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Robust Design to the Combined Array with Multiresponse

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Abstract

The Taguchi parameter design in industry is an approach to reducing performance variation of quality characteristic in products and processes. In the Taguchi parameter design, the product array approach using orthogonal arrays is mainly used. It often requires an excessive number of experiments. An alternative approach, which is called the combined array approach, was studied. In these studies, only single response was considered. In this paper we propose how to simultaneously optimize multiresponse for the robust design using the combined array approach.

Keywords: Taguchi Parameter Design, Product Array Approach, Combined Array Approach, Simultaneously Optimize Multiresponse

1. Introduction

Products and their manufacturing processes are influenced both by control factors that can be controlled by designers and by noise factors that are difficult or expensive to control such as environmental conditions. The basic idea of robust design is to identify, through exploiting interactions between control factors and noise factors, appropriate settings of control factors that make the system's performance robust to changes in the noise factors. Robust design(or Parameter design in a narrow sense) is a quality improvement technique proposed by the Japanese quality expert Taguchi, which was described by Taguchi[1,2], Kackar[3], and others.

In the Taguchi parameter design, the control factors are assigned to an "inner array", which is an orthogonal array. For each row in the inner array, the noise factors are assigned to an "outer array", also an orthogonal array. Because the outer array is run for every row in the inner array, we call this setup a "product array". A large number of experimental trials in Taguchi's product array may be required because the noise array is repeated for every row in the control array.

There have been efforts for integrating Taguchi's important notion of heterogeneous variability the standard experimental design and modeling technology provided by response surface methodology. They combined control and noise factors in a single design matrix, which we call a combined array. The combined array approach was first proposed by Welch, Yu, Kang, and Sacks[4]. The initial motivation of the combined array is the run-size saving. Related approaches were discussed by Vining and Myers[5], Box and Jones[6], Shoemaker, Tsui and Wu[7], and Myers, Khuri and Vining[8], etc. Treatment of the mean and variance responses via a constrained optimization was discussed in Vining and Myers[5].

In many experimental situations, a number of responses are measured for a given setting of design variables. Khuri and Conlon[9] introduced a procedure for the simultaneous optimization of multiple responses using a distance function.

The combined-array approach allows one to provide separate estimates for the mean response and for the variance response. Accordingly, we can apply the primary goal of the Taguchi method which is to minimize the variance while constraining the mean. In this paper, we propose how to simultaneously optimize multiple responses for robust design when data are collected from a combined array. An example is illustrated to show the proposed method.

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2. Simultaneous Optimization of Multiresponse

2.1. Multivariate Linear Model

Suppose the response y, depends on control variables (or factors) and noise variables. Let a set of control variables be denoted by $\underline{x}=(x_1,x_2,\cdots,x_l)'$ and a set of noise variables by $\underline{z}=(z_1,z_2,\cdots,z_m)'$. Suppose that all response functions in a multiresponse system depend on the same set of $\underline{x}$ and $\underline{z}$ and that they can be represented by second order models within a certain region of interest. Let N be the number of experimental runs and r be the number of response functions. The ith second order model is

$$y_i(\underline{x},\underline{z}) = \beta_{i0} + \underline{x}'\underline{\beta}_i + \underline{x}'B_i\underline{x} + \underline{z}'R_i\underline{z} + \underline{z}'\underline{\gamma}_i + \underline{z}'D_i\underline{x} + \epsilon_i, \quad i=1,2,\cdots,r \tag{1}$$

where $\underline{\beta}_i$ is $l\times 1$, $\underline{\gamma}_i$ is $m\times 1$, $B_i'=B_i$ is $l\times l$, $R_i'=R_i$ is $m\times m$, D_i is $l\times m$, which are vectors or matrices of unknown regression parameters, and ϵ_i is a random error associated with the ith response.

Equation (1) can be expressed in matrix notation as

$$\underline{y}_i = X\underline{\theta}_i + \underline{\epsilon}_i, \qquad i=1,2,\cdots,r \tag{2}$$

in which $\underline{y}_i$ is an $N\times 1$ vector of observations on the ith response, X is an $N\times p$ full column rank matrix of known constants, $\underline{\theta}_i$ is the $p\times 1$ column vector of unknown regression parameters, and $\underline{\epsilon}_i$ is a vector of random errors associated with the ith response. We also assume that

$$E(\underline{\epsilon}_i)=\underline{0},\ Var(\underline{\epsilon}_i)=\sigma_{ii}I_N,\ Cov(\underline{\epsilon}_i,\underline{\epsilon}_j)=\sigma_{ij}I_N$$
$$i,j=1,2,\cdots,r,\quad i\neq j.$$

The $r\times r$ matrix whose (i,j)th element is σ_{ij} will be denoted by Σ. An unbiased estimator of Σ is given by

$$\hat{\Sigma} = Y'\,[I_N - X(X'X)^{-1}X']\,Y/(N-p),$$

where $Y=(\underline{y_1},\underline{y_2},\cdots,\underline{y_r})$, and I_N is an identity matrix of order $N\times N$. The r equations given in (2) may be written in a compact form

$$\underline{y} = \begin{pmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \vdots \\ \underline{y}_r \end{pmatrix} = \begin{pmatrix} X & 0 & \cdots & 0 \\ 0 & X & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X \end{pmatrix} \begin{pmatrix} \underline{\theta}_1 \\ \underline{\theta}_2 \\ \vdots \\ \underline{\theta}_r \end{pmatrix} + \begin{pmatrix} \underline{\epsilon}_1 \\ \underline{\epsilon}_2 \\ \vdots \\ \underline{\epsilon}_r \end{pmatrix} = Z\underline{\theta} + \underline{\epsilon}, \tag{3}$$

where $\underline{y}$ is $rN\times 1$, Z is $rN\times rp$, $\underline{\theta}$ is $rp\times 1$, and $\underline{\epsilon}$ is $rN\times 1$. The variance-covariance matrix of $\underline{\epsilon}$ is

$$Var(\underline{\epsilon}) = \Sigma \otimes I = \Omega,$$

where $\otimes$ is a symbol for the direct (or Kronecker) product of matrices.

The BLUE (best linear unbiased estimator) of $\underline{\theta}$ in (3) is

$$\hat{\underline{\theta}} = (Z'\Omega^{-1}Z)^{-1}(Z'\Omega^{-1}\underline{y}) = (Z'Z)^{-1}Z'\underline{y}.$$

Thus, the BLUE of $\underline{\theta}$ is $\hat{\underline{\theta}} = (\hat{\underline{\theta_1}}', \hat{\underline{\theta_2}}', \cdots, \hat{\underline{\theta_r}}')'$, where $\hat{\underline{\theta}}_i = (X'X)^{-1}X'\underline{y}_i$ is the least squares estimator of the $p\times 1$ vector of regression coefficients for the ith response model[10]. The variance-covariance of $\hat{\underline{\theta}}_i$

$$Var(\hat{\underline{\theta}}) = (Z'\Omega^{-1}Z)^{-1} = (X'X)^{-1}\Sigma.$$

The prediction equation for the ith response is given by

$$\hat{y}_i(\underline{x},\underline{z}) = \underline{g}'(\underline{x},\underline{z})\hat{\underline{\theta}}_i, \qquad i=1,2,\cdots,r \tag{4}$$

where $(\underline{x}', \underline{z}')'$ is the vector of coded input variables, $\underline{g}'(\underline{x},\underline{z})$ is a vector of the same form as a row of the matrix X evaluated at the point $(\underline{x},\underline{z})$. From (4) it follows that

$$Var[\hat{y}_i(\underline{x},\underline{z})] = \underline{g}'(\underline{x},\underline{z})(X'X)^{-1}\underline{g}(\underline{x},\underline{z})\sigma_{ii},$$
$$i=1,2,\cdots,r,$$

$$Cov[\hat{y}_i(\underline{x},\underline{z}),\hat{y}_j(\underline{x},\underline{z})] = \underline{g}'(\underline{x},\underline{z})(X'X)^{-1}\underline{g}(\underline{x},\underline{z})\sigma_{ij},$$
$$i,j=1,2,\cdots,r;\ i\neq j.$$

Hence,

$$Var[\underline{\hat{y}}(\underline{x},\underline{z})] = \underline{g}'(\underline{x},\underline{z})(X'X)^{-1}\underline{g}(\underline{x},\underline{z})\Sigma,$$
$$i = 1,2,\cdots,r$$

where $\underline{\hat{y}}(\underline{x},\underline{z}) = (\hat{y}_1(\underline{x},\underline{z}),\hat{y}_2(\underline{x},\underline{z}),\cdots,\hat{y}_r(\underline{x},\underline{z}))'$ is the vector of predicted responses at the point $(\underline{x},\underline{z})$. An unbiased estimator of $Var[\underline{\hat{y}}(\underline{x},\underline{z})]$ is given by

$$\widehat{Var}[\underline{\hat{y}}(\underline{x},\underline{z})] = \underline{g}'(\underline{x},\underline{z})(X'X)^{-1}\underline{g}(\underline{x},\underline{z})\hat{\Sigma}.$$

2.2 Estimated Mean and Variance Models in a Multiresponse

Box and Jones[6] modeled the mean and variance separately in a single response. But, we are interested in showing the estimated mean and variance response models in multiple responses.

The fitted ith second-order model in (4) can be rewritten as

$$\hat{y}_i(\underline{x},\underline{z}) = b_{i0} + \underline{x}'\underline{b}_i + \underline{x}'\hat{B}_i\underline{x} + \underline{z}'\hat{R}_i\underline{z} + \underline{z}'\underline{r}_i + \underline{z}'\hat{D}_i\underline{x},$$
$$i = 1,2,\cdots,r.$$

The noise variables $\underline{z}$ are not controllable and they are random variables. In the absence of other knowledge, $\underline{z}$ would be usually uniformly distributed over R_z.

Let $\hat{m}_i(\underline{x})$ ith estimated mean response at an $\underline{x}$ averaged over the noise variables

$$\hat{m}_i(\underline{x}) = \int_{R_z} \hat{y}_i(\underline{x},\underline{z})p(\underline{z})d\underline{z}, \quad i = 1,2,\cdots,r,$$

where $p(\underline{z})$ is a probability density function of $\underline{z}$, and $\underline{z}$ has a uniform distribution over $R_z(-1 \le z \le 1)$. Box and Jones[6] showed that the ith estimated mean becomes

$$\hat{m}_i(\underline{x}) = b_{i0} + \underline{x}'\underline{b}_i + \underline{x}'\hat{B}_i\underline{x} + \frac{1}{3}tr\hat{R}_i, \quad (5)$$
$$i = 1,2,\cdots,r,$$

where $tr\hat{R}_i$ is the trace of the matrix $\hat{R}_i$. Let us write $\hat{v}_i(\underline{x})$ for the ith mean square variation about the ith mean response

$$\hat{v}_i(\underline{x}) = \int_{R_z} (\hat{y}_i(\underline{x},\underline{z}) - \hat{m}_i(\underline{x}))^2 p(\underline{z})d\underline{z},$$
$$i = 1,2,\cdots,r.$$

Let us call this measure the ith estimated variance, which becomes

$$\hat{v}_i(\underline{x}) = \frac{1}{3}(\underline{r}_i + \hat{D}_i\underline{x})'(\underline{r}_i + \hat{D}_i\underline{x}) + \hat{A}_i, \quad (6)$$
$$i = 1,2,\cdots,r,$$

where $\hat{A}_i = [4\Sigma_{j=1}^{m}(r_{jj}^i)^2 + 5\Sigma_{j=1}^{m-1}\Sigma_{k=j+1}^{m}(r_{jk}^i)^2]/45$ and r_{jk}^i is the jth row and kth column element of the matrix $\hat{R}_i$.

From (2.5) the ith estimated mean can be rewritten as

$$\hat{m}_i(\underline{x}) = \underline{h}'(\underline{x})\underline{\hat{\theta}}_{0i} \quad i = 1,2,\cdots,r \quad (7)$$

where $\underline{h}'(\underline{x}) = (1, x_1, \cdots, x_l, x_1^2, \cdots, x_l^2, x_1x_2, \cdots, x_{l-1}x_l, 1/3, \cdots, 1/3)$ and $\underline{\hat{\theta}}_{0i} = (b_0^i, b_1^i, \cdots, b_{11}^i, \cdots, b_{ll}^i, b_{12}^i, \cdots, b_{l-1}^i, r_{11}^i, \cdots, r_{mm}^i)'$ is a part of $\underline{\hat{\theta}}_i$. From the fact that $\underline{\hat{\theta}}_{0i}$ is a part of $\underline{\hat{\theta}}_i$, the variance-covariance of $\underline{\hat{\theta}}_0$ is given by

$$Var(\underline{\hat{\theta}}_0) = (X'X)_0^{-1}\Sigma,$$

where $\underline{\hat{\theta}}_0 = (\underline{\hat{\theta}}_{01}', \underline{\hat{\theta}}_{02}', \cdots, \underline{\hat{\theta}}_{0r}')'$, $(X'X)^{-1}$ is $p \times p$, and $(X'X)_0^{-1}$ is $q \times q$, where $p = (l+m+n)(l+m+2)/2$, and $q = (l+1)(l+2)/2 + m$. Here $(X'X)_0^{-1}$ is the $q \times q$ submatrix of $(X'X)^{-1}$. From (7) and above, we then have

$$Var[\underline{\hat{m}}(\underline{x})] = \underline{h}'(\underline{x})(X'X)_0^{-1}\underline{h}(\underline{x})\Sigma,$$

where $\underline{\hat{m}}(\underline{x}) = (\hat{m}_1(\underline{x}), \hat{m}_2(\underline{x}), \cdots, \hat{m}_r(\underline{x}))'$ is the vector of estimated mean responces at the point $\underline{x}$. An unbiased estimator of $Var[\underline{\hat{m}}(\underline{x})]$ is given by

$$\widehat{Var}[\underline{\hat{m}}(\underline{x})] = \underline{h}'(\underline{x})(X'X)_0^{-1}\underline{h}(\underline{x})\hat{\Sigma}. \quad (8)$$

2.3. The Proposed D_M measure about the estimated mean responses

Let us find conditions on a set of control variables $\underline{x}$ which optimize a set of estimated mean responses $\underline{\hat{m}}(\underline{x})$ subject to maintaining estimated variance responses $\underline{\hat{v}}(\underline{x})$ within some specified upper bounds. If all the estimated mean $\underline{\widehat{m}}(\underline{x})$ attain their individual optimum values $\underline{\tau}$ at the same set $\underline{x}$ of operating conditions, then the problem of simultaneous optimization is obviously solved. This ideal optimum rarely occurs. In more general situations we might consider finding compromising conditions on the control variables that are somewhat favorable to all mean responses. Such deviation of the compromising conditions from the ideal optimum condition can be evalulated by means of a distance function which measures the distance of $\underline{\widehat{m}}(\underline{x})$, from $\underline{\tau}$.

Let $\underline{\tau}$ be the optimum (or target) value of $\widehat{m}_i(\underline{x})$ over R_x and let $\underline{\tau} = (\tau_1, \tau_2, \cdots, \tau_r)'$. We shall consider a constrained-optimization procedure for each response according to the Taguchi's three basic situations as follows.

1. "nominal-is-best characteristics": target value of $\widehat{m}_i(\underline{x}) = \tau_i$,
2. "larger-the-better characteristics": $Max_{\underline{x}\in R_x} \widehat{m}_i(\underline{x}) = \tau_i$,
3. "smaller-the-better characteristics": $Min_{\underline{x}\in R_x} \widehat{m}_i(\underline{x}) = \tau_i$.

A distance function of $\underline{\widehat{m}}(\underline{x})$ for the target value $\underline{\tau}$ may be expressed as

$$D\left[\underline{\widehat{m}}(\underline{x}), \underline{\tau}\right] = \left[(\underline{\widehat{m}}(\underline{x}) - \underline{\tau})' \left\{ Var[\underline{\widehat{m}}(\underline{x})]\right\}^{-1} (\underline{\widehat{m}}(\underline{x}) - \underline{\tau})\right]^{1/2}$$

Using the estimate given in (8) for the variance-covariance matrix of $\underline{\widehat{m}}(\underline{x})$, we get a distance function

$$\left[\frac{(\underline{\widehat{m}}(\underline{x}) - \underline{\tau})' \widehat{\Sigma}^{-1} (\underline{\widehat{m}}(\underline{x}) - \underline{\tau})}{\underline{h}'(\underline{x})(X'X)_0^{-1}\underline{h}(\underline{x})}\right]^{1/2} \tag{9}$$

If the mean response $\underline{\widehat{m}}(\underline{x})$ takes on different degrees of importance, we can imply weights $w_1, w_2, \cdots, w_r$ where $0 < w_i < 1$ for each i and $\Sigma_{i=1}^{r} w_i = 1$. Then the distance function can be written as

$$\left[\frac{\{ W(\underline{\widehat{m}}(\underline{x}) - \underline{\tau})\}' \widehat{\Sigma}^{-1} \{ W(\underline{\widehat{m}}(\underline{x}) - \underline{\tau})\}}{\underline{h}'(\underline{x})(X'X)_0^{-1}\underline{h}(\underline{x})}\right]^{1/2}$$

(2.9)

where $W = \begin{pmatrix} w_1 & 0 & \cdots & 0 \\ & w_2 & \cdots & 0 \\ & & \ddots & \vdots \\ sym & & & w_r \end{pmatrix}$. From the constrained-optimization procedure and the distance

From the distance measure of $\underline{\widehat{m}}(\underline{x})$ for the target value $\underline{\tau}$, we propose a simultaneous optimization of $\underline{\widehat{m}}(\underline{x})$ over the region of interest R_x. From (9), the proposed simultaneous-optimization measure about the estimated mean responces can be written as

$$D_M = \underset{\underline{x}\in R_x}{Min} D_M(\underline{x}) = \underset{\underline{x}\in R_x}{Min} \frac{\{ W(\underline{\widehat{m}}(\underline{x}) - \underline{\tau})\}' \widehat{\Sigma}^{-1} \{ W(\underline{\widehat{m}}(\underline{x}) - \underline{\tau})\}}{\underline{h}'(\underline{x})(X'X)_0^{-1}\underline{h}(\underline{x})} \tag{10}$$

The D_M measure can be used without a prior knowledge about the estimated mean responses. It takes into consideration the variances and correlations of the estimated mean responses.

2.4. The Proposed D_V Measure About the Estimated Variance Responses

In this section, we propose the simultaneous-optimization measure of multiple responses(quality characteristics) for robust design in a combined array.

If we have a prior knowledge about the estimated mean response $\underline{\widehat{m}}(\underline{x})$, it is possible to minimize the estimated variance response. Let

$$\hat{v}_i^*(\underline{x}) = \frac{\hat{v}_i(\underline{x}) - \underset{\underline{x}\in R_x}{Min} \hat{v}_i(\underline{x})}{\underset{\underline{x}\in R_x}{Max} \hat{v}_i(\underline{x}) - \underset{\underline{x}\in R_x}{Min} \hat{v}_i(\underline{x})}, \quad i = 1, 2, \cdots, r,$$

where $\hat{v}_i(\underline{x})$ is the ith mean square variation about the ith mean response which is in (6). Note that $\hat{v}_i^*(\underline{x})$ is a "standardized" measure of $\hat{v}_i(\underline{x})$. The proposed simultaneous optimization measure about the estimated variance responces can be written as

$$D_V = \underset{\underline{x} \in R_x}{Min} D_V(\underline{x}) = \underset{\underline{x} \in R_x}{Min} \underline{w}' \hat{\underline{v}}^*(\underline{x})$$

$$= \underset{\underline{x} \in R_x}{Min} \sum_{i=1}^{r} w_i \hat{v}_i^*(\underline{x}), \quad i = 1,2,\cdots,r, \tag{11}$$

where $\underline{w} = (w_1, w_2, \cdots, w_r)'$, $\hat{\underline{v}}^*(\underline{x}) = (\hat{v}_1^*(\underline{x}), \hat{v}_2^*(\underline{x}), \cdots, \hat{v}_r^*(\underline{x}))'$, $\Sigma_{i=1}^{r} w_i = 1$. If the variance response $\hat{\underline{v}}(\underline{x})$ takes on different degrees of importance, we can imply weights $w_1, w_2, \cdots, w_r$ where $0 < w_i < 1$ for each i and $\Sigma_{i=1}^{r} w_i = 1$.

2.5. The Proposed Simultaneous-Optimization P_m Measure

Let us find conditions on a set of control variables $\underline{x}$ which optimize simultaneously for a set of estimated mean responses and estimated variance responses. If all the estimated mean and all the estimated variance attain their individual optimum values at the same set $\underline{x}$ of operating conditions, then the problem of simultaneous optimization is obviously solved. This ideal optimum rarely occurs. In more general situations we might consider finding compromising conditions on the control variables that are somewhat favorable to all mean responses and all the estimated variance. Such deviation of the compromising conditions from the ideal optimum condition can be formulated by means of the desirability function.

We propose a simultaneous optimization for a set of estimated mean responses and estimated variance responses over the region of interest R_x using proposed $D_M(\underline{x})$ and $D_V(\underline{x})$. From Equations (10) and (11), the proposed simultaneous-optimization measure can be written as

$$P_m = \underset{\underline{x} \in R_x}{Min} P_m(\underline{x}) \tag{12}$$

$$= \underset{\underline{x} \in R_x}{Min} [\lambda D_M(\underline{x}) + (1 - \lambda) D_V(\underline{x})]$$

where R_x is the region of interest on a set of control variables $\underline{x}$ and $0 \le \lambda \le 1$. This is a criterion in which $D_M(\underline{x})$ and $D_V(\underline{x})$ take on different degrees of importance.

As a way of finding the optimal solution of the control factors according to the proposed formula, genetic algorithm was used in the MATLAB Optimization Toolbox.

3. Numerical Example

In this section we give a numerical example, consisting of a multiresponse system of two response variables, y_1 and y_2, and two control variables, x_1 and x_2, and a noise variable z. The design is somewhat similar to the standard central composite design. The cube portion of the experimental arrangement is chosen to be a 2^3 design and star points are added only for the two control variables. The following Table 1 gives the factor levels and a set of data.

Each of the two responses was fitted to a second order regression model. The estimated response models by the method of least squares are given by

$$\hat{y}_1(\underline{x}, z) = 76.00 - 12.37x_1 - 8.96x_2 - 7.22x_1^2 - 8.45x_2^2 - 8.11x_1x_2 + 5.38z^2 - 1.44z + 2.96x_1z - 1.86x_2z, \tag{13}$$

$$\hat{y}_2(\underline{x}, z) = 103.00 - 12.21x_1 + 6.68x_2 - 13.96x_1^2 - 8.50x_2^2 - 2.93x_1x_2 + 6.23z^2 - 1.38z + 1.75x_1z - 2.95x_2z. \tag{14}$$

From (13) and (14), using the mean and variance response equation (5) and (6), the estimated mean and variance response models are given by

$$\hat{m}_1(\underline{x}) = 77.79 - 12.37x_1 - 8.96x_2 - 7.22x_1^2 - 8.45x_2^2 - 8.11x_1x_2,$$

$$\hat{m}_2(\underline{x}) = 105.08 - 12.21x_1 + 6.68x_2 - 13.96x_1^2 - 8.50x_2^2 - 2.93x_1x_2,$$

$$\hat{v}_1(\underline{x}) = (-1.44 + 2.96x_1 - 1.86x_2)^2/3 + 2.57,$$

$$\hat{v}_2(\underline{x}) = (-1.38 - 1.75x_1 - 2.95x_2)^2/3 + 3.45.$$

The region of interest R_x is given by the inequality

Table 1. Experimental design and response values

Run	x_1	x_2	z	y_1	y_2
1	-1	-1	-1	80.6	81.4
2	-1	-1	1	74.9	95.9
3	-1	1	-1	83.1	105.0
4	-1	1	1	71.2	103.0
5	1	-1	-1	66.8	74.0
6	1	-1	1	74.2	76.8
7	1	1	-1	38.1	81.2
8	1	1	1	36.8	76.9
9	-1.41	0	0	80.9	100.0
10	1.41	0	0	42.4	50.5
11	0	-1.41	0	73.4	71.2
12	0	1.41	0	45.0	101.0
13	0	0	0	77.4	102.0
14	0	0	0	74.6	104.0

Table 2. Simultaneous optimization for P_m

Weight	Location of optima		Simultaneous optima			
λ	x_1	x_2	$\widehat{m}_1(\underline{x})$	$\widehat{m}_2(\underline{x})$	$\hat{v}_1(\underline{x})$	$\hat{v}_2(\underline{x})$
0.10	-0.10	0.18	77.21	107.14	4.00	4.45
0.30	-0.07	0.21	76.49	106.94	3.95	4.62
0.50	-0.05	0.27	75.46	106.88	4.03	4.90
0.70	-0.03	0.29	74.92	106.68	4.00	5.04
0.90	-0.01	0.26	75.03	106.37	3.84	4.96

$-1 \le x_1, x_2 \le 1$. The ranges for $\widehat{m}_1(\underline{x})$, $\widehat{m}_2(\underline{x})$, $\hat{v}_1(\underline{x})$ and $\hat{v}_2(\underline{x})$ over R_x are, respectively, $32.68 \le \widehat{m}_1(\underline{x}) \le 83.25$, $66.66 \le \widehat{m}_2(\underline{x}) \le 109.65$, $2.57 \le \hat{v}_1(\underline{x}) \le 15.63$ and $3.45 \le \hat{v}_2(\underline{x}) \le 15.77$.

Suppose that the quality characteristics for y_1 and y_2 are the nominal-is-best characteristics and the larger-the-better characteristics. Let us assume that the target value of $\widehat{m}_1(\underline{x})$ is taken to be 75.00 and the target value of $\widehat{m}_2(\underline{x})$ is taken to be $Max_{\underline{x} \in R_x} \widehat{m}_2(\underline{x}) = 109.65$.

From (12), we obtained the results of simultaneous optimization based on the minimization of the $P_m(\underline{x})$ measure over R_x. Table 2 indicates that the optimal setting for the λ=0.1, w_1=w_2=1.00 is $x_1 = -0.10$ and $x_2 = 0.18$, which produces a predicted value of 77.21, 107.14, 4.00, and 4.45 for $\widehat{m}_1(\underline{x})$, $\widehat{m}_2(\underline{x})$, $\hat{v}_1(\underline{x})$, and $\hat{v}_2(\underline{x})$, respectively.

If a simultaneous optimum value is much different from its corresponding individual optimum value, we may reoptimize P_m. Also we may analyze P_m sequentially as the acceptable values for λ and weights $w_1, w_2, \cdots, w_r$ are varied.

4. Conclusion

The combined-array approach allows one to provide separate estimates for the mean response and for the variance response. Accordingly, we can apply the primary goal of the Taguchi methodology which is to obtain a target condition on the mean while achieving the variance, or to minimize the variance. In this study we proposed the simultaneous-optimization measure P_m of multiple responses. The proposed concept of P_m measure is minimizing the deviation of the mean responses from the target values and also P_m measure is minimizing the deviation of the variance responses from the target values of the variance responses that is minimum values of the variance responses. The P_m measure is easy to apply, and permits the user to make subjective judgements on the importance of each response.

In this study we assume that the noise variables would be uniformly distributed over the region of interest of noise variables. It will be of interest to consider the case when the noise variables are not uniformly distributed over the region of interest of noise variables.

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