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Estimation and Validation of Taper Equations for Three Major Coniferous Species in Gangwon and North Gyeongsang Provinces of South Korea

Daesung Lee[1], Yeongwan Seo[2], Jungho Lee[1,3] and Jungkee Choi[1,*]

[1]*Department of Forest Management, College of Forest and Environmental Sciences, Kangwon National University, Chuncheon 24345, Republic of Korea*
[2]*Institute of Forest Science, Kangwon National University, Chuncheon 24345, Republic of Korea*
[3]*Department of Forest Welfare, Gangneung Yeongdong University, Gangneung 25521, Republic of Korea*

Abstract

This study was carried out to estimate the parameters of stem taper functions, to figure out the best taper model by species, and to compare with previous studies by species, targeting on the stemmed tree samples collected from the Korean red pine (*Pinus densiflora*), Korean white pine (*Pinus koraiensis*), and Japanese larch (*Larix kaempferi*) stands in Gangwon and North Gyeongsang provinces of South Korea. The seven widely used models were applied in this study, and Muhairwe 1999 model for Korean red pine and Korean white pine and Kozak 2002 model for Japanese larch were evaluated as the best model for each species according to the fit statistics and the predicted stem form comparison. In addition, the predicted diameter was suitably fitted when comparing the previous studies, and the values were more appropriate following stem taper according to neiloid, paraboloid, and cone parts by species. Consequently, the estimation of this study was considered to represent the stem taper well. When comparing stem taper of three species, the diameter was largest in Korean white pine. Overall, the taper models of this study are judged to be useful for estimating stem form and volume computation of Korean red pine, Korean white pine, and Japanese larch.

Key Words: stem taper, nonlinear model, upper diameter, upper height, stem form

Introduction

Stem taper is the rate of decrease in stem diameter with increasing height from ground level to the tree tip (Burkhart and Tomé 2012). In order to predict the stem taper regression analysis was generally used in most cases. Relatively simple regression models are sufficient to solve many forestry problems, but more complex models are needed to adequately describe in the case of stem taper (Max and Burkhart 1976). Many complex models for stem taper were developed and used mainly because of two advantages of using stem taper models.

Taper functions can provide the predicted total and merchantable tree volumes. Moreover, it is highly flexible to predict the log or sectional volumes at any height (Muhairwe 1999; Burkhart and Tomé 2012). On the other hand, in the

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Corresponding author: Jungkee Choi

Department of Forest Management, College of Forest and Environmental Sciences, Kangwon National University, 188-77, Gonji-ro, Chuncheon 24345, Republic of Korea

Tel: 82-33-250-8337, Fax: 82-33-259-5617, E-mail: jungkee@kangwon.ac.kr

situation where the influences of environmental changes and stand treatments were analyzed on stem form, stem taper can provide a great deal of information (Lee et al. 2003). This is because factors which affect tree growth in height and diameter, such as genetic make-up of the tree, climatic fluctuations, site quality, tree age, canopy position, species, and stand density, affect tree taper (Larson 1963; Muhairwe et al. 1994; Muhairwe 1999).

In Korea, some studies have been conducted and introduced stem taper models by providing parameter estimates for major species (Kim et al. 2002; Lee et al. 2003; Son et al. 2004; Son et al. 2013; Kang et al. 2014a, 2014b; Jung et al. 2015). Especially, Korean red pine (*Pinus densiflora* Siebold & Zucc.), Korean white pine (*Pinus koraiensis* Siebold & Zucc.), and Japanese larch (*Larix kaempferi* (Lamb.) Carrière) are regarded as the representative major coniferous species based on the area percentage and volume production for sawtimber (Korea Forest Service 2016; Kim 2017), but the researches on stem taper of these species are still insufficient and restricted. Moreover, a variety of stem taper models should be compared to find the best model by major species.

Therefore, this study was carried out to find out the best taper equation of Korean red pine, Korean white pine, and Japanese larch by comparing the fit statistics and stem form, and to analyze the difference of stem taper between species in Gangwon and North Gyeongsang provinces of South Korea.

Materials and Methods

Data

For this study, sample trees of Korean red pine, Korean white pine, and Japanese larch were collected in the national forests of Gangwon and North Gyeongsang provinces in South Korea. The wood discs of stemmed standard trees were collected according to the stem analysis method from 0.2 m above the ground. After collecting 1 disc at 1.2 m, the following discs was collected at every 2 m intervals. Tree factors such as DBH, total height, upper diameter and height, etc. were collected and more detailed description and methodology about data collection was explained in the preceding studies (Lee and Choi 2016; Lee et al. 2017). The number of sample trees by species ranged from 39 to 47 and the number of wood discs were 366 to 531 by species (Table 1). The samples were diversely distributed to analyze various sizes with small to large trees in DBH and height.

Taper equations and validation

The candidate taper equations were selected to estimate the parameters for this study by reviewing the stem taper models widely used in the previous studies of South Korea (Table 2). Two types of taper equations were analyzed as candidate models: the segment form and variable exponential form. Max and Burkhart 1976 model is the representative taper equation of segment form type to estimate stem taper form by dividing the stem into three parts and by connecting the two inflection points (Max and Burkhart 1976). Kozak models are widely used to estimate stem taper by assuming that stems have an inflection point (Kozak 1988; Kozak 2004). The inflection point, which represents the ratio of relative height over total height, is generally determined by checking the scatter plot of relative diameter over relative height (Kim et al. 2002).

All the taper models were developed to estimate the upper diameter and the tree factors, such as dbh, total height, upper height, the ratio or fractional form of dbh and height, were used as the explanatory variables. The seven candidate taper functions consisted of four to seven parameters by model (Demaerschalk 1972; Max and Burkhart 1976; Kozak 1988; Mulhairwe 1999; Lee et al. 2003; Kozak

Table 1. Summary statistics of stemmed data for taper equations

Species	Statistics	DBH (cm)	Height (m)	No. of trees	No. of wood discs
Pinus densiflora	Mean	30.5	17.0	39	366
	SD	8.4	4.0		
	Minimum	5.2	3.6		
	Maximum	47.3	23.8		
Pinus koraiensis	Mean	29.8	17.2	47	436
	SD	8.0	4.4		
	Minimum	13.9	7.4		
	Maximum	45.9	24.6		
Larix kaempferi	Mean	27.8	22.8	45	531
	SD	8.0	4.5		
	Minimum	17.0	12.2		
	Maximum	47.9	30.6		

Table 2. Selected taper equations fitted in this study

Model	Equation
Demaerschalk (1972)	$d=a_1D^{a_2}(H-h)^{a_3}H^{a_4}$
Max & Burkhart (1976)	$d=D\sqrt{b_1(T-1)+b_2(T^2-1)+b_3(a_1-T)^2I_1+b_4(a_2-T)^2I_2}$
Kozak (1988)	$d=a_1D^{a_2}a_3^DX_i^{b_1T^2+b_2\mathrm{In}(T+0.001)+b_3T^{1/2}+b_4e^T+b_5(D/H)}$
Muhairwe (1999)	$d=a_1D^{a_2}(1-\sqrt{T})^{b_1T+b_2T^2+(b_3/T)+b_4T^3+b_5D+b_6(D/T)}$
Kozak (2001)	$d=a_1D^{a_2}X_j^{b_1+b_2(1/e^{D/H})+b_3D^{X_j}+b_4X_j^{D/H}}$
Kozak (2002)	$d=a_1D^{a_2}H^{a_3}X_k^{b_1T^4+b_2(1/e^{D/H})+b_3X_k^{0.1}+b_4(1/D)+b_5H^Q+b_6X_k}$
Lee (2003)	$d=a_1D^{a_2}(1-T)^{b_1T^2+b_2T+b_3}$

d is an upper diameter outside bark (cm); D is a diameter at breast height (cm); h is an upper height (m); H is a total height (m); *ln* is natural logarithm; *e* is the base of natural logarithm; T=h/H, a proportional height from the ground; $I_i=1$ if $T\leq a_i$ or $I_i=0$ if $T>a_i$; p=0.2, a proportional height of the inflection point; $X_i=[1-T^{1/2}]/[1-p^{1/2}]$; $X_j=(1-T^{1/4})/(1-0.01^{1/4})$; $Q=[1-T^{1/3}]$; $X_k=Q/[(1-(1.3/H)^{1/3}]$; a_1-a_4 and b_1-b_6 are parameters to be estimated in this study.

2004). The parameters were fitted using PROC NLIN on SAS 9.4 software with within 95% confidence limits (SAS Institute Inc. 2013).

To validate the estimated parameters, coefficient of determination (R^2), root mean square error (RMSE), mean deviation (MD), and mean absolute deviation (MAD) were calculated and checked for all models, and the residual plots were displayed to examine the bias estimation or abnormal pattern for final model (Lee and Choi 2016; Lee et al. 2017). The equations are summarized as follows:

$$R^2=1-\left[\sum_{i=1}^{n}(d_i-\hat{d}_i)^2/\sum_{i=1}^{n}(d_1-\overline{d})^2\right]$$

$$\mathrm{RMSE}=\sqrt{\sum_{i=1}^{n}(d_i-\hat{d}_i)^2/n}$$

$$\mathrm{MD}=\sum_{i=1}^{n}(d_i-\hat{d}_i)/n$$

$$\mathrm{MAD}=\sum_{i=1}^{n}|d_i-\hat{d}_i|/n$$

$$\mathrm{Residual}=d_i-\hat{d}_i$$

Where d_i=measured diameter for the ith tree, $\hat{d}_i$= predicted diameter for the ith tree, $\overline{d}$= measured mean tree diameter, n= the total number of trees.

Results and Discussion

Model fitting and validation

Parameters of all models were fitted well at 5% confidence limits and the overestimation or underestimation, which had too small or large parameters, were not found compared to the preceding studies (Table 3). Comparing fit statistics of seven models, to determine the best taper equation by species, Muhairwe 1999 model was shown as the best model for Korean red pine: the lowest RMSE with 1.5826 and the lowest MAD with 1.1798 (Table 4). Fit statistics for Korean white pine also appeared the lowest RMSE with 1.6309 and the lowest MAD in Muhairwe 1999 model.

For Japanese larch, generally, the models were fitted more accurate with the lower fit statistics than the models for Korean red pine and white pine. Especially, Kozak 2002 model was shown as the best model: the lowest RMSE with 0.9722 and the lowest MAD with 0.7205. When the residual plots were checked, the abnormal patterns or outliers were not appeared in any models, but the residuals were slightly different among species and models. Demaerschalk 1972 model, Kozak 2001 model, and Lee 2003 model rela-

Table 3. Parameter estimates of taper equations for models of each species

Species	Model	Parameters									
		a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	b_5	b_6
Pinus densiflora	Demaerschalk (1972)	1.5044	0.8955	0.7952	-0.7768						
	Max & Burkhart (1976)	0.7039	0.1356			-4.5213	2.2779	-2.6475	25.9746		
	Kozak (1988)	1.8921	0.7250	1.0055		-0.1082	0.0526	-1.5978	0.9743	-0.00004	
	Muhairwe (1999)	0.8982	0.9999			1.3842	-2.3648	-0.0532	1.9046	-0.00121	0.000824
	Kozak (2001)	1.3726	0.9536			0.4251	0.2470	0.0183	-0.6851		
	Kozak (2002)	1.0547	0.8940	0.1091		0.6654	-0.2969	0.3107	2.9207	0.0126	0.0154
	Lee (2003)	1.6092	0.9052			2.7752	-3.4307	1.8160			
Pinus koraiensis	Demaerschalk (1972)	1.2793	0.9571	0.7653	-0.7628						
	Max & Burkhart (1976)	0.6541	0.0795			-3.8139	1.8583	-2.1456	66.3002		
	Kozak (1988)	1.4508	0.8276	1.0039		0.8195	-0.1832	1.1853	-0.4488	0.0117	
	Muhairwe (1999)	1.2729	0.8907			0.9481	-0.7222	-0.0305	0.6015	-0.00329	0.000103
	Kozak (2001)	0.9772	1.0479			0.2811	0.7508	0.0584	-1.4884		
	Kozak (2002)	1.1607	0.9489	0.0141		0.4246	-0.5943	0.3326	6.5482	0.0565	-0.4083
	Lee (2003)	1.3128	0.9593			1.7564	-2.1213	1.3739			
Larix kaempferi	Demaerschalk (1972)	1.2384	0.8994	0.7498	-0.6830						
	Max & Burkhart (1976)	0.7291	0.0682			-4.1441	2.0294	-2.0995	107.6		
	Kozak (1988)	0.7796	1.0753	0.9957		0.9594	-0.1849	1.1158	-0.5369	0.1202	
	Muhairwe (1999)	0.9836	0.9798			0.6373	-0.7069	-0.0230	0.7184	0.00221	0.000037
	Kozak (2001)	0.9634	1.0472			0.3902	0.2300	0.0424	-0.9091		
	Kozak (2002)	1.1134	0.9746	-0.0052		0.4258	-0.8668	0.5226	4.8582	0.0616	-0.4874
	Lee (2003)	1.4390	0.9322			2.2122	-2.8443	1.6480			

Table 4. Summary of fit statistics for taper models by species

Species	Model	Fit statistics			
		R^2	RMSE	MD	MAD
Pinus densiflora	Demaerschalk (1972)	0.9568	2.1816	0.3325	1.6128
	Max & Burkhart (1976)	0.9778	1.6931	0.0934	1.2337
	Kozak (1988)	0.9792	1.6162	0.0245	1.1967
	Muhairwe (1999)	0.9803	1.5826	0.0386	1.1798
	Kozak (2001)	0.9472	2.4665	-0.1696	1.9701
	Kozak (2002)	0.9797	1.6023	0.0232	1.2020
	Lee (2003)	0.9771	1.7022	0.0331	1.3357
Pinus koraiensis	Demaerschalk (1972)	0.9683	1.9443	-0.0676	1.5323
	Max & Burkhart (1976)	0.9779	1.6539	0.0160	1.2057
	Kozak (1988)	0.9776	1.6543	-0.0351	1.1832
	Muhairwe (1999)	0.9784	1.6309	-0.0072	1.1739
	Kozak (2001)	0.9548	2.2791	-0.1441	1.8255
	Kozak (2002)	0.9779	1.6542	0.0153	1.1890
	Lee (2003)	0.9739	1.7930	0.0046	1.3402
Larix kaempferi	Demaerschalk (1972)	0.9749	1.5430	-0.0056	1.1695
	Max & Burkhart (1976)	0.9868	1.1545	0.1164	0.9051
	Kozak (1988)	0.9897	0.9986	-0.0393	0.7327
	Muhairwe (1999)	0.9887	1.0439	-0.0124	0.7658
	Kozak (2001)	0.9772	1.4569	-0.0596	1.1421
	Kozak (2002)	0.9902	0.9742	0.0039	0.7205
	Lee (2003)	0.9828	1.2911	0.0275	0.9873

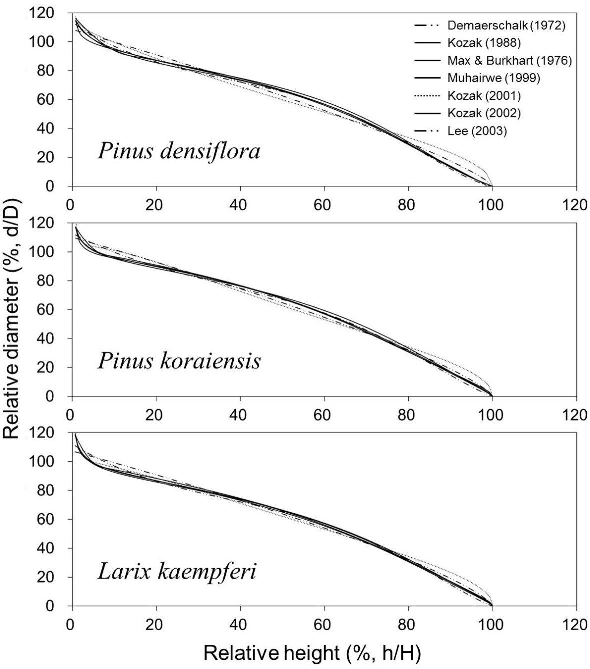

Fig. 1. Stem taper curves of seven candidate models by species.

tively showed high residuals for Korean white pine. Overall, the exceptional fitting were not detected in all models.

Model evaluation

After validating the models through fit statistics, the taper curves of seven models were illustrated in percentage scale with the consideration of growth characteristics according to the summary statistics of the samples: 40 cm in dbh and 25-30 m in height by species. Seven taper functions generally presented the stem form by species, but some distinction was detected in some models (Fig. 1). Stem form of top diameter in cone part were not smooth, but stumpy above 90% relative height in Kozak 2001 model. Also, the inflection points, which presented the neiloid and paraboloid, were not determinate in Demaerschalk 1972 model and Lee 2003 model. The other four models represented ideal stem form, and Muhairwe 1999 model and Kozak 2002 model showed the actual large diameter at stump.

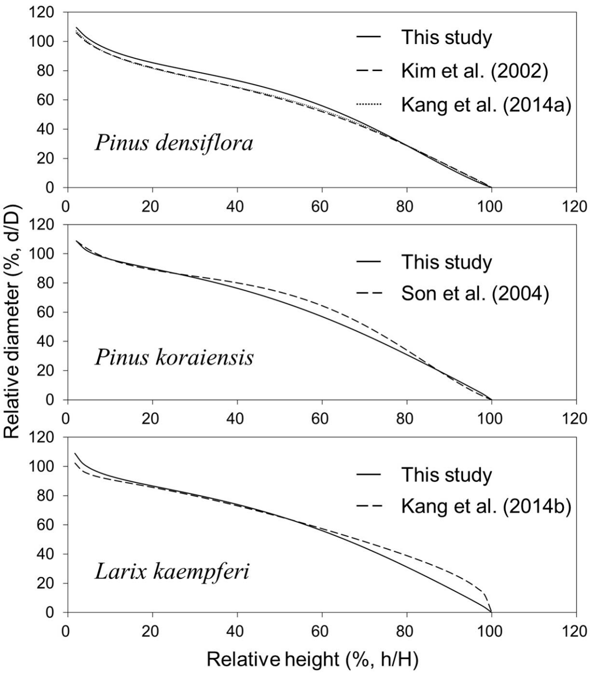

Fig. 2. Comparing stem taper curves of Kozak 1988 model with the previous studies by species.

Model comparison

To examine the accuracy of the estimation for taper models in this study, a taper model were compared with previous studies through the same equation (Fig. 2). For the assessment, the Kozak 1988 model was chosen because the fit statistics in this study were generally suitable and it was referred as the best model in the previous studies. In result, there was no large difference in cone part in Korean red pine, but the predicted diameter of this study were larger below 75% relative height than the value of Kim et al. (2002) and Kang et al. (2014a). In Korean white pine, however, the predicted diameter at the stump and cone part was similar, but showed the difference in paraboloid part of 30-85% relative height. The predicted diameter of this study was lower than the value of Son et al. (2004).

For Japanese larch, the predicted diameter was larger at the stump and similar at paraboloid part of 20-60% relative height. However, the tapering was not shown as paraboloid or cone shape at 65-100% relative height and decreased dramatically above 95% relative height in Kang et al. (2014b). The predicted diameter of this study was more suitable while decreasing gradually with parabola shape. One of the reasons about these different stem forms from the previous studies could be because of the sample data collected in different study sites of each previous studies. Overall, it was judged that the parameters were estimated well in the models of this study and represented the stem taper desirably for the data of this study.

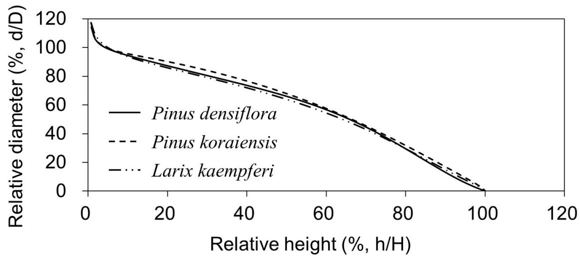

Fig. 3. Estimation of stem taper through the best model of this study by species.

To compare the stem taper by species, finally, the relative diameter over relative height were illustrated using the best model by species: Muhairwe 1999 model for Korean red pine and white pine and Kozak 2002 model for Japanese larch (Fig. 3). Taper shape was not largely different at stump, but the diameter was the largest in Korean white pine at 15-50% relative height, followed by Korean red pine and Japanese larch. Above 85% relative height, however, the diameter size was changed: Korean white pine > Japanese larch > Korean red pine. Overall, the taper equation suitably presented the characteristics of stem form by species.

Conclusion

This study was conducted to predict and compare the best parameter estimates of Korean red pine, Korean white pine, and Japanese larch in Gangwon and North Gyeongsang provinces of South Korea by using the seven well-known stem taper functions. When estimating the parameters of models by species, all models generally were fitted well at 5% confidence limits and over- or under-estimated values were not shown compared to the previous studies.

Best taper equations were determined as Muhairwe 1999 model for Korean red pine and Korean white pine and Kozak 2002 model for Japanese larch, as compared to fit statistics of taper models and displaying the residual plots.

When the predicted diameter was analyzed over height by presenting the taper of the models, cone shape did not appear in top diameter of Kozak 2001 model. Also, the neiloid and paraboloid points were not distinguishable in Demaerschalk 1972 model and Lee 2003 model. The other four models showed the stem taper by species well. Consequently, Muhairwe 1999 model for Korean red pine and white pine and Kozak 2002 for Japanese larch model were selected as the final best models.

When the taper was compared with the preceding studies, the diameter over height was estimated smaller or larger according to neiloid, paraboloid, and cone parts by species. Overall, the prediction of this study was judged to represent the stem taper the best. When compared to the stem form of three species using best models, the diameter was largest in Korean white pine. By applying to the best taper functions by species, henceforward, the merchantable or log volumes at any top height can be precisely calculated for practical use. Consequently, the stem taper models of this study are highly considered to be used for predicting stem form and volume calculation of Korean red pine, Korean white pine, and Japanese larch, especially in Gangwon and North Gyeongsang provinces in South Korea.

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