Journal of the KIMST, Vol. 20, No. 1, pp. 108-118, 2017
ISSN 1598-9127
DOI http://dx.doi.org/10.9766/KIMST.2017.20.1.108

Research Paper | 제어 · 구동 부문

슬라이딩 모드 제어와 스위칭 기법에 기반한 수상함의 경로 추종 제어기 설계

이 준 구*,1)

1) 국방과학연구소 제6기술연구본부

Path Tracking Controller Design for Surface Vessel Based on Sliding Mode Control Method with Switching Law

JunKu Lee*,1)

1) *The 6th Research and Development Institute, Agency for Defense Development, Korea*

(Received 1 June 2016 / Revised 11 October 2016 / Accepted 6 January 2017)

ABSTRACT

In this paper, the path tracking controller for a surface vessel based on the sliding mode control (SMC) with the switching law is proposed. In order to have no restriction on movement and improved tracking performance, the proposed control system is developed as follows: First, the kinematic and dynamic models in Cartesian coordinates are considered to solve the singularity problem at the origin. Second, the new multiple sliding surfaces are designed with the SMC and approach angle concept to solve the under-actuated property. Third, the switching control system is designed to improve tracking performance. To prove the stability of the proposed switching system under the arbitrary switching, the Lyapunov stability analysis method with the common Lyapunov function is used. Finally, the computer simulations are performed to demonstrate the performance, effectiveness and stability of the proposed tracking controller of a surface vessel.

Key Words : Surface Vessel(수상함), Tracking Control(추종 제어), Sliding Mode Control(슬라이딩 제어), Switching Control System(스위칭 제어 시스템), Common Lyapunov Function(공통 리아프노프 함수)

1. Introduction

There have been many researches and developments for tracking controller of a surface vessel. It is because that a surface vessel is a useful and suitable device for environmental surveying, seaside monitoring, military landings, and so on [1-5]. Despite its numerous useful applications, there are several challenging problems in the designing of tracking controller; a highly non-linear system models, stabilization, and under-actuated property, that is a system has smaller number of actuators than

* Corresponding author, E-mail: junkulee86@add.re.kr

degrees of freedom. To solve the problems, many researches have been proposed as follows.

F. A. Papoulias employed the linearized dynamics to design a tracking controller for a surface vessel [1-2], but the proposed controller has the stability problems. To solve this stability problem, J. M. Godhavn proposed the controller without the linearization of dynamics [6], but it has the errors in tracking. To make up for this fault, the improved tracking controllers were proposed in some researches [3-5], but those also have stability problem in case of a straight trajectory. And to simplify the design process, some researchers used the state transformation method [7] which removes the difficulties caused by the off-diagonal terms of the system matrix. The dynamic surface control (DSC) technique [8] was also used to solve the explosion of complexity problem caused by the repeated differentiation of virtual controllers in the back stepping design procedure. In addition to the above mentioned methods, lots of methods have been proposed for the tracking controller of a surface vessel [9-18], and H. Ashrafiuon have focused on the Sliding Mode Control (SMC) for designing the tracking controller of surface vessels [19].

In this paper, I propose a method for designing the path tracking controller of a surface vessel based on the SMC with the switching law. In order to remove a movement restriction and improve a tracking performance, the proposed controller is developed as follows: First, the models of AUV, which has similar kinematic and dynamic behavior with an under-acutaed surface vessel on a horizontal plane [21], in the Cartesian coordinates [20] are considered to remove the singularity problem at the origin. And the water flow disturbance is considered to design the practical tracking controller. Next, to solve the under-actuated property of a surface vessel, the new multiple sliding surfaces are designed with approach angle concept. And to improve the tracking performance, the switching system for tracking control is designed with switching algorithm and law [21-25]. For the stability analysis of the proposed switching control system under arbitrary switching, the Lyapunov stability analysis theory with the common Lyapunov function [26-30] is applied. Finally, the numerical simulations are carried out to demonstrate the performance and effectiveness of the proposed control system.

2. System Modeling

In this chapter, for the kinematic and dynamic models of an under-actuated surface vessel, the models of AUV in the Cartesian coordinates is considered. It is because that AUV presents similar behavior to an under-actuated surface vessel and the AUV model in the polar coordinates has the singularity problem at the origin.

2.1 Kinematic and Dynamic Models

In this paper, the movement of a surface vessel on the horizontal plan is described in the earth fixed frame and the body fixed frame as shown in Fig. 1. To depict the posture of a surface vessel, it is defined that the position (x_c, y_c) as the middle point, the heading angle θ_c as the angle between the X-axis and the heading direction, the current posture as $q_c = [x_c\ y_c\ \theta_c]^T$ and the reference postures as $q_r = [x_r\ y_r\ \theta_r]^T$, respectively. Additionally, the surge velocity, the sway velocity, and the yaw velocity of a surface vessel are defined as *u*, *v*, and *r*, respectively. With the defined terms, the kinematic model of a surface vessel is described as follows:

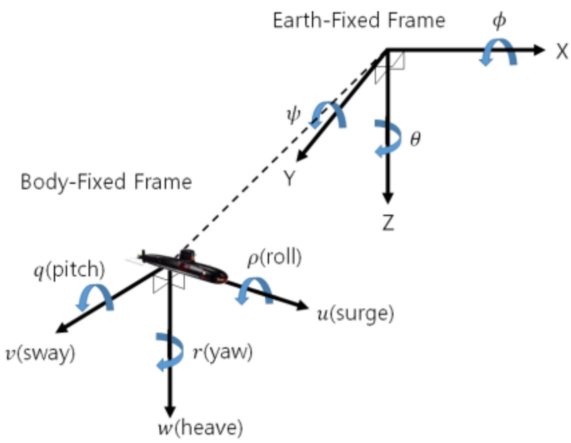

Fig. 1. Earth-fixed and body-fixed frames of system

$$\dot{q_c} = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \end{bmatrix} = \begin{bmatrix} \cos\theta_c & -\sin\theta_c & 0 \\ \sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_c \\ v_c \\ r_c \end{bmatrix}. \quad (1)$$

Then the effect of water flow disturbance is considered with the following assumption.

Assumption 1: The water flow disturbance is described two-dimensionally in the earth-fixed frame with a velocity V_d and a direction θ_d as follows:

$$D_x = V_d \cos\theta_d , \quad D_y = V_d \sin\theta_d . \quad (2)$$

where D_x is the water flow velocity in the X-axis direction, and D_y is the water flow velocity in the Y-axis direction. Since the water flow velocity and direction are not constant, there exist the differences between the disturbance of current and reference position. To apply these disturbance difference to the system model, the following additional terms are defined as shown in Fig. 2.; V_{dc} and θ_{dc} are the water flow velocity and direction of current position, V_{dr} and θ_{dr} are the water flow velocity and direction of reference position. With the water flow disturbance, the kinematics of a surface vessel is transformed as follows:

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \end{bmatrix} = \begin{bmatrix} \cos\theta_c & -\sin\theta_c & 0 \\ \sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_c \\ v_c \\ r_c \end{bmatrix} + \begin{bmatrix} V_{dc}\cos\theta_{dc} \\ V_{dc}\sin\theta_{dc} \\ 0 \end{bmatrix}. \quad (3)$$

Based on the posture error assignment in Fig. 2, the posture error $[x_e \ y_e \ \theta_e]^T$ of a surface vessel is derived as follows:

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_c \\ y_r - y_c \\ \theta_r - \theta_c \end{bmatrix}. \quad (4)$$

By differentiating and transforming (4) with the water disturbance equation and defined terms, the equation for the surge velocity *u*, the sway velocity *v*, and the yaw velocity *r* of a surface vessel is then derived as follows:

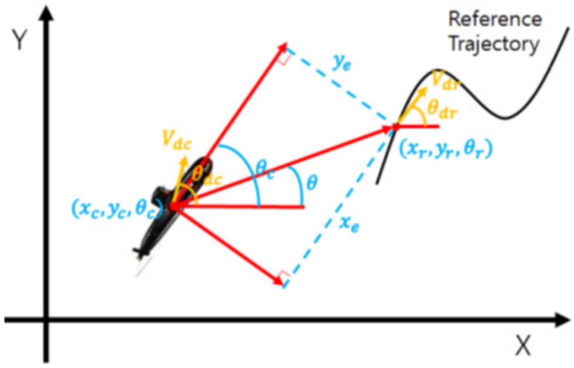

Fig. 2. Coordinate assignments for system modeling

$$\begin{bmatrix} u_c \\ v_c \\ r_c \end{bmatrix} = -\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} + \begin{bmatrix} y_e r_c \\ -x_e r_c \\ 0 \end{bmatrix}$$
$$+ \begin{bmatrix} \cos(\theta_r - \theta_c) & -\sin(\theta_r - \theta_c) & 0 \\ \sin(\theta_r - \theta_c) & \cos(\theta_r - \theta_c) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ r_r \end{bmatrix}$$
$$+ \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dr}\sin\theta_{dr} - V_{dc}\sin\theta_{dc} \\ V_{dr}\cos\theta_{dr} - V_{dc}\cos\theta_{dc} \\ 0 \end{bmatrix}. \quad (5)$$

And the dynamics for a surface vessel is expressed as following differential equations [20] with assumption 2.

Assumption 2: The center of buoyancy coincides with the center of mass, and the mass is distributed homogeneously. The hydrodynamic drag terms, which have higher order than two, heave, pitch, and roll motions can be neglected.

$$\begin{aligned} \dot{u}_c &= \frac{m_{22}}{m_{11}} v_c r_c - \frac{X_u}{m_{11}} u_c - \frac{X_{u|u|}}{m_{11}} u_c |u_c| + \frac{1}{m_{11}} F_u \\ \dot{v}_c &= -\frac{m_{11}}{m_{22}} u_c r_c - \frac{Y_v}{m_{22}} v_c - \frac{Y_{v|v|}}{m_{22}} v_c |v_c| \\ \dot{r}_c &= \frac{m_{11} - m_{22}}{m_{33}} u_c v_c - \frac{N_r}{m_{33}} r_c - \frac{N_{r|r|}}{m_{33}} r_c |r_c| + \frac{1}{m_{33}} F_r \end{aligned} \quad (6)$$

where $m_{11} = m - X_{\dot{u}}$, $m_{22} = m - Y_{\dot{v}}$ and $m_{33} = I_z - N_{\dot{r}}$ Here, m_{11} and m_{22} are the combined rigid body and added mass terms $X_{\dot{u}}$, $Y_{\dot{v}}$. m_{33} is the combined rigid body and added mass moment of inertia I_z about the z-axis with the $N_{\dot{r}}$. And X_u, $X_{u|u|}$, Y_v, $Y_{v|v|}$, N_r,

$N_{r|r|}$ are the linear and quadratic drag coefficients.

In the dynamics of a surface vessel (6), the term F_u denotes the control force along the surge *u* motion of the body-fixed frame, and F_r denotes the control moment for producing the angular motion around the z-axis of the body-fixed frame. In a surface vessel, there exists no side thruster to control the sway motion *v*, thus the sway motion is uncontrollable and a surface vessel is an under-actuated dynamic system.

3. Design of Tracking Controller with Switching

In this chapter, the tracking controller for a surface vessel is designed under the consideration of the under-actuated property and stability problems. For the under-actuated property of a surface vessel, the new multiple sliding surfaces are designed with an approach angle concept. Then, based on the proposed reaching law and sliding surfaces, the controllers are designed. Next, the switching control system is developed with the designed controllers and switching law to improve the tracking performance. Finally, the stability of the proposed control system is proved by using the Lyapunov stability theory with the common Lyapunov function.

3.1 Advanced Reaching Law

Before designing the appropriate sliding surfaces, the reaching law is designed to makes the states converge to the proposed sliding surfaces with the condition as follows:

$$S \cdot \dot{S} < 0, \text{ for any } t \tag{7}$$

Here, to satisfy the above condition, the reaching law is typically chosen as follows:

$$\dot{S} = -k \cdot sgn(S),\ sgn(S) = \begin{cases} 1 & where\ S > 0 \\ 0 & where\ S = 0 \\ -1 & where\ S < 0 \end{cases}. \tag{8}$$

where K is the positive constant. From the above reaching law, the reaching time t_r, the necessary time for each state to converge to the sliding surface, is derived as follows:

$$t_r = \frac{|S(0)|}{K}. \tag{9}$$

According to (9), as K values increases, the reaching time decreases. However, large K value increases the chattering phenomenon and tracking errors, and those worse the tracking performance. In order to improve the tracking performance, this paper considered the following reaching law;

$$\begin{aligned} \dot{S} &= -K_r R(S) sgn(S), \\ R(S) &= \frac{|S|^{\alpha}}{K_p + (1-K_p)e^{-\beta|S|^{\gamma}}}, \end{aligned} \tag{10}$$

where $K_r = [k_{1r}\ k_{2r}]^T$, k_{1r} and k_{2r} are the positive constant, K_p is the strictly positive offset which is less than 1. α, β are γ strictly positive integers. With the above conditions, *R(S)* is always strictly positive. Therefore, *R(S)* does not affect the stability of controller. In the reaching law (10), as $|S|$ increases, *R(S)* converges to $|S|^{\alpha}/K_p$, and consequently t_r converges to $K_p|S(0)|/K_r|S|^{\alpha}$, which is smaller than $|S(0)|/K$ with large $|S|$ value. It means that each state converge to the proposed sliding surface faster. On the other hand, as $|S|$ decreases, *R(S)* converges to $|S|^{\alpha}$, and consequently the reaching law $\dot{S}$ gradually decreases, which decreases the chattering phenomenon and the tracking errors. Therefore, the reaching law (10) is suitable to improve tracking performance of controller.

3.2 Sliding Surface with Approach Angle

In order to make a surface vessel follow the trajectory, the controller needs to make the posture error (x_e, y_e, θ_e) converge to zero as time elapses. To do this, the approach angle concept, respect to the defined coordinate as shown in Fig. 2, is applied. From Fig. 2, it is verified that y_e converges to zero as the heading angle θ_c converges to an approach angle θ. Since the heading angle is tangential to the reference trajectory, the heading angle error θ_e converges to zero as a surface vessel follows the reference trajectory exactly. Therefore, the posture error (x_e, y_e, θ_e) converges to zero as x_e and $\theta-\theta_c$ converge to zero. Then, the sliding surface with the first order filters is defined as follows:

$$S=\begin{bmatrix} s_x \\ s_\theta \end{bmatrix}=\begin{bmatrix} \dot{x}_e+k_1 x_e \\ \dot{\theta}_a+k_2\theta_a \end{bmatrix},$$
$$\theta_a=\theta-\theta_c,\quad \theta=\tan^{-1}\left(\frac{y_r-y_c}{x_r-x_c}\right). \tag{11}$$

where $k_1, k_2>0$. Based on the first order filter property, as s_x and s_θ converge to zero, x_e and θ_a converge to zero. In addition, the sliding surfaces (11) have different properties according to the values of k_1 and k_2. With the large k_1 and k_2 values, each state converge to the sliding surface rapidly. And the small k_1 and k_2 values allows each state converge to the sliding surface with small chattering phenomenon. Based on this property, the multiple sliding surfaces S_1, S_2, $\cdots$, S_n which have different values of k_1 and k_2 are designed as follows:

$$S_n=\begin{bmatrix} s_{xn} \\ s_{\theta n} \end{bmatrix}=\begin{bmatrix} \dot{x}_e+k_{1n}x_e \\ \dot{\theta}_a+k_{2n}\theta_a \end{bmatrix}. \tag{12}$$

where $k_{11}\neq k_{12}\neq\cdots\neq k_{1n}>0$ and $k_{21}\neq k_{22}\neq\cdots\neq k_{2n}>0$. Then, by using the multiple sliding surfaces which have the different properties, the advanced tracking controller can be designed.

3.3 Control Law Design for Surface Vessel

The sliding surfaces S_1, S_2, $\cdots$, S_n have a similar form, only have different values of k_1 and k_2, therefore the each controller for the proposed multiple sliding surfaces can be designed in the same method. Then, to design the appropriate control law for each sliding surfaces, I substitute (1) and (5) into each differentiated sliding surface as follows:

$$\begin{aligned}\dot{S}_n &= -K_r R(S_n)sgn(S_n)=\begin{bmatrix} \ddot{x}_e+k_{1n}\dot{x}_e \\ \ddot{\theta}_a+k_{2n}\dot{\theta}_a \end{bmatrix} \\ &=\begin{bmatrix} \ddot{x}_e+k_{1n}y_e r_c \\ \ddot{\theta}_a+k_{2n}\dot{\theta} \end{bmatrix}-\begin{bmatrix} k_{1n}u_{cn} \\ k_{2n}r_{cn} \end{bmatrix} \\ &\quad+\begin{bmatrix} k_{1n}(u_r\cos(\theta_r-\theta_c)-v_r\sin(\theta_r-\theta_c)) \\ 0 \end{bmatrix} \\ &\quad+\begin{bmatrix} k_{1n}\sin\theta_c(V_{dr}\cos\theta_{dr}-V_{dc}\cos\theta_{dc}) \\ 0 \end{bmatrix} \\ &\quad+\begin{bmatrix} k_{1n}\cos\theta_c(V_{dr}\sin\theta_{dr}-V_{dc}\sin\theta_{dc}) \\ 0 \end{bmatrix}. \end{aligned} \tag{13}$$

Then, by substituting (11) and (13) into (7), the control input $U_{an}(u_{un}, r_{rn})$ for sliding surface S_n is obtained as follows:

$$\begin{aligned} U_{an}&=\begin{bmatrix} u_{cn} \\ r_{cn} \end{bmatrix}=\begin{bmatrix} 1/k_{1n} \\ 1/k_{2n} \end{bmatrix}K_r R(S_n)sgn(S_n) \\ &\quad+\begin{bmatrix} \ddot{x}_e/k_{1n}+(2V_{dr}+2V_{dc}+|u_r|+|v_r|)sgn(s_{xn}) \\ \ddot{\theta}_a/k_{2n}+\dot{\theta} \end{bmatrix}. \end{aligned} \tag{14}$$

In a surface vessel, only F_u and F_r are the controllable inputs, thus the control input $U_{sn}(F_{un}, F_{rn})$ for sliding surface S_n is derived from (6) and (14) as follows:

$$\begin{aligned} U_{sn}&=\begin{bmatrix} F_{un} \\ F_{rn} \end{bmatrix} \\ &=\begin{bmatrix} m_{11}\dot{u}_{cn}-m_{22}v_c r_{cn}+X_u u_{cn}+X_{u|u|}u_{cn}|u_{cn}| \\ m_{33}\dot{r}_{cn}-(m_{11}-m_{22})u_{cn}v_c+N_r r_c+N_{r|r|}r_{cn}|r_{cn}| \end{bmatrix}. \end{aligned} \tag{15}$$

where u_{cn} and r_{cn} are the control input values of U_{an}. Here each controllers have the different control input U_{sn} based on the sliding surface S_n. To operate those multiple controllers as the one unit controller, the switching algorithm for the tracking control of a surface vessel is proposed as shown in Fig. 3.

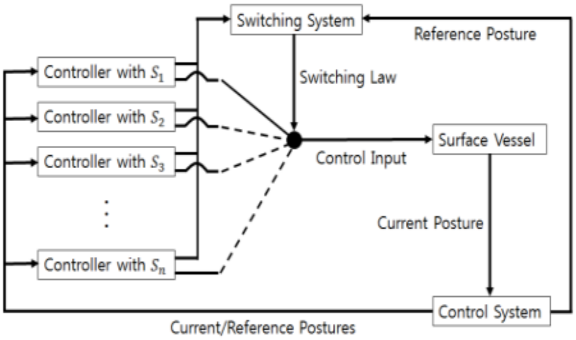

Fig. 3. Switching algorithm for control system

Based on the current posture q_c, the reference posture q_r, and the sliding surface S, each controllers generates the appropriate control inputs which have different control properties. To use those multiple controllers efficiently, the proposed switching system estimates the posture errors with each control input as follows:

$$E.U_{sn} = (k_{e1}(x_r - x_{cn})^2 + k_{e2}(y_r - y_{cn})^2 + k_{e3}(\theta_r - \theta_{cn})^2)^{0.5}, \tag{16}$$

where $x_{cn}, y_{cn}, \theta_{cn}$ are the estimated current posture with the control input U_{sn}, and k_{e1}, k_{e2}, k_{e3} are the positive constant to weight importances for each error. With the defined switching law and control inputs, the control input U_s is designed as follows:

$$U_s = U_{s1} \bullet sw_1 + U_{s2} \bullet sw_2 + \cdots + U_{sn} \bullet sw_n, \tag{17}$$

where $sw_i = 1$ when $E.U_{si} =$ smallest, else $sw_i = 0$ for $i \in 1, 2, \cdots, n$.

3.4 Stability Analysis with Common Lyapunov Function

In the switching system, the stability analysis under the arbitrary switching is one of the most difficult problems because even when all the controllers of switching system are exponentially stable, it is still possible that the overall system can be unstable for a certain switching signals. Thus, it is necessary to show the stability of switching system under the arbitrary switching, and to do so, the common Lyapunov function (CLF) method is used in this paper. It is because if there exists CLF for all its controllers and systems, then the stability of switching system is guaranteed under arbitrary switching.

Theorem: The control input (17) stabilizes the overall tracking control system under Assumption 1 and 2. Thus, the posture errors converge to zero.

Proof: In order to show the existence of CLF for the proposed switching system, the Lyapunov function selected as follows:

$$V = \frac{1}{2} S_i^T S_i > 0 \ (i \in 1, 2, \cdots, n) \text{ for all non-zero matrix} \tag{18}$$

where V is positive definite. To prove the stability of the controllers with the Lyapunov function V, (13) and (15) are substituted as follows:

$$\begin{aligned}
\dot{V} &= S_i^T \dot{S}_i = s_{xi}\dot{s}_{xi} + s_{\theta i}\dot{s}_{\theta i} \\
&= s_{xi}(k_{1i}(u_r \cos(\theta_r - \theta_c) - v_r \sin(\theta_r - \theta_c)) \\
&\quad + k_{1i}\sin\theta_c(V_{dr}\cos\theta_{dr} - V_{dc}\cos\theta_{dc}) \\
&\quad + k_{1i}\cos\theta_c(V_{dr}\sin\theta_{dr} - V_{dc}\sin\theta_{dc}) \\
&\quad + \ddot{x}_e + k_{1i}(y_e r_c - u_{ci})) + s_{\theta i}(\ddot{\theta}_a + k_{2i}\dot{\theta} - k_{2i} r_{ci}) \\
&= k_{1i}(s_{xi}u_r \cos(\theta_r - \theta_c) - |s_{xi}||u_r| \\
&\quad - s_{xi}v_r \sin(\theta_r - \theta_c)) - |s_{xi}||v_r| \\
&\quad + s_{xi}V_{dr}(\sin\theta_c\cos\theta_{dr} + \cos\theta_c\sin\theta_{dr}) - |s_{xi}|2V_{dr} \\
&\quad - s_{xi}V_{dc}(\sin\theta_c\cos\theta_{dc} + \cos\theta_c\sin\theta_{dc}) - |s_{xi}|2V_{dc}) \\
&\quad - k_{1r}R(s_{xi})|s_{xi}| - k_{2r}R(s_{\theta i})|s_{\theta i}| \\
&< k_{1i}(s_{xi}u_r - |s_{xi}||u_r| + s_{xi}v_r - |s_{xi}||v_r| \\
&\quad + s_{xi}2V_{dr} - |s_{xi}|2V_{dr} + s_{xi}2V_{dc} - |s_{xi}|2V_{dc}) \\
&\quad - k_{1r}R(s_{xi})|s_{xi}| - k_{2r}R(s_{\theta i})|s_{\theta i}| \\
&< -k_{1r}R(s_{xi})|s_{xi}| - k_{2r}R(s_{\theta i})|s_{\theta i}| < 0.
\end{aligned} \tag{19}$$

where $i \in 1, 2, \cdots, n$. Since the Lyapunov function V is positive definite and $\dot{V}$ is negative definite regardless, it is proven that all controllers are stable by the Lyapunov function stability theory with CLF, and the proposed switching system is stable under arbitrary switching. Based on the above results, the proposed switching system for a tracking control of a surface vessel makes the posture error (x_e, y_e, θ_e) converge to zero stably with the control input.

4. Simulation Results

In this chapter, the simulation results for the given reference trajectories are presented to demonstrate the performance of the proposed controller for a surface vessel. For the simulations, the rigid body and hydrodynamic parameters [20] are selected as Table 1.

Table 1. Hydrodynamic parameters

Symbol	Parameters	Value	Unit
m	Mass	185	kg
I_z	Rotation inertia	50	kgm^2
$X_{\dot{u}}$	Added mass	-30	kg
$Y_{\dot{v}}$	Added mass	-80	kg
$Y_{\dot{r}}$	Added mass	-1	kg
$N_{\dot{r}}$	Added mass	-30	kgm^2
X_u	surge linear drag	70	kg/s
$X_{u\vert u\vert}$	surge quadratic drag	100	kg/m
Y_v	sway linear drag	100	kg/s
$Y_{v\vert v\vert}$	sway quadratic drag	200	kg/m
N_r	Yaw linear drag	50	kgm^2/s
$N_{r\vert r\vert}$	Quadratic yaw drag	100	kgm^2
x_g	Position of COG	0	m

The initial conditions are selected as $[x_c, y_c, \theta_c]$ = $[0,0,0]$ and $[x_r, y_r, \theta_r] = [5,5,\pi/4]$. Then, the control parameter values are chosen as follows:

$$k_{e1} = 10,\ k_{e2} = 10,\ k_{e3} = 1,\ k_{r1} = 0.4,\ k_{r2} = 0.9,$$
$$K_p = 0.8,\ \alpha = 0.2,\ \beta = 0.3,\ \gamma = 0.2$$

For the water flow disturbance of a surface vessel, the current direction and velocity is designed as follows:

$$V_{dc} = V_c\left[\left|\sin\frac{x_c}{10}\right| + \left|\sin\frac{y_c}{10}\right|\right]/2,$$
$$V_{dr} = V_r\left[\left|\sin\frac{x_r}{10}\right| + \left|\sin\frac{y_r}{10}\right|\right]/2,$$

$$\theta_{dc} = \frac{\pi}{3} + \frac{\pi}{36}\left(\sin\frac{x_c}{10} + \cos\frac{y_c}{10}\right),$$
$$\theta_{dr} = \frac{\pi}{3} + \frac{\pi}{36}\left(\sin\frac{x_r}{10} + \cos\frac{y_r}{10}\right). \quad (20)$$

where $V_c = V_r = 1m/s$, θ_{dr}, θ_{dc} have minimum 50° and maximum 70° values. With the water flow equation and parameters, the disturbances are generated as shown in Figs. 4 and 5.

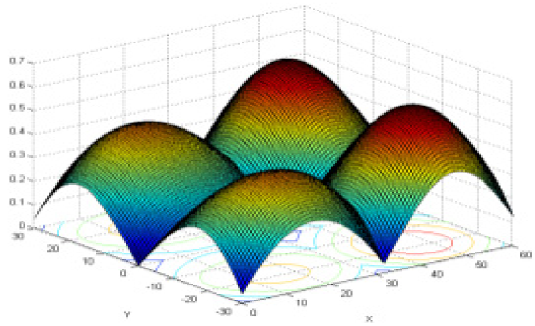

Fig. 4. Water flow velocity in X-axis direction

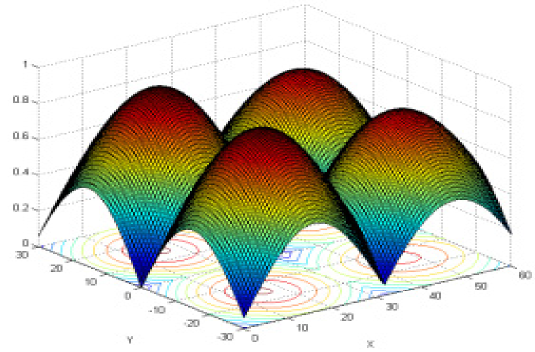

Fig. 5. Water flow velocity in Y-axis direction

Then, to investigate the stability and performance of the proposed controller for a surface vessel, the values for the multiple sliding surfaces and reference condition are selected as follows:

$$S_1 = \begin{bmatrix} \dot{x}_e + x_e \\ \dot{\theta}_a + \theta_a \end{bmatrix},\quad S_2 = \begin{bmatrix} \dot{x}_e + 0.1x_e \\ \dot{\theta}_a + 0.1\theta_a \end{bmatrix},$$
$$S_3 = \begin{bmatrix} \dot{x}_e + 0.1x_e \\ \dot{\theta}_a + 0.1\theta_a \end{bmatrix},\quad S_4 = \begin{bmatrix} \dot{x}_e + 100x_e \\ \dot{\theta}_a + 100\theta_a \end{bmatrix}.$$

Condition : For a complicated line, the linear, angular, surge, and yaw velocities of reference trajectory are selected as follows:

$$
\begin{array}{l}
0s \le t \le 3s : v_r = 4m/s,\ w_r = 0rad/s, \\
3s \le t \le 11s : v_r = 4m/s,\ w_r = -0.4rad/s, \\
11s \le t \le 14s : v_r = 4m/s,\ w_r = 0rad/s, \\
14s \le t \le 22s : v_r = 4m/s,\ w_r = 0.4rad/s, \\
22s \le t \le 30s : v_r = 4m/s,\ w_r = 0rad/s.
\end{array}
$$

Then, to confirm the performance and stability of the proposed switching control system, the simulations are performed with different number of sliding surfaces as follows.

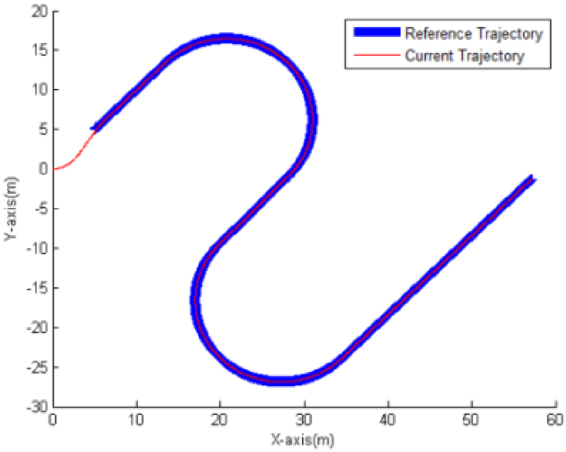

Fig. 6. Tracking control result of the proposed control system with sliding surfaces $S_1 \sim S_4$

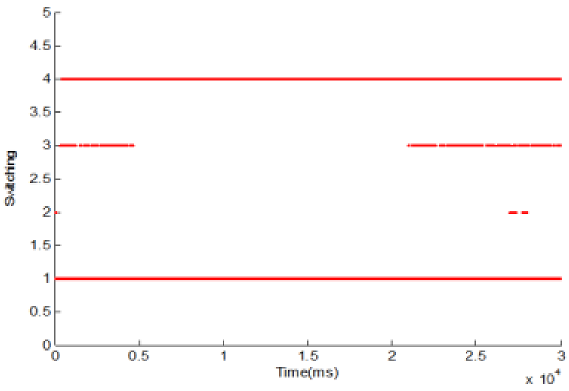

Fig. 7. Switching law result with $S_1 \sim S_4$

Fig. 6 depicts the tracking control result of proposed switching control system with sliding surfaces $S_1 \sim S_4$. As shown in Fig. 6, a surface vessel stably and exactly follows the given reference trajectory from the origin

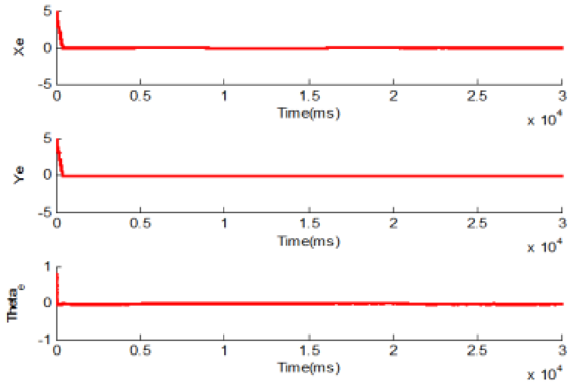

Fig. 8. Tracking errors with sliding surfaces $S_1 \sim S_4$

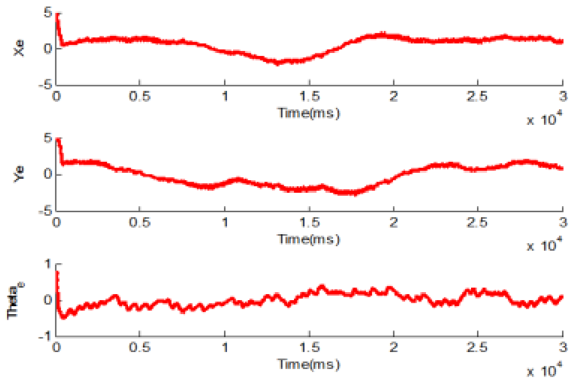

Fig. 9. Tracking errors with sliding surface S_1

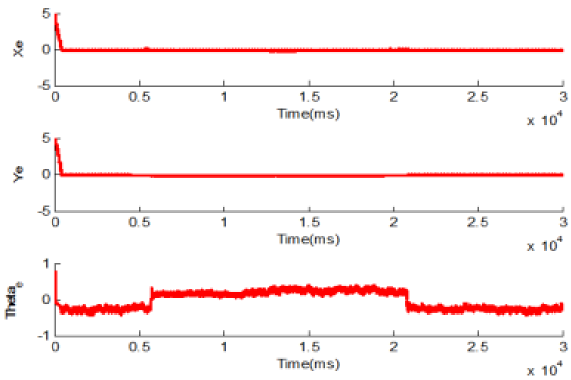

Fig. 10. Tracking errors with sliding surfaces $S_1 \sim S_2$

point, without a singularity problem. Figs. 7 and 8 show switching law result and the tracking errors while a surface vessel follows the given reference trajectory. Fig. 7 shows that the proposed system selects an appropriate controller from the multiple controllers based on the switching algorithm and law. And Fig. 8 shows that the posture errors converge to zero in short time with very small tracking errors and chattering phenomenon. Based on the Figs. 6-8, it is verified that the proposed control system is stable under arbitrary switching with remarkable tracking performance. And Fig. 9 depicts the tracking control result with only one controller (based on S_1), and it shows a large tracking errors. Fig. 10 is with two controllers (based on S_1 and S_2) and shows that the additional controller decreases the tracking errors. And by comparing Figs. 8-10, it is proved that the switching system with multiple controllers improves the tracking performance of proposed control system.

From the simulation results, I verify that the proposed switching control system for surface vessel tracking is stable under arbitrary switching, and make a surface vessel exactly follow the given reference trajectory with the decreased reaching time, tracking errors, and chattering phenomenon.

5. Conclusion

In this paper, a method for designing the path tracking controller for a surface vessel based on the sliding mode control with the switching law is proposed. In order to have no restriction on movement and improved tracking performance, the proposed controller is developed as follows:

First, the kinematic and dynamic model equations in the Cartesian coordinates are considered because the kinematic model in the polar coordinates has the singularity problem at the origin. In addition, the water disturbance is considered to design the practical tracking controller of a surface vessel. Second, in order to solve the under-actuated property, the new multiple sliding surfaces are design by using the SMC with an approach angle. Third, the switching system for tracking control is designed by connecting the multiple controllers with the switching algorithm and law to improve the tracking performance. To prove the stability of the proposed switching control system under the arbitrary switching, the Lyapunov stability analysis method with the common Lyapunov function is used. Finally, the numerical computer simulations are performed to demonstrate the performance and effectiveness of the proposed tracking control system of a surface vessel. From the simulation results, it is verified that the proposed tracking control system is stable under the arbitrary switching, and has the improved tracking performance.

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