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Guidance Law for Agile Turn of Air-to-Air Missile During Boost Phase

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Abstract

This paper proposes the guidance laws for an agile turn of air-to-air missiles during the initial boost phase. Optimal solution for the agile turn is obtained based on the optimal control theory with a simplified missile dynamic model. Angle-of-attack command generating methods for completion of agile turn are then proposed from the optimal solution. Collision triangle condition for non-maneuvering target is reviewed and implemented for update of terminal condition for the agile turn. The performance of the proposed method is compared with an existing homing guidance law and the minimum-time optimal solution through simulations under various initial engagement scenarios. Simulation results verify that transition to homing phase after boost phase with the proposed method is more effective than direct usage of the homing guidance law.

Key words: Missile boost phase guidance, Missile agile turn maneuver, Collision triangle

1. Introduction

An air power is the most important element in recent warfare, and thus having an air supremacy is essential. To achieve such positions, capacities of fighters, which are mainly maneuverability and evasion ability, have progressed enormously over the years. Such improvements have increased the needs for more advanced air-to-air missile capability. Among the various capabilities of air-to-air missiles, an all-aspect capability is required more than anything else to win an air combat. To achieve the all-aspect capability, the air-to-air missile requires an agile turn maneuver.

Due to the importance of the problem, many researchers have dedicated themselves to this area, but most of them have focused on controller design, which enables agile turn maneuvers, rather than agile turn maneuver itself. During the agile turn phase, a body acceleration or angle-of-attack are possible candidates for control parameter, but commanding the body acceleration may not be desirable, compared to angle-of-attack according to [1]. Based on the observation made in [1], researchers [1-8] have investigated it using angle-of-attack. Feedback linearization method with uncertainty adaptation based on neural networks was used for agile missile autopilot design at [2]. Dynamic inversion and the Extended-Mean Assignment control technique were applied to the control of the agile missile using both fin and side thruster in [3]. In addition to these, an adaptive backstepping control based on neural network in [4], the backstepping control methodology in [5], gain scheduled pole placement approach in [6], and a time-delay-control was in [7] were considered as the agile missile control method. Separately, researchers in [8-10] studied the guidance law for varying-velocity missile, but they did not consider the high angle-of-attack dynamics occurred during the agile turn.

A proper controller design is the most critical issue for an agile maneuver capability, and for this reason, almost researches focused on the controller design, and achieved great results. Previous researches study how to generate output actuator commands for a given input guidance command during the boost phase. The given input guidance command

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was simply a constant command (maximum angle-of-attack command) or off-line optimized command. To overcome the limitation, this paper focuses on how to generate input guidance command for a given proper controller in an optimal sense during the boost phase.

This paper covers a velocity maximizing optimal agile turn using only thrust without aerodynamic force, and application with the aerodynamic force. The overall structure of the paper consists of four sections, including this introduction. In addition, the rest of the paper is organized as follows: Detailed description about an agile turn of missile problems, an optimal solution for agile turn without considering aerodynamic forces, and proposing agile turn control law are explained at Section 2. Numerical simulations are conducted under various combat scenarios, and the performance of the proposed law is investigated in Section 3. Finally, Section 4 covers concluding remarks.

2. Agile Turn Control Law

2.1 Problem Definition

Short Range Air-to-Air Missile(SRAAM) must be able to complete an agile turn during the initial boost phase to locate the target within seeker's field of view, so that it can obtain the state of the target required for homing guidance. In addition, maximizing velocity during the agile turn is also important to improve the intercept performance as well as to minimize time to intercept. In that sense, the problem can be formulated as an optimal control problem, which is finding an optimal control history that maximizes terminal velocity at the end of the boost phase while satisfying terminal desired heading angle constraint.

Before constructing the optimal control problem, the equation of pitch planar motion of the missile during the boost phase can be expressed as follows.

$$\begin{aligned}\frac{dV}{dt}&=\frac{1}{m(t)}\left(T\cos\alpha-D-W\sin\gamma\right)\\ \frac{d\gamma}{dt}&=\frac{1}{m(t)V}\left(T\sin\alpha+L-W\cos\gamma\right)\\ \frac{d\alpha}{dt}&=f\left(u,\alpha\right),\ |u|\le u_{\max}\end{aligned}\tag{1}$$

In here, V, r, α and u denote for speed, flight-path angle, angle-of-attack and angle-of-attack command of missile, respectively, and T, L, D, m and W stand for thrust, lift, drag, mass and weight of missile, respectively. Additionally, neglecting gravity term in Eq. (1) represents yaw planar motion of the missile

Under the assumption that an appropriate attitude controller is given, the time derivative of angle-of-attack is expressed as a function of angle-of-attack command and angle-of-attack. Since the angle-of-attack follows the command by proper controller, the function for angle-of-attack dynamics monotonically increases with respect to the command for the given angle-of-attack.

$$u_1\le u_2\rightarrow\forall\alpha,\ f\left(u_1,\alpha\right)\le f\left(u_2,\alpha\right).\tag{2}$$

For example, the most frequently used function is a first-order lag system with time constant τ, which is expressed as Eq. (3) and it satisfies Eq. (2).

$$f\left(u,\alpha\right)=\frac{u-\alpha}{\tau}.\tag{3}$$

2.2 Optimal Thrust Steering Law

From the problem defined at 2.1, the optimal control problem consisting the performance index and the constraint in Eq. (4) can be defined. Note that, $V(t_f)$ and $r(t_f)$ represent velocity and flight-path-angle at the engine burn off time t_f, and r_f denotes for desired flight-path-angle.

$$\begin{aligned}&\mathrm{PI}=-V(t_f)\\ &\gamma_f=\gamma(t_f)\end{aligned}\tag{4}$$

Unfortunately, the above optimal control problem does not provide a closed form solution due to high nonlinearity in aerodynamics. Therefore, the optimal solution without considering aerodynamic force is obtained first, and then the solution is applied to the situation where aerodynamic force exists. Note that for this reason, the agile turn method described in this paper shows better performance in situations where the effect of the aerodynamic force is relatively low, such as a low initial velocity launch and fast attitude control using TVC or side thruster.

Removing aerodynamic forces from Eq. (2) results following simplified dynamic Eq. (5).

$$\begin{aligned}\frac{dV}{dt}&=a(t)\cos\alpha-g\sin\gamma\\ \frac{d\gamma}{dt}&=\frac{1}{V}\left(a(t)\sin\alpha-g\cos\gamma\right),\ \text{where}\quad a(t)=\frac{T}{m(t)}\\ \frac{d\alpha}{dt}&=f\left(u,\alpha\right)\end{aligned}\tag{5}$$

Referring Eq. (1), Hamiltonian and corresponding co-state equations can be derived.

$$H=\lambda_V\left(a\cos\alpha-g\sin\gamma\right)+\lambda_\gamma\frac{a\sin\alpha-g\cos\gamma}{V}+\lambda_\alpha f\left(u,\alpha\right),\tag{6}$$

$$\begin{aligned}\frac{d\lambda_V}{dt}&=-\frac{\partial H}{\partial V}=\frac{\lambda_\gamma}{V^2}\left(a\sin\alpha-g\cos\gamma\right)\\ \frac{d\lambda_\gamma}{dt}&=-\frac{\partial H}{\partial\gamma}=\lambda_V g\cos\gamma-\frac{\lambda_\gamma}{V}g\sin\gamma\\ \frac{d\lambda_\alpha}{dt}&=-\frac{\partial H}{\partial\alpha}=\lambda_V a\sin\alpha-\lambda_\gamma\frac{a\cos\alpha}{V}+\lambda_\alpha\frac{\partial f}{\partial\alpha}.\end{aligned}\tag{7}$$

To completely construct the optimal control problem, boundary conditions must be clearly defined. From explanation of previous problem, boundary conditions for states and corresponding co-state are defined as Eq. (8).

$$\begin{aligned} &V(t_0)=V_0, \quad \gamma(t_0)=\gamma_0, \quad \alpha(t_0)=\alpha_0, \\ &\lambda_V(t_0)=\text{free}, \quad \lambda_\gamma(t_0)=\text{free}, \quad \lambda_\alpha(t_0)=\text{free} \\ &V(t_f)=\text{free}, \quad \gamma(t_f)=\gamma_f, \quad \alpha(t_0)=\text{free}, \\ &\lambda_V(t_f)=-1, \quad \lambda_\gamma(t_f)=\text{free}, \quad \lambda_\alpha(t_f)=0. \end{aligned} \tag{8}$$

Since the control input is bounded, Pontryagin's minimum principles(PMP) Eq (9) is used to derive the optimal condition.

$$H(\mathbf{x}^*,\mathbf{u}^*,\boldsymbol{\lambda}^*,\mathrm{t}) \le H(\mathbf{x}^*,\mathbf{u},\boldsymbol{\lambda}^*,\mathrm{t}). \tag{9}$$

Due to the assumption made at Eq. (2), applying PMP to Eq. (6) results following the optimal control strategy.

$$u^* = \begin{cases} u_{\max} & \lambda_\alpha < 0 \\ -u_{\max} & \lambda_\alpha > 0 \\ \text{Undetermined} & \lambda_\alpha = 0 \end{cases}. \tag{10}$$

As noted in Eq. (10), singular condition occurs when λ_α=0, and terminal conditions Eq. (8) imply existence of the singular interval for reachable problem, at least at the last moment. However, the PMP itself does not give any information about the control command during singular interval, thus following procedures Eqs. (11) to (14) are investigated.

At singular interval $t \in [t_s, t_f]$, Eq. (11) must be held,

$$\lambda_\alpha = 0 \Rightarrow \frac{d\lambda_\alpha}{dt} = 0. \tag{11}$$

Substituting Eq. (11) to Eq. (7) results

$$\lambda_V V \tan\alpha - \lambda_\gamma = 0 \quad \text{for} \quad t \in [t_s, t_f]. \tag{12}$$

Time derivative of Eq. (12) is

$$\begin{aligned} \frac{d}{dt}\left(\lambda_V V \tan\alpha\right) &= \frac{d\lambda_V}{dt} V \tan\alpha + \lambda_V \frac{dV}{dt}\tan\alpha \\ &+ \lambda_V V \sec^2\alpha \frac{d\alpha}{dt} = \frac{d\lambda_\gamma}{dt}. \end{aligned} \tag{13}$$

Rearranging Eq. (13) reveals

$$\begin{aligned} &\tan^2\alpha\left(a\sin\alpha - g\cos\gamma\right) + \left(a\cos\alpha - g\cos\gamma\right)\tan\alpha \\ &+ V\sec^2\alpha f\left(u,\alpha\right) = g\cos\gamma - \tan\alpha\, g\sin\gamma. \end{aligned} \tag{14}$$

Equating Eq. (14) concludes the following nonlinear equation that holds along the singular interval.

$$a\sin\alpha - g\cos\gamma + V f\left(u,\alpha\right) = 0. \tag{15}$$

From dynamic equation, Eq. (15) implies relation between angle-of-attack and flight-path-angle.

$$\frac{d\alpha}{dt} = f\left(u,\alpha\right) = -\frac{a\sin\alpha - g\cos\gamma}{V} = -\frac{d\gamma}{dt}. \tag{16}$$

Integration of Eq. (16) results the main relation (constant attitude) during the singular interval.

$$\alpha + \gamma = \theta = \text{const}. \tag{17}$$

With relation Eq. (17), velocity and flight-path-angle of missile along the singular interval can be expressed explicitly for $t \in [t_s, t_f]$. (Subscript s in Eqs. (18) to (19) means states at $t = t_s$)

$$\begin{aligned} &V_X\left(t\right) = V_X\left(t_s\right) + A\left(t,t_s\right)\cos\theta \\ &V_Y\left(t\right) = V_Y\left(t_s\right) + A\left(t,t_s\right)\sin\theta - g\left(t - t_s\right) \\ &\text{where} \quad A\left(t,t_s\right) = \int_{t_s}^{t} a\left(\tau\right)d\tau. \end{aligned} \tag{18}$$

By terminal flight-path-angle constraint Eq. (4), following must satisfy on singular interval.

$$\begin{aligned} &\gamma_f - \gamma(t_f) = \\ &\gamma_f - \tan^{-1}\left(\frac{V_Y\left(t_s\right) + A\left(t_f,t_s\right)\sin\theta - g\left(t_f - t_s\right)}{V_X\left(t_s\right) + A\left(t_f,t_s\right)\cos\theta}\right) = 0. \end{aligned} \tag{19}$$

Before constructing the switching condition of singular control, overall structure of optimal policy can be estimated by principles of optimality. Since following singular control is optimal from backward from above results, there must be existed moment that enters towards to the singular interval, and the criterion is λ_α. For short, if the singular criterion is satisfied, it follows the singular control, and if not, it follows the maximum control by PMP. However, implementing the criterion directly is impossible, because λ_α is unknown variable. Therefore, we define singular surface $s(X)$ which is only function of states and not of co-states as follow and by condition Eq. (19) λ_α=0 if s=0. In here, $\hat{\gamma}_f$, $\hat{V}_{Xf}$ and $\hat{V}_{Yf}$ are estimated terminal flight-path-angle, inertial X and Y direction speed following singular control from current states.

$$\begin{aligned} &s = \gamma_f - \hat{\gamma}_f \\ &\text{where} \quad \hat{\gamma}_f = \tan^{-1}\left(\frac{V_Y\left(t\right) + A\left(t_f,t\right)\sin\theta\left(t\right) - g\left(t_f - t\right)}{V_X\left(t\right) + A\left(t_f,t\right)\cos\theta\left(t\right)}\right) \\ &\qquad\quad = \tan^{-1}\left(\frac{\hat{V}_{Yf}}{\hat{V}_{Xf}}\right). \end{aligned} \tag{20}$$

With the results above, the following optimal control known as a bang-singular control can be defined as

$$\begin{aligned} &\text{Bang)} \quad u = \operatorname{sgn}\left(\gamma_f - \hat{\gamma}_f\right)u_{\max}, \ \text{until} \ s = 0 \\ &\text{Singular)} \ u \ \text{s.t} \ f\left(u,\alpha\right) = -\frac{d\gamma}{dt}. \end{aligned} \tag{21}$$

In case of yaw planar motion, same policy Eq. (21) is optimal with Eq. (22) instead of Eq. (19).

$$\hat{\gamma}_f = \tan^{-1}\left(\frac{V_Y(t)+A(t_f,t)\sin\theta(t)}{V_X(t)+A(t_f,t)\cos\theta(t)}\right) = \tan^{-1}\left(\frac{\hat{V}_{Yf}}{\hat{V}_{Xf}}\right) \quad (22)$$

Note that, above results turns out to be similar to the rocket ascent optimal guidance [11, 12, 13] with some assumptions.

2.3 Modified Thrust Steering Law

Direct applying previous optimal policy for agile turn is not proper, since there always exist unmodeled forces on the system such as aerodynamic forces. Based on the bang-singular strategy, even if the missile reaches on the singular surface s=0, due to the surface drift made by the unmodeled forces, the maximum correction is required for next moment. This will make control chatter which is not desirable. For this reason, angle-of-attack control command is formulated as Eq. (24) to steer states to the singular surface with positive gain, rather than applying maximum control as Eq. (21).

$$u = K\left(\gamma_f - \hat{\gamma}_f\right) \quad (23)$$

That is, the missile predicts the terminal flight-path-angle error (which is equal to s) as if current states are on the optimal singular interval, and the predicted error is compensated through error feedback command.

To show $\left|\gamma_f - \gamma(t)\right| \to 0$ as $t \to t_f$, it is enough to show $\gamma_f - \hat{\gamma}_f \to 0$ as $t \to t_f$. Assume $\gamma_f - \hat{\gamma}_f \to 0$ as $t \to t_f$ holds. Then by triangular inequality,

$$\left|\gamma_f - \gamma(t)\right| \le \left|\hat{\gamma}_f - \gamma(t)\right| + \left|\hat{\gamma}_f - \gamma_f\right| \quad (24)$$

Limit of first term on right hand is equal to zero as Eq. (25). Limit of second term on right hand is equal to zero by assumption, and proof is complete. (Only yaw planar motion stability is explained for simplicity. Same procedure can be applied to show pitch planar motion stability with minor changes)

$$\begin{aligned}\lim_{t\to t_f}\left|\hat{\gamma}_f - \gamma(t)\right| &= \lim_{t\to t_f}\left|\tan^{-1}\frac{\hat{V}_{Yf}}{\hat{V}_{Xf}} - \tan^{-1}\frac{V_Y}{V_X}\right| \\ &= \lim_{t\to t_f}\left|\tan^{-1}\left(\frac{AV\sin\alpha}{V^2+AV\cos\alpha}\right)\right| \\ &= 0\end{aligned} \quad (25)$$

From above result, we define guidance error as follows. Proving guidance error ε decrease as boost phase end is enough to show the stability of proposed agile turn law.

$$\varepsilon \triangleq \gamma_f - \hat{\gamma}_f \quad (26)$$

Time derivative of guidance error with consideration of aerodynamic effects is

$$\begin{aligned}\frac{d\varepsilon}{dt} &= \eta_T\left(\frac{d\alpha}{dt}+\frac{d\gamma}{dt}\right)+\eta_A \\ \eta_T(t,\alpha) &\triangleq -\frac{A(V\cos\alpha + A)}{\sqrt{\hat{V}_{Xf}^2+\hat{V}_{Yf}^2}} \\ \text{where} \quad \eta_A(t,\alpha) &\triangleq -\frac{L\cos(\hat{\gamma}_f-\gamma)+D\sin(\hat{\gamma}_f-\gamma)}{m\sqrt{\hat{V}_{Xf}^2+\hat{V}_{Yf}^2}}\end{aligned} \quad (27)$$

Assume that missile has axisymmetric shape and maximum angle-of-attack is less than 90 deg; the sign of lift is identical to the sign of angle-of-attack (without consideration of fin and body rate effect). Then sign of η_T and η_A is determined as follow.

$$\begin{aligned}&\operatorname{sgn}(\eta_T) < 0 \\ &\operatorname{sgn}(\eta_A) = -\operatorname{sgn}(\alpha)\end{aligned} \quad (28)$$

Further, if attitude control can be approximated by first order lag system as Eq. (3), the stability of proposed method can be proved indirectly by the following procedure. Closed loop system under guidance law Eq. (23) is as follow.

$$\begin{aligned}\frac{d\alpha}{dt} &= \frac{K\varepsilon-\alpha}{\tau} \\ \frac{d\varepsilon}{dt} &= \eta_T\left(\frac{K\varepsilon-\alpha}{\tau}+\frac{d\gamma}{dt}\right)+\eta_A\end{aligned} \quad (29)$$

Equilibrium point of the closed loop system satisfy subsequent conditions by definition.

$$\begin{aligned}&\frac{K\varepsilon-\alpha}{\tau} = 0 \\ &\eta_T\frac{d\gamma}{dt}+\eta_A = 0\end{aligned} \quad (30)$$

Eq. (30) leaves the only one possible equilibrium point.

$$\alpha = \varepsilon = 0 \quad \left(\because \operatorname{sgn}\left(\eta_T\frac{d\gamma}{dt}+\eta_A\right) = -\operatorname{sgn}(\alpha)\right) \quad (31)$$

Jacobian of closed loop equation (29) at equilibrium point α=ε=0 is

$$J = \begin{bmatrix} -\frac{1}{\tau} & \frac{K}{\tau} \\ -\frac{\eta_T}{\tau}+\eta_T\frac{\partial}{\partial\alpha}\left(\frac{d\gamma}{dt}\right)\Big|_{\alpha=\varepsilon=0}+\frac{\partial\eta_A}{\partial\alpha}\Big|_{\alpha=\varepsilon=0} & \frac{K\eta_T}{\tau}\end{bmatrix} \quad (32)$$

Note that partial derivatives of flight-path-angle rate at equilibrium point is

$$\eta_T\left[\frac{\partial}{\partial\alpha}\left(\frac{d\gamma}{dt}\right)\right]_{\alpha=0} = \eta_T\frac{T+QS_{ref}C_{L_{\alpha=0}}}{mV} < 0 \quad (33)$$

Similarly partial derivatives of η_A equilibrium point is

$$\left[\frac{\partial \eta_A}{\partial \alpha}\right]_{\alpha=0} = -\frac{\left\{QS_{ref}C_{L_{\alpha=0}} + D\dfrac{VA}{V^2+2VA+1}\right\}}{m(V+A)} < 0 \tag{34}$$

From the characteristic equation, sign of both eigenvalues are negative.

$$\left|\lambda I - J\right| = \lambda^2 + \frac{1}{\tau}\left(1-K\eta_T\right)\lambda - \frac{K}{\tau}\left[\eta_T \frac{\partial}{\partial \alpha}\left(\frac{d\gamma}{dt}\right)\bigg|_{\alpha=\varepsilon=0} + \frac{\partial \eta_A}{\partial \alpha}\bigg|_{\alpha=\varepsilon=0}\right] = 0 \tag{35}$$

For the unique equilibrium point, negative eigenvalues of J at the point implies asymptotical stability of closed loop system by nonlinear stability theorem, and therefore both guidance error and angle-of-attack will converge to zero.

2.4 Terminal Condition as Collision Triangle

Although the desired flight-path-angle is treated as a constant until previous section, the value must be updated based on the target information in real time. As mentioned previously, to minimize the engagement time, it is ideal to maximize the speed during the boost phase satisfying intercept condition, and then cruise directly to the target. Therefore, the intercept conditions can be calculated using well-known collision triangle [17]. However, unlike other papers, the proposed method computes the desired flight-path-angle based on the predicted missile and target state information at the end of the boost phase rather than the current time.

The predicted position of the missile at the time can be calculated by integrating the predicted velocity Eq. (18). Assuming a linear mass variation $\dot{m}=-c$, predicted inertial velocity and position of missile at burn off time t_f are computed as follow.

$$\begin{aligned}
\hat{V}_{Xf-M} &= V_{X-M} + \frac{T}{c}\ln\left(\frac{m}{m_f}\right)\cos\theta \\
\hat{V}_{Yf-M} &= V_{Y-M} + \frac{T}{c}\ln\left(\frac{m}{m_f}\right)\sin\theta \\
\hat{X}_{f-M} &= X_M + V_{X-M}\left(t_f - t\right) + \frac{T\cos\theta}{c^2}\left[m_f\ln\left(\frac{m}{m_f}\right) + c\left(t_f - t\right)\right] \\
\hat{Y}_{f-M} &= Y_M + V_{Y-M}\left(t_f - t\right) + \frac{T\sin\theta}{c^2}\left[m_f\ln\left(\frac{m}{m_f}\right) + c\left(t_f - t\right)\right]
\end{aligned} \tag{36}$$

where subscript M denotes the states belong to missile.

Under the assumption of non-maneuvering target, the predicted inertial positions and velocity at burn off time t_f are as follows. Note that subscript T means the target.

$$\begin{aligned}
\hat{V}_{Xf-T} &= V_{X-T} \\
\hat{V}_{Yf-T} &= V_{Y-T} \\
\hat{X}_{f-T} &= X_T + V_{X-T}\left(t_f - t\right) \\
\hat{Y}_{f-T} &= Y_T + V_{Y-T}\left(t_f - t\right)
\end{aligned} \tag{37}$$

From the Eq. (36) and (37), the line of sight angle at the burn out time is computed as

$$\lambda_f = \tan^{-1}\left(\frac{\hat{Y}_{f-T} - \hat{Y}_{f-M}}{\hat{X}_{f-T} - \hat{X}_{f-M}}\right) \tag{38}$$

Then the terminal flight-path-angle satisfying the collision triangle condition is

$$\gamma_f = \lambda_f + \sin^{-1}\left(\frac{\hat{V}_{f-T}}{\hat{V}_{f-M}}\sin\left(\hat{\gamma}_{f-T} - \lambda_f\right)\right), \tag{39}$$

and it is used for the angle-of-attack command Eq. (23).

3. Simulation

To validate the proposed agile turn method, simulations are conducted under various engagement scenarios. Then the performance of the proposed method is analyzed by comparing existing homing guidance law and minimum-time optimal solution through simulation results. The reason to select the minimum-time solution for comparison is that the engagement time shortens due to successful agile turn by the proposed law. Additionally, the maximum sustainable angle-of-attack depends on the missile actuator specification: missile with additional device such as thrust vector control or side thrust or jet vane missile has higher angle-of-attack sustainability than missile with fin control only. The missiles considered in this paper are the ideal one that has such sustainability, so we set the maximum angle-of-attack command to be 90 deg [1, 5, 16].

3.1 Simulation Setup

Referring from [7, 18], parameters of the air-to-air missile are listed in Table 1. For angle-of-attack dynamics, first order lag system Eq. (3) is considered.

Lift and drag coefficients are constructed as a polynomial equation as Eq. (40) and these coefficients are adopted from the same research [14].

$$C_L(\alpha) = \sum_{k=0}^{9} C_{Lk}\alpha^k,\ C_D(\alpha) = \sum_{k=0}^{9} C_{Dk}\alpha^k \tag{40}$$

Proposed methods (with gain K=3), PN guidance law (with navigation gain N=5), and optimal trajectory minimizing total time to intercept are compared for each scenario. In case of simulating proposed algorithm, missile uses the method only in the boost phase, and uses PN guidance law after end of boost phase (Named as Agile Turn and Proportional Navigation Guidance: ATPNG). Lastly, the pseudo-spectral method (PSM) is used to solve a minimum intercept time optimal control problem, and GPOPS-II is used to solve the problem [19].

3.2 Simulation Results

This paper considers three different yaw planar engagement scenarios with high off-boresight angle. Each scenario requires missile to have agile turn capacity for effective interception. For all scenarios, same initial conditions of missiles are used and listed at Table 3. The engagement holds yaw plane at an altitude of h=5000m, and corresponding air density at the altitude is ρ=0.7655kg/m^3. In addition, K=3 is used for simulation.

3.2.1 Scenario Case 1

The first scenario is yaw planar engagement with around 90 deg of aspect and bearing angle, and initial conditions of missiles and the targets are listed in Table 4. Simulation results and missile performance are shown at Fig. 1 and Table 5, respectively.

3.2.2 Scenario Case 2

Initial conditions of target for the second scenario are listed in Table 6. Simulation results and missile performance are shown at Fig. 2 and Table 7, respectively.

3.2.3 Scenario Case 3

The last scenario is engagement with around 180 deg of aspect and bearing angle, and initial conditions of missiles and the targets are listed in Table 8. Simulation results and missile performance are shown at Fig. 3 and Table 9, respectively.

3.3 Simulation Results Analysis

From the results in Section 3.2, using the proposed method to complete agile turn in the boost phase and using the homing guidance law has better results than using the homing guidance law from the beginning in all

Table 1. Missile Parameters

Missile Parameters		Values [Unit]
$\overline{T}$	Average Thrust	34000 [N]
$\overline{c}$	Average burn rate	15 [kg/s]
S_{ref}	Reference area	0.01824 [m^2]
t_f	Thrust burn out time	2.69 [sec]
τ	Angle-of-attack controller time constant	0.15 [sec]

Table 2. Lift and Drag Coefficient

Parameter	Values	Parameter	Values	Parameter	Values	Parameter	Values
C_{L0}	0	C_{L5}	41.207	C_{D0}	0.171	C_{D5}	0
C_{L1}	9.679	C_{L6}	0	C_{D1}	-2.809	C_{D6}	-17.66
C_{L2}	0	C_{L7}	16.34	C_{D2}	11.582	C_{D7}	0
C_{L3}	32.728	C_{L8}	0	C_{D3}	0	C_{D8}	3.2816
C_{L4}	0	C_{L9}	-2.272	C_{D4}	24.809	C_{D9}	0

Table 3. Missile Initial Conditions

V_{M0}	γ_{M0}	X_{M0}	Y_{M0}	α_{M0}
300 [m/s]	0 [deg]	0 [m]	0 [m]	0 [deg]

scenarios. From the results of the scenario 1, the proposed law shows great performance against the existing guidance. This is because, the proposed law constructs collision triangle, which is based on the predicted states. That is, the proposed law performs better than any guidance law that does not consider the boost effect. The results of scenario 2 and 3 show the same tendency. Especially, in the case of the scenario 3, PN guidance law fails to intercept the target.

Also, referring to the minimum time optimal trajectory, the proposed method shows performance close to the optimal result in the respective scenarios. All scenario results show that the proposed law commands to sustain the maximum angle-of-attack more than the optimal. The main reason of the difference is aerodynamic effects; the law is derived without consideration of aerodynamic force. That is, the guidance error is emphasized by neglecting such forces that give positive effect for turning. If the approximated aerodynamic model is given [15], that can be adopted to the proposed law.

3.4 Gain Selection

Referring to the section 2.3, infinite gain is optimal under no aerodynamic forces, and the positivity of the gain is required to make system stable. According to the change of the gain, the performance of the proposed law at scenario #1 and #2 show following tendency.

As the guidance error is emphasized like mentioned in previous section, a slow correction (smaller gain) will show

Table 4. Target Initial Conditions for Scenario 1

V_{T0}	γ_{T0}	X_{T0}	Y_{T0}	$\dot{\gamma}_T$
300 [m/s]	30 [deg]	350 [m]	2000 [m]	11.241 [deg/s]

Table 5. Performance Index of each algorithm for Scenario 1

Index	ATPNG	Optimal	PNG
t_f [sec]	3.9389	3.8870	4.8643
V_f [m/s]	1135.5	1144.3	920.0

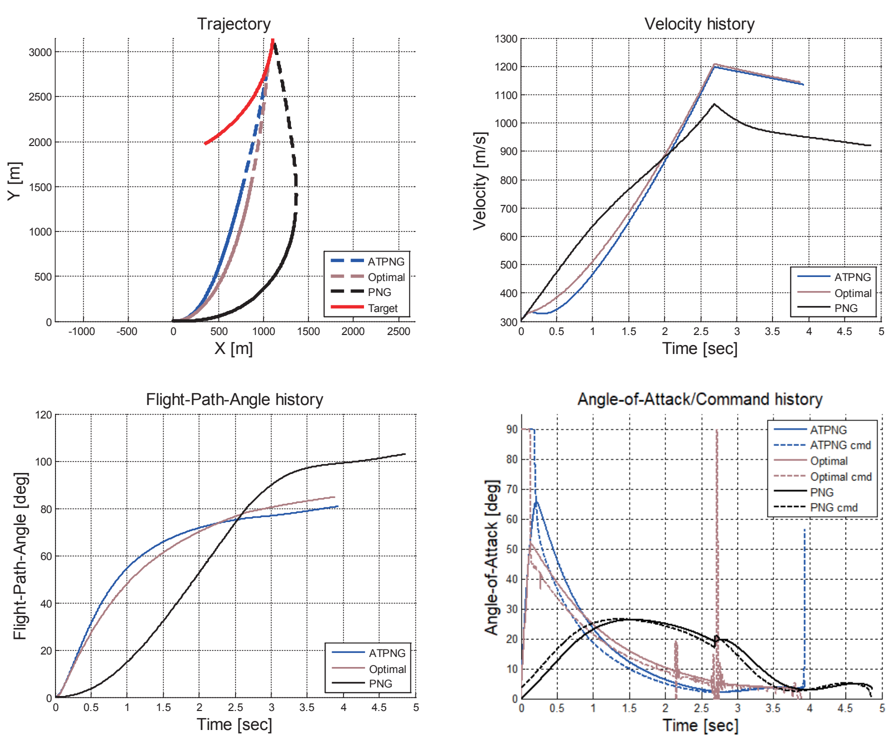

Fig. 1. (a) Trajectory, (b) Velocity, (c) FPA, (d) AoA/AoA Command for Scenario 1

better performance than a fast correction (higher gain) when positive angle-of-attack generates positive lift within the region of interest. However, the closed loop guidance error will converge to zero slowly under the smaller gain, the gain must exceed a certain lower bound. Unfortunately, such tendency is a function of operating environment, missile parameters, initial conditions, etc., and therefore an appropriate gain should be selected through numerical search method.

4. Conclusion

Velocity-maximizing control law for agile turn of missile at the boost phase is proposed, based on the optimal control theory, and its stability is proved. Conditions of collision triangle are reviewed and corresponding results are imposed to terminal constraint for the proposed control law. Various initial engagement scenarios are considered, and simulation results from the proposed law, PN guidance, and time minimization optimal trajectory are analyzed for each scenario. The simulation results verify that using the proposed method during the boost phase is effective.

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Table 6. Target Initial Conditions for Scenario 2

V_{T0}	γ_{T0}	X_{T0}	Y_{T0}	$\dot{\gamma}_T$
300 [m/s]	180 [deg]	0 [m]	700 [m]	5.621 [deg/s]

Table 7. Performance Index of each algorithm for Scenario 2

Index	ATPNG	Optimal	PNG
t_f [sec]	3.5998	3.5870	4.2759
V_f [m/s]	758.2	756.6	650.5

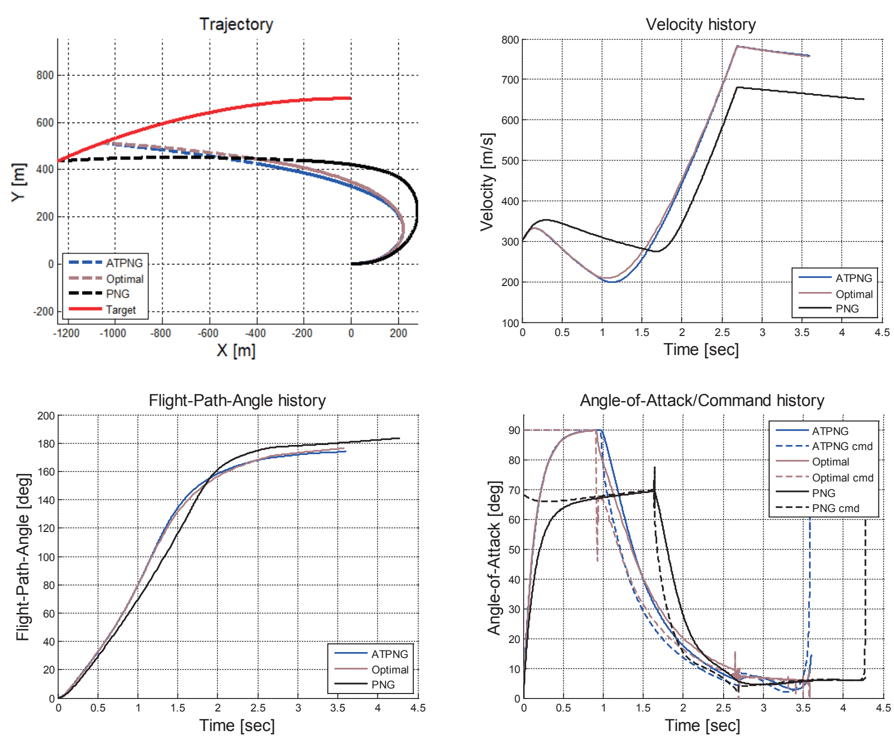

Fig. 2. (a) Trajectory, (b) Velocity, (c) FPA, (d) AoA/AoA Command for Scenario 2

Table 8. Target Initial Conditions for Scenario 3

V_{T0}	γ_{T0}	X_{T0}	Y_{T0}	$\dot{\gamma}_T$
300 [m/s]	–30 [deg]	–1300 [m]	750 [m]	–9.368 [deg/s]

Table 9. Performance Index of each algorithm for Scenario 3

Index	ATPNG	Optimal	PNG
t_f [sec]	3.2035	3.1914	X
V_f [m/s]	675.1	685.2	X

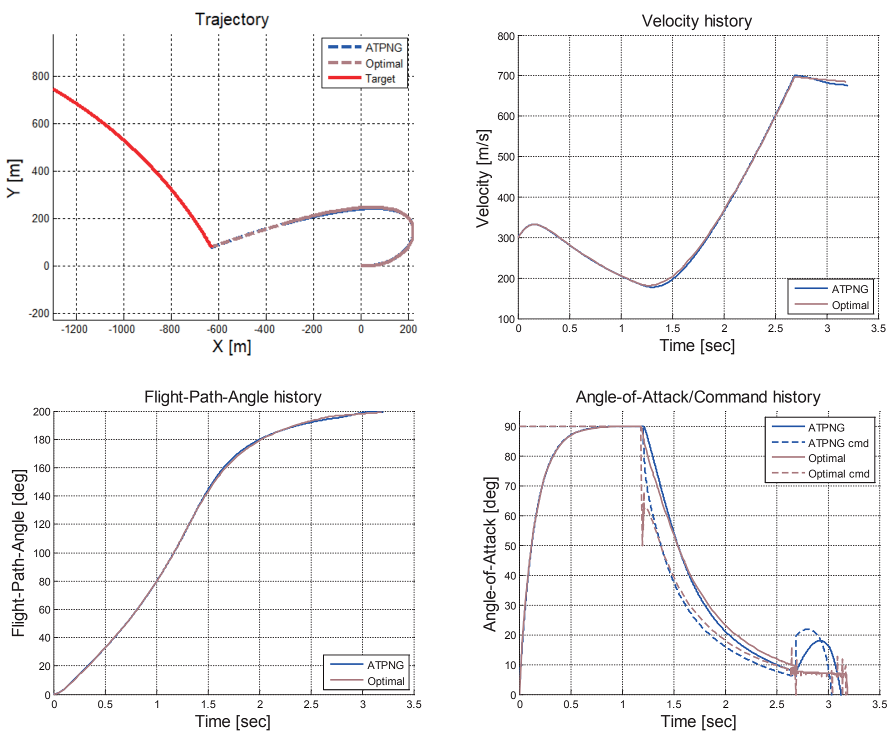

Fig. 3. (a) Trajectory, (b) Velocity, (c) FPA, (d) AoA/AoA Command for Scenario 3

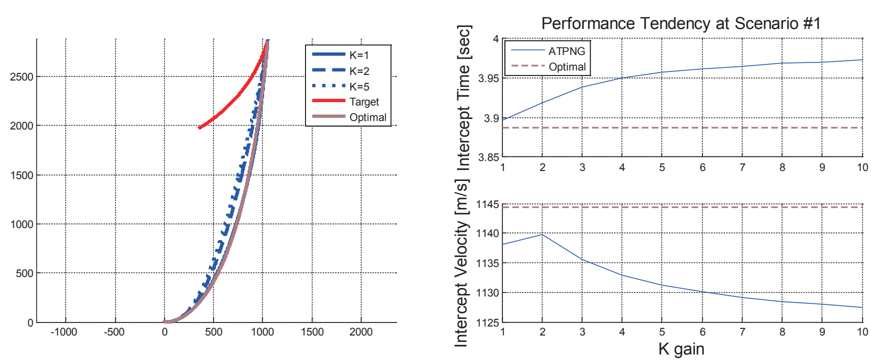

Fig. 4. (a) Trajectory, (b) Performance with various K gain for Scenario 1

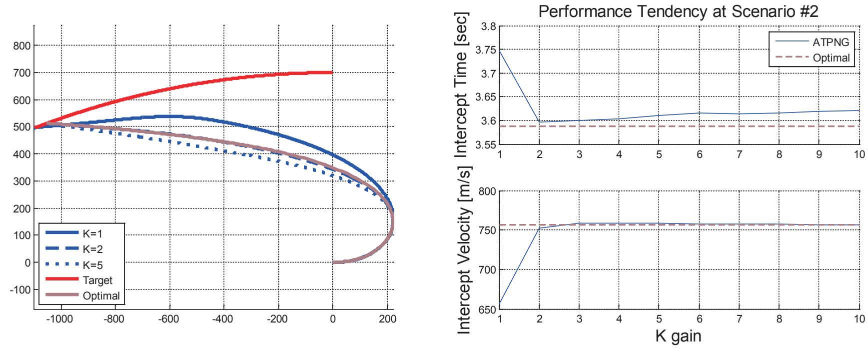

Fig. 5. (a) Trajectory, (b) Performance with various K gain for Scenario 2

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