Adaptive Cooperation for Bidirectional Communication in Cognitive Radio Networks

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Abstract

In the interweave cognitive networks, the interference from the primary user degrades the performance of the cognitive user transmissions. In this paper, we propose an adaptive cooperation scheme in the interweave cognitive networks to improve the performance of the cognitive user transmissions. In the proposed scheme for the bidirectional communication of two end-source cognitive users, the bidirectional communication is completed through the non-relay direct transmission, the one-way relaying cooperation transmission, and the two-way relaying cooperation transmission depending on the limited feedback from the end-sources. For the performance analysis of the proposed scheme, we derive the outage probability and the finite-SNR diversity multiplexing tradeoff (f-DMT) in a closed form, considering the imperfect spectrum sensing, the interference from the primary user, and the power allocation between the relay and the end-sources. The results show that compared with the direct transmissions (DT), the pure one-way relaying transmissions (POWRT), and the pure two-way relaying transmissions (PTWRT), the proposed scheme has better outage performance. In terms of the f-DMT, the proposed scheme outperforms the full cooperation transmissions of the POWRT and PTWRT.

Keywords: Cognitive radio, adaptive cooperation, bidirectional communication, outage probability, diversity-multiplexing tradeoff.

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1. Introduction

Cognitive radio is considered as a promising technique to solve the spectrum efficiency problem which has appeared as a result of the current static spectrum allocation policy[1-3] . Cognitive/unlicensed users in cognitive radio networks access spectrum holes in an opportunistic manner. The spectrum hole is a licensed spectrum unoccupied by its primary network at a particular time and a specific geographic location. The cognitive users (CUs) detect a spectrum hole through the spectrum sensing and then complete the cognitive user transmissions which denote the communication among the CUs over the detected spectrum hole.[1] However, the cognitive user transmissions have to endure the interference from the primary user (PU) due to the miss detection of the presence of PU. The interference degrades the performance of the cognitive user transmissions [4].

Cooperative relaying techniques can build the distributed MIMO systems through distributed antennas on different radio devices to achieve the diversity gain [5-7]. Therefore, they are a propitious mean to improve the performance of the cognitive user transmissions and that of the cognitive security obtained by the cooperative detection of spectrum selfish attacks [8]. The cognitive user transmissions assisted by cooperative relays can be divided into one-way relaying schemes and two-way relaying schemes. In the one-way relaying schemes, one or multiple CUs as the relays help the information transmission from one cognitive source to one cognitive destination. The one-way relaying schemes can reduce the outage probabilities of the cognitive information transmissions [9]. The two-way relaying schemes are proposed for the information exchange between two end-sources, because they have higher spectral efficiency than the one-way relaying schemes in the half-duplex communication scenario [10]. In the two-way relaying schemes, some CUs as two-way relays assist the spectrum sensing to increase the accuracy of detecting the presence of the PU [11]. They can also help other CUs' bidirectional traffic over the detected spectrum hole to decrease the outage probability of the cognitive user transmissions [12]. They assist the PUs' bidirectional traffic to provide better outage performance of the primary system [13-14].

The above cooperative schemes are termed as full cooperation transmissions, because the cooperative relays always assist the cognitive user transmissions without considering the characteristics of the practical networks, such as the limited bandwidth, the requirement of the data rates, and the quality of the direct links between the sources and destinations. The full cooperation transmissions may overuse the network resources, especially when the non-relay direct links between the sources and destinations have a good quality [15]. Moreover, the full cooperation inherits the diversity at the cost of loss of the multiplexing gain, because the relays work all the time. Therefore, adaptive cooperation schemes are studied.

1.1 Related Work

Among many adaptive cooperation strategies, the selection relaying and the incremental relaying protocol proposed in [7] are typical for one-way relaying networks. In the selection relaying protocol, the relay is selected to assist the transmission from the source to the

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[1] There are three different cognitive radio networks: underlay, overlay, and interweave [3]. In the underlay cognitive networks, PU shares its spectrum with CUs whose transmit power is constrained to avoid the interference to the PU. In the overlay cognitive networks, CUs employ the PU's spectrum at the expense of helping the communication between the PUs. For the interweave cognitive networks, in which we are interested in this paper, CUs access the PU's spectrum through the spectrum sensing.

destination only when the quality of the source-relay channel lies above a certain threshold. However, the selection relaying protocol can be loss of the multiplexing gain, especially for the high rates, because it repeats information all the time. In the incremental relaying protocol, whether the relay works or not depends on the feedback from the destination. That means if the non-relay direct transmission fails, the destination feeds back a single bit to the source and relay, and then the relay forwards what it received from the source. Otherwise, a single bit of feedback is transmitted from the destination to indicate the success of the direct transmission, and the relay does nothing. The incremental relaying protocol loses less multiplexing gain, because it repeats only rarely. Based on the incremental relaying protocol, the incremental decode-and-forward (DF) relaying for multi-relay cognitive radio networks is investigated in [4][16], where the best relay with DF is selected only when the non-relay direct transmission fails. The outage analysis in [4] shows that the incremental DF relaying scheme obtains lower outage probability than the full cooperation scheme with DF relay. Similarly, the hybrid cooperation scheme with the incremental amplify-and-forward (AF) relaying is proposed for cognitive radio networks in [17]. It uses the non-relay direct transmission or the AF relaying transmission relying on the SNR of the non-relay direct link. Further, both the incremental DF and the incremental AF relaying for underlay cognitive networks are studied over Nakagami-m fading channels in [18], where the incremental AF relaying outperforms the incremental DF relaying in terms of the diversity-multiplexing tradeoff (DMT). More recently, the opportunistic DF-AF selection scheme is given for cognitive relaying networks in [19], where the relay switches between DF and AF depending on the quality of the source-relay link. Additionally, a novel best cooperative mechanism (BCM) for the wireless energy harvesting and spectrum sharing in 5G networks is proposed in [20], where the PU transmits its information to the PU's receiver directly or asisted by the SU depending on maximization of the throughput of the SU and the PU in each timeslot. The above adaptive cooperation schemes are used in cognitive one-way relaying networks and fully employ the advantages of the non-relay direct transmission and the relay diversity transmission. Thus, they obtain better performance than the full operation schemes.

However, for cognitive two-way relaying networks, the adaptive cooperation transmission schemes have received little attention. An adaptive two-way relaying scheme for overlay cognitive radio networks is investigated in [21], where a two-way relay switches between DF and AF depending on whether it can successfully decode its received data or not. In [22], an adaptive network-coded QAM modulation scheme is proposed over asymmetric two-way relaying channels to maximize the throughput of the relay network. For the adaptive schemes in [21-22], the relay works all the time.

1.2 Our Work

In this paper, we propose an adaptive cooperation transmission based on acknowledgement (ACK) feedback from the destination, called *hybrid relaying cooperation scheme* (HRCS), for the bidirectional communication between two cognitive users CU_1 and CU_2 over the detected spectrum hole. In the HRCS, the relaying cooperation is utilized only when the bidirectional communication through the non-relay direct links fails. Specifically, the one-way relaying cooperation is used if the direct transmission from CU_1 to CU_2 or from CU_2 to CU_1 fails. Otherwise, if the two direct transmissions both fail, the two-way relaying cooperation is applied. The main contributions of this paper can be summarized as follows:

1) It is in cognitive radio networks with the spectrum sensing (i.e., the interweave cognitive radio networks) that the proposed scheme is applied for the information exchange between two cognitive users. Moreover, to our best knowledge, no study has considered the design of

the adaptive cooperation schemes for the bidirectional communication in the interweave cognitive radio networks.

2) Compared with the adaptive schemes in [4][16-22], our proposed scheme exploits the merits of the non-relay direct transmissions, the one-way relaying transmissions, and the two-way relaying transmissions.[2] In the proposed scheme, both the one- and the two-way relaying cooperation are considered as the choice to assist the information exchange, when the non-relay direct transmissions between the two cognitive users are unsuccessful. The adaptive cooperation schemes in the existing literature only use the non-relay direct transmission and the one-way relaying cooperation.

3) We derive the closed-form expression of the outage probability and that of the outage probability floor over Rayleigh fading channels for the proposed scheme, where the interference from the PU to CUs is considered due to the miss detection of the presence of the PU. We also discuss the power allocation between the relay and the end-sources to evaluate the outage probability performance. The power allocation in our proposed scheme is different from that in underlay and overlay cognitive two-way relaying networks, where the SU allocates its power to relay PU's signal and to transmit its own signal [23-25]. We further investigate the f-DMT to achieve the whole performance on the diversity gain and the multiplexing gain.

The results show that our proposed scheme achieves better outage performance than the direct transmissions (DT), pure one-way relaying transmissions (POWRT), and pure two-way relaying transmissions (PTWRT). Moreover, the proposed scheme outperforms the full operation schemes including the POWRT and PTWRT in terms of the finite-SNR DMT (f-DMT) performance.

The remainder of this paper is organized as follows. In Section 2, we describe the system model and the protocol of the HRCS. Section 3 derives the outage probability and the outage probability floor of the HRCS. Then, the f-DMT is discussed in Section 4. Next, in Section 5, we conduct the numerical evaluations. Finally, Section 6 gives some concluding remarks. The main notations used in this paper are listed in **Table 1**.

2. Hybrid Relaying Cooperation for Cognitive User Transmissions

2.1 System Model

A cognitive radio network coexists with a primary network as shown in **Fig. 1.** In the cognitive radio network, there are cognitive users CU_1, CU_2, CR, and an access point (AP). The AP is equivalent to a base station of the cognitive radio network, which controls and coordinates the spectrum allocation and access of cognitive users. CR works as a relay assisting the information exchange between CU_1 and CU_2. We consider CR works at DF mode.[3] To be practically feasible, all nodes operate in a half-duplex mode. The wireless links between two nodes are modeled as Rayleigh fading channels, where the fading process is considered as constant during one time slot. We assume as in [11][14] that cognitive users know the channel

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[2] Non-relay direct transmissions can achieve full multiplexing gain but without any diversity gain. On the contrary, relay diversity transmissions inherit diversity at the cost of loss of multiplexing gain. Moreover, two-way relaying transmissions have high spectral efficiency due to saving transmission time slots for the bidirectional communication.

[3] Compared with the non-regenerative relaying, such as AF and the compress-and-forward (CF), DF relaying does not forward the noise and interference of the signal received at the relay. We select DF relaying for mathematical tractability purposes. Additionally, the HRCS can be also extended to AF relaying.

Table 1. List of the notations used in this paper

Notation	**Description**
H_0, H_1	Events denoting the licensed channel unoccupied and occupied by the PU, respectively.
$H_p(k)$	The status of the licensed channel at the time slot k.
$H_a(k)$	The sensing decision at a fusion center for the time slot k.
$\mathrm{P_d}$, $\mathrm{P_f}$	The probabilities of the detection and the false alarm of the presence of the PU, respectively.
R_1, R_2	The target data rates at CU_1 and CU_2, respectively.
$\lvert h_{ab}\rvert^2$	The channel gain from the node a to b (or abbreviated to a→b). It follows the exponential distribution. The gains of the different channels are assumed to be mutually independent.
$\sigma_{ab}^2 = 1/\lambda_{ab}$	σ_{ab}^2 is the mean of $\lvert h_{ab}\rvert^2$.
$\lambda_{ab\text{-}cd}=\lambda_{ab}/\lambda_{cd}$	The ratio of the channel gain from c→d to that from a→b.
P_1, P_2, P_r, P_p	The transmit power of CU_1, CU_2, CR and PU, respectively.
$\alpha = P_\mathrm{r}/ P_\mathrm{total}$	The ratio of CR's transmit power to the total transmit power of CUs, where $P_\mathrm{total}= P_1+P_2+P_\mathrm{r}$.
$\gamma_i = P_i/N_0$	γ_i is the signal-to-noise ratio (SNR), $i\in\{1, 2, \mathrm{r}, \mathrm{p}\}$.

Define $\gamma_\mathrm{total} = P_\mathrm{total}/N_0$ and assume $P_1 = P_2$. Thus, $\gamma_\mathrm{r} = \alpha\gamma_\mathrm{total}$ and $\gamma = \gamma_1 = \gamma_2 = \beta\gamma_\mathrm{total}$, where $\beta = (1-\alpha)/2$.

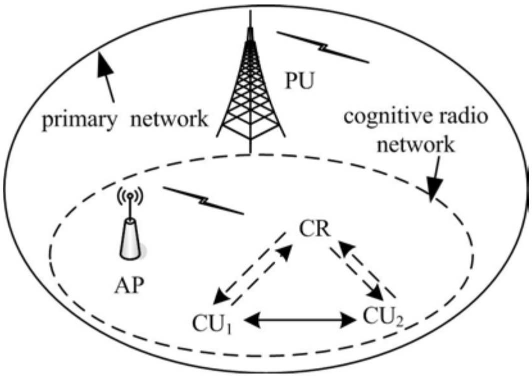

Fig. 1. Coexistence of a cognitive radio network and a primary network.

state information (CSI) of the PU, and the estimation of the channels between the different cognitive users is perfect. The practical implementation of such assumption can be found in [26][27]. In addition, the additive white Gaussian noise (AWGN) at all receivers is modeled as a complex Gaussian random variable with zero mean and variance N_0.

The process of the cognitive user communication in a time slot can be divided into two phases: the spectrum sensing and the information transmission. In the spectrum sensing phase, the cognitive users CU_1, CU_2 and CR utilize a spectrum sensing technique (e.g., the energy detection [28]) to sense the spectrum holes. The sensing results are transmitted from the cognitive users to AP over a common control channel (CCC). Here, the CCC is used to avoid the interference to the PU [29]. The sensing results are fused at the AP to obtain the final sensing decision. If the final sensing decision is that a spectrum hole exists, the AP broadcasts the information of the spectrum hole (e.g., the frequency and bandwidth) to the cognitive users through the CCC. After receiving the spectrum hole information, the cognitive users adjust their transmitter parameters to match the information of the spectrum hole. Then, in the information transmission phase, CU_1 and CU_2 exchange the information over the detected spectrum hole. On the other hand, if the final sensing decision is that a spectrum hole does not exist, the cognitive users repeat the spectrum sensing process in the next time slot.

If there is a spectrum hole at the time slot k, $H_p(k) = H_0$. Otherwise, $H_p(k) = H_1$. $H_p(k)$ can be modeled as a Bernoulli random variable with parameter P_a [4][30-31], i.e., $\Pr\{H_p(k) = H_0\} = P_a$ and $\Pr\{H_p(k) = H_1\} = 1 - P_a$. The detection probability can be represented as $P_d = \Pr\{H_a(k) = H_1 | H_p(k) = H_1\}$. Similarly, the false alarm probability is given by $P_f = \Pr\{H_a(k) = H_1 | H_p(k) = H_0\}$.

2.2 Protocol of the HRCS

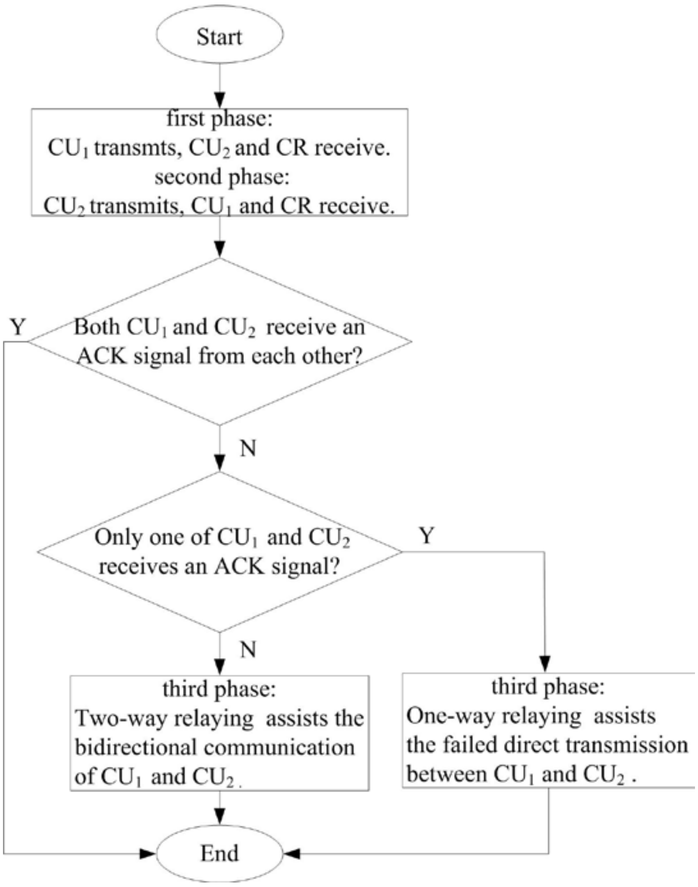

Fig. 2. Flow chart for the hybrid relaying cooperation scheme.

The flow chart of the HRCS is shown in **Fig. 2**. After CU_1 and CU_2 receive the information of the detected spectrum hole from AP, i.e., $H_a(k) = H_0$, they use the first two phases to transmit signals to each other and CR through the non-relay direct links, without involving the CR such that the utilization of additional resource slots is avoided. When they have succeeded in decoding each others data, they broadcast a single bit of an ACK signal to each other and CR. Here, the ACK signal denotes the success of the non-relay direct transmission. If both CU_1 and CU_2 receive the ACK, which represents that the information exchange succeeds, CR does nothing. Otherwise, if only one of CU_1 and CU_2 receives the ACK, which shows only the one-way direct transmission between CU_1 and CU_2 succeeds, the one-way relaying cooperation assisted by CR is used to help the unsuccessful traffic in the third phase. If neither

CU_1 nor CU_2 receives the ACK, which indicates the bidirectional direct transmissions fail, the two-way relaying cooperation assisted by CR is employed to the information exchange in the third phase.

An ACK signal is generally a 1-bit information in an information-theoretical sense, which we assume can be detected reliably, and the data rate and power consumption of which are omitted. Additionally, a cyclic redundancy code (CRC) is used to determine whether the received signals are decoded successfully or not. If the CRC checking passes, the signals are decoded successfully.

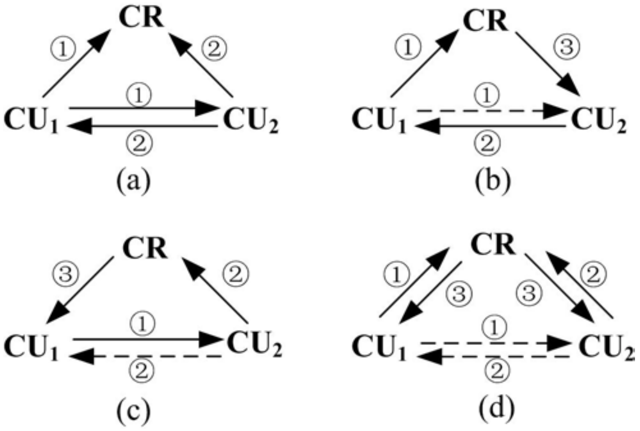

Fig. 3. Four cases of the hybrid relaying cooperation scheme. ①， ② and ③ respectively represent the three phases. The solid and the dotted arrows denote success and failure of non-relay direct transmissions, respectively.

According to the flow chart of the HRCS, the HRCS can be divided into four cases. The first case shown in **Fig. 3** (a) corresponds to the success of the information exchange between CU_1 and CU_2 in the first two phases. For this case, the channel capacity of the direct transmission can be expressed as[4]

$$C_{\text{direct}}^{i-j}=\begin{cases} C_{\text{direct}}^{i-j}(\text{SINR}_j)=\dfrac{1}{2}\log_2\left(1+\dfrac{|h_{ij}|^2\gamma_i}{|h_{pj}|^2\gamma_p+1}\right), & H_p(k)=H_1 \\ C_{\text{direct}}^{i-j}(\text{SNR}_j)=\dfrac{1}{2}\log_2(1+|h_{ij}|^2\gamma_i), & H_p(k)=H_0 \end{cases} \tag{1}$$

where $i,j\in\{1,2\}$ and $i\neq j$. C_{direct}^{i-j} denotes the capacity of the direct transmission from CU_i to CU_j. SINR_j and SNR_j represent the signal to interference-plus-noise ratio and the signal-to-noise ratio at CU_j, respectively. $|h_{ij}|^2$ and $|h_{pj}|^2$ are the channel gains from CU_i and PU to CU_j, respectively. The factor 1/2 in front of the log-function is because each traffic flow takes two phases.

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[4] Although the final sensing decision is $H_a(k)=H_0$, PU may still occupy the spectrum hole in the time slot k due to the miss detection of the presence of PU. That means under the conditions of $H_a(k)=H_0$, there are two possible cases, i.e., $H_p(k)=H_1$ and $H_p(k)=H_0$. Therefore, Eq.(1) is given for the two cases. Similarly, Eqs.(2)-(5) are also expressed with the two cases.

The second and the third case shown in **Fig. 3** (b) and (c), respectively, represent that the non-relay direct transmission of $\mathrm{CU}_i \rightarrow \mathrm{CU}_j$ fails and that of $\mathrm{CU}_j \rightarrow \mathrm{CU}_i$ succeeds in the first two phases ($i, j \in \{1,2\}$ and $i \neq j$). In the third phase, CR is invoked to assist the transmission of $\mathrm{CU}_i \rightarrow \mathrm{CU}_j$, where CR decodes CU_i's signal and forwards it to CU_j. According to the coding theorem, the event of successful decoding at CR occurs as the channel capacity from CU_i to CR is larger than R_i. Thus, the event is given by

$$\mathrm{E}_i = \begin{cases} \dfrac{1}{3}\log_2\left(1+\dfrac{|h_{ir}|^2 \gamma_i}{|h_{pr}|^2 \gamma_p + 1}\right) > R_i, & H_p(k) = H_1 \\ \dfrac{1}{3}\log_2\left(1+|h_{ir}|^2 \gamma_i\right) > R_i, & H_p(k) = H_0 \end{cases} \tag{2}$$

where $|h_{ir}|^2$ and $|h_{pr}|^2$ denote the channel gains from CU_i and PU to CR, respectively. The pre-log factor 1/3 is because each traffic flow takes three phases. Then, using the maximum ratio combining (MRC) method, CU_j combines the direct and the relay signal. Hence, the channel capacity of the one-way relaying cooperation transmission of $\mathrm{CU}_i \rightarrow \mathrm{CU}_j$ is expressed as

$$C_{\mathrm{oneway}}^{i-j} = \begin{cases} C_{\mathrm{oneway}}^{i-j}(\mathrm{SINR}_\mathrm{j}) = \dfrac{1}{3}\log_2\left(1+\dfrac{|h_{ij}|^2 \beta\gamma_{total} + |h_{rj}|^2 \alpha\gamma_{total}}{|h_{pj}|^2 \gamma_p + 1}\right), & H_p(k) = H_1 \\ C_{\mathrm{oneway}}^{i-j}(\mathrm{SNR}_\mathrm{j}) = \dfrac{1}{3}\log_2\left(1+|h_{ij}|^2 \beta\gamma_{total} + |h_{rj}|^2 \alpha\gamma_{total}\right), & H_p(k) = H_0 \end{cases} \tag{3}$$

where $|h_{rj}|^2$ denotes the channel gain from CR to CU_j. On the other hand, if CR fails to decode CU_i 's signal, the outage event occurs and is defined as $\overline{\mathrm{E}}_i$ which is the complementary event of E_i.

The fourth case shown in **Fig. 3** (d) corresponds to the failure of the bidirectional direct transmissions between CU_1 and CU_2 in the first two phases. In this case, CR is activated to assist the bidirectional communication in the third phase, where CR decodes CU_1's and CU_2's signals received in the first two phases. When CR succeeds in decoding both CU_1's and CU_2's signals, the event is defined as E, i.e.,

$$\mathrm{E} = \mathrm{E}_1 \cap \mathrm{E}_2. \tag{4}$$

After decoding successfully, CR encodes the decoded signals through the network coding, and then broadcasts the encoded signals to both CU_1 and CU_2.[5] Subsequently, CU_2 (CU_1) combines the direct and the relayed signal [32]. Thus, the channel capacity of the two-way relaying cooperation transmission is given by

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[5] As stated in [32-34], the process of the network coding is that CR jointly encodes the decoded CU_1's and CU_2's data (i.e., D_1 and D_2) through a bitwise XOR operation into a composite data D_3. Accordingly, the XOR operation is $d_3(l) = d_1(l) \oplus d_2(l)$ where the bit $d_k(l)$ is the lth bit in D_k. After receiving D_3, CU_1 (CU_2) can retrieve D_2 (D_1) through a bitwise XOR of its own data D_1 (D_2) and the composite data D_3.

$$C_{\text{twoway}}^{i-j}=\begin{cases} C_{\text{twoway}}^{i-j}(\text{SINR}_{\text{j}})=\frac{1}{3}\log_2\left(1+\frac{|h_{ij}|^2\beta\gamma_{total}+|h_{rj}|^2\alpha\gamma_{total}}{|h_{pj}|^2\gamma_p+1}\right), & H_p(k)=H_1 \\ C_{\text{twoway}}^{i-j}(\text{SNR}_{\text{j}})=\frac{1}{3}\log_2\left(1+|h_{ij}|^2\beta\gamma_{total}+|h_{rj}|^2\alpha\gamma_{total}\right), & H_p(k)=H_0 \end{cases}, \tag{5}$$

where $i, j \in \{1,2\}$ and $i \neq j$. On the other hand, if CR fails to decode, which is denoted by $\overline{\text{E}}$, the transmission outage would happen.

According to the above explanation of the HRCS, the essential difference between the one-way relaying and the two-way relaying cooperation is that the method difference of processing the received signals at the relay. Compared with the one-way relaying cooperation, the two-way relaying cooperation can jointly process the two signals received from CU_1 and CU_2, which saves the transmission time and makes the bidirectional communication complete in the three phases.

3. Outage Events and Probabilities Analysis

3.1 Outage Events and Outage Probabilities

An outage event is said to occur when the channel capacity falls below a target data rate [7]. Since the transmission aim is to exchange the information between CU_1 and CU_2, an outage event is declared when the channel capacity of either $\text{CU}_1 \rightarrow \text{CU}_2$ or $\text{CU}_2 \rightarrow \text{CU}_1$ falls below the target data rate. According to such a definition of an outage event, we investigate the outage probability for the HRCS in this section. Additionally, the outage probability floor is also derived to discuss the effect of the interference from the PU on the outage performance of the cognitive user transmissions.

According to the protocol of the HRCS, the information exchange between CU_1 and CU_2 occurs after the spectrum hole is detected, i.e., $H_a(k) = H_0$. Moreover, the outage of the information exchange occurs when CR assisted transmissions fail. Therefore, the outage probability of the HRCS is written as

$$\text{P}_{\text{out}}=\Pr(\text{E}_{\text{out-1}}^{\text{one}}\mid H_a(k)=H_0)+\Pr(\text{E}_{\text{out-2}}^{\text{one}}\mid H_a(k)=H_0)+\Pr(\text{E}_{\text{out}}^{\text{two}}\mid H_a(k)=H_0), \tag{6}$$

where $\text{E}_{\text{out-1}}^{\text{one}}$ and $\text{E}_{\text{out-2}}^{\text{one}}$ denote the outage events of the one-way relaying cooperation transmission of $\text{CU}_1 \rightarrow \text{CU}_2$ and $\text{CU}_2 \rightarrow \text{CU}_1$, respectively. $\text{E}_{\text{out}}^{\text{two}}$ represents the outage event of the two-way relaying cooperation transmission. Obviously, each two of the three events $\text{E}_{\text{out-1}}^{\text{one}}$, $\text{E}_{\text{out-2}}^{\text{one}}$ and $\text{E}_{\text{out}}^{\text{two}}$ can not happen simultaneously.

From the total probability and conditional probability theorems, $\Pr(\text{E}_{\text{out-1}}^{\text{one}}\mid H_a(k)=H_0)$ in Eq.(6) can be rewritten as

$$\begin{aligned}&\Pr(\text{E}_{\text{out-1}}^{\text{one}}\mid H_a(k)=H_0)\\&=\eta_1\Pr\left[\text{E}_{\text{out-1}}^{\text{one}}\mid H_p(k)=H_0,H_a(k)=H_0\right]+\eta_2\Pr\left[\text{E}_{\text{out-1}}^{\text{one}}\mid H_p(k)=H_1,H_a(k)=H_0\right]\end{aligned} \tag{7}$$

where $\eta_1=\Pr[H_p(k)=H_0\mid H_a(k)=H_0]=\text{P}_\text{a}(1-\text{P}_\text{f})/[\text{P}_\text{a}(1-\text{P}_\text{f})+(1-\text{P}_\text{a})(1-\text{P}_\text{d})]$ and $\eta_2=\Pr[H_p(k)=H_1\mid H_a(k)=H_0]=(1-\text{P}_\text{a})(1-\text{P}_\text{d})/[\text{P}_\text{a}(1-\text{P}_\text{f})+(1-\text{P}_\text{a})(1-\text{P}_\text{d})]$. Since the outage event occurs when either CR fails to decode the received signal, or CU_i fails to decode the relayed signal, $\Pr\left[\mathrm{E}_{\text{out-1}}^{\text{one}} \mid H_p(k)=H_0, H_a(k)=H_0\right]$ in (7) can be expanded into[6]

$$\begin{aligned}
&\Pr(\mathrm{E}_{\text{out-1}}^{\text{one}} \mid H_a(k)=H_0)\\
&=\eta_1\Pr\left[C_{\text{direct}}^{1-2}(\mathrm{SNR}_2)<R_1, C_{\text{direct}}^{2-1}(\mathrm{SNR}_1)>R_2\right]\Pr\left[\left(\mathrm{E}_1, C_{\text{oneway}}^{1-2}(\mathrm{SNR}_2)<R_1\right)\cup\overline{\mathrm{E}}_1\right]\\
&\quad+\eta_2\Pr\left[C_{\text{direct}}^{1-2}(\mathrm{SINR}_2)<R_1, C_{\text{direct}}^{2-1}(\mathrm{SINR}_1)>R_2\right]\Pr\left[\left(\mathrm{E}_1, C_{\text{oneway}}^{1-2}(\mathrm{SINR}_2)<R_1\right)\cup\overline{\mathrm{E}}_1\right].
\end{aligned} \tag{8}$$

Similarly, $\Pr(\mathrm{E}_{\text{out-2}}^{\text{one}} \mid H_a(k)=H_0)$ and $\Pr(\mathrm{E}_{\text{out}}^{\text{two}} \mid H_a(k)=H_0)$ in Eq.(6) are given by Eq.(9) and (10), respectively.

$$\begin{aligned}
&\Pr(\mathrm{E}_{\text{out-2}}^{\text{one}} \mid H_a(k)=H_0)\\
&=\eta_1\Pr\left[C_{\text{direct}}^{1-2}(\mathrm{SNR}_2)>R_1, C_{\text{direct}}^{2-1}(\mathrm{SNR}_1)<R_2\right]\Pr\left[\left(\mathrm{E}_2, C_{\text{oneway}}^{2-1}(\mathrm{SNR}_1)<R_2\right)\cup\overline{\mathrm{E}}_2\right]\\
&\quad+\eta_2\Pr\left[C_{\text{direct}}^{1-2}(\mathrm{SINR}_2)>R_1, C_{\text{direct}}^{2-1}(\mathrm{SINR}_1)<R_2\right]\Pr\left[\left(\mathrm{E}_2, C_{\text{oneway}}^{2-1}(\mathrm{SINR}_1)<R_2\right)\cup\overline{\mathrm{E}}_2\right].
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\Pr(\mathrm{E}_{\text{out}}^{\text{two}} \mid H_a(k)=H_0)\\
&=\eta_1\Pr\left[C_{\text{direct}}^{1-2}(\mathrm{SNR}_2)<R_1, C_{\text{direct}}^{2-1}(\mathrm{SNR}_1)<R_2\right]\Pr\left[\left(\mathrm{E}, C_{\text{twoway}}^{1-2}(\mathrm{SNR}_2)<R_1\cup C_{\text{twoway}}^{2-1}(\mathrm{SNR}_1)<R_2\right)\cup\overline{\mathrm{E}}\right]\\
&+\eta_2\Pr\left[C_{\text{direct}}^{1-2}(\mathrm{SINR}_2)<R_1, C_{\text{direct}}^{2-1}(\mathrm{SINR}_1)<R_2\right]\Pr\left[\left(\mathrm{E}, C_{\text{twoway}}^{1-2}(\mathrm{SINR}_2)<R_1\cup C_{\text{twoway}}^{2-1}(\mathrm{SINR}_1)<R_2\right)\cup\overline{\mathrm{E}}\right].
\end{aligned} \tag{10}$$

From the above analysis, we can obtain the closed-form expression of the outage probability for the HRCS as the following theorem.

Theorem 1: The outage probability for the HRCS has an exact closed-form solution as

$$\begin{aligned}
\mathrm{P}_{\text{out}} &= \eta_1[t_{12}(1-y_1m_{12})+t_{21}(1-y_2m_{21})+t_{12}t_{21}(y_1m_{12}+y_2m_{21}-y_1y_2m_{12}m_{21}-1)]\\
&\quad+\eta_2[u_{12}(1-v_1n_{12})+u_{21}(1-v_2n_{21})+u_{12}u_{21}(v_1n_{12}+v_2n_{21}-sn_{12}n_{21}-1)]
\end{aligned} \tag{11}$$

where

$$t_{ij}=\Pr(|h_{ij}|^2<\Theta_i)=1-e^{-\lambda_{ij}\Theta_i}, \tag{12}$$

$$y_i=\Pr(|h_{ir}|^2>\Lambda_i)=e^{-\lambda_{ir}\Lambda_i}, \tag{13}$$

$$u_{ij}=\Pr\left(\frac{|h_{ij}|^2}{|h_{pj}|^2\gamma_p+1}<\Theta_i\right)=1-\frac{\lambda_{pj}e^{-\lambda_{ij}\Theta_i}}{\gamma_p\lambda_{ij}\Theta_i+\lambda_{pj}}, \tag{14}$$

$$v_i=\Pr\left(\frac{|h_{ir}|^2}{|h_{pr}|^2\gamma_p+1}>\Lambda_i\right)=\frac{\lambda_{pr}e^{-\lambda_{ir}\Lambda_i}}{\gamma_p\lambda_{ir}\Lambda_i+\lambda_{pr}}, \tag{15}$$

$$s=\Pr\left(\frac{|h_{1r}|^2}{|h_{pr}|^2\gamma_p+1}>\Lambda_1, \frac{|h_{2r}|^2}{|h_{pr}|^2\gamma_p+1}>\Lambda_2\right)=\frac{\lambda_{pr}e^{-\lambda_{1r}\Lambda_1-\lambda_{2r}\Lambda_2}}{\lambda_{pr}+\gamma_p\lambda_{1r}\Lambda_1+\gamma_p\lambda_{2r}\Lambda_2}, \tag{16}$$

---

[6] In Eqs.(8)-(10), the channel capacity $C_{\bullet}^{i-j}(\mathrm{SNR}_j)$ ($i,j\in\{1,2\}$ and $i\neq j$) corresponds to the case $H_p(k)=H_0$, $H_a(k)=H_0$, while $C_{\bullet}^{i-j}(\mathrm{SINR}_j)$ corresponds to $H_p(k)=H_1, H_a(k)=H_0$.

$$m_{ij} = \Pr(\alpha |h_{rj}|^2 + \beta |h_{1j}|^2 > \beta\Lambda_i) = \begin{cases} \dfrac{\beta\lambda_{rj} e^{-\lambda_{ij}\Lambda_i} - \alpha\lambda_{ij} e^{-\beta\lambda_{rj}\Lambda_i/\alpha}}{\beta\lambda_{rj} - \alpha\lambda_{ij}}, & \dfrac{\lambda_{rj}}{\alpha} \neq \dfrac{\lambda_{ij}}{\beta} \\ (1+\lambda_{ij}\Lambda_i)e^{-\lambda_{ij}\Lambda_i}, & \dfrac{\lambda_{rj}}{\alpha} = \dfrac{\lambda_{ij}}{\beta} \end{cases}, \tag{17}$$

$$n_{ij} = \Pr\left(\frac{\alpha |h_{rj}|^2 + \beta |h_{ij}|^2}{|h_{pj}|^2 \gamma_p + 1} > \beta\Lambda_i \right) = \begin{cases} \dfrac{\lambda_{pj}}{\alpha\lambda_{ij} - \beta\lambda_{rj}} \left[\dfrac{\alpha^2 \lambda_{ij} e^{-\beta\lambda_{rj}\Lambda_i/\alpha}}{\alpha\lambda_{pj} + \beta\gamma_p\lambda_{rj}\Lambda_i} - \dfrac{\beta\lambda_{rj} e^{-\lambda_{ij}\Lambda_i}}{\lambda_{pj} + \gamma_p\lambda_{ij}\Lambda_i} \right], & \dfrac{\lambda_{rj}}{\alpha} \neq \dfrac{\lambda_{ij}}{\beta} \\ \dfrac{\lambda_{pj} e^{-\lambda_{ij}\Lambda_i} \left[\lambda_{pj}(1+\lambda_{ij}\Lambda_i) + \lambda_{ij}\gamma_p\Lambda_i(2+\lambda_{ij}\Lambda_i) \right]}{(\lambda_{pj} + \lambda_{ij}\gamma_p\Lambda_i)^2}, & \dfrac{\lambda_{rj}}{\alpha} = \dfrac{\lambda_{ij}}{\beta} \end{cases}, \tag{18}$$

with $\Theta_i = (2^{2R_i} - 1)/\gamma$, $\Lambda_i = (2^{3R_i} - 1)/\gamma$, $i, j \in \{1,2\}$ and $i \neq j$. The closed-form expressions of Eqs.(14)-(18) are obtained by referring to Eqs.(A.1)-(A.4) in Appendix A.

Proof: Substituting Eqs.(1)-(3) into (8) and (9), and considering the mutual independence of RVs, we can respectively rewrite Eq.(8) and (9) as

$$\begin{aligned} &\Pr(\mathrm{E}_{\text{out-1}}^{\text{one}} \mid H_a(k) = H_0) \\ &= \eta_1 t_{12}(1-t_{21})[y_1(1-m_{12}) + (1-y_1)] + \eta_2 u_{12}(1-u_{21})[v_1(1-n_{12}) + (1-v_1)], \end{aligned} \tag{19}$$

$$\begin{aligned} &\Pr(\mathrm{E}_{\text{out-2}}^{\text{one}} \mid H_a(k) = H_0) \\ &= \eta_1 t_{21}(1-t_{12})[y_2(1-m_{21}) + (1-y_2)] + \eta_2 u_{21}(1-u_{12})[v_2(1-n_{21}) + (1-v_2)]. \end{aligned} \tag{20}$$

Similarly, we substitute Eqs.(1), (4) and (5) into (10), then rewrite (10) as

$$\begin{aligned} &\Pr(\mathrm{E}_{\text{out}}^{\text{two}} \mid H_a(k) = H_0) \\ &= \eta_1 t_{12} t_{21}[y_1 y_2((1-m_{12}) + (1-m_{21}) - (1-m_{12})(1-m_{21})) + (1-y_1 y_2)] \\ &\quad + \eta_2 u_{12} u_{21}[v_1 v_2((1-n_{12}) + (1-n_{21}) - (1-n_{12})(1-n_{21})) + (1-s)]. \end{aligned} \tag{21}$$

By simplifying the sum of Eqs.(19)-(21), the exact closed-form solution of the outage probability for the HRCS is obtained as Eq.(11). ■

3.2 Outage Probability Analysis

From Eq.(11), we find the closed-form expression of the outage probability for the HRCS is tedious. In order to get insight into Eq.(11), we calculate it when the cognitive user SNR γ approaches infinity.

Theorem 2: Suppose the channel gains between CR and the end-sources are independent identically distributed (i.i.d), i.e., $\lambda_{r1} = \lambda_{1r}$ and $\lambda_{r2} = \lambda_{2r}$. Let the primary transmit SNR $\gamma_p = a\gamma$, where a is a constant. Assume $R = R_1 = R_2$. Then, as $\gamma \to \infty$, the outage probability of the HRCS is given by

$$\mathrm{P}_{\text{outfloor}} = \frac{\eta_2 \left[a\tilde{u}\lambda_{12-p2}(1 - \frac{\xi_{12}}{\Pi_1 \Pi_{12}}) + a\tilde{u}\lambda_{21-p1}(1 - \frac{\xi_{21}}{\Pi_2 \Pi_{21}}) + a^2\tilde{u}^2\lambda_{12-p2}\lambda_{21-p1}(1 - \frac{\xi_{12}\xi_{21}}{\Pi_3 \Pi_{12} \Pi_{21}}) \right]}{(1 + a\tilde{u}\lambda_{12-p2})(1 + a\tilde{u}\lambda_{21-p1})} \tag{22}$$

where $\tilde{u} = 2^{2R} - 1$, $\Pi_3 = 1 + au\lambda_{1r-pr} + au\lambda_{2r-pr}$, $\Pi_i = 1 + au\lambda_{ir-pr}$, $\Pi_{ij} = 1 + au\lambda_{ij-pj}$, and

$$\xi_{ij} = \begin{cases} \dfrac{\alpha^2 \Pi_{ij} \lambda_{ij-pj} - \beta \lambda_{rj-pj} \rho_j}{(\alpha \lambda_{1j-pj} - \beta \lambda_{rj-pj}) \rho_j}, & \dfrac{\lambda_{1j-pj}}{\beta} \neq \dfrac{\lambda_{rj-pj}}{\alpha} \\ 2 - \dfrac{1}{\Pi_{ij}}, & \dfrac{\lambda_{1j-pj}}{\beta} = \dfrac{\lambda_{rj-pj}}{\alpha} \end{cases}$$

with $u = 2^{3R} - 1$, $\rho_j = \alpha + \beta au\lambda_{rj-pj}$, $i, j \in \{1, 2\}$ and $i \neq j$.

Proof: Through substituting the conditions of $\lambda_{r1} = \lambda_{1r}$, $\lambda_{r2} = \lambda_{2r}$, $\gamma_p = a\gamma$, $R = R_1 = R_2$ into Eq.(11), and considering $\gamma \to \infty$, Eq.(22) can be easily obtained. ■

As indicated from Eq.(22), when SNRs γ and γ_p approach infinity, the term multiplied by η_1 in Eq.(11), denoted by T_1, approaches zero, and the term multiplied by η_2 in Eq.(11), denoted by T_2, approaches a constant irrelevant to SNRs. Since T_1 denotes the outage probability under the conditions that a spectrum hole exists and is detected correctly, there is no interference from the PU to CUs during the cognitive user transmissions, so that T_1 can decrease and approach zero with the infinite γ. On the contrary, T_2 represents the outage probability when a spectrum hole does not exist but CUs sense it existing, which results in the interference from the PU to CUs. The interference increases with the increase of γ_p, thus becomes the main factor leading to an outage event of the cognitive user transmissions. Therefore, when the cognitive user SNR increases to a certain value, the outage performance of the cognitive user transmissions is no longer improved. This phenomenon is called an outage probability floor. However, it can be decreased by the efficient cooperation schemes as shown in **Fig. 4** and **Fig. 5**, where the HRCS obtains lower outage probability floor than the DT, POWRT and PTWRT.

4. Diversity-multiplexing tradeoff

DMT provides a whole view on the diversity gain and multiplexing gain for communication systems. According to the definition of DMT at the asymptotically high SNR in [35], the diversity gain is given by $d = -\lim_{\mathrm{SNR} \to \infty} \log P_e(\mathrm{SNR}) / \log \mathrm{SNR}$. Considering that the packet error rate P_e equals the outage probability under the assumption of the capacity-achieving codes applied in per packet, we find that the diversity gain with such a definition would always be zero for the HRCS. The reason is that the outage probability floor exists, as shown in Eq. (22), which means the outage probability approaches a non-zero constant in the high SNR region. Based on [35], the definition of the DMT at the finite SNR is presented in [36], where the diversity gain is given by

$$d(r, \mathrm{SNR}) = -\frac{\partial \ln \mathrm{P}_{\mathrm{out}}(r, \mathrm{SNR})}{\partial \ln \mathrm{SNR}} = -\frac{\mathrm{SNR}}{\mathrm{P}_{\mathrm{out}}(r, \mathrm{SNR})} \frac{\partial \mathrm{P}_{\mathrm{out}}(r, \mathrm{SNR})}{\partial \mathrm{SNR}} \tag{23}$$

with SNR and the multiplexing gain $r = R_t / \log_2(1 + \mathrm{SNR})$. R_t is the target data rate. $\mathrm{P}_{\mathrm{out}}(r, \mathrm{SNR})$ is the outage probability of the transmission link. The definition of the finite-SNR DMT provides the finite-SNR tradeoff of the data rate (represented by the multiplexing gain) and the reliability (represented by the diversity gain). Therefore, it is suitable to the practical cognitive radio systems which operate in the low to moderate SNR region due to the limited transmit power of CUs for avoiding the interference to the PU.

For the bidirectional communication, $R_t = R_1 + R_2$ is the target sum rate of the bidirectional channels. We define $R_1 = \delta R_t$ and $R_2 = (1-\delta)R_t$, where $\delta < 1$ is a rate allocation parameter. $\mathsf{P}_{\text{out}}(r, \text{SNR})$ represents the outage probability of the bidirectional channels. Substituting Eq.(11) into Eq.(23), we can obtain the corresponding f-DMT expression for the HRCS. The f-DMT expression, however, is tedious due to the complexity of Eq.(11). Considering a length limit of the paper, we only give the f-DMT expression under the specific conditions as the following Theorem 3.

Theorem 3: Under the specific conditions: (a) All the channel gains between each two CUs are i.i.d. with the parameter λ; (b) The channel gains from the PU to CUs are i.i.d. with the parameter λ_p; (c) $\alpha = \beta$; (d) $\delta = 1/2$; the f-DMT of the HRCS is given by

$$d(r,\gamma) = -\frac{\gamma}{\mathsf{P}_{\text{out}}(r,\gamma)}\left(\eta_1 \frac{\partial A}{\partial \gamma} + \eta_2 \frac{\partial B}{\partial \gamma}\right) \tag{24}$$

where

$$\begin{aligned} \mathsf{P}_{\text{out}}(r,\gamma) = {} & \eta_1[2t_{12}(1-y_1 m_{12}) + t_{12}^2(2y_1 m_{12} - y_1^2 m_{12}^2 - 1)] \\ & + \eta_2[2u_{12}(1-v_1 n_{12}) + u_{12}^2(2v_1 n_{12} - s n_{12}^2 - 1)], \end{aligned} \tag{25}$$

$$\frac{\partial A}{\partial \gamma} = 2\lambda\left[1 - t_{12} + y_1 m_{12} t_{12}\right]\left[\frac{\partial \Theta}{\partial \gamma}(1-t_{12})(1-y_1 m_{12}) + \frac{\partial \Lambda}{\partial \gamma} y_1 t_{12}\left(m_{12} + \lambda \Lambda y_1\right)\right],$$

$$\begin{aligned} \frac{\partial B}{\partial \gamma} = \frac{2\lambda_p}{A_1 A_2 A_3}\Bigg\{ & y_1 u_{12}\Bigg[\frac{\partial \Lambda}{\partial \gamma}\frac{n_{12}\lambda}{A_2 A_3}\left(A_3^2 \lambda_p e^{-\lambda\Theta}\left(A_2 + \gamma_p\right) - A_2^2 n y_1\left(A_3 + \gamma_p\right)\left(\lambda_p e^{-\lambda\Theta} - A_1\right)\right) \\ & + \frac{\partial n}{\partial \gamma}\left(A_2 n_{12} y_1 \lambda_p e^{-\lambda\Theta} - A_3 \lambda_p e^{-\lambda\Theta} - A_1 A_2 n_{12} y_1\right)\Bigg] + \frac{\partial \Theta}{\partial \gamma}\frac{2\lambda}{A_1^2} e^{-\lambda\Theta}(A_1 + \gamma_p) \\ & \times\left[A_3 n_{12} e^{-\lambda\Lambda}(2\lambda_p e^{-\lambda\Theta} - A_1) - A_2 n_{12}^2 y_1^2(\lambda_p e^{-\lambda\Theta} - A_1) - A_2 A_3 e^{-\lambda\Theta}\right]\Bigg\}, \end{aligned}$$

with $t_{12} = 1 - e^{-\lambda\Theta}$, $y_1 = e^{-\lambda\Lambda}$, $u_{12} = 1 - \lambda_p e^{-\lambda\Theta}/A_1$, $v_1 = \lambda_p y_1/A_2$, $s = \lambda_p e^{-2\lambda\Lambda}/A_3$, $m_{12} = (1+\lambda\Lambda)y_1$, $n_{12} = \lambda_p\left(A_2 m_{12} + \lambda\gamma_p \Lambda y_1\right)/A_2^2$, $A_1 = \lambda_p + \gamma_p \lambda\Theta$, $A_2 = \lambda_p + \gamma_p \lambda\Lambda$, $A_3 = \lambda_p + 2\gamma_p\lambda\Lambda$, $\frac{\partial \Theta}{\partial \gamma} = [r\gamma(1+\gamma)^{r-1} - (1+\gamma)^r + 1]/\gamma^2$, $\frac{\partial \Lambda}{\partial \gamma} = [\frac{3}{2} r\gamma(1+\gamma)^{\frac{3}{2}r-1} - (1+\gamma)^{\frac{3}{2}r} + 1]/\gamma^2$, and $\frac{\partial n_{12}}{\partial \gamma} = \frac{\partial \Lambda}{\partial \gamma}\lambda\lambda_p e^{-\lambda\Lambda}\left(2 + \lambda\Lambda - 2\gamma_p^2\lambda\Lambda/A_2^2\right)/A_2$.

Proof: Considering the specific condition (d) $\delta = 1/2$, we can derive $R_1 = R_2 = r\log_2(1+\gamma)/2$, $\theta_1 = \theta_2 = \theta = [(1+\gamma)^r - 1]/\gamma$, and $\Lambda_1 = \Lambda_2 = \Lambda = [(1+\gamma)^{3r/2} - 1]/\gamma$. Substituting the specific conditions (a)-(d) into Eq.(11), we can obtain Eq.(25). Then, Eq.(24) are derived out by substituting Eq.(25) into Eq.(23). ∎

It is difficult to obtain an insight of Eq.(24), because the characteristic can not be extracted from it. Additionally, due to the existence of the outage probability floor in the cognitive user transmissions, the asymptotic expression of the f-DMT with $\gamma \to \infty$ and that with $r \to 0$ have no closed-form solution. However, we can utilize the numerical results to further observe the property of the f-DMT for the HRCS. As shown in **Fig. 8** where the lines with the legend HRCS are given by Eq.(24), the diversity gain for the HRCS is larger than one in the moderate

SNR region when the multiplexing gain approaches zero, and the multiplexing gain is about 0.8 when the diversity gain approaches zero. Moreover, the HRCS always has better f-DMT performance than the POWRT and PTWRT.

5. Numerical Results

In this section, we presents the numerical results to evaluate the outage and f-DMT performance for the HRCS. In order to show the performance gains, the performance of the HRCS is also compared with that of the DT, POWRT and PTWRT.[7]

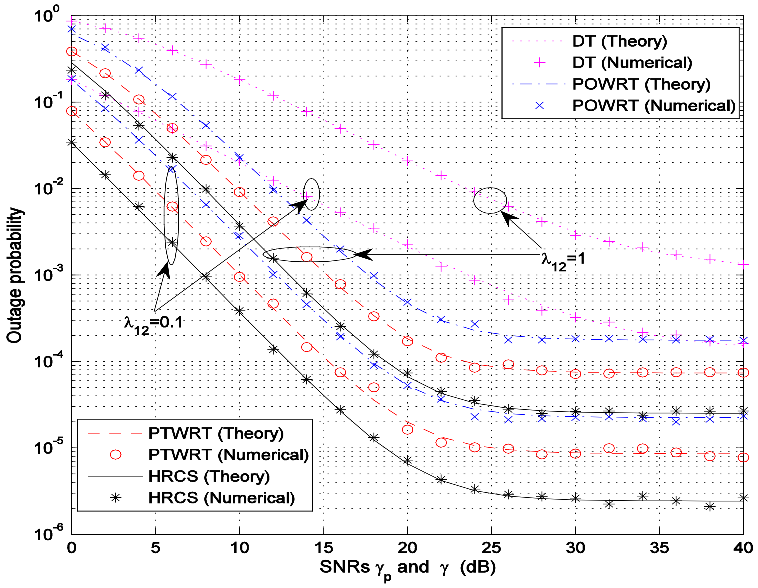

Fig. 4. Outage probability versus primary and cognitive users SNRs with P_a =0.6, P_d=0.99, P_f=0.01, a=1, $\alpha=\beta$=1/3, $\lambda_{1r}=\lambda_{2r}$=0.1, $\lambda_{p1}=\lambda_{p2}=\lambda_{pr}$=10, $\lambda_{12}=\lambda_{21}$, and $R_1=R_2$=0.5 bit/s/Hz.

Fig. 4 shows the outage probability versus SNRs (γ and γ_p) for the different channel gains of the direct links between CU_1 and CU_2, i.e., λ_{12}=0.1 and λ_{12}=1. It is clear that the theoretical results fit with the numerical simulation. As expected, the outage floors exist in the high SNR region for the four transmission schemes. The reason is that as the primary transmit SNR γ_p increases, the interference from the PU becomes the dominant factor causing the outage of the cognitive user transmissions. However, for both $\lambda_{1r}=\lambda_{2r}$=0.1, λ_{12}=1 which means the channel gains of the relay channels are better than those of the direct links, and $\lambda_{1r}=\lambda_{2r}=\lambda_{12}$=0.1 which means the relay channels have the same channel gains with the direct links, the HRCS always

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[7] For the DT, CU_1 and CU_2 exchange the information through the non-relay direct links, which is similar to the first case of the HRCS. Hence, the performance of the DT is derived using Eq.(1). For the POWRT, CR assists the information exchange between CU_1 and CU_2 in four phases [37]. For the PTWRT, CR helps the information exchange in three phases, which is the time-division broadcast (TDBC) protocol in [38]. Notice that different from non-cognitive radio networks, the cognitive radio networks exist with the spectrum sensing and the mutual interference between the PU and CUs. Therefore, the closed-form expressions of the outage probability for the one- and the two-way relaying transmissions in non-cognitive radio networks can not be straightforwardly used in cognitive radio networks. Since our early work in [39] has given the closed-form expressions of the outage probability for the POWRT and PTWRT in cognitive radio networks, we directly use those expressions in this paper.

has lower outage probability than the other three transmission schemes across the whole SNR region. This phenomenon shows that the proposed scheme improves the outage performance of the cognitive user transmissions, no matter whether the channel gains of the direct links are worse or not than those of the relay channels. Notice that an ACK from CU_1 or CU_2 is required for the HRCS, which results in the communication delay. Thus, for the case of the real-time communication, the PTWRT which has the suboptimal outage performance is more suitable than the HRCS.

Additionally, the outage probabilities of the four transmission schemes are lower if λ_{12}=0.1 than those if λ_{12}=1 in **Fig. 4**,. This is because the direct links between CU_1 and CU_2 can be utilized on the bidirectional communication for the four transmission schemes. Therefore, when the channel quality of the direct links becomes better from λ_{12}=1 to λ_{12}=0.1, the outage possibility of the transmissions through the direct links is less so that the outage probability decreases.

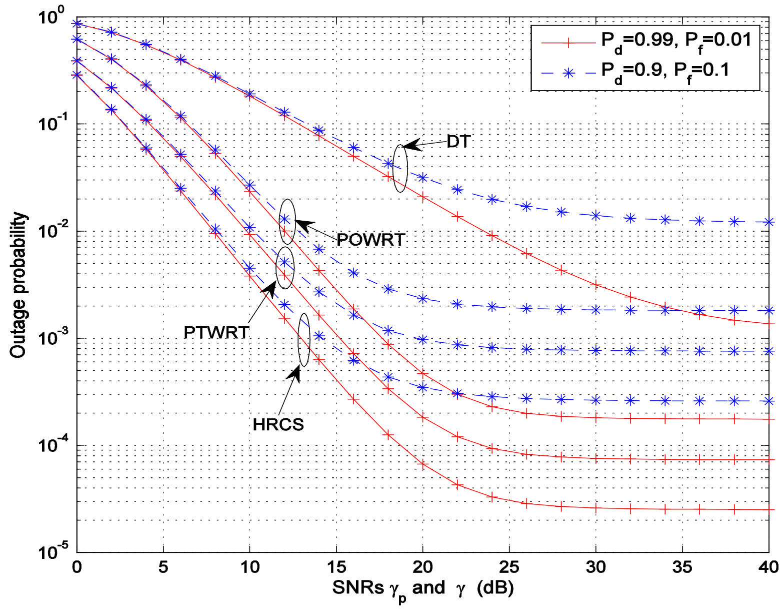

Fig. 5. Outage probability versus primary and cognitive users SNRs with P_a =0.6, a=1, $\alpha=\beta$=1/3, $\lambda_{1r}=\lambda_{2r}$=0.1, $\lambda_{p1}=\lambda_{p2}=\lambda_{pr}$=10, $\lambda_{12}=\lambda_{21}$=1 and $R_1=R_2$=0.5 bit/s/Hz.

In **Fig. 5**, we compare the outage probabilities of the four transmission schemes considering two results of the spectrum sensing, i.e., P_d =0.99, P_f =0.01 and P_d =0.9, P_f =0.1. As shown in **Fig. 5**, the outage probability for P_d =0.99, P_f =0.01 is lower than that for P_d =0.9, P_f =0.1, no matter which transmission scheme is considered. The reason is that more exact sensing for the primary spectrum reduces the possibility of the interference from the PU to CUs, so that the cognitive user transmissions can be less interrupted. Therefore, it is necessary for the performance improvement of the cognitive user transmissions to increase the accuracy of the spectrum sensing. Additionally, the outage probability floor exists in the high SNR region for each scheme. Moreover, no matter which case of the sensing results is considered, the HRCS always outperforms the other three transmission schemes in terms of the outage probability.

Fig. 6 plots the outage probability floor as a function of the channel gain ratio λ_{12-p2}^{-1}, where $\lambda_{12-p2}^{-1}=\sigma_{12}^2/\sigma_{p2}^2$ denotes the channel gain ratio of the non-relay direct link to the interference link. It is clear that the outage probability floor reduces with the increase of λ_{12-p2}^{-1} for each scheme. The increase of λ_{12-p2}^{-1} means that σ_{p2}^{2} drops or σ_{12}^{2} grows, which leads to the decrease of the interference from the PU to CUs or the increase of the successful transmission through the non-relay direct links, respectively. It follows that the transmission outage decreases for the bidirectional communication. It is also clear in **Fig. 6** that the HRCS outperforms the other three schemes in terms of the outage probability floor over the wide λ_{12-p2}^{-1} region. This phenomenon shows that the HRCS is an efficient adaptive cooperation scheme to reduce the outage probability floor.

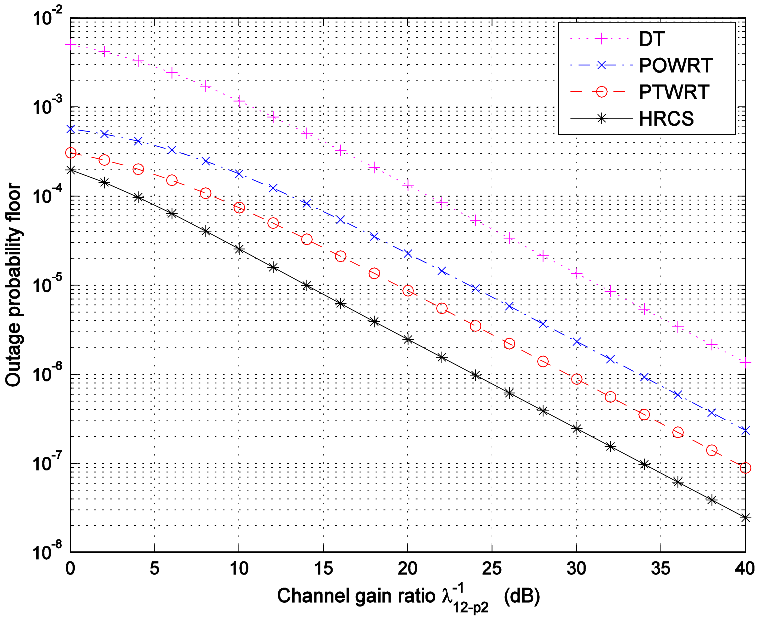

Fig. 6. Outage probability floor versus the channel gain ratio λ_{12-p2}^{-1} with P_a =0.6, P_d=0.99, P_f=0.01, a=1, $\alpha=\beta$=1/3, $\lambda_{r1-p1}=\lambda_{r2-p2}$=0.01, $\lambda_{12-p2}=\lambda_{21-p1}$, and $R_1=R_2$=0.5 bit/s/Hz.

Next, the outage probability versus power allocation factor α is given in **Fig. 7** for two cases (R_1=0.5 bit/s/Hz, R_2=1 bit/s/Hz) and (R_1=1 bit/s/Hz, R_2=1 bit/s/Hz), where α represents the power allocation between the relay node and the end-sources. As observed from **Fig. 7**, under the conditions of the fixed total power P_{total}, the optimal α corresponding to the lowest outage probability exists for the POWRT, PTWRT, and HRCS. Specifically, for the full cooperation transmission schemes including the POWRT and PTWRT, the best outage performance is obtained at α=0.5, which means the transmit power of the relay is equal to the sum of the transmit power of CU_1 and CU_2. For the HRCS, the optimal outage probability corresponds to α=0.4, which means the transmit power of the relay is less than the sum of the transmit power of CU_1 and CU_2. The reason is that for the full cooperation, the relay links and the direct links have the identical opportunity involved in the information exchange between CU_1 and CU_2. In contrast, for the HRCS, the direct links have higher priority to undertake the bidirectional communication than the relay links. Therefore, the relatively high transmit power of the two end-sources is required to obtain the least outage probability for the HRCS.

Additionally, the case (R_1=0.5 bit/s/Hz, R_2=1 bit/s/Hz) obtains lower outage probability than the case (R_1=1 bit/s/Hz, R_2=1 bit/s/Hz) for each scheme, because the decrease in R_1 leads to the probability decrease of decoding failure at CU$_2$ and CR. Meanwhile, **Fig. 7** shows that for the identical data rate as (R_1=1 bit/s/Hz, R_2=1 bit/s/Hz), or the different data rate as (R_1=0.5 bit/s/Hz, R_2=1 bit/s/Hz), the HRCS achieves better outage performance than the POWRT and PTWRT.

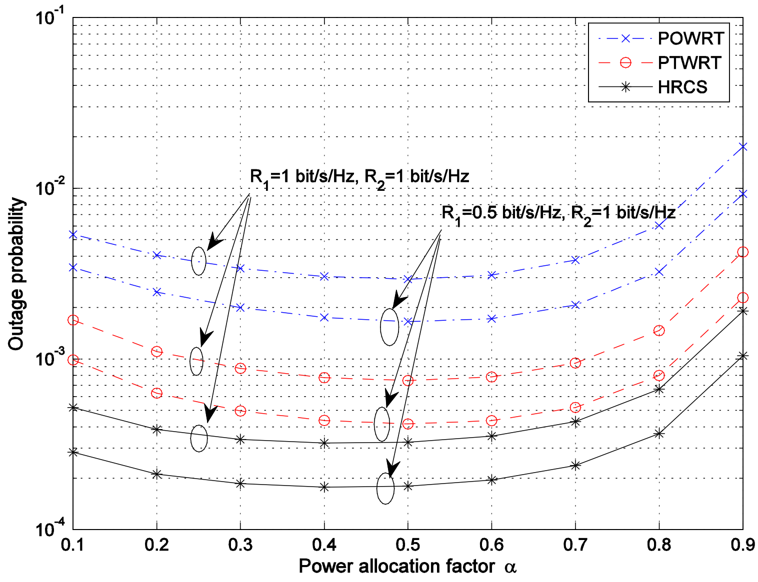

Fig. 7. Outage probability versus power allocation factor α with P_a =0.6, P_d=0.99, P_f=0.01, a=1, λ_{1r}=λ_{2r}=0.1, λ_{p1}=λ_{p2}=λ_{pr}=10, λ_{12}=λ_{21}=1 and γ_{total} =30 dB.

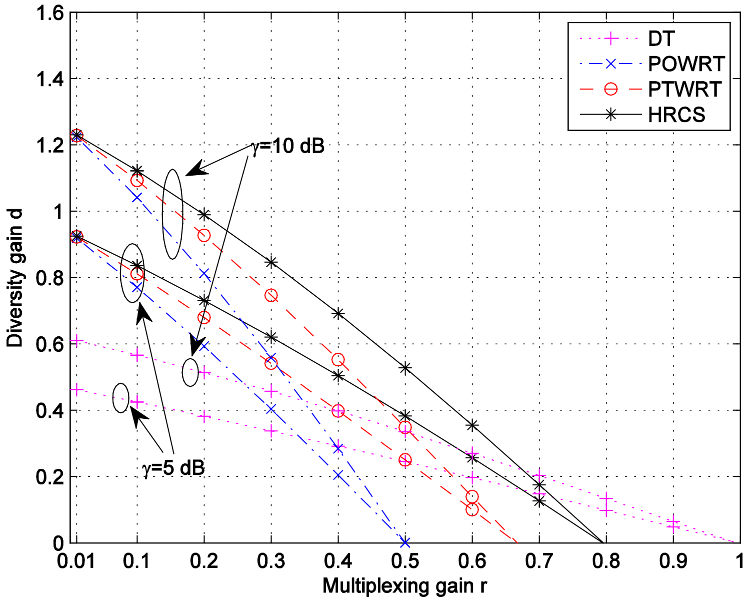

Fig. 8. f-DMT curves with P_a =0.6, P_d=0.99, P_f=0.01, a=1, α =β =1/3, λ_{1r}=λ_{2r}=0.1, λ_{p1}=λ_{p2}=λ_{pr}=10, λ_{12}=λ_{21}=0.1, and δ=0.5.

Last, we investigate the f-DMT for the four transmission schemes. The f-DMT curves are given in **Fig. 8** for γ=5 dB and γ=10 dB. As expected, the DT obtains higher maximal multiplexing gain than the other three transmission schemes. Meanwhile, the HRCS always achieves better f-DMT performance than the POWRT and PTWRT for γ=5dB and γ=10dB. Furthermore, a crossover point between the HRCS and DT appears as r approaches 0.7. Thus, given r>0.7, the HRCS is worse than the DT in terms of the diversity gain. In addition, no matter which transmission scheme is considered, the f-DMT curve for γ=10dB is always above the curve for γ=5dB. This indicates that as the SNR increases from the low to the moderate, the outage probability with a fixed multiplexing gain r falls fast, which leads to the increase of the negative slope of the log-log plot of the outage probability versus SNR, i.e., the increase of the diversity gain.

6. Conclusion

In this paper, we investigate the information exchange between two CUs in cognitive radio networks with the HRCS. The HRCS, where the relay works only when the non-relay direct transmissions fail, combines the advantages of the non-relay direct transmissions and the one- and the two-way relaying diversity transmissions. Therefore, as the numerical results show, the HRCS obtains better outage and f-DMT performance than the full cooperation transmission schemes including the POWRT and PTWRT. Also, the HRCS decreases the outage probability floor, which indicates the HRCS is efficient for improving the performance of the cognitive user transmissions.

Appendix A

In this Appendix, we present the formulas used to derive the closed-form expressions of outage probabilities..

Let $X_1 = |h_{ab}|^2$, $X_2 = |h_{cd}|^2$ and $X_3 = |h_{ef}|^2$. Then, the probability of $X_1 - \gamma_p x X_2 < x$ can be calculated as

$$\begin{aligned}\Pr(X_1 - \gamma_p x X_2 < x) &= \int_0^{\infty} \lambda_{cd} e^{-\lambda_{cd} x_2} dx_2 \int_0^{(\gamma_p x_2 + 1)x} \lambda_{ab} e^{-\lambda_{ab} x_1} dx_1 \\ &= 1 - \frac{\lambda_{cd} e^{-\lambda_{ab} x}}{\gamma_p \lambda_{ab} x + \lambda_{cd}}.\end{aligned} \tag{A.1}$$

Likewise, the probability of the event $\alpha X_1 + \beta X_2 < \beta x$ is derived by

$$\begin{aligned}\Pr(\alpha X_1 + \beta X_2 < \beta x) &= \int_0^{\frac{\beta x}{\alpha}} \lambda_{ab} e^{-\lambda_{ab} x_1} dx_1 \int_0^{x - \frac{\alpha}{\beta} x_1} \lambda_{cd} e^{-\lambda_{cd} x_2} dx_2 \\ &= \begin{cases} 1 - \dfrac{\beta \lambda_{ab} e^{-\lambda_{cd} x} - \alpha \lambda_{cd} e^{-\beta \lambda_{ab} x / \alpha}}{\beta \lambda_{ab} - \alpha \lambda_{cd}}, & \dfrac{\lambda_{ab}}{\alpha} \neq \dfrac{\lambda_{cd}}{\beta} \\ 1 - (1 + \lambda_{cd} x) e^{-\lambda_{cd} x}, & \dfrac{\lambda_{ab}}{\alpha} = \dfrac{\lambda_{cd}}{\beta}. \end{cases}\end{aligned} \tag{A.2}$$

Referring to [12, Eq.(A.1)], we can easily write the probability of

$[X_1-\gamma_p xX_2>x]\cap[X_3-\gamma_p yX_2>y]$ as

$$\Pr(X_1-\gamma_p xX_2>x,X_3-\gamma_p yX_2>y)=\frac{\lambda_{cd}e^{-\lambda_{ab}x-\lambda_{ef}y}}{\lambda_{cd}+\gamma_p\lambda_{ab}x+\gamma_p\lambda_{ef}y}. \tag{A.3}$$

Furthermore, we derive the probability of $\alpha X_1+\beta X_2-\gamma_p\beta xX_3>\beta x$ as

$$\begin{aligned}
&\Pr(\alpha X_1+\beta X_2-\gamma_p\beta xX_3>\beta x)\\
&=\int_0^{\frac{\beta x}{\alpha}}\lambda_{ab}e^{-\lambda_{ab}x_1}dx_1\int_{x-\frac{\alpha}{\beta}x_1}^{\infty}\lambda_{cd}e^{-\lambda_{cd}x_2}dx_2\int_0^{\frac{\alpha x_1+\beta x_2-\beta x}{\beta\gamma_p x}}\lambda_{ef}e^{-\lambda_{ef}x_3}dx_3\\
&\quad+\int_{\frac{\beta x}{\alpha}}^{\infty}\lambda_{ab}e^{-\lambda_{ab}x_1}dx_1\int_0^{\infty}\lambda_{cd}e^{-\lambda_{cd}x_2}dx_2\int_0^{\frac{\alpha x_1+\beta x_2-\beta x}{\beta\gamma_p x}}\lambda_{ef}e^{-\lambda_{ef}x_3}dx_3\\
&=\begin{cases}\dfrac{\lambda_{ef}}{\alpha\lambda_{cd}-\beta\lambda_{ab}}\left(\dfrac{\alpha^2\lambda_{cd}e^{-\beta\lambda_{ab}x/\alpha}}{\alpha\lambda_{ef}+\beta\gamma_p\lambda_{ab}x}-\dfrac{\beta\lambda_{ab}e^{-\lambda_{cd}x}}{\lambda_{ef}+\gamma_p\lambda_{cd}x}\right), & \dfrac{\lambda_{ab}}{\alpha}\neq\dfrac{\lambda_{cd}}{\beta}\\[2ex]
\dfrac{\lambda_{ef}e^{-\lambda_{cd}x}\left[\lambda_{ef}(1+\lambda_{cd}x)+\lambda_{cd}\gamma_p x(2+\lambda_{cd}x)\right]}{(\lambda_{ef}+\lambda_{cd}\gamma_p x)^2}, & \dfrac{\lambda_{ab}}{\alpha}=\dfrac{\lambda_{cd}}{\beta}.\end{cases}
\end{aligned} \tag{A.4}$$

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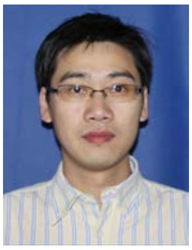

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