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REPRESENTATION OF INTUITIONISTIC FUZZY SOFT SET USING COMPLEX NUMBER

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Abstract. Soft sets are fantastic mathematical tools to handle imprecise and uncertain information in complicated situations. In this paper, we defined the hybrid structure which is the combination of soft set and complex number representation of intuitionistic fuzzy set. We defined basic set theoretic operations such as complement, union, intersection, restricted union, restricted intersection etc. for this hybrid structure. Moreover, we developed this theory to establish some more set theoretic operations like Disjunctive sum, difference, product, conjugate etc.

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1. Introduction

Most of the real life problems have various uncertainties. The Theory of Probability, Evidence Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. are mathematical tools to deal with such problems. Soft sets theory was introduced by Molodstov [26] in 1999 and proven the important results associated to this theory. In the fields of social science, economics, medical sciences etc., it is a wide-ranging mathematical tool for allocating with difficulties. In 2003, Maji, Biswas and Roy [21] studied the theory of soft sets initiated by Molodtsov.

Fuzzy sets and fuzzy logic were introduced by Zadeh in 1965 in order to handle uncertainty and ambiguity ([33] ,[34]). A fuzzy set is characterized by a membership degree whose range is the unit interval. Fuzzy logic is a multilevel extension to the classical logic such that proposition can get any value in the unit interval instead of one of the two values 'True' or 'False'. Based on the theory of fuzzy sets, several additional concepts such as interval valued fuzzy sets [35], type-2 fuzzy sets [14] and intuitionistic fuzzy sets ([4],[5]) have been developed

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in order to efficiently handle uncertainty. Fuzzy sets and fuzzy logic have applications in signal processing ([24],[25]), control theory ([18],[31]), reasoning [19], and data mining [15]. Intuitionistic fuzzy set was introduced by Atanassove in 1986 as a generalization of fuzzy set by adding the degree of non-membership into the fuzzy set [4]. Thus, an intuitionistic fuzzy set is characterized by a degree of membership and a degree of non-membership. Intuitionistic fuzzy sets has been successfully applied in the fields of modeling imprecision [16], decision making problems [14], pattern recognition [31], computational intelligence [13] and medical diagnosis [30].

Complex fuzzy set and logic, which is the extension of fuzzy sets and logic respectively, was first proposed by Ramot et al. ([28],[29]). According to their definition, a complex fuzzy set is characterized by a complex grade of membership which is a combination of a traditional fuzzy degree of membership referred to as the amplitude term with the addition of an extra term, the phase term. In similarity to the case of intuitionistic fuzzy set, a complex intuitionistic fuzzy set is characterized by a complex grade of membership and complex grade of non-membership [2]. And complex intuitionistic fuzzy sets [3] have been applied in multi attribute decision making problems.

Combining fuzzy sets with soft sets, Maji et al. [22] introduced the notion of fuzzy soft sets. This work were further revised and improved by Ahmad and Kharal [1]. They defined arbitrary fuzzy soft union and intersection and proved De Morgan Inclusions and De Morgan laws in fuzzy soft set theory. In 2011, Neog and Sut [27] put forward some propositions regarding fuzzy soft set theory. Thereafter, Maji and his coauthor [23] introduced the notion of intuitionistic fuzzy soft set which is based on the combination of the intuitionistic fuzzy sets and soft set models and they studied the properties of intuitionistic fuzzy soft set. M. Bora et al. [9] defined disjunctive sum and difference of two intuitionistic fuzzy soft sets and studied their basic properties. Additional literature on intuitionistic fuzzy soft sets have been studied ([6],[8],[10],[11],[12],[17],[32],[36]). Kumar and Bajaj [20] introduced the concept of complex intuitionistic fuzzy soft sets which is parametric in nature. However, the theory of complex fuzzy sets and complex intuitionistic fuzzy sets are independent of the parametrization tools. Some real life problems, for example, multi-criteria decision making problems, involve the parametrization tools.

In this paper, we define the hybrid structure which is the combination of soft set and complex number representation of intuitionistic fuzzy set. The organization of this paper is followed. In section 1, we presented the literature review while in section 2, we discussed basic concepts. In section 3, intuitionistic fuzzy soft set based on complex numbers have been introduced with their basic set theoretic operations. Section 4 is further dedicated to some more set theoretic operations. De Morgan laws have been studied in section 5. Conclusion is given in section 6.

2. Basic Concepts

Definition 2.1. [2] Let X be an initial universe set and $\tilde{A}$ be an intuitionistic fuzzy soft set characterized by the complex number $z = \mu + j\nu$, where μ is the degree of **membership**, ν is the degree of non-membeship and j being the imaginary number which satisfies $j^2 = -1$ and $\mu, \nu \in [0, 1]$.

Definition 2.2. [33] Let X be a non-empty set. A fuzzy set $\tilde{F}$ in X is characterized by its membership function

$\tilde{F} : X \to [0, 1]$. And $\tilde{F}(\tilde{a})$ is interpreted as the degree of membership of an element $\tilde{a}$ in fuzzy set $\tilde{F}$ for each $\tilde{a} \in X$.

Definition 2.3. [26] A pair $(\tilde{F}, E)$ is called a soft set (over X) if and only if $\tilde{F}$ is a mapping of E into the set of all subsets of the set X.

In other words, the soft set is a parameterized family of subsets of the set X. Every $\tilde{F}(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε- elements of the soft set $(\tilde{F}, E)$, or as the set of ε- approximate elements of the soft set.

Definition 2.4. [23] Let X be an initial universe set and E be the set of parameters such that $\tilde{A} \subseteq E$. Let IF^X denote the collection of all intuitionistic fuzzy sets of X. A pair $(\tilde{F}, \tilde{A})$ is called an intuitionistic fuzzy soft over X where $\tilde{F}$ is a mapping given by $\tilde{F} : \tilde{A} \to IF^X$ such that $\tilde{\mathbf{F}}(\varepsilon)$ is a subset of $IF^X$for all $\varepsilon \in \tilde{A}$ such that

$\tilde{F}(\varepsilon) = \{(\tilde{a}, \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), \nu_{\tilde{\mathbf{F}}_{(\varepsilon)}}(\tilde{\mathbf{a}})) : \tilde{a} \in X, \varepsilon \in \tilde{A}\}$, where μ, ν denote the degrees of membership and non-membership respectively.

Definition 2.5. [9] A soft set $(\tilde{F}, \tilde{A})$ over X is said to be absolute intuitionistic fuzzy soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in \tilde{A}, \tilde{F}(\varepsilon)$ is the absolute intuitionistic fuzzy set 1of X, where $1(x) = 1, \forall x \in X$.

Definition 2.6. [9] A soft set $(\tilde{F}, \tilde{A})$ over X is said to be null intuitionistic fuzzy soft set denoted by ϕ if $\forall \varepsilon \in \tilde{A}, \tilde{\mathbf{F}}(\varepsilon)$ is the null intuitionistic fuzzy set 0 of X, where $0(x) = 0, \forall x \in X$.

Definition 2.7. [9] Union of two intuitionistic fuzzy soft sets $(\tilde{F}, \tilde{A})$and $(\tilde{G}, \tilde{B})$ over (X, E) is an intuitionistic fuzzy soft set $(\tilde{H}, \tilde{C})$, where $\tilde{C} = (\tilde{A} \cup \tilde{B})$ and $\forall \varepsilon \in \tilde{C}$,

$$\tilde{H}(\varepsilon) = \begin{cases} \tilde{F}(\varepsilon),\ if\ \varepsilon \in \tilde{A} - \tilde{B}, \\ \tilde{G}(\varepsilon), if\ \varepsilon \in \tilde{B} - \tilde{A}, \\ \tilde{F}(\varepsilon) \cup \tilde{G}(\varepsilon), if\ \varepsilon \in \tilde{A} \cap \tilde{B}. \end{cases}$$

Definition 2.8. [9] Let $(\tilde{F}, \tilde{A})$ and $(\tilde{G}, \tilde{B})$ be two fuzzy soft sets in a soft class (X, E) with $\tilde{A} \cap \tilde{B} \neq \emptyset$. Then the intersection of two intuitionistic fuzzy soft sets $(\tilde{F}, \tilde{A})$ and $(\tilde{G}, \tilde{B})$ over (X, E) is an intuitionistic fuzzy soft set $(\tilde{H}, \tilde{C})$, where

$\tilde{C} = (\tilde{A} \cap \tilde{B})$ and $\forall \varepsilon \in \tilde{C}$,

$$\tilde{H}(\varepsilon) = \begin{cases} \tilde{F}(\varepsilon),\ if\ \varepsilon \in \tilde{A} - \tilde{B}, \\ \tilde{G}(\varepsilon), if\ \varepsilon \in \tilde{B} - \tilde{A}, \\ \tilde{F}(\varepsilon) \cap \tilde{G}(\varepsilon), if\ \varepsilon \in \tilde{A} \cap \tilde{B}. \end{cases}$$

Definition 2.9. [9] If $(\tilde{F}, \tilde{A})$ and $(\tilde{G}, \tilde{B})$ be two intuitionistic fuzzy soft sets, then $(\tilde{F}, \tilde{A})\ AND\ (\tilde{G}, \tilde{B})$ is an intuitionistic fuzzy soft set denoted by $(\tilde{F}, \tilde{A}) \wedge (\tilde{G}, \tilde{B}) = (\tilde{H}, \tilde{A} \times \tilde{B})$, where $\tilde{H}\,(a_1, a_2) = \tilde{F}\,(a_1) \cap \tilde{G}\,(a_2) \,\forall\ a_1 \in \tilde{F}$ and $a_2 \in \tilde{G}$.

Definition 2.10. [9] Let $(\tilde{F}, \tilde{A})$and $(\tilde{G}, \tilde{B})$ be two intuitionistic fuzzy soft sets, then $(\tilde{F}, \tilde{A})\ OR\ (\tilde{G}, \tilde{B})$ is an intuitionistic fuzzy soft set denoted by $(\tilde{F}, \tilde{A}) \vee (\tilde{G}, \tilde{B}) = (\tilde{H}, \tilde{A} \times \tilde{B})$, where $\tilde{H}\,(a_1, a_2) = \tilde{F}\,(a_1) \cup \tilde{G}\,(a_2) \,\forall\ a_1 \in \tilde{F}$ and $a_2 \in \tilde{G}$.

3. Intuitionistic fuzzy soft set based on complex numbers and their set theoretic operations

In this section, we introduced intuitionistic fuzzy soft **set** based on complex numbers and studied their set theoretic operations and properties.

Definition 3.1. Let X be an initial universe set and E be the set of parameters such that $\tilde{A} \subseteq E$. Let IF^X denote the collection of all intuitionistic fuzzy sets of X represented by complex numbers. A pair $(\tilde{F}, \tilde{A})$ is called an intuitionistic fuzzy soft set based on complex numbers over X where $\tilde{F}$ is a mapping given by $\tilde{F} : \tilde{A} \to IF^X$ such that $\tilde{F}(\varepsilon)$ is a subset of $IF^X$for all $\varepsilon \in \tilde{A}$.

Definition 3.2. Complement : The compliment of an intuitionistic fuzzy soft set $(\tilde{F}, \tilde{A})$, characterized by a complex number $z = \mu + j\nu$, where μ and ν are the degrees of membership and non-membership respectively, is denoted by $\left(\tilde{F}, \tilde{A}\right)^C$ and is defined by $\left(\tilde{F}, \tilde{A}\right)^C = (\tilde{F}^C, -\tilde{A})$ where $\tilde{F}^C : -\tilde{A} \to IF^X$ is the mapping given by $\tilde{\mathbf{F}}^C(\varepsilon) = [\tilde{\mathbf{F}}(\varepsilon)]^C$ for all $\tilde{a} \in \tilde{A}$. Thus if $\tilde{F}(\varepsilon) = \{\tilde{a}, \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) + j\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})) : \tilde{a} \in X\}$, then $\varepsilon \in \tilde{A}, \tilde{F}^C(\varepsilon) = \{\tilde{a}, \nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) + j(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}))\colon \tilde{a} \in X\}$.

Example 3.3. Let $X = \{x_1, x_2, x_3\}$and $E = \{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5\}$ and $\tilde{A} = \{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4\} \subseteq E$.

Suppose that

$$(\tilde{F}, \tilde{A}) = \begin{cases} \tilde{F}\,(\tilde{a}_1) = \{(x_1, 0.4 + 0.3j)\,, (x_2, 0.5 + 0.1j)\,, (x_3, 0.6 + 0.2j)\}\,, \\ \tilde{F}\,(\tilde{a}_2) = \{(x_1, 0.7 + 0.1j)\,, (x_2, 0.8 + 0.1j)\,, (x_3, 0.5 + 0.3j)\}\,, \\ \tilde{F}\,(\tilde{a}_3) = \{(x_1, 0.5 + 0.3j)\,, (x_2, 0.4 + 0.2j)\,, (x_3, 0.9 + 0.01j)\}\,, \\ \tilde{F}\,(\tilde{a}_4) = \{(x_1, 0.3 + 0.2j)\,, (x_2, 0.4 + 0.02j)\,, (x_3, 0.7 + 0.03j)\}\,. \end{cases}$$

Then, $(\tilde{F},\tilde{A})^C = (\tilde{F}^C, -\tilde{A}) =$

$$\begin{cases} \tilde{F}^C(-\tilde{a}_1) = \{(x_1, 0.3+0.4j), (x_2, 0.1+0.5j), (x_3 0.2+0.6j)\}, \\ \tilde{F}^C(-\tilde{a}_2) = \{((x_1, 0.1+0.7j), (x_2, 0.01+0.8j), (x_3, 0.3+0.5j)\}. \\ \tilde{F}(\tilde{a}_3) = \{(x_{1,}0.3+0.5j), (x_2, 0.2+0.4j), (x_3, 0.01+0.9j)\}, \\ \tilde{F}(\tilde{a}_4) = \{(x_{1,}0.2+0.3j), (x_2, 0.02+0.4j), (x_3, 0.03+0.7j)\}. \end{cases}$$

Definition 3.4. Absolute: A soft set $(\tilde{F},\tilde{A})$ over X is said to be absolute intuitionistic fuzzy soft set described by complex number $z = \mu + j\nu$, where μ and ν are the degrees of membership and non-membership respectively, and is denoted by $\tilde{A}$ if $\forall\varepsilon \in \tilde{A}$ and is defined as $\tilde{F}(\varepsilon) = \{(\tilde{a}, \mu_{\tilde{F}(\varepsilon)}(\tilde{a}) + 0j) : \tilde{a} \in X\}$.

Definition 3.5. Null: A soft set $(\tilde{F},\tilde{A})$ over X is said to be null intuitionistic fuzzy soft set described by complex number $z = \mu + j\nu$, where μ and ν are the degrees of membership and non-membership respectively, and is denoted by Φ if $\forall\varepsilon \in (\tilde{A}).\tilde{F}(\varepsilon) = \{(\tilde{a}, 0 + j\nu_{\tilde{F}(\varepsilon)}(\tilde{a})) : \tilde{a} \in X\}$.

Definition 3.6. Union: Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets over the same universe set X which are characterized by two complex numbers $z_1 = \mu_1 + j\nu_1$ and $z_2 = \mu_2 + j\nu_2$, where μ_1, μ_2 and ν_1, ν_2 denotes the degrees of membership and non-membership respectively. Then the union of $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ denoted by $(\tilde{F},\tilde{A}) \cup (\tilde{G},\tilde{B})$ and is defined as $(\tilde{F},\tilde{A}) \cup (\tilde{G},\tilde{B}) = (\tilde{H},\tilde{C})$, where $\tilde{C} = (\tilde{A} \cup \tilde{B})$ and the membership and non-membership of $(\tilde{H},\tilde{C})$ are as follows: $\tilde{H}(\varepsilon)$

$$= \begin{cases} \left\{\tilde{a}, \mu_{\tilde{F}(\varepsilon)}(\tilde{a}) + j\nu_{\tilde{F}(\varepsilon)}(\tilde{a}) : \tilde{a} \in X\right\}, if\ \varepsilon \in \tilde{A} - \tilde{B}, \\ \left\{\tilde{a}, \mu_{\tilde{G}(\varepsilon)}(\tilde{a}) + j\nu_{\tilde{G}(\varepsilon)}(\tilde{a}) : \tilde{a} \in X\right\}, if\ \varepsilon \in \tilde{B} - \tilde{A}, \\ \left\{\max\left(\mu_{\tilde{F}(\varepsilon)}(\tilde{a}), \mu_{\tilde{G}(\varepsilon)}(\tilde{a})\right) + j\min\left(\nu_{\tilde{F}(\varepsilon)}(\tilde{a}), \nu_{\tilde{G}(\varepsilon)}(\tilde{a})\right)\right\}, if\ \varepsilon \in \tilde{A} \cap \tilde{B}. \end{cases}$$

where $\mu_{\tilde{H}_{(\varepsilon)}} = \max\left(\mu_{\tilde{F}(\varepsilon)}(\tilde{a}), \mu_{\tilde{G}(\varepsilon)}(\tilde{a})\right)$ and $\nu_{\tilde{H}_{(\varepsilon)}} = j\min\left(\nu_{\tilde{F}(\varepsilon)}(\tilde{a}), \nu_{\tilde{G}(\varepsilon)}(\tilde{a})\right)$.

Definition 3.7. Extended Union: The extended union of two intuitionistic fuzzy soft set $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ over a common universe X which are characterized by two complex numbers $z_1 = \mu_1 + j\nu_1$ and $z_2 = \mu_2 + j\nu_2$where, $\mu_1, \mu_2$and ν_1, ν_2 denotes the degrees of membership and non-membership respectively. Then the extended union of $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ is $(\tilde{H},\tilde{C})$, where $\tilde{C} = (\tilde{A} \cup \tilde{B})$ and $\forall\varepsilon \in \tilde{\mathbf{C}}$, $\tilde{\mathbf{H}}(\varepsilon) =$

$$\begin{cases} \{(\tilde{a}, \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) + j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})) : \tilde{a} \in X\}, if\ \varepsilon \in \tilde{A} - \tilde{B}, \\ \{(\tilde{a}, \mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}) + j(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})) : \tilde{a} \in X\}, if\ \varepsilon \in \tilde{B} - \tilde{A}, \\ \{max(\mu_{\tilde{\mathbf{F}}_{(\varepsilon)}}(\tilde{\mathbf{a}}), \mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})) + jmin(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), \nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})) : \tilde{a} \in X\}, if\ \varepsilon \in \tilde{A} \cap \tilde{B}. \end{cases}$$

We write, $(\tilde{F},\tilde{A}) \cup_E (\tilde{G},\tilde{B}) = (\tilde{H},\tilde{C})$.

Definition 3.8. Restricted Union: Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets over the same universe set X such that $\tilde{A}\cap\tilde{B}\neq 0$, is characterized by complex numbers $z_1=\mu_1+j\nu_1$ and $z_2=\mu_2+j\nu_2$, where $\mu_1,\mu_2$and ν_1,ν_2denotes the degrees of membership and non-membership respectively. The restricted union of $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ is denoted by $((\tilde{F},\tilde{A})\cup_R(\tilde{G},\tilde{B}))=(\tilde{H},\tilde{C})$, and is defined as $((\tilde{F},\tilde{A})\cup_R(\tilde{G},\tilde{B}))=(\tilde{H},\tilde{C})$, where $\tilde{C}=(\tilde{A}\cap\tilde{B})$ and $\forall\varepsilon\in\tilde{\mathbf{C}},\tilde{H}(\varepsilon)=\tilde{F}(\varepsilon)\cup\tilde{G}(\varepsilon)$.

Definition 3.9. Intersection: Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets over the same universe set X such that $\tilde{A}\cap\tilde{B}\neq 0$, characterized by complex numbers $z_1=\mu_1+j\nu_1$ and $z_2=\mu_2+j\nu_2$,where $\mu_1,\mu_2$and ν_1,ν_2 denotes the degrees of membership and non-membership respectively. Then the intersection of $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ denoted by $(\tilde{F},\tilde{A})\cap(\tilde{G},\tilde{B})$ and is defined as $(\tilde{F},\tilde{A})\cap(\tilde{G},\tilde{B})=(\tilde{K},\tilde{C})$, where $\tilde{C}=(\tilde{A}\cap\tilde{B})$ and the membership and non-membership of $(\tilde{K},\tilde{C})$ are as follows: $\tilde{K}(\varepsilon)=$

$$\begin{cases} \{(\tilde{a},\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{A}-\tilde{B},\\ \{(\tilde{a},\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{B}-\tilde{A},\\ \{\min(\mu_{\tilde{\mathbf{F}}_{(\varepsilon)}}(\tilde{\mathbf{a}}),\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))+j\max(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{A}\cap\tilde{B}. \end{cases}$$

Definition 3.10. Extended intersection: The extended intersection of two intuitionistic fuzzy soft sets $(\tilde{F},\tilde{A})and(\tilde{G},\tilde{B})$ over a common universe X is characterized by two complex numbers $z_1=\mu_1+j\nu_1$ and $z_2=\mu_2+j\nu_2$, where $\mu_1,\mu_2$and ν_1,ν_2, denotes the degrees of membership and non-membership respectively. The extended intersection of $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})is(\tilde{K},\tilde{C})$, where $\tilde{C}=(\tilde{A}\cup\tilde{B})$ and $\forall\varepsilon\in\tilde{C}$, $\tilde{K}(\varepsilon)=$

$$\begin{cases} \{(\tilde{a},\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{A}-\tilde{B},\\ \{(\tilde{a},\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{B}-\tilde{A},\\ \max(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))+j\min(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{A}\cap\tilde{B}. \end{cases}$$

We write, $(\tilde{F},\tilde{A})\cap_E(\tilde{G},\tilde{B})=(\tilde{K},\tilde{C})$.

Definition 3.11. Restricted intersection: Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets over the same universe set X such that $\tilde{A}\cap\tilde{B}\neq 0$ characterized by complex numbers $z_1=\mu_1+j\nu_1$ and $z_2=\mu_2+j\nu_2$, where $\mu_1,\mu_2$and ν_1,ν_2, denotes the degrees of membership and non-membership respectively. Then the restricted intersection of $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ is denoted by$(\tilde{F},\tilde{A})\cap_R(\tilde{G},\tilde{B})=(\tilde{K},\tilde{C})$, where $\tilde{C}=\tilde{A}\cap\tilde{B}$ and $\forall\varepsilon\in\tilde{C},\tilde{K}(\varepsilon)=\tilde{F}(\varepsilon)\cap\tilde{G}(\varepsilon)$.

Example 3.12. Let X be the initial universe and E be the set of parameters, $X=\{x_1,x_2,x_3,x_4\}$, $E=\{\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,\tilde{a}_4,\tilde{a}_5\}$. Suppose $\tilde{A}=\{\tilde{a}_1,\tilde{a}_2,\tilde{a}_3\}\subseteq E$, and $\tilde{B}=\{\tilde{a}_1,\tilde{a}_2,\tilde{a}_4,\tilde{a}_5\}\subseteq E$. We consider the IFSSs: $(\tilde{F},\tilde{A})=$

$$\begin{cases} \tilde{F}(\tilde{a}_1)=\{(x_1,0.8+0.1j),(x_2,0.7+0.2j),(x_3,0.4+0.3j),(x_4,0.5+0.4j)\},\\ \tilde{F}(\tilde{a}_2)=\{(x_1,0.6+0.22j),(x_2,0.4+0.5j),(x_3,0.3+0.2j),(x_4,0.1+0.06j)\},\\ \tilde{F}(\tilde{a}_3)=\{(x_1,0.7+0.03j),(x_2,0.8+0.05j),(x_3,0.4+0.3j),(x_4,0.2+0.01j)\}, \end{cases}$$

And $(\tilde{G}, \tilde{B}) =$

$$\begin{cases} \tilde{G}(\tilde{a}_1) = \{(x_1, 0.7 + 0.3j), (x_2, 0.5 + 0.44j), (x_3, 0.6 + 0.33j), (x_4, 0.3 + 0.22j)\}, \\ \tilde{G}(\tilde{a}_2) = \{(x_1, 0.5 + 0.04j), (x_2, 0.7 + 0.05j), (x_3, 0.3 + 0.04j), (x_4, 0.2 + 0.02j)\}, \\ \mathbf{\tilde{G}(\tilde{a}_4)} = \{(x_1, 0.2 + 0.05j), (x_2, 0.6 + 0.01j), (x_3, 0.9 + 0.01j), (x_4, 1 + 0j)\}, \\ \mathbf{\tilde{G}(\tilde{a}_5)} = \{(x_1, 0.8 + 0.02j), (x_2, 0.7 + 0.03j), (x_3, 0.6 + 0.04j), (x_4, 0.1 + 0.5j)\}, \end{cases}$$

(i) *For union*, $(\tilde{F}, \tilde{A}) \cup (\tilde{G}, \tilde{B}) = (\tilde{H}, \tilde{C})$, where $\tilde{C} = (\tilde{A} \cup \tilde{B}) = \{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5\}$ and, $(\tilde{H}, \tilde{C}) =$

$$\begin{cases} \tilde{H}(\tilde{a}_1) = \{(x_1, 0.8 + 0.1j), (x_2, 0.7 + 0.2j), (x_3, 0.6 + 0.3j), (x_4, 0.5 + 0.4j)\}, \\ \tilde{H}(\tilde{a}_2) = \{(x_1, 0.6 + 0.02j), (x_2, 0.7 + 0.05j), (x_3, 0.3 + 0.2j), (x_4, 0.2 + 0.02j)\}, \\ \tilde{H}(\tilde{a}_3) = \{(x_1, 0.7 + 0.03j), (x_2, 0.8 + 0.05j), (x_3, 0.9 + 0.01j), (x_4, 1 + 0j)\}, \\ \tilde{H}(\tilde{a}_4) = \{(x_1, 0.8 + 0.02j), (x_2, 0.7 + 0.03j), (x_3, 0.6 + 0.04j), (x_4, 0.1 + 0.5j)\}, \\ \tilde{H}(\tilde{a}_5) = \{(x_1, 0.9 + 0.1j), (x_2, 0.7 + 0.2j), (x_3, 0.5 + 0.3j), (x_4, 0.4 + 0.6j)\}. \end{cases}$$

(ii) *For intersection*, $(\tilde{F}, \tilde{A}) \cap (\tilde{G}, \tilde{B}) = (\tilde{K}, \tilde{C})$,where $\tilde{C} = (\tilde{A} \cap \tilde{B}) = \{\tilde{a}_1, \tilde{a}_2\}$, $(\tilde{K}, \tilde{C}) =$

$$\begin{cases} \tilde{K}(\tilde{a}_1) = \{(x_1, 0.7 + 0.3j), (x_2, 0.5 + 0.44j), (x_3, 0.4 + 0.3j), (x_4, 0.3 + 0.22j)\}, \\ \tilde{K}(\tilde{a}_2) = \{(x_1, 0.5 + 0.04j), (x_2, 0.4 + 0.05j), (x_3, 0.3 + 0.04j), (x_4, 0.1 + 0.06j)\}. \end{cases}$$

Example 3.13. Let X be the initial universe and E be the set of parameters, $X = \{x_1, x_2, x_3, x_4\}, E = \{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5\}$. Suppose $\tilde{A} = \{\tilde{a}_1, \tilde{a}_2\} \subseteq E$, and $\tilde{B} = \{\tilde{a}_2, \tilde{a}_4\} \subseteq E$. Let $(\tilde{F}, \tilde{A})$ and $(\tilde{G}, \tilde{B})$ be the intuitionistic fuzzy soft sets over X characterized by complex numbers is defined by the following: $(\tilde{F}, \tilde{A}) =$

$$\begin{cases} \tilde{F}(\tilde{a}_1) = \{(x_1, 0.6 + 0.3j), (x_2, 0.8 + 0.1j), (x_3, 0.5 + 0.4j), (x_4, 0.4 + 0.1j)\}, \\ \tilde{F}(\tilde{a}_2) = \{(x_1, 0.3 + 0.5j), (x_2, 0.2 + 0.4j), (x_3, 0.9 + 0.01j), (x_4, 1 + 0j)\}, \end{cases}$$

And $(\tilde{G}, \tilde{B}) =$

$$\begin{cases} \tilde{G}(\tilde{a}_2) = \{(x_1, 0.5 + 0.1j), (x_2, 0.6 + 0.02j), (x_3, 0.4 + 0.5j), (x_4, 0.2 + 0.6j)\}, \\ \tilde{G}(\tilde{a}_4) = \{(x_1, 0.9 + 0.1j), (x_2, 0.7 + 0.2j), (x_3, 0.6 + 0.3j), (x_4, 0.5 + 0.06j)\}, \end{cases}$$

(i) *For Extended union*, $(\tilde{F}, \tilde{A}) \cup_E (\tilde{G}, \tilde{B}) = (\tilde{H}, \tilde{C})$, where $(\tilde{H}, \tilde{C}) =$

$$\begin{cases} \tilde{H}(\tilde{a}_1) = \{(x_1, 0.6 + 0.3j), (x_2, 0.8 + 0.1j), (x_3, 0.5 + 0.4j), (x_4, 0.4 + 0.1j)\}, \\ \tilde{H}(\tilde{a}_2) = \{(x_1, 0.5 + 0.1j), (x_2, 0.6 + 0.02j), (x_3, 0.9 + 0.01j), (x_4, 1 + 0j)\}, \\ \tilde{H}(\tilde{a}_4) = \{(x_1, 0.9 + 0.1j), (x_2, 0.7 + 0.2j), (x_3, 0.6 + 0.3j), (x_4, 0.5 + 0.06j)\}. \end{cases}$$

(ii) *For Restricted union*, $(\tilde{F}, \tilde{A}) \cup_R (\tilde{G}, \tilde{B}) = (\tilde{H}, \tilde{C})$, where $(\tilde{H}, \tilde{C}) =$

$$\Big\{\tilde{H}(\tilde{a}_2) = \{(x_1, 0.5 + 0.1j), (x_2, 0.6 + 0.02j), (x_3, 0.9 + 0.01j), (x_4, 1 + 0j)\}.$$

(iii) *For Extended intersection*, $(\tilde{F}, \tilde{A}) \cap_E (\tilde{G}, \tilde{B}) = (\tilde{K}, \tilde{C})$, where $(\tilde{K}, \tilde{C}) =$

$$\begin{cases} \tilde{K}(\tilde{a}_1) = \{(x_1, 0.6 + 0.3j), (x_2, 0.8 + 0.1j), (x_3, 0.5 + 0.4j), (x_4, 0.4 + 0.1j)\}, \\ \tilde{K}(\tilde{a}_2) = \{(x_1, 0.3 + 0.5j), (x_2, 0.2 + 0.4j), (x_3, 0.4 + 0.5j), (x_4, 0.2 + 0.6j)\}, \\ \tilde{K}(\tilde{a}_4) = \{(x_1, 0.9 + 0.1j), (x_2, 0.7 + 0.2j), (x_3, 0.6 + 0.3j), (x_4, 0.5 + 0.06j)\}. \end{cases}$$

(iv) *For Restricted intersection*, $(\tilde{F},\tilde{A}) \cup_R (\tilde{G},\tilde{B}) = (\tilde{K},\tilde{C})$, where $(\tilde{K},\tilde{C}) =$

$$\Big\{\{\tilde{K}(\tilde{a}_2) = \{(x_1, 0.3+0.5j), (x_2, 0.2+0.4j), (x_3, 0.4+0.5j), (x_4, 0.2+0.6j)\}.$$

Definition 3.14. Conjugate: The conjugate of an intuitionistic fuzzy soft set $(\tilde{F},\tilde{A})$ characterized by a complex number $z = \mu + j\nu$, where μ and ν are the degrees of membership and non-membership respectively, is denoted by $(\tilde{F},\tilde{A})^-$ is defined as $(\tilde{F},\tilde{A})^- = (\tilde{F},\tilde{A}^-)$ where $\tilde{F} : \tilde{A}^- \to IF^X$ is given by $\tilde{F}[(\varepsilon)]^- = \tilde{F}(\varepsilon)^-$ for all $\varepsilon^- \in \tilde{A}^-$. Thus if $\tilde{F}(\varepsilon) = \{\tilde{a}, \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) + j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})) : \tilde{a} \in X\}$.Then $\tilde{F}(\varepsilon)^- = \{\tilde{a}, \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) - j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})) : \tilde{a} \in X\}$.

Definition 3.15. OR: Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets characterized by two complex numbers $z_1 = \mu_1 + j\nu_1$ and $z_2 = \mu_2 + j\nu_2$, where μ_1, μ_2 and ν_1, ν_2denote the degrees of membership and non-membership respectively, then $(\tilde{F},\tilde{A})\ OR\ (\tilde{G},\tilde{B})$ is an intuitionistic fuzzy soft set described by complex numbers denoted by $(\tilde{F},\tilde{A}) \vee (\tilde{G},\tilde{B})$ and is defined by $(\tilde{F},\tilde{A}) \vee (\tilde{G},\tilde{B}) = (\tilde{H},\tilde{A}\times\tilde{B})$ where $\tilde{K}(\tilde{a},\tilde{b}) = \tilde{F}(\tilde{a}) \cup \tilde{G}(\tilde{b}) \forall \tilde{a} \in \tilde{A}$ or $\tilde{b} \in \tilde{B}$, where $\cup$ is the operation between two intuitionistic fuzzy soft set described by complex numbers.

Definition 3.16. AND: Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets characterized by two complex numbers $z_1 = \mu_1 + j\nu_1$ and $z_2 = \mu_2 + j\nu 2$, where μ_1, μ_2 and ν_1, ν_2 denotes the degrees of membership and non-membership respectively, then $(\tilde{F},\tilde{A})\ AND\ (\tilde{G},\tilde{B})$ is an intuitionistic fuzzy soft set described by complex numbers denoted by $(\tilde{F},\tilde{A}) \wedge (\tilde{G},\tilde{B})$ and is defined by $(\tilde{F},\tilde{A}) \wedge (\tilde{G},\tilde{B}) = (\tilde{K},\tilde{A}\times\tilde{B})$ where $\tilde{K}(\tilde{a},\tilde{b}) = \tilde{F}(\tilde{a}) \cap \tilde{G}(\tilde{b}) \forall \tilde{a} \in \tilde{A}$ and $\tilde{b} \in \tilde{B}$, where $\cap$ is the operation between two intuitionistic fuzzy soft set described by complex numbers.

Example 3.17. Let X be the initial universe and E be the set of parameters, $X = \{x_1, x_2, x_3, x_4\}, E = \{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5\}$. Suppose $\tilde{A} = \{\tilde{a}_1, \tilde{a}_2\} \subseteq E$, and $\tilde{B} = \{\tilde{a}_2, \tilde{a}_3\} \subseteq E$. $\tilde{F} : \tilde{A} \to IF^X$, $(\tilde{F},\tilde{A}) =$

$$\begin{cases} \tilde{F}(\tilde{a}_1) = \{(x_1, 0.6+0.2j), (x_2, 0.5+0.4j), (x_3, 0.7+0.1j), (x_4, 0.4+0.3j)\}, \\ \tilde{F}(\tilde{a}_2) = \{(x_1, 0.2+0.3j), (x_2, 0.4+0.5j), (x_3, 0.8+0.02j), (x_4, 0.3+0.3j)\}, \end{cases}$$

And $\tilde{G} : \tilde{A} \to IF^X$, $(\tilde{G},\tilde{B}) =$

$$\begin{cases} \tilde{G}(\tilde{a}_2) = \{(x_1, 0.7+0.2j), (x_2, 0.9+0.1j), (x_3, 0.3+0.1j), (x_4, 0.6+0.3j)\}, \\ \tilde{\mathbf{G}}(\tilde{\mathbf{a}}_3) = \{(x_1, 0.2+0.4j), (x_2, 0.6+0.4j), (x_3, 0.5+0.2j), (x_4, 0.1+0.01j)\}, \end{cases}$$

(i) OR, Now we approximate the resulting intuitionistic fuzzy soft sets characterized by complex number is obtained by applying the above mentioned on $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$. We have $(\tilde{F},\tilde{A}) \vee (\tilde{G},\tilde{B}) = (\tilde{H},\tilde{A}\times\tilde{B})$, $\tilde{H} : \tilde{A}\times\tilde{B} \to IF^X$,

$$\tilde{H}(\tilde{A}\times\tilde{B}) = \begin{cases} \tilde{H}(\tilde{a}_1,\tilde{a}_2) = \{(x_1, 0.7+0.2j), (x_2, 0.9+0.1j), (x_3, 0.7+0.1j), \\ \qquad (x_4, 0.6+0.3j)\}, \\ \tilde{H}(\tilde{a}_1,\tilde{a}_3) = \{(x_1, 0.6+0.2j), (x_2, 0.6+0.4j), (x_3, 0.7+0.1j), \\ \qquad (x_4, 0.4+0.3j)\}, \\ \tilde{H}(\tilde{a}_2,\tilde{a}_2) = \{(x_1, 0.7+0.2j), (x_2, 0.9+0.1j), (x_3, 0.8+0.02j), \\ \qquad (x_4, 0.6+0.3j)\}, \\ \tilde{H}(\tilde{a}_2,\tilde{a}_3) = \{(x_1, 0.2+0.4j), (x_2, 0.6+0.4j), (x_3, 0.8+0.02j), \\ \qquad (x_4, 0.3+0.3j)\}. \end{cases}$$

(ii) AND, Now we approximate the resulting intuitionistic fuzzy soft sets characterized by complex number is obtained by applying the above mentioned on $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$. We have $(\tilde{F},\tilde{A})\wedge(\tilde{G},\tilde{B}) = (\tilde{K},\tilde{A}\times\tilde{B})$, $\tilde{H}:\tilde{A}\times\tilde{B}\to IF^X$,

$$\tilde{K}(\tilde{A}\times\tilde{B}) = \begin{cases} \tilde{K}(\tilde{a}_1,\tilde{a}_2) = \{(x_1, 0.6+0.2j), (x_2, 0.5+0.1j), (x_3, 0.3+0.1j), \\ \qquad (x_4, 0.4+0.3j)\}, \\ \tilde{K}(\tilde{a}_1,\tilde{a}_3) = \{(x_1, 0.2+0.4j), (x_2, 0.5+0.4j), (x_3, 0.5+0.2j), \\ \qquad (x_4, 0.1+0.01j)\}, \\ \tilde{K}(\tilde{a}_2,\tilde{a}_2) = \{(x_1, 0.2+0.3j), (x_2, 0.4+0.5j), (x_3, 0.3+0.1j), \\ \qquad (x_4, 0.3+0.3j)\}, \\ \tilde{K}(\tilde{a}_2,\tilde{a}_3) = \{(x_1, 0.2+0.3j), (x_2, 0.4+0.5j), (x_3, 0.5+0.2j), \\ \qquad (x_4, 0.1+0.01j)\}. \end{cases}$$

4. Some more set theoritic operations

Definition 4.1. Disjunctive sum of intuitionistic fuzzy soft sets: Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets over (X,E) characterized by two complex numbers $z_1 = \mu_1 + j\nu_1$ and $z_2 = \mu_2 + j\nu_2$, where μ_1,μ_2 and ν_1,ν_2 denotes the degrees of membership and non-membership respectively, we define the disjunctive sum of $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ as the intuitionistic fuzzy soft set $(\tilde{H},\tilde{C})$ over (X,E) which is characterized by complex numbers, written as $(\tilde{F},\tilde{A})\oplus(\tilde{G},\tilde{B}) = (\tilde{H},\tilde{C})$, where $\tilde{C} = \tilde{A}\cap\tilde{B}$ and $\forall\varepsilon\in\tilde{C}, \tilde{a}\in X$,

$$\mu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) = max(min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1-\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})), min(1-\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), \mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))),$$

$$\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) = jmin(max(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1-\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})), max(1-\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), \nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))).$$

Definition 4.2. Difference: Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets over (X,E) characterized by two complex numbers $z_1 = \mu_1 + j\nu_1$ and $z_2 = \mu_2 + j\nu_2$, where μ_1,μ_2 and ν_1,ν_2 denotes the degrees of membership and non-membership respectively, we define the difference of $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ as the intuitionistic fuzzy soft set $(\tilde{H},\tilde{C})$ over (X,E) which is characterized by complex numbers, written as $(\tilde{F},\tilde{A})\ominus(\tilde{G},\tilde{B}) = (\tilde{H},\tilde{C})$, where $\tilde{C} = \tilde{A}\cap\tilde{B}$ and $\forall\varepsilon\in\tilde{C}, \tilde{a}\in X$,

$$\mu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) = min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1-\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})),$$

$$\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) = jmax(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})).$$

Definition 4.3. Product: Let $(\tilde{F}, \tilde{A})$ and $(\tilde{G}, \tilde{B})$ be two intuitionistic fuzzy soft sets over (X, E) characterized by two complex numbers $z_1 = \mu_1 + j\nu_1$ and $z_2 = \mu_2 + j\nu_2$, where μ_1, μ_2 and ν_1, ν_2 denotes the degrees of membership and non-membership respectively.

(i) The extended product of $(\tilde{F}, \tilde{A})$ and $(\tilde{G}, \tilde{B})$ be two intuitionistic fuzzy soft sets over (X, E) characterized by two complex numbers $z_1 = \mu_1 + j\nu_1$ and $z_2 = \mu_2 + j\nu_2$ where μ_1, μ_2 and ν_1, ν_2 denote the degrees of membership and non-membership respectively, is defined as $(\tilde{F}, \tilde{A}) \odot (\tilde{G}, \tilde{B}) = (\tilde{H}, \tilde{C})$,where $\tilde{C} = \tilde{A} \cup \tilde{B}$ and $\forall \varepsilon \in \tilde{C}$,

$$\tilde{H}(\varepsilon) = \begin{cases} (\tilde{F}(\varepsilon), if\ \varepsilon \in \tilde{A} - \tilde{B}, \\ (\tilde{G}(\varepsilon), if\ \varepsilon \in \tilde{B} - \tilde{A}, \\ \tilde{F}(\varepsilon) \odot \tilde{G}(\varepsilon), if\ \varepsilon \in \tilde{A} \cap \tilde{B}. \end{cases}$$

(ii) The restricted product of $(\tilde{F}, \tilde{A})$ and $(\tilde{G}, \tilde{B})$ be two intuitionistic fuzzy soft sets over (X, E) characterized by two complex numbers $z_1 = \mu_1 + j\nu_1$ and $z_2 = \mu_2 + j\nu_2$, where μ_1, μ_2 and ν_1, ν_2 denotes the degrees of membership and non-membership respectively, is defined as $(\tilde{F}, \tilde{A}) \odot (\tilde{G}, \tilde{B}) = (\tilde{H}, \tilde{C})$,where $\tilde{C} = \tilde{A} \cap \tilde{B} \neq \emptyset$ and $\forall \varepsilon \in \tilde{C}$,

$$\tilde{H}(\varepsilon) = \begin{cases} (\tilde{F}(\varepsilon), if\ \varepsilon \in \tilde{A} - \tilde{B}, \\ (\tilde{G}(\varepsilon), if\ \varepsilon \in \tilde{B} - \tilde{A}, \\ \tilde{F}(\varepsilon) \odot \tilde{G}(\varepsilon), if\ \varepsilon \in \tilde{A} \cap \tilde{B}. \end{cases}$$

Example 4.4. Let X be the initial universe and E be the set of parameters, $X = \{x_1, x_2, x_3, x_4\}, E = \{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5\}$. Suppose $\tilde{A} = \{\tilde{a}_1, \tilde{a}_2\} \subseteq E$, and $\tilde{B} = \{\tilde{a}_2, \tilde{a}_3\} \subseteq E$. $\tilde{F} : \tilde{A} \to IF^X$, $(\tilde{F}, \tilde{A}) =$

$$\begin{cases} \tilde{F}(\tilde{a}_1) = \{(x_1, 0.6 + 0.2j), (x_2, 0.5 + 0.4j), (x_3, 0.7 + 0.1j), (x_4, 0.4 + 0.3j)\}, \\ \tilde{F}(\tilde{a}_2) = \{(x_1, 0.2 + 0.3j), (x_2, 0.4 + 0.5j), (x_3, 0.8 + 0.02j), (x_4, 0.3 + 0.3j)\}. \end{cases}$$

And $\tilde{G} : \tilde{A} \to IF^X$, $(\tilde{G}, \tilde{B}) =$

$$\begin{cases} \tilde{G}(\tilde{a}_2) = \{(x_1, 0.7 + 0.2j), (x_2, 0.9 + 0.01j), (x_3, 0.3 + 0.1j), (x_4, 0.6 + 0.3j)\} \\ \tilde{\mathbf{G}}(\tilde{\mathbf{a}}_3) = \{(x_1, 0.2 + 0.4j), (x_2, 0.6 + 0.04j), (x_3, 0.5 + 0.2j), (x_4, 0.1 + 0.01j)\} \end{cases}$$

Now we approximate the resulting intuitionistic fuzzy soft sets characterized by complex number is obtained by applying the above mentioned on $(\tilde{F}, \tilde{A})$ and $(\tilde{G}, \tilde{B})$. We have

(i) $(\tilde{F}, \tilde{A}) \odot (\tilde{G}, \tilde{B}) = (\tilde{H}, \tilde{C})$ where $\tilde{C} = \tilde{A} \cup \tilde{B} = \{\tilde{a}_1, \tilde{a}_2, \tilde{a}_3\}$,

$\tilde{H}:\tilde{A}\cup\tilde{B}\to IF^X$. Now, $\tilde{H}(\tilde{C})=$

$$\begin{cases} \tilde{H}(\tilde{a}_1)=\{(x_1,0.6+0.2j),(x_2,0.5+0.4j),(x_3,0.7+0.1j),\\ (x_4,0.4+0.3j)\},\\ \tilde{H}(\tilde{a}_2)=\{(x_1,0.38+0.26j),(x_2,0.355+0.454j),(x_3,0.238+0.086j),\\ (x_4,0.9+0.27j)\},\\ \tilde{H}(\tilde{a}_3)=\{(x_1,0.2+0.4j),(x_2,0.6+0.04j),(x_3,0.5+0.2j),\\ (x_4,0.1+0.01j)\}.\end{cases}$$

(ii) $(\tilde{F},\tilde{A})\odot(\tilde{G},\tilde{B})=(\tilde{H},\tilde{C})$, where $\tilde{C}=\tilde{A}\cap\tilde{B}=\{\tilde{a}_2\},\tilde{H}:\tilde{A}\cap\tilde{B}\to IF^X$. Now, $\tilde{H}(\tilde{C})=\{\tilde{H}(\tilde{a}_2)=$

$$\{(x_1,0.38+0.26j),(x_2,0.355+0.454j),(x_3,0.238+0.086j),(x_4,0.9+0.27j)\}.$$

Proposition 4.5. *Let $(\tilde{F},\tilde{A})$ be an intuitionistic fuzzy soft set over X characterized by a complex number $z=\mu+j\nu$, where μ is the degree of membership and ν is the degree of non-membership respectively, then the following results hold.*
(i) $[(\tilde{F},\tilde{A})^C]^C=(\tilde{F},\tilde{A})$
$(ii)(\tilde{F},\tilde{A})\cup(\tilde{F},\tilde{A})=(\tilde{F},\tilde{A})$
$(iii)(\tilde{F},\tilde{A})\cap(\tilde{F},\tilde{A})=(\tilde{F},\tilde{A})$

Proof. (i) is straight forward.
(ii) Let $(\tilde{F},\tilde{A})\cup(\tilde{F},\tilde{A})=(\tilde{H},\tilde{C})$ where $\tilde{C}=\tilde{A}\cup\tilde{A}=\tilde{A}$ and $\forall\ \varepsilon\in\tilde{C}$,

$$\tilde{H}(\varepsilon)=\{(\tilde{a},\mu_{\tilde{H}(\varepsilon)}(\tilde{a})+j(\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X,\varepsilon\in\tilde{A}\}\}.$$

Consider the right hand side of the equality $(\tilde{F},\tilde{A}),\forall\varepsilon\in\tilde{A}$,

$$\tilde{F}(\varepsilon)=\{(\tilde{a},\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X,\varepsilon\in\tilde{A}\}\}.$$

Since $\tilde{H}(\varepsilon)=\tilde{F}(\varepsilon)$. Thus $(\tilde{F},\tilde{A})\cup(\tilde{F},\tilde{A})=(\tilde{F},\tilde{A})$.
(iii) Similar as (ii) □

Proposition 4.6. *Let $(\tilde{F},\tilde{A}),(\tilde{G},\tilde{B})$ and $(\tilde{H},\tilde{C})$ are three intuitionistic fuzzy soft sets in the same universe (X,E) characterized by three complex numbers $z_1=\mu_1+j\nu_1,z_2=\mu_2+j\nu_2$and $z_3=\mu_3+j\nu_3$ where μ_1,μ_2,μ_3 and ν_1,ν_2,ν_3 denote the degrees of membership and non-membership respectively, then the following holds.*
(i) $(\tilde{F},\tilde{A})\cup(\tilde{G},\tilde{B})=(\tilde{G},\tilde{B})\cup(\tilde{F},\tilde{A})$
$(ii)(\tilde{F},\tilde{A})\cap(\tilde{G},\tilde{B})=(\tilde{G},\tilde{B})\cap(\tilde{F},\tilde{A})$
$(iii)(\tilde{F},\tilde{A})\cap((\tilde{G},\tilde{B})\cap(\tilde{H},\tilde{C}))=((\tilde{F},\tilde{A})\cap(\tilde{G},\tilde{B}))\cap(\tilde{H},\tilde{C})$
$(iv)(\tilde{F},\tilde{A})\cup((\tilde{G},\tilde{B})\cup(\tilde{H},\tilde{C}))=((\tilde{F},\tilde{A})\cup(\tilde{G},\tilde{B}))\cup(\tilde{H},\tilde{C})$

Proof. (i) $(\tilde{F},\tilde{A})\cup(\tilde{G},\tilde{B})=(\tilde{G},\tilde{B})\cup(\tilde{F},\tilde{A})$. Let $(\tilde{F},\tilde{A})\cup(\tilde{G},\tilde{B})=(\tilde{H},\tilde{C})$ where $\tilde{C}=\tilde{A}\cup\tilde{B}$ and $\forall\varepsilon\in\tilde{C}$, $\tilde{H}(\varepsilon)=$

$$\begin{cases} \{(\tilde{a},\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\},if\ \varepsilon\in\tilde{A}-\tilde{B},\\ \{(\tilde{a},\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\},if\ \varepsilon\in\tilde{B}-\tilde{A},\\ \{max(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),(\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))+jmin(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\},if\ \varepsilon\in\tilde{A}\cap\tilde{B}.\end{cases}$$

Again let$(\tilde{G},\tilde{B})\cup(\tilde{F},\tilde{A})=(\tilde{K},\tilde{D})$ where $\tilde{D}=\tilde{B}\cup\tilde{A}$ and $\forall\ \varepsilon\in\tilde{D}$, $\tilde{K}(\varepsilon)=$

$$\begin{cases} \{(\tilde{a},\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{B}-\tilde{A}, \\ \{(\tilde{a},\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{F}_{(v)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{A}-\tilde{B}, \\ \{max(\mu_{\tilde{F}_{(v)}}(\tilde{a}),\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))+jmin(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}),\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ v\in\tilde{A}\cap\tilde{B}). \end{cases}$$

Since $\tilde{C}=\tilde{D}$ and $\tilde{H}(\varepsilon)=\tilde{K}(\varepsilon)$. Thus $(\tilde{F},\tilde{A})\cup(\tilde{G},\tilde{B})=(\tilde{G},\tilde{B})\cup(\tilde{F},\tilde{A})$.

(ii) Similar as (i) .

(iii) $(\tilde{F},\tilde{A})\cap((\tilde{G},\tilde{B})\cap(\tilde{H},\tilde{C}))=((\tilde{F},\tilde{A})\cap(\tilde{G},\tilde{B}))\cap(\tilde{H},\tilde{C})$. Let $(\tilde{G},\tilde{B})\cap(\tilde{H},\tilde{C})=(\tilde{K}_1,\tilde{D}_1)$ where $\tilde{D}_1=\tilde{B}\cap\tilde{C}$ and for all $\varepsilon\in\tilde{D}_1$, $\tilde{K}_1(\varepsilon)=$

$$\begin{cases} \{(\tilde{a},\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{B}-\tilde{C}, \\ \{(\tilde{a},\mu_{\tilde{H}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ v\in\tilde{C}-\tilde{B}, \\ \{max(\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}),\mu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}))+jmin(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}),\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{B}\cap\tilde{C}. \end{cases}$$

And $(\tilde{F},\tilde{A})\cap(\tilde{K}_1,\tilde{D}_1)=(\tilde{K}_2,\tilde{D}_2)$ where $\tilde{D}_2=\tilde{A}\cap\tilde{D}_1=\tilde{A}\cap(\tilde{B}\cap\tilde{C})$ and for all $\varepsilon\in\tilde{D}_2$, $\tilde{K}_2(\varepsilon)=$

$$\begin{cases} \{(\tilde{a},\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{A}-\tilde{D}_{1,} \\ \{(\tilde{a},\mu_{\tilde{K}_{1(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{K}_{1(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{D}_1-\tilde{A}, \\ \{max(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),\mu_{\tilde{K}_{1(\varepsilon)}}(\tilde{a}))+jmin(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),\nu_{\tilde{K}_{1(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{A}\cap\tilde{D}_1\}. \end{cases}$$

Taking right hand side, let $(\tilde{F},\tilde{A})\cap(\tilde{G},\tilde{B})=(\tilde{K}_3,\tilde{D}_3)$ where $\tilde{D}_3=\tilde{A}\cap\tilde{B}$ and for all $\varepsilon\in\tilde{D}_3$, $\tilde{K}_3(\varepsilon)=$

$$\begin{cases} \{(\tilde{a},\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})+j\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{A}-\tilde{B}, \\ \{(\tilde{a},\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{B}-\tilde{A}, \\ \{max(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))+jmin(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{A}\cap\tilde{B})\}. \end{cases}$$

And $(\tilde{K}_3,\tilde{D}_3)\cap(\tilde{H},\tilde{C})=(\tilde{K}_4,\tilde{D}_4)$ where $\tilde{D}_4=\tilde{D}_4\cap\tilde{C}=(\tilde{A}\cap\tilde{B})\cap\tilde{C}$ and for all $\varepsilon\in\tilde{D}_4$, $\tilde{K}_4\,(\varepsilon)=$

$$\begin{cases} \{(\tilde{a},\mu_{\tilde{K}_{3(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{K}_{3(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{D}_3-\tilde{C}, \\ \{(\tilde{a},\mu_{\tilde{H}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{C}-\tilde{D}_3, \\ \{max(\mu_{\tilde{K}_{3(\varepsilon)}}(\tilde{a}),\mu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}))+jmin(\nu_{\tilde{K}_{3(\varepsilon)}}(\tilde{a}),\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{D}_3\cap\tilde{C}. \end{cases}$$

Since $\tilde{D}_2=\tilde{D}_4$ and $\tilde{K}_2(\varepsilon)=\tilde{K}_4(\varepsilon)$. Thus $(\tilde{F},\tilde{A})\cap((\tilde{G},\tilde{B})\cap(\tilde{H},\tilde{C}))=((\tilde{F},\tilde{A})\cap(\tilde{G},\tilde{B}))\cap(\tilde{H},\tilde{C})$. □

5. De Morgan's law in intuitionistic fuzzy soft set characterized by complex numbers

Theorem 5.1. *Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets in (X,E) characterized by two complex numbers $z_1=\mu_1+j\nu_1$and $z_2=\mu_2+j\nu_2$ where μ_1,μ_2 and ν_1,ν_2 denote the degrees of membership and non-membership respectively, then we the following.*

(i) $((\tilde{F},\tilde{A})\cup(\tilde{G},\tilde{B}))^C=(\tilde{F},\tilde{A})^C\cap(\tilde{G},\tilde{B})^C$
(ii) $((\tilde{F},\tilde{A})\cap(\tilde{G},\tilde{B}))^C=(\tilde{F},\tilde{A})^C\cup(\tilde{G},\tilde{B})^C$

Proof. (i) Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets in (X,E) characterized by two complex numbers $z_1=\mu_1+j\nu_1$and $z_2=\mu_2+j\nu_2$ where μ_1,μ_2 and ν_1,ν_2 denote the degrees of membership and non-membership respectively and $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})=(\tilde{H},\tilde{C})$ where $\tilde{C}=\tilde{A}\cup\tilde{B}$ and $\forall\varepsilon\in\tilde{C}$, $\tilde{H}(\varepsilon)=$

$$\begin{cases}\{\tilde{a},\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{A}-\tilde{B},\\ \{\tilde{a},\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \varepsilon\in\tilde{B}-\tilde{A},\\ \{max(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))+jmin(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})):\tilde{a}\in X\},\\ if\ \varepsilon\in\tilde{A}\cap\tilde{B})\}.\end{cases}$$

Now $((\tilde{F},\tilde{A})\cup(\tilde{G},\tilde{B}))^C=(\tilde{H},\tilde{C})^C=(\tilde{H}^C,\neg\tilde{C})$ for all $\neg\varepsilon\in\neg\tilde{C}=\neg\tilde{A}\cup\neg\tilde{B}$, $\tilde{H}^C(\neg\varepsilon)=$

$$\begin{cases}\{\tilde{a},\mu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}^C, if\ \neg\varepsilon\in\neg\tilde{A}-\neg\tilde{B},\\ \{\tilde{a},\mu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}^C, if\ \neg\varepsilon\in\neg\tilde{B}-\neg\tilde{A},\\ \{max(\mu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a}),\mu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a}))+jmin(\nu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a}),\nu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}^C,\\ if\ \neg\varepsilon\in\neg\tilde{A}\cap\neg\tilde{B})\}.\end{cases}$$

$$=\begin{cases}\{(\tilde{a},\nu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})+j(\mu_{\tilde{F}(\neg\varepsilon)}(\tilde{a}):\tilde{a}\in X\}, if\ \neg\varepsilon\in\neg\tilde{A}-\neg\tilde{B},\\ \{(\tilde{a},\nu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})+j(\mu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \neg\varepsilon\in\neg\tilde{B}-\neg\tilde{A},\\ \{min(\mu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a}),\mu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a}))+jmax(\nu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a}),\nu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})):\tilde{a}\in X\},\\ if\ \neg\varepsilon\in\neg\tilde{A}\cap\neg\tilde{B}\}.\end{cases}$$

Consider the right hand side of the equality $(\tilde{F},\tilde{A})^C\cap(\tilde{G},\tilde{B})^C=(\tilde{F}^C,\neg\tilde{A})\cap(\tilde{G}^C,\neg\tilde{B})=(\tilde{K},\tilde{L})$ where $\tilde{L}=\neg\tilde{A}\cup\neg\tilde{B}$ and for all $\neg\varepsilon\in\tilde{L}$, $\tilde{\mathbf{K}}(\neg\varepsilon)=$

$$\begin{cases}\{\tilde{a},\mu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}^C, if\ \neg\varepsilon\in\neg\tilde{A}-\neg\tilde{B},\\ \{\tilde{a},\mu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a})+j(\nu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}^C, if\ \neg\varepsilon\in\neg\tilde{B}-\neg\tilde{A},\\ \{max(\mu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a}),\mu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a})+jmin(\nu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a}),\nu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a}):\tilde{a}\in X\}^C,\\ if\ \neg\varepsilon\in\neg\tilde{A}\cap\neg\tilde{B})\}.\end{cases}$$

$$=\begin{cases}\{\tilde{a},\nu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})+j(\mu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \neg\varepsilon\in\neg\tilde{A}-\neg\tilde{B},\\ \{\tilde{a},\nu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})+j(\mu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})):\tilde{a}\in X\}, if\ \neg\varepsilon\in\neg\tilde{B}-\neg\tilde{A},\\ \{min(\mu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a}),\mu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a}))+jmax(\nu_{\tilde{G}_{(\neg\varepsilon)}}(\tilde{a}),\nu_{\tilde{F}_{(\neg\varepsilon)}}(\tilde{a})):\tilde{a}\in X\},\\ if\ \neg\varepsilon\in\neg\tilde{A}\cap\neg\tilde{B})\}.\end{cases}$$

Since $\tilde{H}^C(\neg\varepsilon)=\tilde{\mathbf{K}}(\neg\varepsilon)$. We conclude that $((\tilde{F},\tilde{A})\cup(\tilde{G},\tilde{B}))^C=(\tilde{F},\tilde{A})^C\cap(\tilde{G},\tilde{B})^C$.

(ii) Straightforward as (i), so we omit it. □

Proposition 5.2. *Let $(\tilde{F},\tilde{A})$ and $(\tilde{G},\tilde{B})$ be two intuitionistic fuzzy soft sets in (X,E) characterized by two complex numbers $z_1=\mu_1+j\nu_1$and $z_2=\mu_2+j\nu_2$*

where μ_1, μ_2 and ν_1, ν_2 denote the degrees of membership and non-membership respectively, then we have the following result hold:
$(i)(\tilde{F}, \tilde{A})(\tilde{G}, \tilde{B}) = (\tilde{G}, \tilde{B}) \oplus (\tilde{F}, \tilde{A})$
$(ii)(\tilde{F}, \tilde{A}) \oplus ((\tilde{G}, \tilde{B}) \oplus (\tilde{H}, \tilde{C})) = ((\tilde{G}, \tilde{A}) \oplus (\tilde{F}, \tilde{B})) \oplus (\tilde{H}, \tilde{C})$

Proof. (i) As we know that

$$(\tilde{F}, \tilde{A}) = \{\tilde{a}, \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) + j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), \forall \tilde{a} \in X, \forall\ \varepsilon \in \tilde{A}\}$$

$$(\tilde{G}, \tilde{B}) = \{\tilde{a}, \mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}) + j(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}), \forall \tilde{a} \in X, \forall \varepsilon \in \tilde{B}\}$$

Let $(\tilde{F}, \tilde{A}) \oplus (\tilde{G}, \tilde{B}) = (\tilde{H}, \tilde{C})$ where $\tilde{C} = \tilde{A} \cap \tilde{B}$ and $\forall \varepsilon \in \tilde{C}, \tilde{a} \in X$,

$$\mu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) = max(min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})), min(1 - \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), \mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))),$$

$$\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) = jmin(max(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a})), max(1 - \nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), \nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))).$$

. Let $(\tilde{G}, \tilde{B}) \oplus (\tilde{F}, \tilde{A}) = (\tilde{K}, \tilde{L})$ where $\tilde{L} = \tilde{A} \cap \tilde{B}$ and $\forall \varepsilon \in \tilde{L}, \tilde{a} \in X$,

$$\mu_{\tilde{K}_{(\varepsilon)}}(\tilde{a}) = max(min(\mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}), 1 - \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})), min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \mu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))),$$

$$\nu_{\tilde{K}_{(\varepsilon)}}(\tilde{a}) = jmin(max(\nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}), 1 - \nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})), max(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \nu_{\tilde{G}_{(\varepsilon)}}(\tilde{a}))).$$

It follows that $(\tilde{H}, \tilde{C}) = (\tilde{K}, \tilde{L})$. Therefore $(\tilde{F}, \tilde{A}) \oplus (\tilde{G}, \tilde{B}) = (\tilde{G}, \tilde{B}) \oplus (\tilde{F}, \tilde{A})$.

(ii) Straightforward, so we omit it. □

Proposition 5.3. $(\tilde{F}, \tilde{A}) \oplus (\phi, \tilde{A}) = (\tilde{F}, \tilde{A})$.

Proof. As we know that $(\tilde{F}, \tilde{A}) = \{(\tilde{a}, \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) + j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})), \forall \tilde{a} \in X, \forall \varepsilon \in \tilde{A})\}$. $(\phi, \tilde{A}) = \{(\tilde{a}, (0 + j(\nu_{\phi_{(\varepsilon)}}(\tilde{a})), \forall \tilde{a} \in X, \forall \varepsilon \in \tilde{A})\}$. $(\tilde{F}, \tilde{A}) \oplus (\phi, \tilde{A}) = (\tilde{H}, \tilde{C})$ where $\tilde{C} = \tilde{A} \cap \tilde{A} = \tilde{A}$, for all $\varepsilon \in \tilde{C}, \tilde{a} \in X$,

$$\begin{aligned}
\mu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) &= max(min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \mu_{\phi_{(\varepsilon)}}(\tilde{a})), min(1 - \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), \mu_{\phi_{(\varepsilon)}}(\tilde{a}))) \\
&= max(min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - 0), min(1 - \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 0)) \\
&= max(min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1), min(1 - \mu\tilde{F}(\varepsilon)(\tilde{a}), 0)) \\
&= max(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 0) \\
&= \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}),
\end{aligned}$$

$$\begin{aligned}
\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) &= jmin(max(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \nu_{\phi_{(\varepsilon)}}(\tilde{a})), max(1 - \nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), \nu_{\phi_{(\varepsilon)}}(\tilde{a})) \\
&= jmin(max(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - 1), max(1 - \nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1)) \\
&= jmin(max(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 0), max(1 - \nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1)) \\
&= jmin(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1) \\
&= j\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}).
\end{aligned}$$

Therefore, $(\tilde{H}, \tilde{C}) = \{(\tilde{a}, \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) + j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})), \forall \tilde{a} \in X, \forall \varepsilon \in \tilde{C} = \tilde{A})\}$. Thus $(\tilde{F}, \tilde{A}) \oplus \tilde{(}\phi, \tilde{A}) = (\tilde{F}, \tilde{A})$. □

Proposition 5.4. *(i)* $(\tilde{F}, \tilde{A}) \ominus (\phi, \tilde{A}) = (\tilde{F}, \tilde{A})$
(ii) $(\tilde{F}, \tilde{A}) \ominus (\tilde{X}, \tilde{A}) = (\phi, \tilde{A})$

Proof. *(i)* $(\tilde{F}, \tilde{A}) = \{(\tilde{a}, (\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) + j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})), \forall \tilde{a} \in X, \forall \varepsilon \in \tilde{A})\}$. $(\phi, \tilde{A}) = \{(\tilde{a}, (0 + j(\nu_{\phi_{(\varepsilon)}}(\tilde{a})), \forall \tilde{a} \in X, \forall \varepsilon \in \tilde{A})\}$. Now $(\tilde{F}, \tilde{A}) \ominus (\phi, \tilde{A}) = (\tilde{H}, \tilde{C})$ where $\tilde{C} = \tilde{A} \cap \tilde{A} = \tilde{A}$, for all $\varepsilon \in \tilde{C}, \tilde{a} \in X$,

$$\begin{aligned}
\mu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) &= min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \mu_{\phi_{(\varepsilon)}}(\tilde{a})) \\
&= min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - 0) \\
&= min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1) \\
&= \mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})
\end{aligned}$$

$$\begin{aligned}
\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) &= jmax(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \nu_{\phi_{(\varepsilon)}}(\tilde{a})) \\
&= jmax(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - 1) \\
&= jmax(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 0) \\
&= j\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})
\end{aligned}$$

Therefore, $(\tilde{H}, \tilde{C}) = \{(\tilde{a}, (\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) + j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})), \forall \tilde{a} \in X, \forall \varepsilon \in \tilde{C} = \tilde{A})\}$. Thus, $(\tilde{F}, \tilde{A}) \ominus (\phi, \tilde{A}) = (\tilde{F}, \tilde{A})$.

(ii) $(\tilde{F}, \tilde{A}) \ominus (X, \tilde{A}) = (\phi, \tilde{A})$, $(\tilde{F}, \tilde{A}) = \{(\tilde{a}, (\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}) + j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})), \forall \tilde{a} \in X, \forall \varepsilon \in \tilde{A})\}$. $(\tilde{X}, \tilde{A}) = \{(\tilde{a}, (\mu_{X_{(\varepsilon)}}(\tilde{a}) + 0j), \forall \tilde{a} \in X, \forall \varepsilon \in \tilde{A})\}$. Now $(\tilde{F}, \tilde{A}) \ominus \tilde{(}X, \tilde{A}) = (\tilde{H}, \tilde{C})$ where $\tilde{C} = \tilde{A} \cap \tilde{A} = \tilde{A}$, for all $\varepsilon \in \tilde{C}, \tilde{a} \in X$,

$$\begin{aligned}
\mu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) &= min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \mu_{\mathbf{X}_{(\varepsilon)}}(\tilde{a})) \\
&= min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - 1) \\
&= min(\mu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 0) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\nu_{\tilde{H}_{(\varepsilon)}}(\tilde{a}) &= jmax(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - \nu_{\mathbf{X}_{(\varepsilon)}}(\tilde{a})) \\
&= jmax(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1 - 0) \\
&= jmax(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a}), 1) \\
&= j
\end{aligned}$$

Therefore, $(\tilde{H}, \tilde{C}) = \{(\tilde{a}, (0 + j(\nu_{\tilde{F}_{(\varepsilon)}}(\tilde{a})), \forall \tilde{a} \in X, \forall \varepsilon \in \tilde{C} = \tilde{A})\}$. Thus $(\tilde{F}, \tilde{A}) \ominus (X, \tilde{A}) = (\phi, \tilde{A})$. □

6. Conclusion

In this paper, we defined the hybrid structure which is the combination of soft set and complex number representation of intuitionistic fuzzy set. It is basically intuitionistic fuzzy soft set but with representation of membership and non-membership by complex numbers. Further, we defined basic set theoretic operations such as complement, union, intersection, restricted union, restricted intersection etc. for this hybrid structure. Moreover, we developed this theory to establish some more set theoretic operation s like Disjunctive sum, difference, product, conjugate etc. We also studied de Morgan laws and their properties in this paper. In the future, we will define algebraic soft structures based on this approach of complex numbers.

References

1. B. Ahmad and A. Kharal, "*On Fuzzy Soft Sets*", Advances in Fuzzy Systems, **2009** (2009).
2. A. Alkouri and A. Salleh, "*Complex intuitionistic fuzzy sets*," in International Conference on Fundamental and Applied Sciences, AIP Conference Proceedings **1482** (2012), 464-470.
3. A. Alkouri and A.R. Salleh, *Some operations on complex Atanassov's intuitionistic fuzzy sets*, AIP Conference Proceedings 1571, **987** (2013).
4. K.T. Atanassov, "*Intuitionistic fuzzy sets*," Fuzzy Sets and Systems **20** (1986), 87-96.
5. K.T. Atanassov, *Intuitionistic fuzzy sets*, Theory and applications of Studies in Fuzziness and Soft Computing, Physica, Heidelberg, Germany **35**, 1999.
6. K.V. Babitha and Sunil Jacob John, *Generalized Intuitionistic Fuzzy Soft sets and its applications*, Gen. Math. Notes **7** (2011), 1-14.
7. T.M. Basu, N.K. Mahapetra, S.K. Mondal, *On Some New Operations in Fuzzy soft Set and Intuitionistic Fuzzy soft Set Theor*y, Department of Applied Mathematics with Oceanology and computer Programming, Vidyasagar University, Midnapure -721102, W.B., India.
8. M. Bashir, A.R. Salleh, and S. Alkhazaleh, *Possibility Intuitionistic Fuzzy Soft Set*, Advances in Decision Sciences **2012** (2012), Article ID 404325, 24 pages, doi:10.1155/2012/404325.
9. M. Bora, T. Jyoti Neog, D. Kumar Sut, *Some New Operations of Intuitionistic Fuzzy Soft Set*, International Journal of Soft Computing and Engineering (IJSCE) ISSN: 2231-2307, **2** (2012)..
10. S. Broumi, F. Smarandache, *Intuitionistic Fuzzy Soft Expert Sets and its Application in Decision Making*, Journal of New Theory, Number **1** (2015), 89-105.
11. S. Broumi, F. Smarandache, *Mapping on Intuitionistic Fuzzy Soft Expert Sets and its Application in Decision Making*, Journal of New Results in science, **9** (2015), 1-10.
12. S. Broumi, *Q-Intuitionistic Fuzzy Soft Sets*, Journal of New Theory **5** (2015), 80-91.
13. M.D. Cock, C. Cornelis and E.E. Kerre, "*Intuitionistic Fuzzy Relational Images*," Studies in Computational Intelligence **2** (2005), 129-145.
14. S. Coupland, R. John, *New geometric inference techniques for type-2 fuzzy sets*, International Journal of Approximate Reasoning **49** (2008), 198-211.
15. S.P. De, R.P. Krishna, *A new approach to mining fuzzy databases using nearest neighb or classification by exploiting attribute hierarchies*, Int J. Intell Syst **19** (2004), 1277-1290.
16. G. Deschrijive and E.E. Kerre, "*On the Position of Intuitionistic Fuzzy Set Theory in the Framework of Theories Modelling Imprecision*," Information Sciences **177** (2007), 1860-1866.
17. B. Dinda and T.K. Samanta, *Relations on Intuitionistic Fuzzy Soft Sets*, Gen. Math. Notes, **l** (2010), 74-83.
18. D. Drianko, H. Hellendorf, M. Reinfrank, *An introduction to fuzzy control*, Berlin: Springer-Verlag, 1993.
19. A. Kandel, *Fuzzy mathematical techniques with applications*, New York: Addison Wesley, 1987.

20. T. Kumar and R.K. Bajaj, *On Complex Intuitionistic Fuzzy Soft sets with Distance Measure and Entropies*, Hindawi Publishing Corporation, Journal of Mathematics **2014** (2014), Article ID 972198, 12 pages.
21. P.K. Maji, R. Biswas, A. Roy, *Soft set theory*, Department of Mathematics, Indian Institute of Technology, Kharagpur – 721302, West Bengal, India.
22. P.K. Maji, R. Biswas, A. Roy, *Fuzzy Soft sets*, Journal of Fuzzy Mathematics **9** (2001), 589-602.
23. P.K. Maji, R. Biswas, A.R. Roy, *Intuitionistic fuzzy soft sets*, The journal of fuzzy mathematics **9** (2001), 677-692.
24. J.M. Mendel, *Uncertainty, Fuzzy logic, and signal processing*, Signal Processing **80** (2000), 913-933.
25. A. Mhalla, N. Jerbi, S.C. Dutilleul, E. Craye, M. Benrejeb, *Fuzzy Filtering of Sensors Signals in Manufacturing Systems with Time, Constraints*, International Journal of Computers Communications & Control **5** (2010), 362-374.
26. D. Molodtsov, *Soft set theory, First result*, Computing center of the Russian Academy of Science 40 Vavilova Street, Moscow 117967, Russia.
27. T.J. Neog, D.K. Sut, "*On Union and Intersection of Fuzzy Soft Sets*", Int. J. Comp. Tech. Appl. **2** , 1160-1176.
28. D. Ramot, R. Milo, M. Friedman, A. Kandel, *Complex fuzzy sets*, IEEE Transaction on Fuzzy Systems **10** (2002), 171-186.
29. D. Ramot, M. Friedman, G. Langholz, A. Kandel, *Complex fuzzy logic*, IEEE Transaction on Fuzzy Systems **11** (2003), 450-461.
30. E. Szmidt, J. Kacprzyk, *Medical diagnostic reasoning using a similarity measure for intuitionistic fuzzy sets*, Note on IFS **10** (2004), 61-69.
31. L.K. Vlachos and G.D. Sergiadis, "*Intuitionistic Fuzzy Information - Applications to Pattern Recognition*,"Pattern Recognition Letters **28** (2007), 197- 206.
32. N. Yaqoob, M. Akram and M. Aslam, *Intuitionistic Fuzzy Soft Groups Induced by (t, s)-norm*, Indian Journal of Science and Technology, 2010.
33. L.A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965), 338-353.
34. L.A Zadeh, *The concept of a linguistic variable and its application to approximate reasoning- Part I*, Information Sciences **7** (1975), 199-249.
35. L.A. Zadeh, *Fuzzy algorithms*, Information and Control **12** (1968), 94–102.
36. J. Zhao, Y. Li and Y. Yin, *Intuitionistic Fuzzy Soft Semigroup*, Mathematical Aeterna **1** (2011).

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