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Replacement model under warranty with age-dependent minimal repair

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Abstract. In this paper, we consider a renewable repair-replacement warranty strategy with age-dependent minimal repair service and propose an optimal maintenance model during post-warranty period. Such model implements the repair time limit under warranty and follows with a certain form of system maintenance strategy when the warranty expires. The expected cost rate is investigated per unit time during the life period of the system as for the standard for optimality. Based on the cost design defined for each failure of the system, the expected cost rate is derived during the life period of the system, considering that a renewable minimal repair-replacement warranty strategy with the repair time limit is provided to the customer under warranty. When the warranty is finished, the maintenance of the system is the customer's responsibility. The life period of the system is defined and the expected cost rate is developed from the viewpoint of the customer's perspective. We obtain the optimal maintenance strategy during the maintenance period by minimizing such a cost rate after a warranty expires. Numerical examples using field data are shown to exemplify the application of the methodologies proposed in this paper.

Key Words: *Age-dependent minimal repair, Maintenance cost, Renewable warranty, Repair time limit*

1. INTRODUCTION

The optimal maintenance strategy needs to be developed from the manufacturer's viewpoint to maintain the system operating without failure before the warranty expires so that the warranty cost is considered. On the contrary, the main concern would be to develop a maintenance policy from the customer's viewpoint minimizing the maintenance cost after the warranty expires. From this point of view, there have been a number of maintenance studies from both the manufacturer's and the customer's point of view in the literature.

Repair service, which is one of the maintenance actions, has been studied in the literature by many authors, including Park and Pham (2009, 2010, 2010), Shafiee, Chukova, Yun and Niaki (2011), and many others. Though the replacement models have their own

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usefulness in certain conditions, it would be more reasonable to consider that the repair service can also be provided when the system fails. Because the repair cost is generally less than the replacement cost in most circumstances, the manufacturer would naturally try to repair the failed product initially before providing a replacement service. Consider the following situations: A product failure occurs under warranty and the failed product is delivered to the customer center to make it operate again. For most of the non-fatal failures, the center will first attempt to repair it. However, if they are unable to fix it until the pre-specified time limit, they are advised to stop the repair service and to replace the failed product by a new one instead. Such a time limit of repair services would be determined by the manufacturer to satisfy customer so that the repaired product or the brand new one can be returned back to the customer. Supposing the center provides the replacement service, the warranty will be renewed. In this regard, it would be invaluable to consider a warranty which can provide both repair and replacement services simultaneously when the system failure occurs. Sahin and Polatoglu (1996) consider two forms of replacement policies when the warranty expires subject to certain conditions and discuss the cost incurred during the life period of the system under the customer's viewpoint. Jung and Park (2003) develop the optimal maintenance strategy when the warranty expires to minimize the expected cost rate during the life period of the system. Later, Jung, Park and Park (2010) study the life period anew from the customer's viewpoint and develop new optimal system maintenance policies when the warranty is expired, assuming the renewing warranty strategy. In their study, the life period of the system is defined to start when the new system is set up and to end when the system is replaced by a new one at the expense of the customer when the warranty is expired, while in many other maintenance policies, the life period ends when the new system installed originally fails. Other maintenance policies including those from the manufacturer’s viewpoint are provided therein.

Under warranty, in respect to the rectification for the failure occurrence, three basic forms of warranties are usually offered: free repair-replacement warranty (FRW), pro-rata warranty (PRW) and combination warranty (CMW). Under a FRW, the customers receive a repair/replacement warranty service for free and the PRW is such that the repair/replacement is not given for free, but at a prorated cost, counting on the amount of usage or service time provided prior to its current failure. The CMW contains both features of FRW and PRW. See Blischke (1994) and Park and Pham (2011).

In this paper, a renewable minimal repair-replacement(MRR) warranty policy is investigated to consider age-dependent minimal repair service and replacement service. Although a two-factor warranty policy is considered in this study, this is quite different from the usual two-dimensional models which use the product’s usage and age as two factors affecting the warranty. The renewable MRR warranty policy we consider in this study works as follows: When a system failure occurs under warranty, the manufacturer or the repair shop promptly starts the minimal repair for the failed system. In case the repair can’t be accomplished within a pre-specified repair time limit, the replacement service is provided free of charge to the customer instead and the original warranty terms are renewed. Thus, under the MRR warranty policy, both repair and replacement services are simultaneously considered with the repair service time pre-specified. During the maintenance period after the warranty expires, the customer maintains the system at its

own expenses and a minimal repair is taken whenever a failure occurs. We develop the mathematical formulation to obtain the expected cost rate (ECR) during the life period of the system and propose a new optimal maintenance strategy under such a two-factor warranty policy. The uniqueness of such an optimal maintenance strategy is verified for a repairable system with an increasing malfunction rate. In this study, we extend Park et al.'s model (2013) considering age-dependent minimal repair.
The remainder of this paper is organized as follows. In Section 2, the research problem of the study is described. Section 3 derives the cost models under the renewable MRR warranty policy, which takes both repair service and replacement service into account for each system failure. The mathematical formulas to compute the ECR's are derived as well. Section 4 discusses the method to optimize the ECR under the proposed renewable warranty policy and the optimal solution for decision variables of the maintenance strategy after the warranty expires is obtained by minimizing the ECR derived in Section 3. Finally, concluding remarks are shown in Section 5.

Nomenclature

RFRW, RPRW : Renewable Free Repair Warranty, Renewable Pro Rata Warranty
NHPP : Non-homogeneous Poisson Process
ECR : Expected Cost Rate
MRR : Minimal Repair-Replacement
$\lambda(\cdot)$: intensity function
T, Y: failure time and repair time, respectively
$f(t), F(t)$: density function of T and its distribution function, respectively
$g(y), G(y)$: pdf and cdf of repair times, respectively
δ : fixed length of maintenance period
C_r : replacement cost in the warranty period
C_m : minimal repair cost in the warranty period
C_f : failure cost during the life period of the system
c_r: unit cost of replacement
c_m: unit cost of minimal repair
c_{fm}: unit failure cost during the maintenance period
N_R : number of replacements in the warranty period
N_ψ: number of minimal repair services in the area Ψ
N_T: total number of system failures under warranty
N_δ: total number of system failures when the warranty is expired

2. PROBLEM DESCRIPTION

When the warranty expires, the system is kept by the customer for a fixed length of time, which is referred to as a maintenance period of length δ. The system is minimally repaired at each system failure during the maintenance period, and is replaced with a brand new one at the end of the maintenance period. Therefore, the life period is defined from the purchasing time of the system to the restoration time at which the system is replaced at the expense of the customer after the warranty expires. The expected maintenance cost during the life period is investigated under the customer's point of view under warranty and the fixed length of maintenance period. Since all the maintenance costs incurred during the maintenance period are paid entirely at the expense of the customer, the customer's main concern would be to develop an optimized maintenance strategy minimizing the maintenance cost after the warranty period.

Age-dependent minimal repair service and replacement service are dealt with simultaneously in this paper. The main goal of this paper is to propose an optimized maintenance strategy during the maintenance period after the warranty expires by determining the length of maintenance period depending on the renewable MRR warranty policy with the repair time limit. Under the renewable MRR warranty considered in this study, the system is either age-dependent minimally repaired or replaced when the repair time goes beyond the repair time limit for each failure under warranty. The ECR is used as the criterion for the optimality and the total maintenance cost is evaluated from the customer's viewpoint during the life period of the system. The warranty period and the repair time limit are pre-specified by the manufacturer in this study.

3. COST MODEL UNDER MINIMAL REPAIR-REPLACEMENT WARRANTY STRATEGY (Park, Jung et al. 2013)

In this section, we consider a MRR warranty strategy, which is renewable when the replacement is provided, to derive a formula to evaluate the ECR. Under this model, the manufacturer specifies the repair time limit which works as the time limit for the repair service. That is, if the repair service can't be accomplished within the threshold, the failed system will be replaced by a new one, instead of minimal repair and the warranty is renewed. Both minimal repair and replacement are performed free of charge to the customer under warranty. Such warranty strategy could be more desirable and realistic in practice for satisfaction of the customers. Should the warranty period expire, the customer must be responsible for the relevant costs incurred due to the system failure or mal-functioning. Under the renewable warranty strategy, the warranty strategy would be renewed automatically anew for the replaced system and the same warranty terms will be in effective again. In the following, we first explain a renewable MRR warranty strategy using the system failure time and repair time as two factors recorded at the time of system failure. Then the mathematical formulas to obtain the ECR are derived by consideration of several cost designs which are defined for the system failures during and after the warranty period under the customer's point of view.

3.1. Minimal repair-replacement warranty policy

The warranty strategy under study is the renewable MRR warranty with a pre-specified length of warranty period, denoted by w. Under such a warranty strategy, the manufacturer is in charge for providing the minimal repair-replacement services upon the system failures under warranty, which works as follows. When the system failure occurs, the failed system receives an immediate consideration of the customer center, which set the repair time limit. In case the customer center can't repair the failed system within the pre-specified threshold, then the center should supply the replacement service instead of minimal repair. In this situation, the warranty strategy is renewed for the replaced system with absolutely the same warranty terms as the original one. On the other hand, if the center successfully provides the minimal repair to restore the failed system within the repair time limit, the warranty would be effective only in the remaining warranty period and it would not be renewed.

The graphical representation for such a warranty strategy is shown in Fig. 1. For $j = 1, 2, \cdots,$ we let $T_{R_j}\left(< w\right)$ denote the inter-replacement time interval elapsed between $j-1^{\text{th}}$ replacement and the j^{th} replacement of the system under warranty. If $T_{R_1} < w,$ then under the renewable FRW strategy, the replacement service will be granted free of charge to the customer, and the manufacturer will be in charge for the replacement of the failed system. Starting from T_{R_1}, the restored system will have the same renewable warranty with the length of period of w again as shown in Figure 1. Let W_0 denote the length of warranty period, defined as the time interval starting from the point of sale and ending at the expiration time of warranty. It is clear that a warranty period coincides with a warranty period w for a non-renewable warranty. However, for a renewable strategy, W_0 is a *r.v.* depending on the total number of system replacements, the inter-arrival times between two successive replacements under warranty and the length of warranty period. Let $N_T = N_\Psi + N_R$ be the total number of system failures under warranty, where N_ψ and N_R are the number of minimal repairs and the number of replacements, respectively. Then, W_0 can be expressed as (Park, Jung et al. 2013)

$$\mathrm{W_0 = T_{R_1} + T_{R_2} + \cdots + T_{R_{N_R}} + w} \tag{1}$$

Fig. 1 depicts the case in which, if the first replacement occurs at time $\mathrm{t_1}$ and the second replacement does not occur until the warranty expires, then it is clear that $w_0 = t_1 + w$, where w_0 is the realization of W_0. In Fig. 1, T_j and Y_j, $\mathrm{j} = 1,2,\dots$, denote the time at which the j^{th} failure occurs and the length of its repair time, respectively.

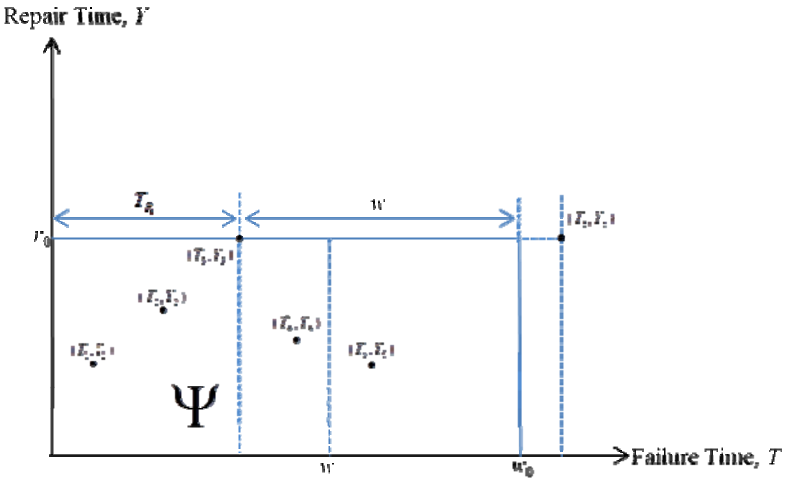

Figure 1. Renewable warranty policy under minimal repair-replacement strategy (Park, Jung et al. 2013)

3.2. Length of life period (Park, Jung et al. 2013)

In this section we obtain the expected length of life period of the system in the situations where the renewable MRR warranty strategy is applied under warranty and the Sahin and Polatoglu's (1996) replacement model is considered during the maintenance period following the expiration of warranty period. Under the renewable MRR, the manufacturer provides two kinds of warranty services on each system failure under warranty, the minimal repair initially and the replacement service if the repair work can't be accomplished within the repair time limit. In consideration of both repair and replacement services by the manufacturer under warranty and the maintenance work by the customer during the maintenance period, the expected length of life period is derived. Jung, Park and Park (2010) define the life period of the system anew from the customer's viewpoint and study the optimized maintenance strategy after the renewing warranty is expired. The life period we consider in this study is defined similarly to that of Jung, Park and Park (2010). A life period of the system begins with the installation of a new system. If the system fails under warranty, it is either minimally repaired or replaced by a new one under the same warranty terms. When the replacement service is granted due to the excess of the repair time limit, then the life of the system is further extended. Once the system survives beyond the warranty period, the customer applies a minimal repair for each failure for a fixed length of time and changes the system with a brand new one at the end of this period. The life period ends when the system installed initially is replaced at the expense of the customer during the maintenance period. Let T be a *r.v.* denoting the failure time of the system having $f(t)$ and $F(t)$ as its pdf and cdf, respectively and let $T_1, T_2, \cdots$ denote the failure times of the system. Let $Y_1, Y_2, \cdots$ denote the *r.v.*'s representing the lengths of repair times which are considered to be *i.i.d.* continuous having g(y) and G(y) as its pdf and cdf, respectively. Consider that the system is replaced N_R times under warranty under the renewable MRR warranty strategy. Although other system failures occur N_ψ times within the warranty period, these failures are minimally repaired and do not affect the length of life period. Thus, the length of life period can be expressed as $L(w,\delta)=W_0+\delta=\sum_{j=1}^{N_R} T_{R_j}+(w+\delta)$, where W_0 is the warranty period given in Eq. (1). It follows that given $N_R=n$, the expected period length can be represented as

$$\begin{aligned} E\left(L(w,\delta)\middle|N_R=n\right)&=\sum_{j=1}^{n} E\left(T_{R_j}\middle|T_{R_j}\le w, N_R=n\right)+(w+\delta) \\ &=\sum_{j=1}^{n} E\left(T_j\middle|T_j\le w, Y_j\ge r_0, N_R=n\right)+(w+\delta) \\ &=n\cdot\frac{\int_0^w t\cdot f(t)\,dt}{F(w)\bar{G}(r_0)}+(w+\delta) \end{aligned} \quad (2)$$

If r_0 approaches zero, then Eq. (2) becomes Eq. (3) below which is the same as the length of life period given in the Sahin and Polatoglu's (1996) replacement model.

$$E\left(L(w,\delta)\middle|N_R=n\right)=n\cdot\frac{\int_0^w t\cdot f(t)\,dt}{F(w)}+(w+\delta) \quad (3)$$

The replacement service is provided by the manufacturer only when the system fails under warranty and the repair service goes beyond the pre-specified repair time limit. Thus, the number of system replacements under warranty has the following geometric distribution.

$$P(N_R=n)=\left\{\bar{F}(w)+F(w)\cdot G(r_0)\right\}\cdot\left\{F(w)\cdot\bar{G}(r_0)\right\}^n, \quad n=0,1,2,\cdots. \quad (4)$$

By taking the expectation for the conditional expectation of Eq. (2) with respect to N_R, we obtain the following expected period length under the renewable MRR warranty policy.

$$\begin{aligned} E\left(L(w,\delta)\right)&=\sum_{j=0}^{\infty}\left\{\bar{F}(w)+F(w)\cdot G(r_0)\right\}\cdot\left\{F(w)\cdot\bar{G}(r_0)\right\}^j\cdot\left\{j\cdot\int_0^w t\cdot f(t)\,dt\,/\left(F(w)\bar{G}(r_0)\right)\right\}+(w+\delta) \\ &=\frac{\int_0^w t\cdot f(t)\,dt}{\left\{1-F(w)\cdot\bar{G}(r_0)\right\}}+(w+\delta) \end{aligned} \quad (5)$$

The life span of a system is considered finished when the system is replaced by a new one at the customer's expense at the end of maintenance period. Under the maintenance model considered in this paper, the length of maintenance period is considered to be fixed at δ following the expiration of the renewing warranty term.

3.3. Expected maintenance cost

In this section, we obtain the expectations of the total maintenance cost incurred during the life period of the system and thereby derive the expected cost rate per unit time during the life period of the system. Let C_r, C_m and C_f denote the *r.v.*'s representing the replacement cost that the customer is in charge for under warranty, minimal repair cost and failure cost during the life period of the system, respectively. Further, let c_r be the fixed unit cost of replacement at the end of the life period of the system. In order to maintain the system during its life period, the customer would be charged the total amount of cost equaling $C_r + C_m + C_f + c_r$, where the last term c_r needs to be added because the system is replaced at the customer's expense at the end of maintenance period. In this study, we consider both FRW and PRW based on two factors of failure time and repair time. The detailed discussion on FRW and PRW is given in Section 1.

Under the renewing minimal repair-replacement FRW, both minimal repair and replacement are performed at no charge to the customer under warranty and thus, $P(C_r = 0) = 1$. However, under the renewing minimal repair-replacement PRW, the customer is responsible for the pro-rated replacement cost under warranty and thus, the customer's replacement cost can be expressed as a function of T_{R_j} 's as follows (Park, Jung et al. 2013):

$$C_r = \sum_{j=1}^{N_R} c_r \frac{T_{R_j}}{w}. \tag{6}$$

During the maintenance period after the warranty is expired, the repair service is minimal for each failure and no repair time limit is set. As a result, the failure intensity is considered to follow the NHPP of rate $\lambda(t)$. Let N_δ denote the number of failures during the maintenance period following the expiration of warranty. Then the pdf of N_δ is given by (Park, Jung et al. 2013)

$$P(N_\delta = n_\delta) = \frac{e^{-\int_w^{w+\delta} \lambda(s)ds} \left[\int_w^{w+\delta} \lambda(s)\, ds \right]^{n_\delta}}{n_\delta !} \tag{7}$$

and thus, it follows that

$$E(N_\delta) = \int_w^{w+\delta} \lambda(s)\, ds \tag{8}$$

Now, we evaluate the expected total cost, which is charged to the customer during the life period of the system, for our replacement model. The expected total cost can be expressed as

$$\mathrm{EC}(w, \delta) = \mathrm{E}(C_r) + \mathrm{E}(C_m) + \mathrm{E}(C_f). \tag{9}$$

Each of the expected costs, given in Eq. (9), can be obtained as follows:

$$\mathrm{E}(C_f) = c_{fw} \left(\frac{F(w)\overline{G}(r_0)}{1 - F(w)\overline{G}(r_0)} \right) + c_{fm} \int_w^{w+\delta} \lambda(t)\, dt,$$

$$\mathrm{E}(C_M) = \int_w^{w+\delta} C_m(t) \lambda(t)\, dt\,,$$

$$E(C_r) = \begin{cases} 0\,, & \text{FMRR} \\ \dfrac{c_r}{w}\left\{\displaystyle\int_0^w tf(t)\,dt/(1-F(w)\bar{G}(r_0))\right\}, & \text{PMRR} \end{cases}$$

Here $C_m(t)$ is the cost of minimal repair for $t \geq 0$, where t denotes the time to failure of the system and $C_m(t)$ is considered to be a non-decreasing continuous function of t. As the system ages, it costs more to perform minimal repair. Thus, the expected total cost during the life period of the system, given in (9), can be obtained as follows.

$$EC(w,\delta) = \begin{cases} c_1 + c_{fm}\displaystyle\int_w^{w+\delta}\lambda(t)\,dt + \int_w^{w+\delta} C_m(t)\lambda(t)\,dt + c_r\,, \text{FMRR} \\ c_2 + c_{fm}\displaystyle\int_w^{w+\delta}\lambda(t)\,dt + \int_w^{w+\delta} C_m(t)\lambda(t)\,dt + c_r\,, \text{PMRR} \end{cases} \tag{10}$$

where,

$c_1 = c_{fw}\left(\frac{F(w)\bar{G}(r_0)}{1-F(w)\bar{G}(r_0)}\right)$ and

$$c_2 = \frac{c_r}{w}\left\{\int_0^w tf(t)\,dt/(1-F(w)\bar{G}(r_0))\right\} + c_{fw}\left(\frac{F(w)\bar{G}(r_0)}{1-F(w)\bar{G}(r_0)}\right)$$

The expression, given in Eq. (10), is the total expected cost for which the customer is responsible to maintain the system during the life period of the system until the system is replaced by a new one at the expense of the customer. Dividing $EC(\omega,\delta)$ of Eq. (10) by the expected length of life period, $E(L(\omega,\delta))$, given in Eq. (5), we obtain the expected cost rate (ECR) per unit time during the life period of the system under the renewing minimal repair-replacement PRW as

$$ECR(w,\delta) = \begin{cases} \dfrac{c_1 + c_{fm}\int_w^{w+\delta}\lambda(t)\,dt + \int_w^{w+\delta} C_m(t)\lambda(t)\,dt + c_r}{\int_0^w tf(t)\,dt/\left(1-F(w)\bar{G}(r_0)\right) + (w+\delta)}\,, \text{FMRR} \\ \dfrac{c_2 + c_{fm}\int_w^{w+\delta}\lambda(t)\,dt + \int_w^{w+\delta} C_m(t)\lambda(t)\,dt + c_r}{\int_0^w tf(t)\,dt/\left(1-F(w)\bar{G}(r_0)\right) + (w+\delta)}\,, \text{PMRR} \end{cases} \tag{11}$$

In our model, we consider that the cost of minimal repair depends on the age of system. Thus we can use the special form of function. Ja *et al.* (2001) consider the cost of minimal repair is a function of product age. Especially, they select the linearly increasing function for consumer products such as washers, dryers, or refrigerators and the non-linearly increasing function for industrial products such as boring machine. Thus, we can consider the special case of repair costs as follows.

(a) Linearly increasing with age

Firstly, we consider the linearly increasing function as follows (Ja *et al.* (2001)).

$$C_m(t) = a + bt$$

where a and b are constants, whose values can be estimated from historical cost-estimates.

(b) Non-linearly increasing with age

We can consider the non-linearly increasing function as follows (Ja *et al.* (2001)).

$$C_m(t) = a + bt^2$$

where a and b are constants. Above function is one of non-linearly increasing function of cost versus product age, an appropriate functional form could be determined by regression modeling using historical observations.

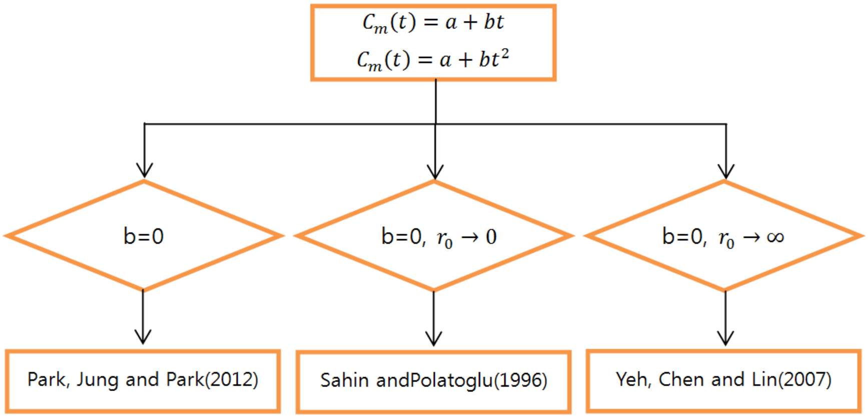

Figure 2. Relationships for age-dependent minimal repair cost models from existing literature

4. OPTIMAL SOLUTION (Park, Jung et al. 2013)

This section discusses the optimal solutions for the ECR to be minimized under the renewable MRR warranty strategy and compare both the FRW and the PRW policies with respect to the minimized ECR which is proposed in this study. For the optimization approach, we apply the well-known Nelder-Mead downhill simplex method (Li and Pham 2005), which does not need the calculation of derivatives and is one of the most frequently used direct search method for obtaining the optimum solution of a nonlinear function. The optimized maintenance strategy which adapts a renewable MRR warranty under warranty and the Sahin and Polatoglu's (1996) replacement model during the maintenance period is established by determining the optimized value of δ so that the $ECR(w,\delta)$ is minimized. Various unit costs, the length of warranty period and the repair time limit, denoted by r_0, are considered to be fixed for the purpose of minimizing the ECR.

5. CONCLUSION

This paper deals with a renewable warranty strategy with the repair time limit under warranty considering both repair time and failure time. Once the warranty period expires, then the customer will be in charge for maintaining the system by using Sahin and Polatoglu (1996)'s replacement model during the maintenance period. The system is changed with a new one at the end of the maintenance period which is considered to have a fixed length of δ. The life period of a system is considered to start with the installation of a new system and to end when the system installed initially is replaced at the expense of the customer at the end of the maintenance period.

Both age-dependent minimal repair service and replacement service upon the system failure are considered simultaneously to conduct the cost analysis under warranty and the fixed-length replacement model after the warranty expires. If a system sustains after the warranty expires, the system undergoes a age-dependent minimal repair on each failure until it is replaced by a brand new one. When the warranty is expired, the maintenance cost is entirely borne by the customer.

The goal of this paper is to consider the repair time limit under the renewable warranty strategy so that the warranty strategy is affected by both the failure time and the repair time. In case the repair takes too much time beyond a certain pre-specified time limit, the system is replaced by a new one with the same length of original warranty terms instead of continuing the repair works. Such a repair time limit would be fixed by the manufacturer for the customer's satisfaction. Applying the repair time limit on the failed system under warranty has not been discussed in the literature regarding the optimized maintenance model, which would be more reasonable in several practical situations.

This paper proposes an optimized maintenance model during the maintenance period under the two- factor renewable warranty strategy. We utilize the expected cost rate per unit time as the criterion for optimality during the warranty period of the system from the customer's viewpoint.

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