A Structural Engineer's Approach to Differential Vertical Shortening in Tall Buildings

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Abstract

Vertical shortening in tall buildings would be of little concern if all vertical elements shortened evenly. However, vertical elements such as walls and columns may shorten different amounts due to different service axial stress levels. With height, the differential shortening may become significant and impact the strength design and serviceability of the building. Sometimes column transfers or other vertical structural irregularities may cause differential shortening. If differential shortening is not addressed properly, it can impact the serviceability of the building. This paper takes the perspective of a structural engineer in planning the design, predicting the shortening and its effects, and communicating the information to the contractor.

Keywords: Differential shortening, Analysis, Specification, Tolerances, Elevators

1. Introduction

Many years ago, LERA was called upon to review the design of a sixty story building that was already built. The as-built elevation of the perimeter of the upper floors of the building varied by as much as 100mm between individual columns due to differential shortening. The owner wanted to know how that happened and what could be done about it. Through our review and investigation, we found that the engineer of record and contractor had recognized vertical shortening as an issue in the design and construction of tall buildings, but had failed to address it adequately.

Vertical shortening occurs in all buildings but the effects of shortening are usually only considered in the design and construction of tall buildings where the effects become more measurable. Vertical shortening in itself would be of little concern if all vertical elements shortened evenly. However, vertical elements such as walls and columns will typically shorten different amounts because they have different axial stress levels. With height, this differential shortening may become significant and impact the serviceability of the building. The most obvious and direct consequence of differential shortening is uneven floors as was the case in the sixty story building in the opening paragraph. In certain cases, where the geometry or the structure of the building is asymmetrical, differential shortening may also lead to the building leaning under its self-weight. In buildings designed with an intentional lean, or even a sloped side, there will be additional unintentional lean which could be significant enough to require mitigation in buildings of as little as 20 floors.

This paper relates LERA's experience over the years with regard to shortening and differential shortening in tall buildings. Through our involvement in the design and peer review of tall buildings, we have encountered a broad range of challenges and solutions. We will start with a brief description of vertical shortening, followed by design considerations, methods of predicting and compensating for the shortening, and then describe options for documenting and communicating information to contractor.

2. Vertical Shortening

The main cause of vertical shortening in tall buildings is the compression of vertical elements under the load they carry. This is known as elastic shortening. In reinforced concrete buildings, creep and shrinkage of the concrete cause additional shortening of vertical elements beyond the elastic shortening. Foundation settlements and deflections of structural transfers technically are not shortening, but they may contribute to the perceived shortening of vertical elements. Their effects should be considered where appropriate.

2.1. Elastic Shortening

Elastic shortening is explained by Hooke's Law which states that the force F needed to extend or compress a spring by some distance X is proportional to that distance. That is $F = kX$, where k is the spring stiffness. If we substitute change in length L of a vertical element for X, and the axial stiffness of the vertical element AE/L for k,

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we obtain the following useful equation ΔL=FL/AE which we can apply to structural elements in the low deformation elastic range.

Where vertical elements in a building are made of the same material (i.e., same young's modulus E) and same length L, ΔL becomes proportional to the axial stress level F/A. Therefore, vertical elements of different sizes having the same axial stress level will shorten similarly, whereas vertical elements of the same size but different axial levels will shorten differently. We can also derive from the equation above that ΔL increases linearly with length L even if the axial stress level F/A is constant. Therefore the taller the building, the greater the shortening amount which is why shortening considerations become more critical in tall buildings.

2.2. Creep and Shrinkage

In concrete structures, creep and shrinkage of the concrete increase the amount of vertical shortening, often more than doubling the amount of the elastic shortening. In simple terms, creep represents the tendency of concrete to continue to compress over time under sustained loads, while shrinkage represents the decrease in the volume of concrete during hardening and drying. The actual behavior is quite complex and beyond the scope of this paper. ACI 209.2R-08[1] provides a good discussion on the topic and documents various mathematical models for predicting deformations due to creep and shrinkage.

2.3. Foundation settlements

Foundation settlements contribute to the perceived shortening of elements. In high rise buildings, usually the core walls shorten less than the perimeter columns. Conversely, the foundation under the core of the building will usually settle more than the foundation under the perimeter columns. This may offset some of the differential shortening caused by the columns shortening more than the core, and should be considered when estimating corrections for vertical shortening.

2.4. Structural Transfers of Vertical Elements

Sometimes, transfers of vertical elements are required for architectural reasons or to clear existing below grade infrastructure. Depending on the type of transfer, the transfer system may deflect and cause the supported vertical element to "shorten". This effect on the overall shortening of the supported column or wall can be compensated by cambering the transfer element for the amount of predicted deflection; however, one needs also consider the potential local effects of such deflections of transfers on the adjacent finish construction.

3. Design Planning

Put into practice, the above implies that to limit differential shortening, a structural engineer should strive to make the service level axial stresses in the vertical elements uniform at each floor to ensure a uniform shortening of the structure. However, there are often structural and non-structural considerations that may make this proposition challenging. It is therefore good to be aware of how design decisions may impact differential shortening early on in design and take appropriate measures to mitigate the differential shortening through design or construction.

Following are some examples of design challenges related to differential shortening and solutions that we have used at LERA. Other situations or solutions may be possible, this is not intended to be an exhaustive list:

3.1. Equalizing Stresses between the Core and Perimeter Columns

This is one of the more typical problems one encounters in tall buildings. Where concrete shear walls in the core are the main lateral load resisting system in a high rise building, the core wall thickness will often be driven by the building lateral stiffness requirements and the walls may be at low service axial stress levels compared to the perimeter columns. Additionally, architects and owners usually prefer smaller columns for planning purposes and to increase saleable or rentable area, which often results in smaller columns at a higher service level axial stresses than the core. This will cause the perimeter columns to shorten more than the core. The typical solution for this problem is to super-elevate the perimeter columns as compared to the core during construction such that when the columns shorten more than the core, the resulting floor is level.

3.2. Asymmetrical Structural Plan Layout

In narrow floor plate buildings, it is often more efficient architecturally to locate the core at one side of the floor plate. Where this is the case, the core wall on the exterior face receives little gravity loads directly, whereas the core wall near the center of the floor plate receive a large portion of the gravity load. As a result, the central wall will shorten more than the exterior wall and will result in the building leaning in the direction of the center wall, i.e. away from the core towards the column supported side of the floor plate. Some of the lean may be mitigated by tuning the structure to account for the asymmetry, such as making the central wall thicker than the exterior wall. Where there remains a significant lean under self-weight, the solution is to camber the building during construction such that when the building deflects laterally it deflects into a plumb position.

3.3. Buildings with Asymmetrical Massing

When the massing of the building is asymmetrical, with taller and shorter portions, the vertical elements supporting the taller portions support more load and may compress more than the elements supporting the shorter por-

tions of the building. This may result in the building leaning under its self-weight toward the taller portion of the building. As for the asymmetrical floor plan condition, some of the lean may be mitigated by tuning the structure to account for the asymmetry in the massing, such as making the elements supporting the high rise components larger than the elements supporting the low rise components. Where there remains a significant lean under self-weight, the solution is to camber the building during construction.

3.4. Buildings with Stepped Setbacks

Tall buildings often incorporate setbacks. Where the setback is stepped as opposed to tapered, it tends to pose structural challenges at the setback location. For example, in a two tiered structure, with a low rise section and a high rise section, the vertical elements at the top of the low rise portion support little in terms of gravity loads, whereas the adjacent vertical elements supporting the high rise portion may have significant loads. In this case, the building structure will try to equalize stresses between the high rise and low rise vertical elements at the setback level, introducing large forces in members that interconnect the two, and often causing overstressing of those members. Stepped setbacks between the first quarter and middle of the height of the building present the greatest challenges as the combination of load differential and shortening differential is usually largest in this range. Where stepped setbacks are desired, we recommend that they be located either high in the tower where the load differential is least or very low in the tower where the differential shortening potential is least. Where possible, it is preferable to taper the structural transition within the step. Where a step remains, the structural elements above and below the step should be examined carefully during the early stages of design as to size them adequately. This can mean either strengthening the structure which interconnects the adjacent columns or walls, conversely, designing this structure to be flexible, or using other techniques. Staged vertical post-tensioning of the shorter columns is an applicable technique.

3.5. Buildings with Outriggers

When outriggers are used to stiffen a high rise building by engaging perimeter columns in resisting lateral loads, the columns connected to the outrigger may attract significant transient axial loads due to wind, but may otherwise have similar loading as other columns due to gravity loads. The wind loads may require the columns connected to the outriggers to be larger than the gravity only columns. Additionally, increasing the size of the columns connected to the outriggers is often an effective way of improving the building lateral stiffness. The result is that the columns connected to the outriggers may be significantly larger and at a lower service axial stress level than the gravity only columns, therefore the columns connected

to the outrigger will shorten less than other columns. Going back to the 60 story building in the introductory paragraph, this was one of the main reasons columns shortened differently along the perimeter. The columns connected to the outriggers were significantly larger than the gravity columns and shortened less under the self-weight of the structure, however, during construction all columns were super elevated by the same amount. This type of challenge is best addressed during design. One solution is to even out the column sizes in a building with outriggers by using belt trusses or belt walls to engage all perimeter columns in resisting wind loads transferred from an outrigger. Another, is the use of a megastructure concept where outriggers connect to mega-columns. LERA has been a proponent of this system particularly when it comes to supertall office buildings. In this system, belt trusses transfer loads from gravity only columns to mega-columns at regular intervals along the height of the building. By transferring the load of gravity only columns to mega-columns at regular intervals, the mega-columns carry most of the gravity load on the perimeter frame. This has several benefits. Collecting the gravity loads in the mega-columns helps resist uplift forces from wind loads. Also, by carrying most of the perimeter frame gravity loads, the mega-column become large and provide lateral stiffness to the building. Finally, because the mega-columns may carry substantial wind loads, they tend to be at a lower service axial stress level than gravity only columns would be, and therefore may have less of a differential stress level with the core than would typical gravity only columns.

4. Predicting Shortening Effects and Construction Corrections

Today, the prediction of shortening and shortening effects is most effectively achieved with the use of advanced structural analysis software. However it is important to understand the behavior of the software and the types of analysis option available as compared with the physical behavior of the actual building. It is also useful and recommended to do some more simple calculations using spreadsheets to estimate the anticipated shortening amounts and validate the results from the analysis software.
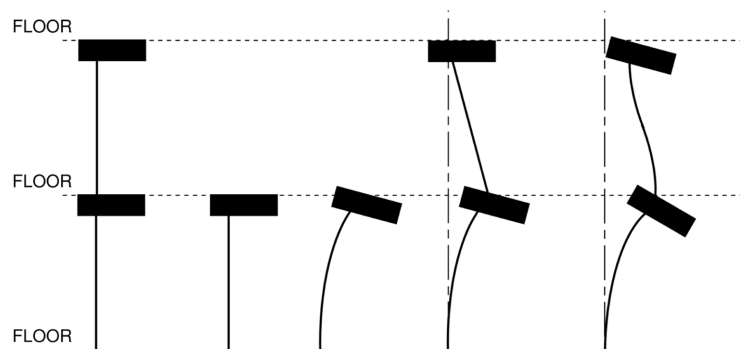
4.1. Analysis Model Behavior

The most typical analysis option used in the analysis and design of buildings is an "instant on" elastic analysis. By "instant on", we refer to the assumption in the model that the entire structure materializes instantly, and the forces and deformations in the model are calculated on that basis, which in turn means that the shortening and differential shortening of vertical elements is also calculated based on the "instant on" analysis. This type of analysis is adequate for most applications but can be misleading when designing elements that may be impacted by

differential shortening such as outriggers. In such a case, deformations that happen before the outriggers are built and connected need not be considered as inducing stresses in the outriggers. Using a "staged construction" analysis can resolve this.

As the name implies, "staged construction" analysis allows the user to activate different parts of the structure in a sequential manner more representative of the actual construction sequence. It is available in several advanced analysis software packages. Using this type of analysis provides a better prediction of the actual building behavior considering shortening effects. The user can choose the fineness or coarseness of the staging depending on the types of results desired. The finer the staging, the longer the analysis time. Therefore, the user will look to use the minimum amount of stages that provides the necessary results.

Within staged construction analysis, there are two possible software behaviors with regard to how new stages are added to the model. Understanding the software behavior and choosing the appropriate modeling option becomes important when predicting corrections required during construction to compensate for shortening.

In the analysis software that LERA uses, the default software behavior is to introduce new construction stages at their theoretical location, correcting for any shortening or lateral displacement that may have happened below. This software behavior can be useful as it mimics what would happen in actual construction where contractor is casting floors to target elevations and correcting for shortening that may have happened below, or where contractor is plumbing the building correcting for any lateral displacement or out of plumbness that may have happened below. However, the amount of correction that the software is making at each stage is not always clearly measurable. This may not be too much of an issue when considering the effects of shortening on the levelness of a floor and predicting super elevation amounts required at vertical elements, but may be problematic when looking to predict the required lateral camber required for correcting the lean of a building under its self-weight. Fig. 1

below illustrates this default modeling behavior for a building that leans under its self-weight.

In the same analysis software that LERA uses, it is possible to model a so called "ghost structure", a replica of the real structure with almost zero stiffness and no mass/weight, that displaces and moves as the lower stages of the structure shorten or lean. In this options, new stages are introduced in the model relative to the stages below without adjustments to the length of vertical elements, or adjustments to their plan location. This option allows the user to understand the behavior of the structure if no corrections are made, and then calculate the total amount of correction required as opposed to a super elevation beyond casting the floor level. Fig. 2 below illustrates the "ghost structure" model behavior for a building that deflects under its self-weight.

4.2. Factors Affecting Shortening Predictions

In structural steel buildings, predicting the shortening of vertical elements is relatively straight forward as compared to reinforced concrete due to the linear behavior of the structural steel in the service level range of stresses. However, even with steel structures, our experience has been that actual structures are often stiffer than predicted by the analysis model. This is due to the fact that analysis models do not model all elements in a building. In our experience, the elastic shortening may be as little as 0.7 times the predicted shortening and as high as 1.0 times the predicted amount.

To predict the effects of differential shortening that may be experienced at critical stages during construction, such as when the lifts are to be installed, the construction schedule and the sequence of loading become the main factors affecting the predictions while recognizing that actual amounts may be less than predicted. Since the construction schedule is usually being developed throughout the design phases and into construction, and since the contractor may not be engaged until the design is effectively complete, one usually needs to make estimates during the design phases based on best available schedule information and reasonable assumptions about the schedule based

Figure 1. Staged Construction Default Model Behavior.
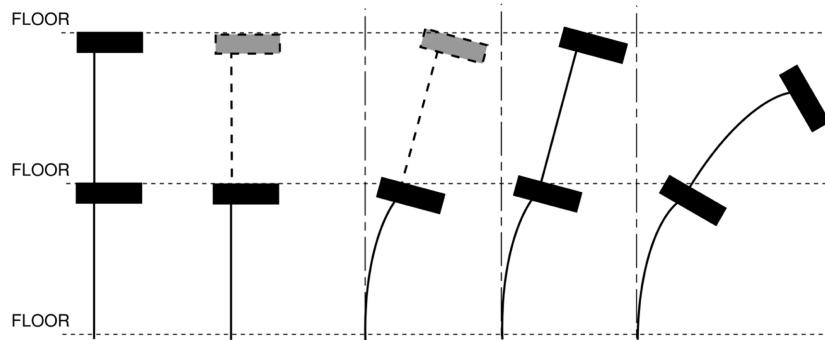
**Figure 2.** Staged Construction "Ghost Structure" Model Behavior.

on past experience.

With reinforced concrete buildings, predicting the shortening of vertical elements is more complex due to the creep and shrinkage of the concrete. There are various mathematical prediction models yielding different results, each with a coefficient of variation as summarized in ACI 209.2R-08. To predict the amount of differential shortening that may be experienced by certain elements, the construction schedule, the sequence of loading, and the creep and shrinkage properties of the mix design become the main factors affecting the predictions. For the best

predictions, it is desirable to have the creep and shrinkage properties of the actual concrete mix design tested. Usually, this is not an option during the design phase and one has to account for the variation in the prediction models.

4.3. Analysis Results and Interpretation for Level Floors

As mentioned earlier, differential shortening is a bigger concern than shortening. Therefore when looking at the analysis results one usually looks at the shortening of vertical elements across a floor plate and identify areas
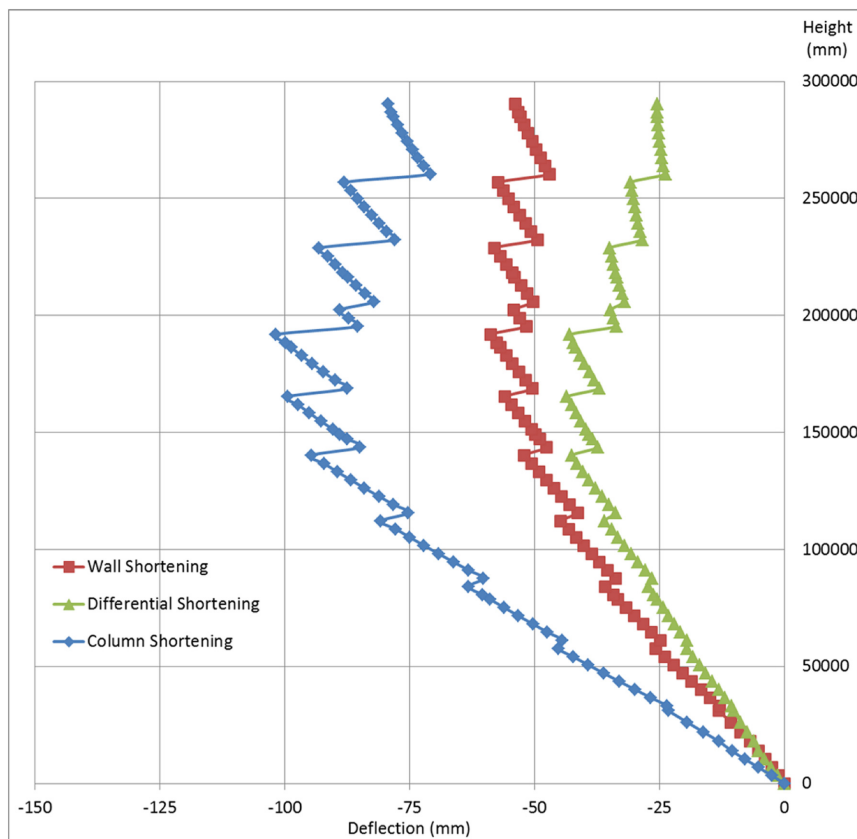
Figure 3. Differential Shortening Between Perimeter and Core - Default Behavior.
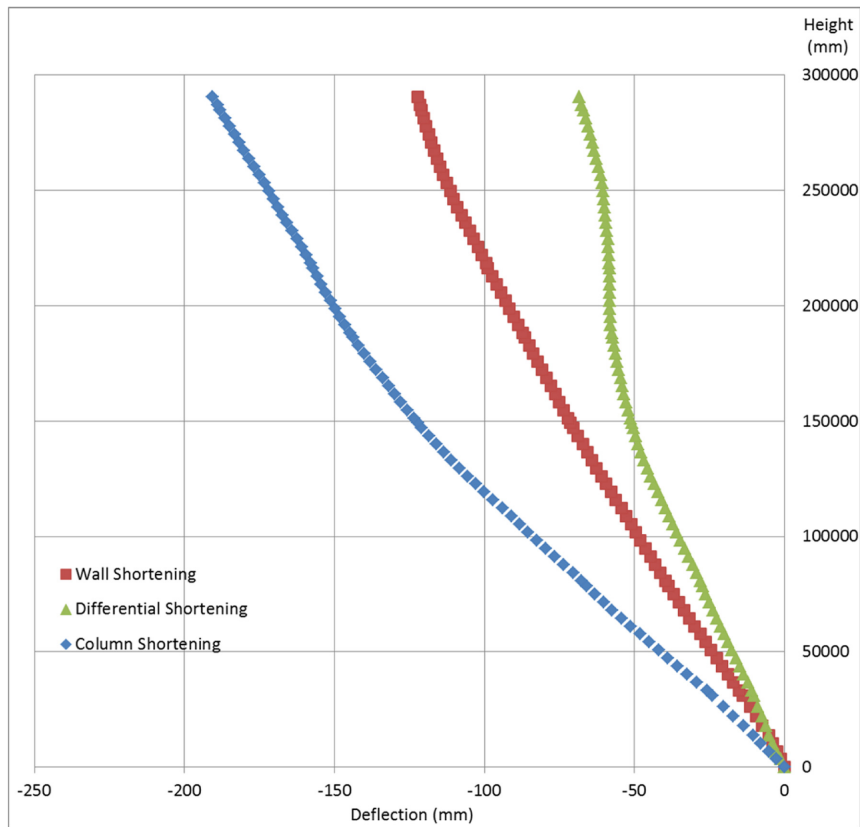
Figure 4. Differential Shortening Between Perimeter and Core - "Ghost Model" Behavior.

with differential shortening. One usually looks at the differential slope between adjacent columns and walls to see where it exceeds levelness tolerances such as distance/500 or 25 mm whichever is smaller. Once this is done, one considers whether there are practical design solutions to mitigate the differential. If yes, one may propose revising the design to stakeholders. Where the differences are large and not practically resolvable by a design change, the solution will usually involve a correction during construction such as super elevating the perimeter columns with respect to the core.

Figs. 3 and 4 below are representative graphs comparing the shortening of a perimeter column to the shortening of the core at the closest distance between the core and perimeter for a sample 80-story 280 m tall building. Fig. 3 is a graph of the results using the program default staged analysis method, while Fig. 4 is a graph of the results using the "ghost structure" modeling technique. Each graph plots the shortening of the perimeter column (blue line), the shortening of the core (red line), and the differential between the two (green line).

You will note that the Fig. 3 graph steps at various points along the height. Each step represents the introduction of a new construction stage where the software corrects the new stage for the amount of shortening that has occurred up to the introduction of the stage. In this case,

10 construction stages were modeled resulting in 9 steps. The graph indicates that if floors are cast to their theoretical elevations, the greatest amount of column shortening will be measured somewhere around the 2/3 height of the tower and will be approximately 100 mm, the greatest amount of core shortening will be around the 2/3-height of the building and decrease slightly towards the top of the building, and the greatest differential shortening between the perimeter column and core will be from approximately the mid-height to the 2/3-height of the tower at a little over 40 mm.

Fig. 4 gives the shortening amounts if no correction to the height of vertical elements are made during construction. In this case, the maximum shortening and differential shortening amounts are at the top of the building, with the perimeter column shortening approximately 190 mm, the core shortening approximately 120 mm, and the maximum differential amount being approximately 70 mm.

Both figures provide valuable information and depending on the construction materials and methods, one set of information may be more informative than the other.

In the case of this building, it was a reinforced concrete building where if nothing was specified, the contractor would have cast each floor to a target elevation similar to the analysis model for Fig. 3. Fig. 3 was therefore used to determine the amount of super elevation to specify for the
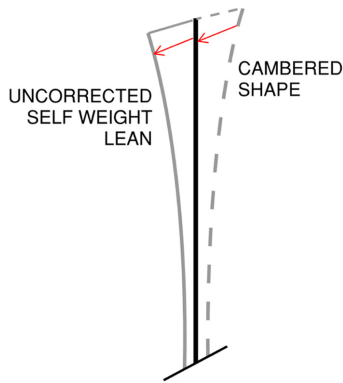
Figure 5. Cambering Building Concept.

perimeter columns relative to the core. In specifying the amount of super-elevation, we took into account the fact that the actual shortening amounts may be less than predicted using a factor of 0.7. We chose to err on the side of under correction because if the perimeter concrete at the columns is a little low, it is easily rectified with additional screed or fill on top of the slabs, whereas if the column heights are over corrected and the perimeter concrete is high, chipping of the concrete may be required which is a much more difficult correction to make.

4.4. Analysis Results and Interpretation for Lean Correction

As mentioned before, uneven shortening of elements can cause a building to lean under its self-weight when the building is asymmetrical. The asymmetry could be in the massing of the building or in the structural layout such as an offset core in a narrow floor plate building. Where there is a significant lean under self-weight, the solution

is to camber the building during construction such that when the building deflects laterally it deflects into a plumb position. In its most basic form, the concept is illustrated by Fig. 5.

However due to P-Delta effects, the solution is not as simple as reversing the deflected shape. If there is no camber and the building is leaning under its self-weight, the P-Delta effects contribute to increasing the lean. When the structure is cambered opposite the lean direction, the P-Delta effects oppose the lean. The process of finding the cambered shape involves some iteration. The first iteration involves taking the deflected shape and inverting it to use as an initial camber. The cambered building will lean towards the plumb position but not reach the vertical position. The difference between the deflected shape and vertical position is applied as a correction to the initial camber to obtain a new camber. The process can be repeated until the cambered shape leans towards a vertical position. Fig. 6 below illustrates this iterative process.

Fig. 7 below shows the predicted lean of a sample 100-story concrete building with an eccentric core. The deformation of approximately 290 mm is likely imperceptible to the naked eye given the height of the tower, but it may impact the installation and operation of the elevators, the typical variation from plumb tolerance for elevator shafts being +/- 50 mm. Given the building's tendency to lean and knowing that a correction is required, LERA would work with the elevator consultant to coordinate the structural specifications with the elevator specifications. In a similar project, LERA specified that the tower elevator shaft plumbness should be within +/- 75mm, relaxing our standard tolerance on verticality to acknowledge the added complexity in the construction, at the same time, the elevator consultant specified that the elevators needed to accommodate a +/- 100 mm tolerance on verticality, and
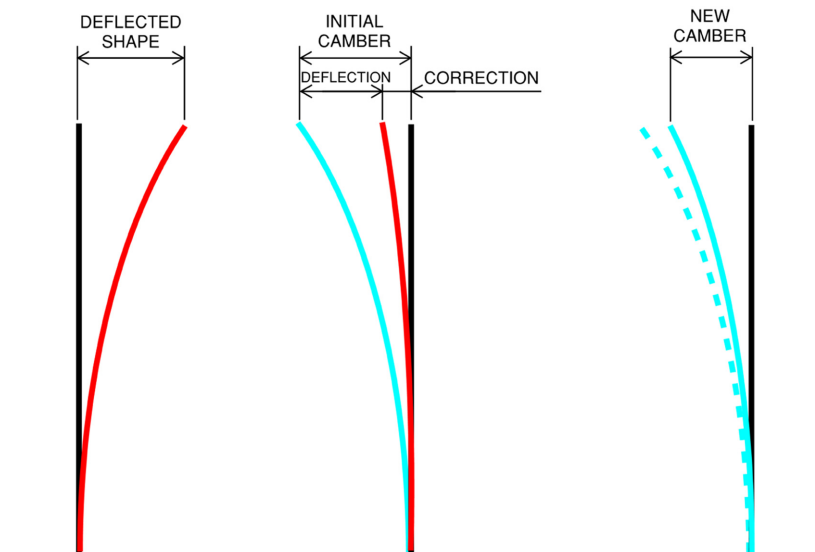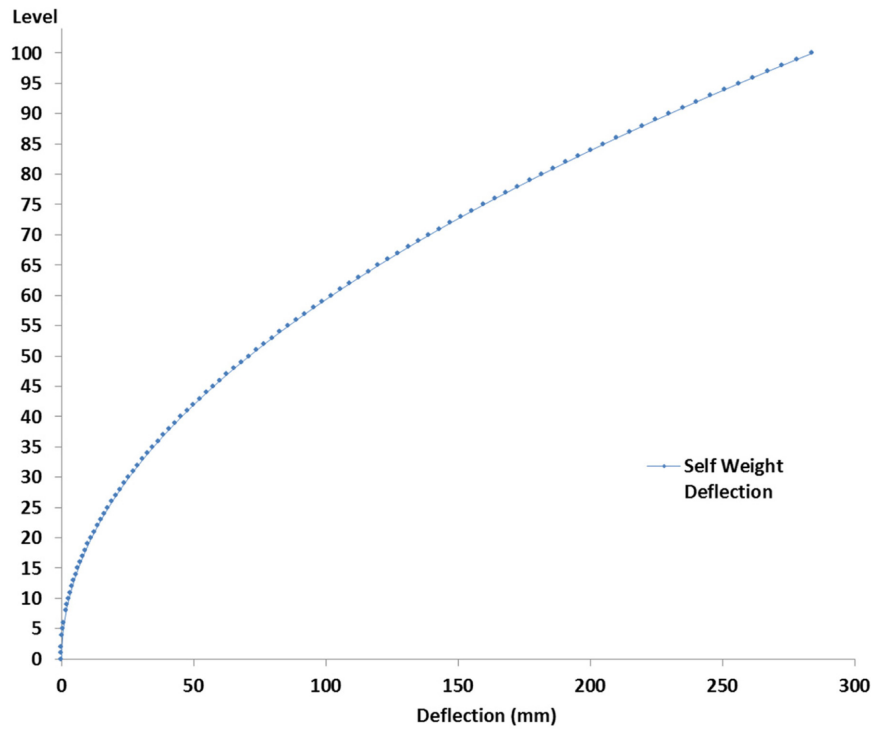

**Figure 6.** Iterative Cambered Shape Finding.

Figure 7. Predicted Lean of a Sample 100-Story Concrete Building.

oversized the shafts accordingly. This provided an added tolerance margin should the structural tolerance slightly miss the mark.
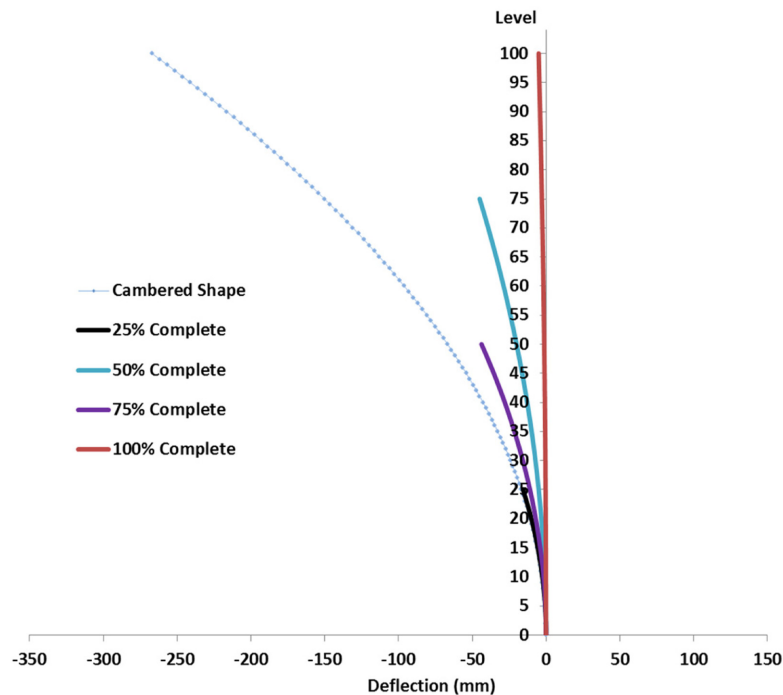
LERA calculated the required camber using the iterative process described above, and verified that it would fall within the specified tolerances considering the possible variations on the prediction. For bounding the results, we took 0.7 times the elastic deformations and 0.75 times
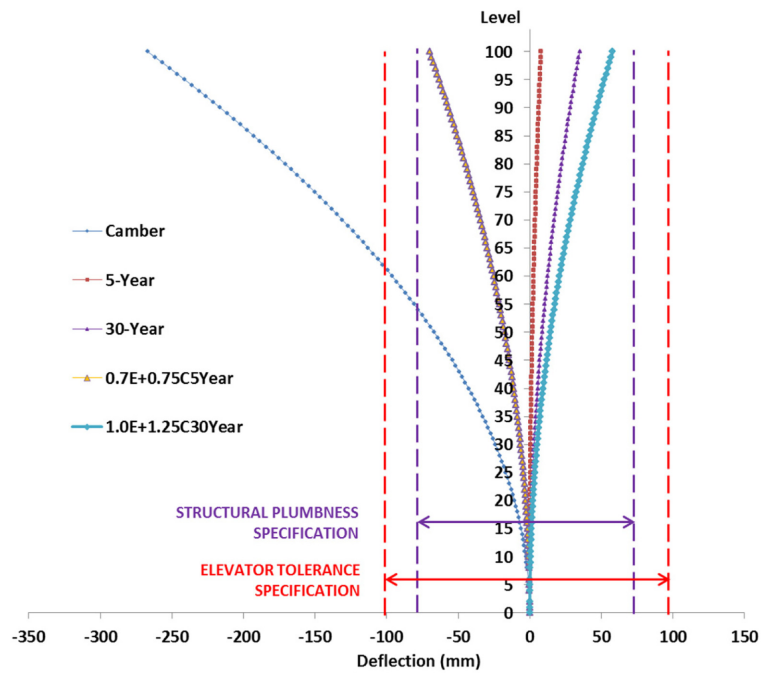
Figure 8. Building Camber and Verticality Tolerance.

Figure 9. Target Building Shape at Various stages of Construction.

the 5 year creep deformations on the low end, and 1.0 times the elastic deformations and 1.25 times the 30 year creep deformation on the upper end. The same is illustrated for the sample 100 story building in Fig. 8. For the actual project, LERA also specified periodic surveying of the structure to compare to analysis predictions and to allow correction of the specified camber as required to bring the tower within tolerances.

It is worth noting that the cambered shape in Fig. 8 is a theoretical shape showing the relative position of each floor with respect to the floor below, and not an actual target shape at any point in time. As the building is getting built and starts leaning towards the vertical position, new floors are added relative to the floor below based on the theoretical cambered shape. It is possible to plot the building position at different heights and times during construction to have as target geometries. See Fig. 9.

5. Communicating Information to Contractor

Once the amount and type of correction are identified, it is important to communicate the information to Contractor for implementation. We have seen three different project delivery methods when it comes to correcting for differential shortening and or lean.

Option 1, is where Owner's Structural Engineer identifies to Contractor in the project specifications the types of corrections expected and the expected tolerances and specify that Contractor calculate and implement the correc-

tions needed during construction. Where appropriate, we try to relax the standard structural tolerance specifications in coordination with other trades as to make Contractor's task in achieving the project tolerances reasonable. The advantage of this approach for project Owners is that the prediction and implementation of corrections lies with one entity, the Contractor who also has the most control over the end product, by controlling the construction quality, construction schedule and the accuracy of the surveying. Many international contractors have in-house engineering departments with the technical ability to predict and correct for the effects of shortening and use sophisticated surveying tools to verify and inform their predictions.

Option 2, is basically the same as Option 1, except that Contractor submits calculations and correction predictions for review by the structural engineer. The advantage to the Owner in this case is that the structural engineer who is most familiar with the structure of the building reviews Contractor's calculations for reasonableness in assumptions and predicted corrections. The drawback is that if the construction is out of tolerance, the responsibility is not as clear cut as in Option 1 as the Owner's structural engineer reviewed and approved the calculations.

Option 3 is where the Owner's Structural Engineer calculates required corrections based on the construction schedule provided by contractor, and adjust predictions based on the regular surveys provided by Contractor. This is sometimes required when the Contractor does not have the technical capability to do the required analysis, or in

markets where Contractors are unwilling to take on the responsibility of the prediction. In this option, the responsibilities are clearly split between the Owner and the Contractor, as the structural engineer has no control over the construction quality, construction schedule and survey accuracy, but is making the prediction on behalf of Owner. Where the project specifications are not met, it will inevitably be argued as to where the responsibility lies as the calculations and implementation are done by two different parties.

6. Conclusion

Predicting the effects of differential shortening in tall buildings and addressing them through design and construction is an important part in the delivery of a service-able building to Owners. The structural engineer has to account for the shortening in the design and where appropriate tune the structure to mitigate differential shortening. Where differential shortening remains, or where the structure leans under its self-weight, the structural engineer needs to identify to contractor the mitigations required in construction. At the same time, the structural engineer should consider whether the standard tolerances can be relaxed in coordination with other trades as to make Contactor's task more achievable.

References

ACI Committee 209. (2008). ACI 209.2R-08 - Guide for Modeling and Calculating Shrinkage and Creep in Hardened Concrete.