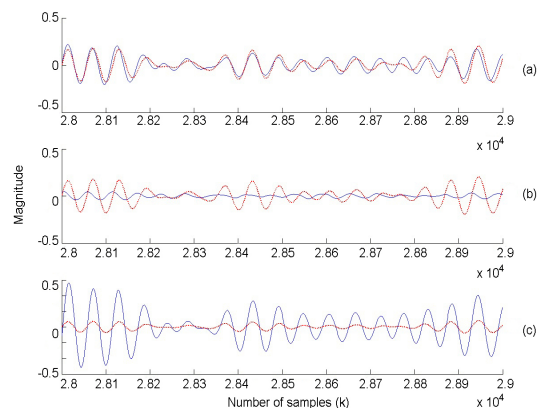

Fig. 8. Comparison of array output (solid line) and desired signal (dotted line) for two-coherent interference case: (a) g = 0.29, (b) Frost's, (c) g = 2.0, for $28001 \leq k \leq 29000$.

around the two incident angles -54.3° and 57.5° of the coherent interferences.

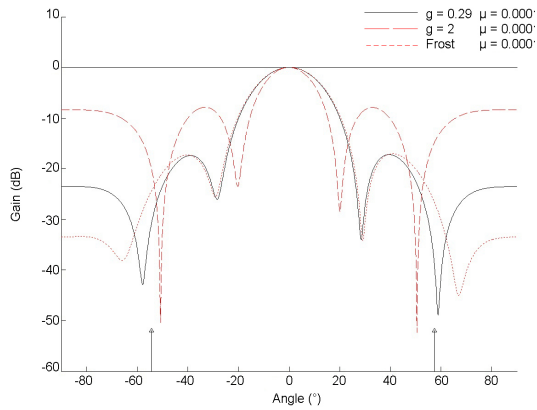Fig. 9. Comparison of beam patterns for two-coherent interference case at −54.3°, 57.5°.

C) Case for one noncoherent interference

It is assummed that a noncoherent interference is incident at -48.5°. The variation of the error power between the array output and the desired signal is displayed in Fig. 10. The optimum value of g is shown to be 0.09. The comparison of the array performance for $g = 0.09$, the conventional linearly constrained adaptive array proposed by Frost, and the case for $g = 2.0$ are shown in Figs. 11 and 12 with respect to the array output and the desired signal for $k = 1 \sim 1000$ and $28001 \sim 29000$.

It is shown for $k = 28001 \sim 29000$ that the case for $g = 0.09$ and the Frost's array yield a similar performance while both of them performs better than the case for $g = 2.0$. The beam patterns are shown in Fig. 13, in which the case for $g = 0.09$ and the Frost's array yields a similar
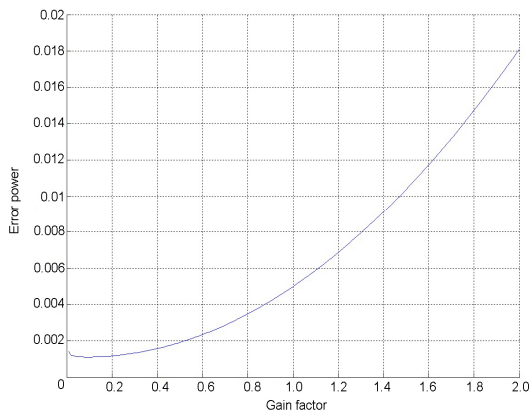

Fig. 10. Variation of the error power in terms of gain factor for one noncoherent interference case.
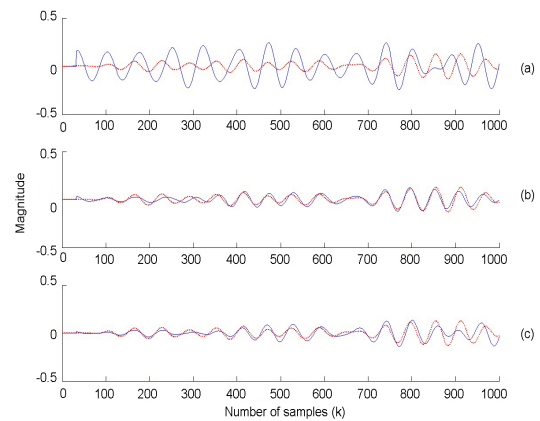

Fig. 11. Comparison of array output (solid line)and desired signal (dotted line) for one coherent interference case; (a) g = 0.09, (b) Frost's, (c) g = 2.0, for $1 \le k \le 1000$.
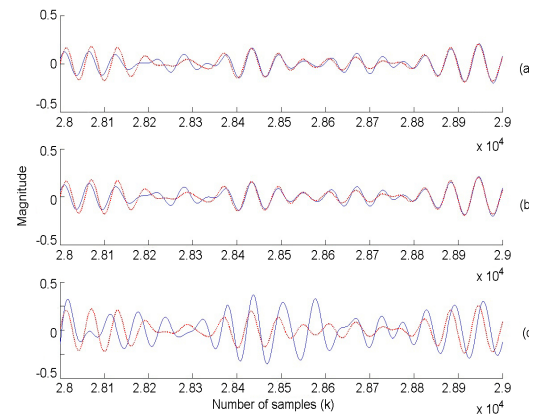

Fig. 12. Comparison of array output (solid line) and desired signal (dotted line) for one coherent interference case; (a) g = 0.09, (b) Frost's, (c) g = 2.0, for $28001 \le k \le 29000$.
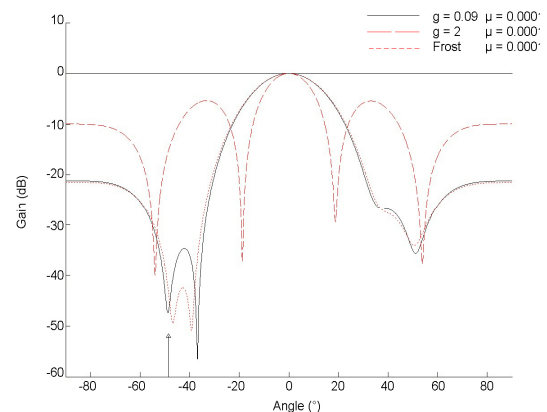

Fig. 13. Comparison of beam patterns for one noncoherent interference case at −48.5°.

gain at the incident angle of the noncoherent interference. It is observed that a more exact null is formed at the incident angle of the noncoherent interference for the case of $g = 0.09$ than for the Frost's.

V. Conclusions

A general linearly constrained adaptive array is proposed to improve the nulling performance in coherent and noncoherent signal environments. The nulling performance is examined in the array weight vector space. It is observed that the constraint plane is shifted to the origin perpendicularly by the value of the gain factor such that the increase of the gain factor results in the decrease of the distance from the constraint plane to the origin.

Thus the variation of the gain factor has an effect on the extent of orthogonality between the weight vector and the steering vectors for the interference signals such that the orthogonality between the weight vector and the steering vectors for the interference signals is improved at an optimum gain factor. Therefore, the nulling performance of the general linearly constrained adaptive array with an optimum gain factor is improved compared to the conventional linearly constrained adaptive array.

It is demonstrated in the computer simulation that the general linearly constrained adaptive array performs better at the optimal gain factor than the conventional linearly constrained adaptive array in coherent environment while it yields a similar performance to the conventional linearly constrained adaptive array in noncoherent environment.

Acknowledgments

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Profile

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