Current Optics and Photonics
Vol. 1, No. 2, April 2017, pp. 85-89

ISSN: 2508-7266(Print) / ISSN: 2508-7274(Online)
DOI: https://doi.org/10.3807/COPP.2017.1.2.085

Generation of *U* and *P* Singularities in Partially Coherent Beams Using Intensity Control

Xinyu Peng, Dong Ye, Guo Zheng, Qi Zhao*, and Minmin Song

Department of Optical Engineering, School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China

(Received July 11, 2016 : revised November 28, 2016 : accepted February 25, 2017)

It is shown that polarization singularities of a new type, namely *U* and *P* singularities, arise at the transverse cross section of a partially coherent beam, instead of the common singularities such as *C* points and *L* lines in a completely coherent vector field. A relationship between the two kind of singularities with respect to intensity is proposed. We also present a setup that can generate the new singularities, and any desired distribution of degree of polarization, using intensity control.

Keywords : Polarization, Coherence, Singular optics, Spatial light modulator
OCIS codes : (030.1640) Coherence, (260.5430) Polarization, (260.6042) Singular optics

I. INTRODUCTION

Since 1974 [1], when Nye and Berry first found dislocations in optical fields, singular optics have widely attracted attention. While phase singularities (wave dislocations, or optical vortices) are frequently encountered in the interference of scalar waves [2, 3], they evolve into polarization singularities when the vector nature of light is retained. The main subject of coherent singular optics often refers to the singular optics of vector fields [4-10]. Within the framework of vector singular optics, one considers the set of *L* lines (lines along which polarization is linear, with smoothly varying azimuth of polarization and the handedness of the electric field undetermined) and *C* points (where polarization is circular and the azimuth of polarization is undetermined). Note that *L* lines and *C* points are defined at the observation plane; one considers *L* surfaces (“envelopes”) and *C* lines (analogs of “snake-like distortions” of the wave front in scalar singular optics [11]) in three dimensions. The significance of the aforementioned elements of vector optical fields is that they constitute the particular skeleton of the field, so that by knowing the characteristics of such elements, one can very reliably predict the behavior of polarization parameters in the areas between them.

Since both *L* lines and *C* points are defined in a completely coherent field, in partially spatially coherent fields polarization singularities of a new type arise at the transverse cross section, composed of an incoherent superposition of the orthogonally circularly polarized laser beams. There are *U* contours, along which the degree of polarization is zero and the state of polarization is undetermined (singular), and isolated *P* points, where the degree of polarization is unity and the state of polarization is determined by the nonvanishing component of the combined beam. These are known as *singularities of the degree of polarization (DOP)* and have been elaborated upon both theoretically and experimentally [12-16].

In this paper, two mutually incoherent and orthogonally linearly polarized beams with different distributions of intensity are coaxially mixed. Then, the condition for the occurrence of *U* and *P* singularities is proposed. According to this condition, an experiment is set up to generate *U* and *P* singularities, and any desired distribution of DOP.

II. THEORETICAL ANALYSIS

Let us consider vector singularities in partially coherent optical beams by mixing two mutually incoherent and orthogonally linearly polarized beams with different distributions of intensity.

To analyze the conditions for the occurrence of optical

*Corresponding author: zhaoqi@njust.edu.cn

Color versions of one or more of the figures in this paper are available online.

singularities, we need to proceed from the Jones vectors of two orthogonally linear polarized beams, an *x*-axis linearly polarized wave and a *y*-axis wave,

$$\mathbf{E}_1 = \begin{bmatrix} E_1 e^{i\varphi_1} \\ 0 \end{bmatrix}, \quad \mathbf{E}_2 = \begin{bmatrix} 0 \\ E_2 e^{i\varphi_2} \end{bmatrix}, \tag{1}$$

where E_i and $\varphi_i (i=1,2)$ are the amplitudes and phases of two components. The Jones vector of the combined beam can then be written as

$$\mathbf{E}_{total} = \mathbf{E}_1 + \mathbf{E}_2 = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_1 e^{i\varphi_1} \\ E_2 e^{i\varphi_2} \end{bmatrix}. \tag{2}$$

The coherence matrix of the beam is defined as [17]

$$\mathbf{J} \equiv \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} = \begin{bmatrix} \langle E_x^* E_x \rangle & \langle E_x^* E_y \rangle \\ \langle E_y^* E_x \rangle & \langle E_y^* E_y \rangle \end{bmatrix}. \tag{3}$$

The angular brackets denote an ensemble average, and an asterisk denotes a complex conjugate.

For our purpose, we first consider the limiting case when the two beams are completely mutually coherent. With the disappearance of any optical-path difference between two beams, we believe that two components are completely mutually coherent. The coherence matrix can then be written as

$$\mathbf{J} \equiv \begin{bmatrix} E_1^2 & E_1 E_2 e^{-i(\varphi_1 - \varphi_2)} \\ E_1 E_2 e^{i(\varphi_1 - \varphi_2)} & E_2^2 \end{bmatrix}. \tag{4}$$

Combining the elements of the coherence matrix, one can find the full Stokes parameters:

$$\begin{aligned} S_0 &= J_{xx} + J_{yy} = E_1^2 + E_2^2; \\ S_1 &= J_{xx} - J_{yy} = E_1^2 - E_2^2; \\ S_2 &= J_{xy} + J_{yx} = 2E_1 E_2 \cos(\Delta); \\ S_3 &= i(J_{xy} - J_{yx}) = 2E_1 E_2 \sin(\Delta). \end{aligned} \tag{5}$$

where $\Delta = \varphi_1 - \varphi_2$ is the phase difference between E_x and E_y. Note that, these parameters satisfy the relationship $S_0^2 = S_1^2 + S_2^2 + S_3^2$. The degree of polarization is defined in terms of the Stokes parameters:

$$P = \sqrt{s_1^2 + s_2^2 + s_3^2} = 1. \tag{6}$$

where $s_i = S_i / S_0$, $(i = 1,2,3)$ are the normalized Stokes parameters. Thus the beam with two components is completely polarized. Any state of polarization can be represented in the Stokes space, i.e. the Poincaré sphere (see Fig. 1).

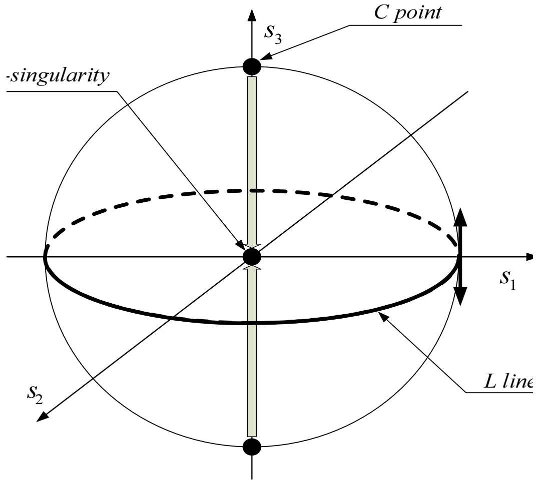

FIG. 1. Representation of optical singularities on the Poincaré sphere.

The azimuthal angle θ and ε ellipticity are two important parameters of the polarization ellipse. Using Eq. (5), one can obtain the following relationships:

$$\theta = \frac{1}{2}\arctan\left(\frac{S_2}{S_1}\right) = \frac{1}{2}\arctan\left(\frac{2E_1E_2\cos(\Delta)}{E_1^2 - E_2^2}\right), \tag{7}$$

$$\varepsilon = \frac{1}{2}\arcsin\left(\frac{S_3}{\sqrt{S_1^2 + S_2^2 + S_3^2}}\right) = \frac{1}{2}\arcsin\left(\frac{2E_1E_2\sin(\Delta)}{E_1^2 + E_2^2}\right). \tag{8}$$

Here we only consider the polarization ellipse with respect to the distribution of intensities of two components. To simplify, we set $\Delta = \frac{\pi}{2}$.

When the intensities of the two components are equal, *i.e.* $E_1 = E_2$, according to Eq. (5) we can obtain the normalized Stokes parameters 1,0,0,1, *i.e* the polarization is circular and the azimuth of polarization is undetermined, which means the singularity of a *C* point occurs. In Fig. 1, the poles of the Poincaré sphere represent *C* points.

Analogously, when one of the intensities of the mixed components is zero, *i.e.* $E_1 = 0$ (or $E_2 = 0$), proceeding from Eqs. (5), (7), and (8), we find that the ellipticity is zero, which means the state of polarization is linear. The *L* line consists of points satisfying this condition. In Fig. 1, the equator of the Poincaré sphere represents the *L* line.

Before considering the most general case of partial mutual coherence of the mixed orthogonally polarized beams, let us consider another limiting case, *viz.* completely incoherent mixing of them. In this case Eq. (6) becomes:

$$\mathbf{J} \equiv \begin{bmatrix} E_1^2 & 0 \\ 0 & E_2^2 \end{bmatrix}. \tag{9}$$

It is clear that when two components become equal in intensity, the normalized Stokes parameters of the combined beam become $\{1,0,0,0\}$. The point for such elements is completely unpolarized, and a U singularity occurs. This case is represented as the center of the sphere in Fig. 1, and all the other points inside the sphere image partially polarized fields. For those points, the length of a vector drawn from the center of the Poincaré sphere to the imaging point equals the degree of polarization.

If one of the intensities of the mixed components is equal to zero, then the degree of polarization equals unity, which is referred to as a P (completely polarized) point. Its location is determined by the vanishing component of the combined beam. The set of P points and U contours corresponding to extremes of a field's DOP are the singularities of DOP that form the vector skeleton of the two-component mixture of orthogonally polarized beams.

We emphasize that the conditions under which U and P singularities occur in the completely incoherent case are respectively the same as the conditions for C and L singularities in the completely coherent limit. This means that C and L singularities gradually transform into U and P singularities, respectively, with decreasing coherence of the components.

III. EXPERIMENT

According to the analysis above, we know that the degree of polarization is just related to the intensities of the mixed, completely incoherent beams. The degree of polarization can be represented in terms of the coherence matrix:

$$P = \sqrt{1 - \frac{4 \det \mathbf{J}}{(\mathrm{tr}\,\mathbf{J})^2}}. \tag{10}$$

where det and tr respectively denote the determinant and trace of the coherence matrix. Substituting Eq. (9) into Eq. (10), we obtain the following relationship:

$$P = \left| \frac{I_1 - I_2}{I_1 + I_2} \right|. \tag{11}$$

where $I_i = E_i^2 \ (i = 1, 2)$ is the intensity of the mixed beam. From Eq. (11), we can also come to the conclusion following Eq. (9). We can use this relationship to generate U and P singularities, or any desired distribution of degree of polarization. The setup we propose is shown in Fig. 2.

The laser emitted from polarizer P_1 is x-axis linearly polarized. The rotating ground-disk makes the beam incoherent. The purpose of the three lenses plus circular aperture is to remove the stray light at the edge generated by the RGGD. Next, the spatial light modulator (SLM, HOLOEYE PLUTO 1080p) acts as a phase grating designed by the method of computer-generated holograms. The pattern of the phase grating loaded onto the SLM is obtained by computing the interference pattern between a plane wave (reference wave) and a desired wave. To generate different distributions of beam intensities, two different lights are required. The distribution of normalized amplitude is shown in Fig. 3, and the pattern loaded onto the SLM is shown in Fig. 4. The first-order diffraction pattern is selected by the 4f-system. Then, the two mutually incoherent beams B_1 and B_2 are split by the beam splitter (BS), one part of which is y-axis linearly polarized by a half-wave plate. A quarter-wave plate and polarizer at the receiving end, together with a CCD camera, serve for Stokes polarimetric analysis of the combined beams.

Figures 5 and 6 show the one-dimensional intensity distributions for the two mixed beams, and the degree of polarization of the combined beam. We can see that when the

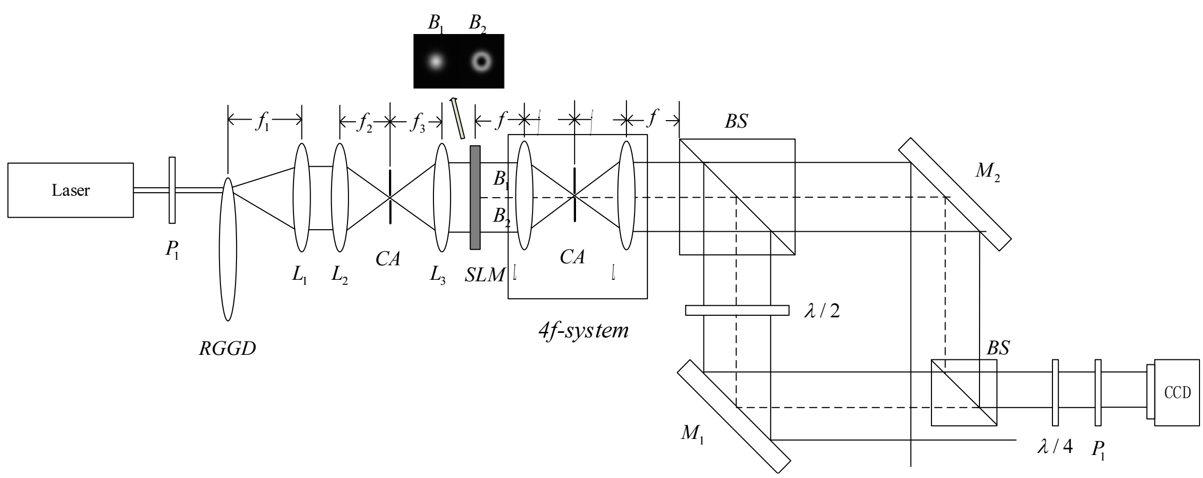

FIG. 2. Schematic illustration for generating and detecting singularities in the degree of polarization: P_1, P_1, polarizers; RGGD, rotating ground-disk; L_1, L_2, L_3, L thin lenses; CA, circular aperture; SLM, spatial light modulator; BS, beam splitter; M_1, M_2, mirrors; $\lambda/2$, half-wave plate; $\lambda/4$, quarter-wave plate; CCD, charge-coupled device.

intensities of the components are equal, the degree of polarization is zero-a U singularity. A P singularity occurs at the point {0,0,2,0}, where one of the intensities of the mixed components is zero.

Also, comparing Figs. 5 and 6, the distribution of degree of polarization in experiment is in quite satisfactory qualitative agreement with the simulation result, except for the edge of the light caused by the fluctuating intensity, which is caused by the rotating ground-disk. The intensity on the edge is weak. According to Eq. (11), when small fluctuations occur, the degree of polarization may experience large fluctuations. We can see, though, that the U and P singularities are generated very well. So, we can generate any desired two-dimensional distribution of degree of polarization by combining two mutually incoherent beams.

At last, we can obtain the vector skeleton of the combined beam, as shown in Fig. 7. The contour of the U singularity shows some fluctuation, because of the use of a rotating ground-disk to generate complete incoherent light. The two U contours separate the plane into three parts. By studying the vector skeleton, we can determine the behavior

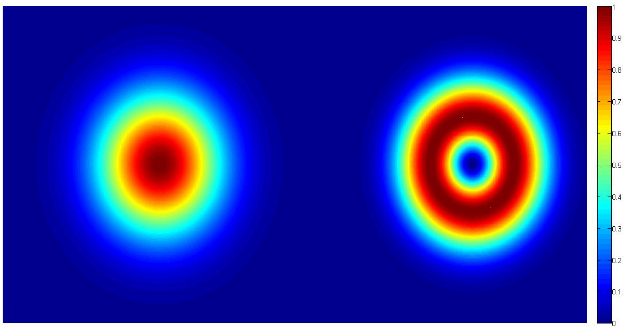

FIG. 3. Distributions of the normalized amplitude.

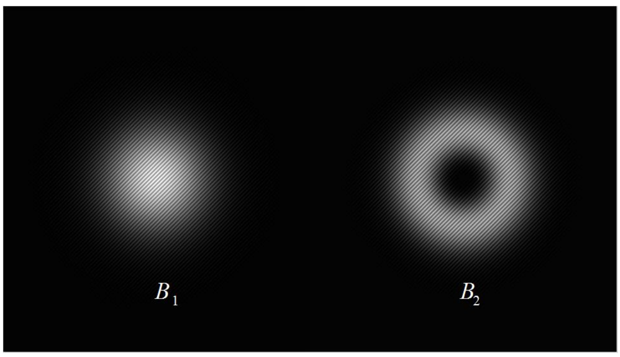

FIG. 4. Phase grating for generating two different beams B_1 and B_2.

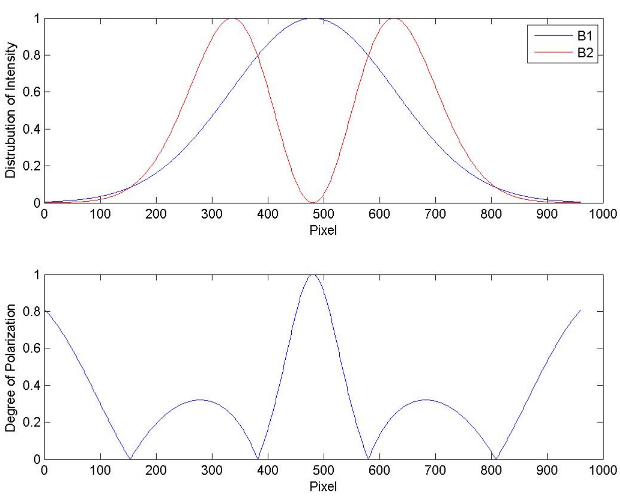

FIG. 5. Simulation of the one-dimensional intensity distributions for the two mixed beams, and the degree of polarization of the combined beam.

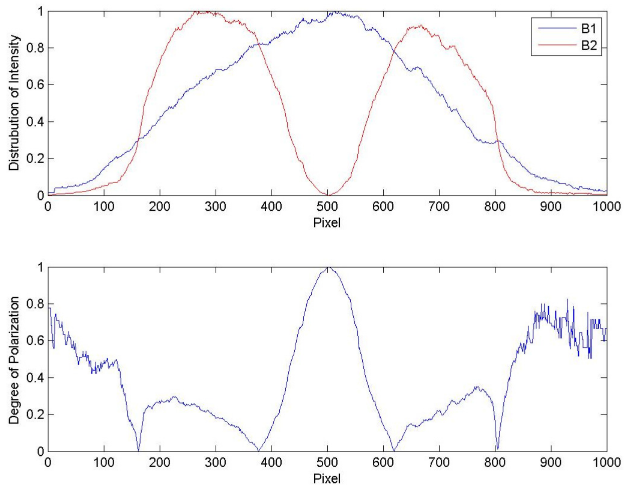

FIG. 6. Experimentally obtained intensity distributions for the two mixed beams, and the degree of polarization of the combined beam.

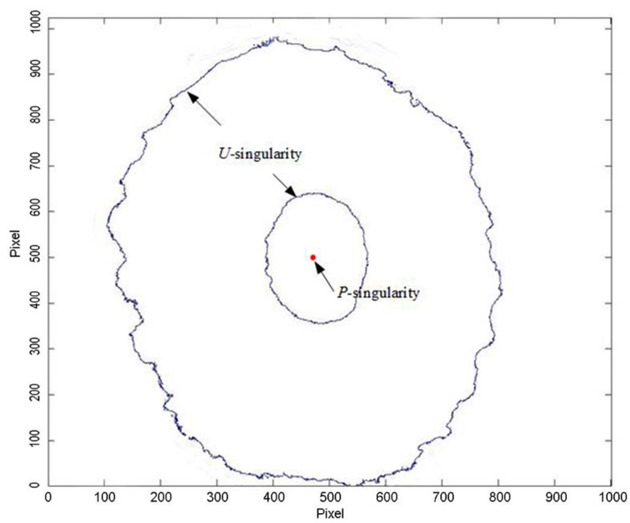

FIG. 7. The vector skeleton formed by P and U singularities, for completely incoherent mixing of linear polarized components.

of the beam's degree of polarization in transmission through free space or turbulence, which could be widely used in polarimetric detection.

IV. CONCLUSIONS

In this paper, we first introduced new polarization singularities *U* and *P*. Then, we proceeded from the Jones vectors of a mixture of two orthogonally linearly polarized beams, *x*-axis and *y*-axis linearly polarized waves, to analyze the conditions for the occurrence of optical singularities. The relationship between common polarization singularities and the new singularities were obtained: When the intensities of two components are equal, *C* and *U* singularities occur in a completely mutually coherent field and an incoherent one, separately, from which come *L* and *P* singularities in two limiting cases, respectively, if one of the intensities of the two components becomes zero. So, we can see a gradual transformation of *C* and *L* singularities into *P* and *U* singularities, respectively, accompanying a decreasing degree of mutual coherence of the components. This conclusion offers a method to generate *P* and *U* singularities, and is promising for generating any desired distribution of the degree of polarization in experiments.

ACKNOWLEDGMENT

This study was supported by grants from the National Natural Science Foundation of China (Grant Nos. 61377003), the Ph.D. Programs Foundation of Ministry of Education of China (20123219110021); the Innovation Fund for the Shanghai Aerospace Science and Technology (SAST201350).

REFERENCES

1. J. F. Nye and M. V. Berry, "Dislocations in wave trains," Proc. R. Soc. London **336**, 165-190 (1974).
2. J. Masajada, A. Popiolek-Masajada, and D. M. Wieliczka, "The interferometric system using optical vortices as phase markers," Opt. Commun. **207**, 85-93 (2002).
3. D. S. Lim and E. H. Lee, "Structural characteristics and properties of phase singularities in optical fibers," J. Opt. Soc. Korea **1**, 81-89 (1997).
4. J. F. Nye, Natural Focusing and Fine Structure of Light: Caustics and Wave Dislocations (CRC Press, 1999).
5. I. Freund and D. A. Kessler, "Singularities in speckled speckle," Opt. Lett. **33**, 479-481 (2008).
6. J. F. Nye, "Lines of circular polarization in electromagnetic wave fields," Proc. R. Soc. Lond. A **389**, 279-290 (1983).
7. E. J. Galvez, B. L. Rojec, and K. Beach, "Mapping of all polarization-singularity C-point morphologies," Proc. SPIE **8999**, 89990I (2014).
8. E. J. Galvez, B. L. Rojec, and K. R. McCullough, "Imaging optical singularities: Understanding the duality of C-points and optical vortices," Proc. SPIE **8637**, 863706 (2013).
9. M. R. Dennis, "Fermionic out-of-plane structure of polarization singularities," Opt. Lett. **36**, 3765-3767 (2011).
10. T. Bauer, P. Banzer, E. Karimi, S. Orlov, A. Rubano, L. Marrucci, E. Santamato, R. W. Boyd, and G. Leuchs, "Observation of optical polarization Möbius strips," Science **347**, 964-966 (2015).
11. N. B. Baranova, A. V. Mamaev, H. F. Pilipetskii, V. V. Shkunov, and B. Y. Zel'dovich, "Wave-front dislocations: topological limitations for adaptive systems with phase conjugation," J. Opt. Soc. Am. **73**, 525-528 (1983).
12. C. V. Felde, A. A. Chernyshov, H. V. Bogatyryova, P. V. Polyanskii, and M. S. Soskin, "Polarization singularities in partially coherent combined beams," JETP Lett. **88**, 418-422 (2008).
13. A. A. Chernyshov, C. V. Felde, H. V. Bogatyryova, and P. V. Polyanskii, "Vector singularities at superposition of mutually incoherent orthogonally polarized beams," Opt. Spectrosc. **107**, 645-650 (2009).
14. A. A. Chernyshov, C. V. Felde, H. V. Bogatyryova, P. V. Polyanskii, and M. S. Soskin, "Vector singularities of the combined beams assembled from mutually incoherent orthogonally polarized components," J. Opt. Pure Appl. Opt. **11**, 094010 (2009).
15. P. V. Polyanskii, C. V. Felde, and A. A. Chernyshov, "Polarization degree singularities," Proc. SPIE **7388**, 73880A (2009).
16. M. S. Soskin and P. V. Polyanskii, "New polarization singularities of partially coherent light beams," Proc. SPIE **7613**, 76130G (2010).
17. L. Mandel and E. Wolf, Optical Coherence and Quantum Optics (Cambridge University Press, Cambridge, UK, 1995).