Target Birth Intensity Estimation Using Measurement-Driven PHD Filter

Huanqing Zhang, Hongwei Ge, and Jinlong Yang

The probability hypothesis density (PHD) filter is an effective means to track multiple targets in that it avoids explicit data associations between the measurements and targets. However, the target birth intensity as a prior is assumed to be known before tracking in a traditional target-tracking algorithm; otherwise, the performance of a conventional PHD filter will decline sharply. Aiming at this problem, a novel target birth intensity scheme and an improved measurement-driven scheme are incorporated into the PHD filter. The target birth intensity estimation scheme, composed of both PHD pre-filter technology and a target velocity extent method, is introduced to recursively estimate the target birth intensity by using the latest measurements at each time step. Second, based on the improved measurement-driven scheme, the measurement set at each time step is divided into the survival target measurement set, birth target measurement set, and clutter set, and meanwhile, the survival and birth target measurement sets are used to update the survival and birth targets, respectively. Lastly, a Gaussian mixture implementation of the PHD filter is presented under a linear Gaussian model assumption. The results of numerical experiments demonstrate that the proposed approach can achieve a better performance in tracking systems with an unknown newborn target intensity.

Keywords: Multi-target tracking, Measurement-driven scheme, Target birth intensity, Gaussian mixture PHD.

Manuscript received Jan. 22, 2016; revised May 4, 2016; accepted May 16, 2016.

Huanqing Zhang (hsj_hqzhang@126.com), Hongwei Ge (corresponding author, ghw8601@163.com) and Jinlong Yang (corresponding author, yjlgedeng@163.com) are with the School of Internet of Things Engineering, Jiangnan University and with the Key Laboratory of Advanced Process Control for Light Industry (Jiangnan University), Ministry of Education, Jiangsu, China.

I. Introduction

In recent years, tracking based on the random finite set (RFS) theory [1] as an alternative to a classical data association-based target-tracking algorithm has attracted considerable attention. To obtain a computational tractable solution, three suboptimal approximations are the probability hypothesis density (PHD) [2], cardinality PHD [3], and multi-target multi-Bernoulli [4]. Two implementations of the three suboptimal approximations are applied through a Sequential Monte Carlo (SMC) method [5] and a Gaussian mixture (GM) [4], [6]. More recently, the concept of labeled RFSs has been introduced to cope with a multi-target tracking problem, and its implementations include labeled multi-Bernoulli [7] and generalized labeled multi-Bernoulli [8], [9] approximations. These analytic approximations of a multi-target Bayes filter through an RFS and labeled RFSs have a various scope of applications including radar target tracking [10], [11], computer vision [12], [13], and sensor networks [14], [15].

Owing to the advantage of target state extraction and track generation, the Gaussian mixture PHD as a closed-form implementation for the PHD recursion has been extensively applied in a multi-target tracking field under a linear Gaussian model assumption. However, the target birth intensity is assumed to be known as a prior before tracking in a traditional PHD algorithm, which is inapplicable to real engineering applications. To overcome the drawback of the PHD filter, some improved approaches have been reported. In [16] and [17], an adaptive target birth intensity PHD filter using SMC approximation was presented, where the initial target birth intensity is formed by utilizing both the position and likelihood of the measurement. In the update step, the survival and birth targets are updated using different PHD update formula with the latest measurement set. However, the algorithm proposed in [17] has a significant bias in terms of the target number under a dense clutter scenario. In addition, the birth target intensity may cover the entire state space, which makes the computational load of the adaptive algorithm relatively heavy. In [18], a novel detection-guided multi-target Bayesian filter is proposed, where the positions of unknown newborn targets are detected using a sequential probability ratio test-based track initiation method. The target birth intensity is formed through position estimates of the newborn targets. Owing to the fact that some measurements in the latest measurement set are forbidden to initialize newborn target tracks, the proposed approach in [18] cannot deal with the target birth problem under a scenario in which multiple targets move near each other. In [19], the unknown newborn target-tracking problem is solved from the aspect of the Gaussian component of the GM-PHD filter. Every measurement in the latest measurement set at each time step is related to at least one Gaussian component in the new fusion scheme of the Gaussian component. Although the method proposed in [19] can obtain better newborn target intensity estimations, the proposed method suffers from a heavy computational load, which is nearly double that of the GM-PHD filter. Moreover, the proposed method is unable to track a newborn target under dense clutter conditions in that the computational burden is significantly increased. In [20] and [21], Zhou and others proposed a target birth intensity estimation algorithm for tracking visual targets, where the entropy distribution and coverage rate are introduced to model a newborn target intensity. However, an entropy distribution and coverage rate-based newborn target estimation scheme is only suitable to computer vision because the estimates of the birth target rely on both the intersection rate and area rate of different birth targets. The proposed algorithm cannot obtain a satisfactory level of performance when applied to radar point target tracking.

Both PHD pre-filter technology [22] and velocity estimations [23], [24] have been previously proposed for track initialization in multi-target tracking. Therefore, the unknown newborn target intensity estimation for the PHD filter can be solved through the comprehensive use of these two methods. In addition, the PHD filter has an obvious cardinality bias under a dense clutter scenario. In [25] and [26], a measurement-driven scheme is used to decrease the bias in the number of targets. However, both algorithms also assume that the target birth intensity is known prior to the tracking. Moreover, in [25] the authors suppose that only one newborn target can appear at each time step, and the bias in the number of targets is still greater in [26] owing to the defect in its update process.

In this paper, a novel target birth intensity estimation algorithm based on a measurement-driven Gaussian mixture PHD is proposed for a multi-target tracking system. First, a novel newborn target intensity approach was developed based on PHD pre-filter technology and a target velocity extent scheme. In particular, the PHD algorithm is utilized as a clutter pre-filter, where measurements irrelevant to newborn targets are discarded from the original measurement set. The remaining measurements are again purified using the target velocity extent scheme, and the possible target birth intensity can finally be obtained. Second, for the purpose of decreasing the disturbance in both clutter measurements and measurements originating from survival and newborn targets, an improved measurement-driven scheme is incorporated into the PHD filter, and its implementation is obtained through a Gaussian mixture scheme. Simulation results demonstrate that the proposed algorithm not only obtains better newborn target intensity estimations compared with another existing algorithm, but also achieves less computational load.

The remainder of this paper is structured as follows. Section II provides a brief review of an RFS-based PHD filter and its Gaussian mixture implementation. The proposed target birth intensity scheme and improved measurement-driven scheme are then detailed in Section III. In Section IV, performance comparisons of different algorithms are presented under several different tracking scenarios. Finally, some concluding remarks are given in Section V.

II. Background

1. Random Finite Set and PHD Filter

In RFS-based multi-target tracking, the multi-target states and multi-target observations defined as random finite sets are $X_k = \{x_{k,1}, \dots, x_{k,N_k}\}$ and $Z_k = \{z_{k,1}, \dots, z_{k,M_k}\}$, where N_k and M_k denote the target number and measurement number at time k, respectively. Letting $p_{k-1}(X|Z_{1:k-1})$ be the multi-target intensity function at time $k-1$, the optimal multi-target Bayesian iterative formulas can then be computed as

$$p_{k|k-1}(X_k|Z_{1:k-1}) = \int f_{k|k-1}(X_k|X) p_{k-1}(X|Z_{1:k-1}) \mu_s(dX), \quad (1)$$

$$p_k(X_k|Z_{1:k}) = \frac{g_k(Z_k|X_k) p_{k|k-1}(X_k|Z_{1:k-1})}{\int g_k(Z_k|X) p_{k|k-1}(X|Z_{1:k-1}) \mu_s(dX)}, \quad (2)$$

where $f_{k|k-1}(X_k|X)$ is the state transition probability density function of multiple targets, $g_k(x)$ is the multi-target likelihood function, and μ_s denotes the approximate Lebesgue measure of the state space.

As a suboptimal alternative to a multi-target Bayesian filter, the PHD filter is a first-order statistical moment of a multi-target posterior distribution, which is composed of a prediction step and an update step. The prediction equation is

$$\begin{aligned}\mathcal{V}_{k|k-1}(x) = \int p_{S,k}(\zeta) f_{k|k-1}(x|\zeta)\mathcal{V}_{k-1}(\zeta)d\zeta \\ + \int \beta_{k|k-1}(x|\zeta)\mathcal{V}_{k-1}(\zeta)d\zeta + \gamma_k(x).\end{aligned} \tag{3}$$

When up-to-date measurements are available at each time step, the update equation can be described as

$$\begin{aligned}\mathcal{V}_k(x) = \left[1 - p_{D,k}(x)\right]\mathcal{V}_{k|k-1}(x) + \\ \sum_{z\in Z_k} \frac{p_{D,k}(x) g_k(z|x)\mathcal{V}_{k|k-1}(x)}{\mathcal{K}_k(z) + \int p_{D,k}(\zeta) g_k(z|\zeta)\mathcal{V}_{k|k-1}(\zeta)d\zeta},\end{aligned} \tag{4}$$

where $p_{S,k}$ and $p_{D,k}$ denote the survival probability and detection probability, respectively; $\mathcal{K}_k(z)$ is the clutter intensity; $\gamma_k(x)$ is the intensity function of newborn targets; and $\beta_{k|k-1}(x|\zeta)$ is the spawn intensity.

2. Gaussian Mixture PHD Filter

The Gaussian mixture PHD (GM-PHD) filter uses mixed Gaussian components to approximate the distribution of multiple target states, which is more suitable for a linear Gaussian system. Let $\mathcal{N}(\cdot; m, P)$ illustrate a Gaussian density with mean m and covariance P. Based on some necessary assumptions [6], if the posterior intensity of multi-target states can be represented in a Gaussian mixture form at time $k-1$, then

$$D_{k-1}(x) = \sum_{i=1}^{J_{k-1}} w_{k-1}^i \mathcal{N}\left(x; m_{k-1}^i, P_{k-1}^i\right), \tag{5}$$

where w_{k-1}^i is the weight of the *ith* Gaussian mixture, and J_{k-1} is the number of Gaussian components. The predicted intensity is also a Gaussian mixture at time k, and can be approximated as

$$D_{k|k-1}(x) = D_{k|k-1}^s(x) + D_{k|k-1}^\beta(x) + \gamma_k(x), \tag{6}$$

$$D_{k|k-1}^s(x) = p_{S,k} \sum_{i=1}^{J_{k-1}} w_{k-1}^i \mathcal{N}\left(x; F_{k-1} m_{k-1}^i, F_{k-1} P_{k-1}^i F_{k-1}^T\right), \tag{7}$$

$$D_{k|k-1}^\beta(x) = \sum_{i=1}^{J_{k-1}} \sum_{j=1}^{J_k^\beta} w_{k-1}^i w_k^{\beta,j} \mathcal{N}\left(x; m_{k|k-1}^{i,j}, P_{k|k-1}^{i,j}\right), \text{ and} \tag{8}$$

$$\gamma_k(x) = \sum_{i=1}^{J_{\gamma,k}} w_{\gamma,k}^i \mathcal{N}\left(x; m_{\gamma,k}^i, P_{\gamma,k}^i\right), \tag{9}$$

where $D_{k|k-1}^s(x)$ is the predicted intensity of the survival targets, and $D_{k|k-1}^\beta(x)$ is the predicted intensity of the spawned targets.

The given predicted intensity in Gaussian mixture form with $J_{k|k-1}$ components is

$$D_{k|k-1}(x) = \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^i \mathcal{N}\left(x; m_{k|k-1}^i, P_{k|k-1}^i\right). \tag{10}$$

The posterior intensity is also then a Gaussian mixture, which can be described as

$$D_k(x) = (1 - p_{D,k}) D_{k|k-1}(x) + \sum_{z\in Z_k} D_{D,k}(x; z), \tag{11}$$

$$D_{D,k}(x; z) = \sum_{i=1}^{J_{k|k-1}} w_k^i \mathcal{N}\left(x; m_{k|k}^i, P_{k|k}^i\right). \tag{12}$$

For simplicity, the main iterative framework is briefly summarized as above. The whole iterative process and details of how to update the associated parameters in the PHD iteration are available in [6].

III. Proposed Algorithm

In this section, a novel target birth intensity estimation approach is proposed, where the PHD pre-filter technology and target velocity extent schemes are utilized to obtain the possible newborn target intensity. An improved measurement-driven scheme is then adopted into the Gaussian mixture PHD framework to improve the performance of the original GM-PHD filter.

1. Target Birth Intensity Estimation Approach

At each time step, the target birth intensity is initialized according to the estimated survival target states and measurements obtained at the last time step. Owing to the fact that the measurements may be generated from the survival targets, birth targets, or clutter, some clutter measurements may be used to model the initial birth target intensity. To eliminate the disturbance of these clutter measurements, the PHD filter is used as a clutter pre-filter to obtain the possible target birth intensity. Unfortunately, some clutter measurements also exist in the estimated target birth intensity. Therefore, the target velocity extent scheme is utilized to purify the estimated target birth intensity, and the most likely newborn target intensity can finally be obtained.

To distinguish an individual target, a unique label is assigned for each target, which is denoted by ℓ. Each Gaussian component of the individual target has the same label. Assume that the measurement set $Z_k = \left\{z_k^i\right\}_{i=1}^{M_k}$ is available at time k, and that the target set Ξ_E, in which the weights of the targets are bigger than the given threshold at time $k-1$, can be obtained as

$$\Xi_E = \left\{\ell_k^i : w_{k-1}^i > w_{Th}, \ i = 1, \ldots, J_{k-1}\right\}, \tag{13}$$

where w_{Th} is a preset state extraction threshold defined in the original GM-PHD filter, and J_{k-1} is the number of Gaussian components.

For all targets in Ξ_E, a possible associated measurement set $Z_{E,k}$ can be obtained at time step k as

$$Z_{E,k} = \bigcup_{i=1}^{M_{E,k}} \hat{z}_k^i, \tag{14}$$

$$\hat{z}_k^i = \left(z_{k,j} : \underset{j}{\operatorname{argmin}} \varphi^{\mathrm{T}} \left(S_{E,k}^i \right)^{-1} \varphi \right), z_{k,j} \in Z_k, \forall j = 1:M_k, \tag{15}$$

$$\varphi = z_{k,j} - H_k m_{E,k|k-1}^i, \tag{16}$$

$$S_{E,k}^i = H_k P_{E,k|k-1}^i H_k^{\mathrm{T}} + R_k, \text{ and} \tag{17}$$

$$M_{E,k} = numel\left(unique\left(\Xi_E\right)\right), \tag{18}$$

where H_k is the measurement matrix, S_k is the measurement residual covariance matrix, and R_k is the measurement noise covariance. *Unique*(x) is a function that can extract different targets without repeat targets from a set, and *numel*(x) is also a function that can compute the cardinality of a set. Both the residual measurement set $Z_{R,k}$ and the cardinality of $Z_{R,k}$ for the birth targets, spawning targets, and clutter can be obtained as

$$Z_{R,k} = Z_k - Z_{E,k} \quad \text{and} \tag{19}$$

$$M_{R,k} = M_k - M_{E,k}. \tag{20}$$

In the proposed target birth intensity estimation approach, the newborn target intensity is initialized with all measurements in $Z_{R,k}$, which can be approximated as

$$\gamma_{\mathrm{init},k} = \sum_{i=1}^{M_{R,k}} w_{\gamma,k}^i \mathcal{N}\left(x; m_{\gamma,k}^i, P_{\gamma,k}^i\right), \tag{21}$$

$$w_{\gamma,k}^i = 0.1, \tag{22}$$

$$m_{\gamma,k}^i = H_k^{-1} z_k^i, \quad \text{and} \tag{23}$$

$$P_{\gamma,k}^i = H_k^{-1} R_k \left(H_k^{-1}\right)^{\mathrm{T}}. \tag{24}$$

Each birth target is assigned with a unique label, and a target birth intensity label set $\aleph_{\mathrm{init},k}$ can be obtained by

$$\aleph_{\mathrm{init},k} = \left\{ \ell_{\gamma,k}^1, \ldots, \ell_{\gamma,k}^i, \ldots, \ell_{\gamma,k}^{M_{R,k}} \right\} \quad \forall i = 1:M_{R,k}. \tag{25}$$

In the initial process of the target birth intensity, some clutter measurements exist in the residual measurement set, and are used to model the initial target birth intensity. To obtain the possible newborn targets from the initial birth target intensity at time k, PHD technology is used to eliminate the initial target birth intensity originating from the clutter measurements. Assume that the latest measurement set $Z_{k+1} = \left\{ z_{k+1}^j \right\}_{j=1}^{M_{k+1}}$ is available at time $k+1$, which is utilized to update the initialized target birth intensity $\gamma_{\mathrm{init},k}$. Using the PHD algorithm as a clutter pre-filter, some initialized birth targets coming from the clutter measurements can be deleted from $\gamma_{\mathrm{init},k}$.

Let $w_{k+1}^{i,j}$ denote the weight of the target state $x_{k+1}^{i,j}$ at time $k+1$, which can be calculated as

$$w_{\gamma,k+1}^{i,j} = \frac{p_{D,k+1} w_{\gamma,k+1|k}^i \eta_{k+1}^i}{\mathcal{K}(z_{j,k+1}) + \sum_{i=1}^{M_{R,k}} p_{D,k+1} w_{\gamma,k+1|k}^i \eta_{k+1}^i}, \forall j = 1:M_{k+1}, \tag{26}$$

$$\eta_{k+1}^i = \mathcal{N}\left(z_{j,k+1}; H_k m_{\gamma,k+1|k}^i, R_k + H_k P_{\gamma,k+1|k}^i (H_k)^{\mathrm{T}}\right). \tag{27}$$

After the initialized target birth intensity $\gamma_{\mathrm{init},k}$ has been updated using the latest M_{k+1} measurements, a weight matrix W_m with a size of $M_{R,k} \times M_{k+1}$ can be formed, which is composed of the updated weights of the target birth intensity $\gamma_{\mathrm{init},k}$. The label $\ell_{\gamma,k+1}^{i,j}$ of the target state $x_{k+1}^{i,j}$ is the same as $\ell_{\gamma,k}^i$.

The false initialized target birth intensity, originating from the clutter measurements in $\gamma_{\mathrm{init},k}$, is removed under the condition that the maximum weight of each initialized birth target in weight matrix W_m is less than a given threshold σ_γ. Meanwhile, the residual target birth intensity $\gamma_{\mathrm{rema},k}$, its cardinality $M_{\mathrm{rema},k}$, and the residual target birth intensity label set $\aleph_{\mathrm{rema},k}$ can be obtained as

$$\gamma_{\mathrm{rema},k} = \sum_{i=1}^{M_{\mathrm{rema},k}} w_{\gamma,k}^i \mathcal{N}\left(x; m_{\gamma,k}^i, P_{\gamma,k}^i\right), \quad \ell_k^i \in \aleph_{\mathrm{rema},k}, \gamma_{\mathrm{rema},k} \subset \gamma_{\mathrm{init},k}, \tag{28}$$

$$\aleph_{\mathrm{rema},k} = \aleph_{\mathrm{init},k} - \aleph_{\mathrm{phd},k}, \tag{29}$$

$$\aleph_{\mathrm{phd},k} = \left\{ \ell_{\gamma,k}^i \,|\, w_{\max,k+1}^i < \sigma_\gamma, \ \forall i = 1:M_{R,k} \right\}, \tag{30}$$

$$\psi_{\mathrm{rema},k} = \left\{ i \,|\, w_{\max,k+1}^i < \sigma_\gamma, \ \forall i = 1:M_{R,k} \right\}, \tag{31}$$

$$w_{\max,k+1}^i = \underset{\forall j=1:M_{k+1}}{\arg\max} \left(w_{\gamma,k+1}^{i,j} \right), \text{ and} \tag{32}$$

$$M_{\mathrm{rema},k} = numel(\aleph_{\mathrm{rema},k}). \tag{33}$$

After using the PHD filter as clutter pre-filter step, a false initialized target birth intensity, relevant to most of the clutter measurements, is deleted from $\gamma_{\mathrm{init},k}$. Unfortunately, some clutter measurements still exist in the residual birth target intensity $\gamma_{\mathrm{rema},k}$, in that a few clutter measurements are close to each other between the successive time steps. To further eliminate such noises, the target velocity extent scheme is introduced, and the most possible target birth intensity can be ultimately approximated.

Assume that the velocity of a target is represented by parameter v_k at time k. The maximum velocity of the ith survival target between time 1 and k can be obtained as

$$v_{\max}^i = \arg\max\left(\left|v_1^i\right|, \ldots, \left|v_j^i\right|, \ldots, \left|v_k^i\right| \right), \ \forall j = 1:k, \tag{34}$$

where $|x|$ is the absolute value function.

Under the assumption that the maximum velocity of each target is similar, the maximum velocity of the ith target can be utilized as the maximum velocity of all targets, that is, $\upsilon_{\max} = v^i_{\max}$. Using the residual target birth intensity $\gamma_{\text{rema},k}$ and weight matrix W_m, the most probable target birth intensity $\gamma_{\text{bir},k}$, its cardinality $M_{\text{bir},k}$ and its label set $\aleph_{\text{bir},k}$ can be approximated as

$$\gamma_{\text{bir},k} = \sum_{i=1}^{M_{\text{bir},k}} w^i_{\gamma,k} \mathcal{N}\left(x; m^i_{\gamma,k}, P^i_{\gamma,k}\right), \quad \ell^i_k \in \aleph_{\text{bir},k}, \gamma_{\text{bir},k} \subset \gamma_{\text{rema},k}, \tag{35}$$

$$\aleph_{\text{bir},k} = \left\{\ell^i_{\gamma,k} | f^i_{\text{bir}} \equiv \text{true}, \ \ell^i_{\gamma,k} \in \gamma_{\text{rema},k}, \forall i \in \psi_{\text{rema},k}\right\}, \tag{36}$$

$$\mu = \varepsilon\sqrt{\begin{bmatrix}1 & 0\end{bmatrix} R_k}, \tag{37}$$

$$f^i_{\text{bir}} = \begin{cases} \text{false}, & \text{otherwise}, \\ \text{true}, & \upsilon_{\max} - \mu \le \left|v^{i,j}_{k+1}\right| \le \upsilon_{\max} + \mu, \end{cases} \tag{38}$$

$$v^{i,j}_{k+1} = \left\| z^j_{k+1} - H_k m^i_{\gamma,k} \right\|, \ z^j_{k+1} \in Z_{k+1}, \quad \forall j = 1{:}M_{k+1}, w^{i,j}_{\gamma,k+1} \ge \sigma_k, \ \text{and} \tag{39}$$

$$M_{\text{bir},k} = numel(\gamma_{\text{bir},k}), \tag{40}$$

where ε and $\|x\|$ denote a scaling factor and the Euclid distance function, respectively.

2. Measurement-Driven Gaussian Mixture PHD Scheme

The original GM-PHD filter updates all targets using each measurement in measurement set Z_k at time k. The measurement set is composed of survival-target originated measurements, birth-target originated measurements, and clutter measurements at each time step. Not only can the clutter measurements degrade the performance of the GM-PHD filter, but a misuse of the survival-target originated measurements or birth-target originated measurements can also disturb the precision of the target estimates. In this section, an improved measurement-driven scheme under the GM-PHD framework is proposed to improve the performance of the GM-PHD filter in terms of both reducing the number of clutter measurements and degrading the possibility of misusing the target-originated measurements in the update process of the PHD filter.

Based on the assumption that there are no spawned targets, the multi-target predicted intensity (6) can be approximated by

$$D_{k|k-1}(x) = D_{S,k|k-1}(x) + \gamma_k(x), \tag{41}$$

$$D_{S,k|k-1}(x) = \sum_{i=1}^{J_{k|k-1}} w^i_{k|k-1} \mathcal{N}\left(x; m^i_{S,k|k-1}, P^i_{S,k|k-1}\right), \ \text{and} \tag{42}$$

$$\gamma_k(x) = \sum_{j=1}^{J_{\gamma,k}} w^j_{\gamma,k} \mathcal{N}\left(x; m^j_{\gamma,k}, P^j_{\gamma,k}\right). \tag{43}$$

Given the predicted targets states and the newborn target intensity, the latest measurement set obtained at each time step is divided into the survival target measurement set, birth target measurement set, and clutter measurement set using a gating method. That is, when the newest measurement set Z_k and its cardinality M_k are obtained, according to (42), the survival target measurement set $Z_{S,k}$, composed of survival-target originated measurements, can be computed through (47). Similarly, based on (43), the birth target measurement set $Z_{\gamma,k}$, consisting of birth-target originated measurements, can also be approximated by (49).

$$U_{p,k} = \left\{d^{(1)}_{1,k}(z), \ldots, d^{(1)}_{t,k}(z), \ldots, d^{(i)}_{1,k}(z), \ldots, d^{(i)}_{t,k}(z) | \ z \in Z_k, \forall i = 1{:}J_{k|k-1}, \forall t = 1{:}M_k\right\}, \tag{44}$$

$$d^{(i)}_{t,k}(z) = \left(z - H_k m^i_{S,k|k-1}\right)^{\mathrm{T}} \left(S^i_{t,k}\right)^{-1} \left(z - H_k m^i_{S,k|k-1}\right), \tag{45}$$

$$S^i_{t,k} = H_k P^i_{S,k|k-1} H^{\mathrm{T}}_k + R_k, \tag{46}$$

$$Z_{S,k} = \left\{z : \arg\min_i \left(d^{(i)}_{t,k}(z)\right) | \ d^{(i)}_{t,k} \in U_{p,k}, z \in Z_k, \forall i = 1{:}J_{k|k-1}\right\}, \tag{47}$$

$$Z_{b,k} = Z_k - Z_{S,k}, \tag{48}$$

$$Z_{\gamma,k} = \left\{z : \left(z - H_k m^j_{\gamma,k}\right)^{\mathrm{T}} \left(S^j_{\gamma,k}\right)^{-1} \left(z - H_k m^j_{\gamma,k}\right) \le \eta, \ z \in Z_{b,k}, \forall j = 1{:}J_{\gamma,k}\right\}, \tag{49}$$

$$S^j_{\gamma,k} = H_k P^j_{\gamma,k} H^{\mathrm{T}}_k + R_k, \ \text{and} \tag{50}$$

$$\eta = -2\ln(1 - p_G) \quad \text{if } n_z = 2, \tag{51}$$

where the parameters η and p_G represent a gating threshold and the preset probability of target-originated measurements in the elliptical region, respectively, and n_z is the measurement dimension.

With the survival target measurement set $Z_{S,k}$ and the birth target measurement set $Z_{\gamma,k}$, the main steps of the measurement-driven GM-PHD algorithm can be described as follows.

Prediction: Given that the multi-target posterior intensity is represented by (5), and the Gaussian mixture survival target intensity is computed as

$$D_{S,k-1}(x) = \sum_{i=1}^{J_{k-1}} w^i_{k-1} \mathcal{N}\left(x; m^i_{S,k-1}, P^i_{S,k-1}\right), \tag{52}$$

at time k, the target birth intensity can be approximated by (35). The multi-target predicted intensity $D_{k|k-1}$ can then be obtained by

$$D_{k|k-1} = \sum_{i=1}^{J_{k|k-1}} w^i_{k|k-1} \mathcal{N}\left(x; m^i_{S,k|k-1}, P^i_{S,k|k-1}\right) + \sum_{j=1}^{M_{\text{bir},k}} w^j_{\gamma,k} \mathcal{N}\left(x; m^j_{\gamma,k}, P^j_{\gamma,k}\right), \tag{53}$$

$$w^i_{k|k-1} = p_{S,k} w^i_{k-1}, \quad (54)$$

$$m^i_{S,k|k-1} = F_{k-1} m^i_{S,k-1}, \quad \text{and} \quad (55)$$

$$P^i_{S,k|k-1} = Q_{k-1} + F_{k-1} P^i_{S,k-1} F^{\mathrm{T}}_{k-1}, \quad (56)$$

where $J_{k|k-1}$ is the predicted number of Gaussian components of the survival targets.

Measurement-driven: When the latest measurement set Z_k is available at time k, the survival target measurement set $Z_{S,k}$ and birth target measurement set $Z_{\gamma,k}$ are extracted by (47) and (49), respectively. Given sets $Z_{S,k}$ and $Z_{\gamma,k}$, a clutter measurement set $Z_{C,k}$ can be constructed using the remaining measurements, which have not been extracted in measurement set Z_k.

$$Z_{C,k} = Z_k - Z_{S,k} - Z_{\gamma,k}, \quad (57)$$

For the purpose of degrading the disturbance of clutter measurements in the target update process and achieving a lighter computational load, the clutter measurement set $Z_{C,k}$ is not used in the update step of the PHD filter.

Update: Given that the multi-target predicted intensity is computed using (53) at time k, the Gaussian-mixture multi-target posterior intensity can be approximated through (58), where the survival targets are updated using the survival target measurement set $Z_{S,k}$, and the newborn targets are updated using the birth target measurement set $Z_{\gamma,k}$.

$$D_k(x) = D_{S,k}(x) + D_{\gamma,k}(x), \quad (58)$$

$$D_{S,k}(x) = (1 - p_{D,k}) D_{S,k|k-1}(x) + \sum_{z \in Z_{S,k}} \sum_{i=1}^{J_{S,k|k-1}} w^{(i)}_{S,k}(z)\, \mathcal{N}\left(x; m^{(i)}_{S,k|k}(z), P^{(i)}_{S,k|k}\right), \quad (59)$$

$$D_{\gamma,k}(x) = (1 - p_{D,k}) \gamma_k(x) + \sum_{z \in Z_{\gamma,k}} \sum_{j=1}^{J_{\gamma,k}} w^{(j)}_{\gamma,k}(z)\, \mathcal{N}\left(x; m^{(j)}_{\gamma,k|k}(z), P^{(j)}_{\gamma,k|k}\right), \quad (60)$$

$$w^{(\tau)}_{\alpha,k}(z) = \frac{p_{D,k} w^{(\tau)}_{\alpha,k|k-1} q^{(\tau)}_k(z)}{\mathcal{K}_k(z) + \vartheta(z)}, \quad \tau \in \{i, j\}, \alpha \in \{S, \gamma\}, \quad (61)$$

$$q^{(\tau)}_k(z) = \mathcal{N}\left(z; H_k m^{(\tau)}_{\alpha,k|k-1}, H_k P^{(\tau)}_{\alpha,k|k-1} H^{\mathrm{T}}_k + R_k\right), \quad (62)$$

$$\vartheta(z) = p_{D,k} \sum\nolimits_{\varphi=1}^{\varepsilon} w^{(\varphi)}_{\alpha,k|k-1} q^{(\tau)}_k(z), \ \varepsilon \in \left\{J_{S,k|k-1}, J_{\gamma,k}\right\}, \quad (63)$$

$$m^{(\tau)}_{\alpha,k|k}(z) = m^{(\tau)}_{\alpha,k|k-1} + K^{(\tau)}_{\alpha,k}\left(z - H_k m^{(\tau)}_{\alpha,k|k-1}\right), \quad (64)$$

$$P^{(\tau)}_{\alpha,k|k} = \left[I - K^{(\tau)}_{\alpha,k} H_k\right] P^{(\tau)}_{\alpha,k|k-1}, \quad \text{and} \quad (65)$$

$$K^{(\tau)}_{\alpha,k} = P^{(\tau)}_{\alpha,k|k-1} H^{\mathrm{T}}_k \left(H_k P^{(\tau)}_{\alpha,k|k-1} H^{T}_k + R_k\right)^{-1}, \quad (66)$$

where $K^{(\tau)}_{\alpha,k}$ is the Kalman gain of the individual target at time step k.

IV. Simulation Results

To validate the effectiveness and robustness of the proposed algorithm, it is compared with the ABI-GM-PHD filter [17] by tracking a variable number of targets under several different multi-target tracking scenarios. Suppose a simulation scenario in which the targets move in a two-dimensional area with [–1,000, 1,000](m) × [–1,000, 1,000](m). At time k, the state vector of each target $x_k = [x_{1,k}, x_{2,k}, x_{3,k}, x_{4,k}]^{\mathrm{T}}$ is composed of both the position $[x_{1,k}, x_{2,k}]^{\mathrm{T}}$ and velocity $[x_{3,k}, x_{4,k}]^{\mathrm{T}}$ of the target. The sampling interval is configured using 1, and all scenarios are simulated 100 times. Each target moves following a dynamic model as described in (67), and the sensor generates measurements according to the measurement model using (68).

$$x_k = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_{k-1} + \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{bmatrix} w_k, \quad \text{and} \quad (67)$$

$$z_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x_k + v_k, \quad (68)$$

where the process noise w_k and measurement noise v_k are zero mean Gaussian white noise with covariance matrix Q = *diag*([0.4, 0.4]) and R = *diag*([225, 225]), respectively.

The probabilities of detection and target survival are set to $p_{D,k} = 0.99$ and $p_{S,k} = 0.99$, respectively. The thresholds of the pruning and merging scheme of the Gaussian components are the same as those adopted in [6]. The thresholds $\sigma_\gamma = 0.5$ and $\varepsilon = 3$ are selected empirically during the experiments.

For each simulation-tracking scenario, 100 Monte Carlo runs are performed, where the mean number of target estimate (NTE) errors [27] and the optimal sub-pattern assignment (OSPA) [28] are used to assess the performance of the proposed algorithm.

$$\text{NTE}\ \{X_k, \hat{X}_k\} = E\left\{\left|\hat{X}_k\right| - \left|X_k\right|\right\}, \quad \text{and} \quad (69)$$

$$\text{OSPA}_{p,c}(X_k, \hat{X}_k) = \left(\frac{1}{\left|\hat{X}_k\right|} \min_{\pi \in \Pi_{|\hat{X}_k|}} \sum\nolimits_{i=1}^{|X_k|} \left(d_c\left(x^i, \hat{x}^{\pi(i)}\right)\right)^p + c^p \times \left(\left|\hat{X}_k\right| - \left|X_k\right|\right)\right)^{\frac{1}{p}}, \quad (70)$$

http://dx.doi.org/10.4218/etrij.16.0116.0040

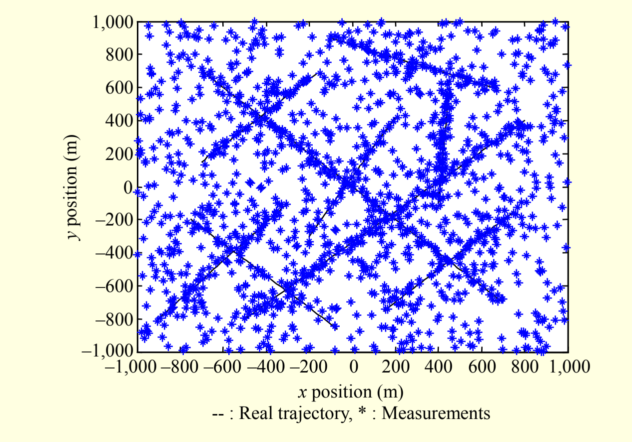

Fig. 1. Real trajectories of nine targets and measurements.

where the true target set and estimated target set are denoted by X_k and $\hat{X}_k$, respectively. The cut-off parameter and order parameter for the OSPA distance are set to $c = 100$ and $p = 2$, respectively.

Scenario: As shown in Fig. 1, nine targets are tracked in the current tracking scenario, and the clutter rate is modeled as a Poisson RFS with the mean $\lambda_c = 10 \times 10^{-6} m^{-2}$. The initial states of targets 1 and 2 are set to $m_S^{(1)} = [-700, 700, 0, 0]^{\mathrm{T}}$ and $m_S^{(2)} = [-500, -800, 0, 0]^{\mathrm{T}}$, and the two targets persist from 1 to 100 s in the surveillance region. The other seven targets, with unknown target intensity at each time step, appear and disappear at random times of between 1 and 100 s.

The comparison results of the two algorithms are shown in Fig. 2, which demonstrate that the proposed algorithm achieves a better level of performance in the unknown target birth intensity scenario. From Fig. 2(a), it can be seen that there are some large peaks in the OSPA distance obtained by the proposed algorithm and ABI-GM-PHD filter because both algorithms cannot immediately detect newborn targets at the moment when the birth targets appear. However, the proposed algorithm achieves a better overall OSPA distance than that of the ABI-GM-PHD filter because the proposed algorithm can better estimate the newborn target intensity, and eliminate the disturbance of target-originated measurements in the update step. The NTE of the two algorithms shown in Fig. 2(b) also illustrates the better performance of the proposed algorithm, where the mean number of target estimate errors of the proposed algorithm is relatively low.

To evaluate the effectiveness of the proposed algorithm in a complex tracking scenario, varied clutter rates are considered to compare the performance of the different algorithms. The clutter rate λ_c varies from $1 \times 10^{-6} m^{-2}$ to $30 \times 10^{-6} m^{-2}$ with an interval of $5 \times 10^{-6} m^{-2}$ in each clutter rate experiment, whereas the detection probability $p_{D,k}$ is set to 0.99. A

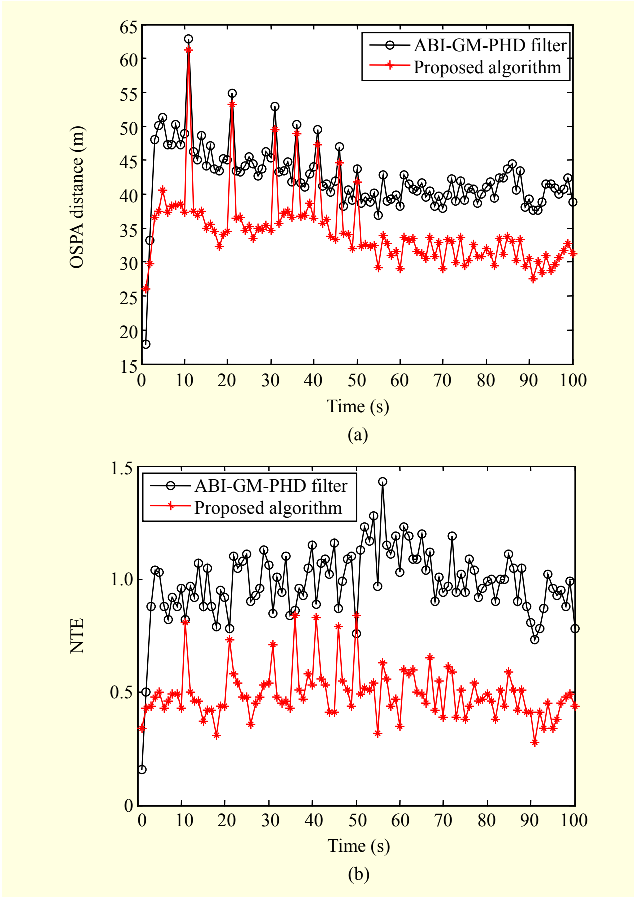

Fig. 2. Comparison between different algorithms: (a) OSPA distance and (b) NTE.

performance comparison of the proposed algorithm and the ABI-GM-PHD filter is given in Fig. 3. The simulation results show that the efficiency of the two algorithms declines as the clutter rate increases. However, both the NTE and the running time obtained from the proposed algorithm maintain a lower increase compared with the ABI-GM-PHD filter because the proposed target birth intensity approach can accurately estimate the newborn target intensity before each iteration of the PHD filter, and the improved measurement-driven scheme degrades the disturbance of the measurements when updating both the survival and newborn targets. Owing to the fact that most clutter measurements are eliminated from the measurement set at each time step, and do not participate in each update step of the PHD filter, the running time of the proposed algorithm maintains a very low increase in high clutter rate scenarios.

The performance comparison of the two algorithms was also studied from the aspect of various detection probabilities. The detection probability is initially set to $p_{D,k} = 0.8$, and terminated at $p_{D,k} = 1$, where the experimental interval of the detection probability is 0.5. Moreover, the clutter rate $\lambda_c = 10 \times 10^{-6} m^{-2}$ is fixed for each detection probability experiment. Figure 4

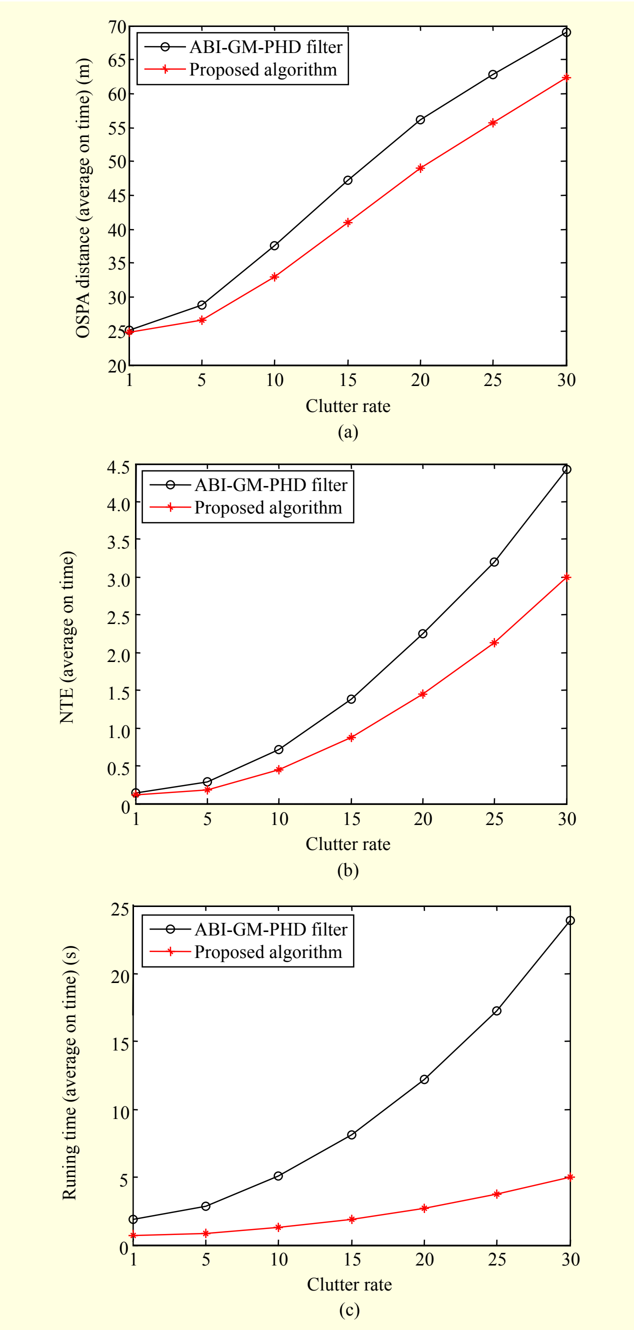

Fig. 3. Results of different algorithms for varied clutter rates with detection probability $p_{D,k}$ = 0.99: (a) OSPA distance, (b) NTE, and (c) running time.

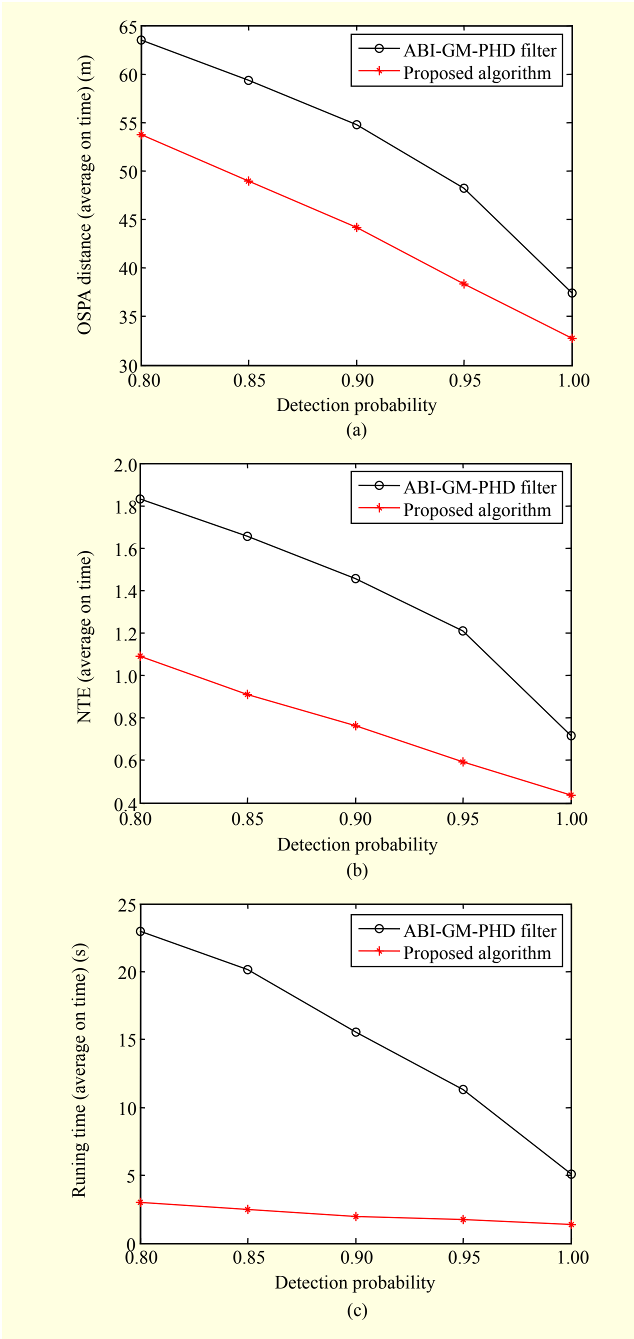

Fig. 4. Results of different algorithms for varied detection probabilities with clutter rate $\lambda_c = 10 \times 10^{-6}\ m^{-2}$: (a) OSPA distance, (b) NTE, and (c) running time.

illustrates the performance comparison between the proposed algorithm and the ABI-GM-PHD filter, where the performance of both algorithms has been improved to a certain extent as the detection probability increases. Not only does the proposed algorithm clearly achieve a lower OSPA distance, its NTE also decreases to the lowest level.

In addition, the running time obtained from the proposed algorithm remains at a low level, far below than that of the ABI-GM-PHD filter. From the results of various detection probability experiments, it can be concluded that the proposed approach can be applied to track the unknown prior target birth intensity under relatively low-detection probability scenarios.

For a better study on the proposed algorithm, different measurement noises were applied. Each measurement noise varied from a range of 5 to 30 m with an experimental interval

http://dx.doi.org/10.4218/etrij.16.0116.0040

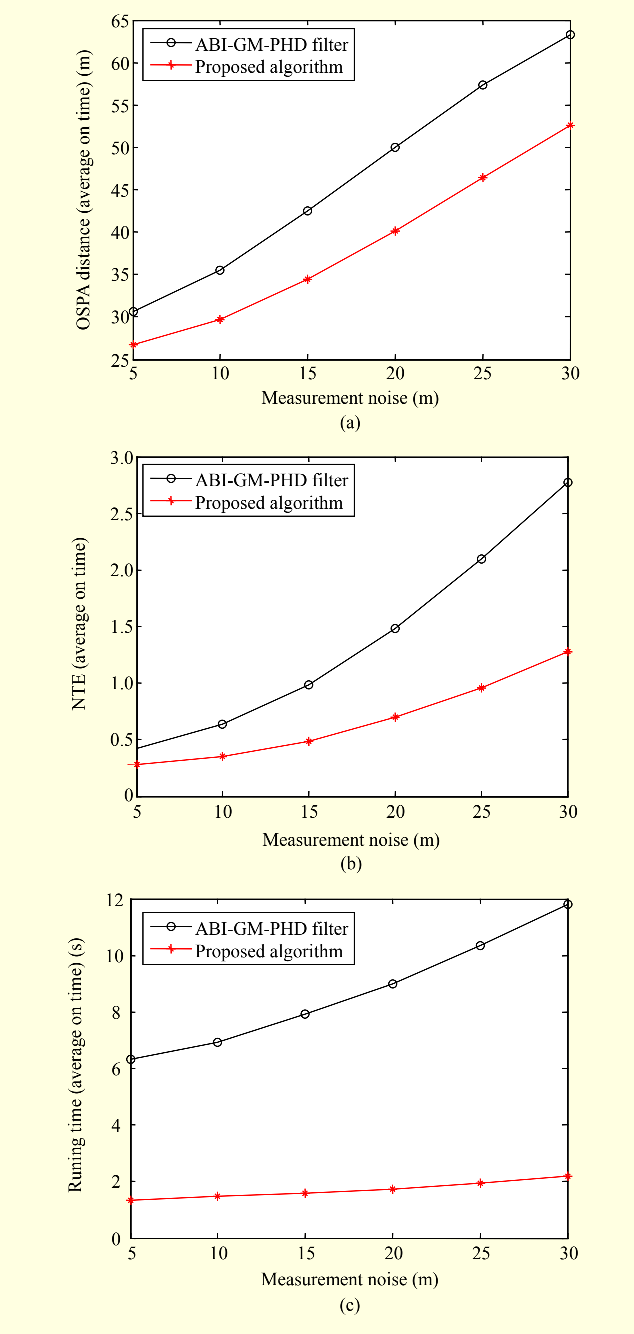

Fig. 5. Results of different algorithms for varied measurement noises with clutter rate $\lambda_c = 10 \times 10^{-6} m^{-2}$ and detection probability $p_{D,k} = 0.99$: (a) OSPA distance, (b) NTE, and (c) running time.

of 5 m, where the clutter rate and detection probability were set to $\lambda_c = 10 \times 10^{-6} m^{-2}$ and $p_{D,k}$ = 0.99, respectively, and remained unchanged in each measurement noise simulation. Figure 5 illustrates the comparison results of the proposed algorithm and the ABI-GM-PHD filter under different measurement noises. As each measurement noise increases, the efficiency of the two algorithms decreases. However, it is clear that both the OSPA distance and NTE of the proposed algorithm are lower than those of the ABI-GM-PHD filter, and that the increment speed of the NTE of the proposed algorithm changes more slowly than that of the ABI-GM-PHD filter. In particular, the running time obtained from the proposed algorithm remains substantially unchanged, whereas the running time of the ABI-GM-PHD filter has a significant increase.

V. Conclusion

In this paper, within the framework of-the Gaussian mixture probability hypothesis density, an improved multi-target tracking algorithm was proposed to cope with a multi-target tracking system where prior knowledge of the target birth intensity is unknown. The proposed algorithm is composed of a newborn target intensity scheme and an improved measurement-driven scheme. For newborn target intensity estimation, both a PHD pre-filter technique and a target velocity extent method are employed to recursively estimate the target birth intensity at each time step. Specifically, the PHD filter is first adopted as a clutter pre-filter to remove possible clutter from the latest measurement set at the current time step, and the target velocity extent method is then utilized to obtain the most likely newborn target measurements, which are used to approximate the ultimate target birth intensity. To reduce the computational burden of the original PHD filter and improve the estimation accuracy, an improved measurement-driven scheme was proposed, where the measurement set at each time step is divided into the survival target measurement set, birth target measurement set, and clutter measurement set. In the update step of the proposed algorithm, the survival targets are updated using the survival target measurement set, newborn targets are updated using the target birth measurement set, and the clutter measurement set is prohibited to participate in the update process. Simulation results demonstrate that the proposed algorithm can effectively achieve multi-target tracking in tracking systems with unknown newborn target intensity, and that it has a strong robustness.

As future work, the proposed algorithm will be utilized to estimate the clutter measurement intensity, and a more complicated environment, that is, spawning targets, will also be considered to verify the performance of the proposed algorithm.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (No. 61305017), the Natural Science Foundation of Jiangsu Province (No. BK20130154), the

Graduate Innovation Foundation of Jiangsu Province (No. KYLX_1122), the 111 Project (No. B12018), and PAPD of Jiangsu Higher Education Institutions.

References

[1] R. Mahler, *Statistical Multisource-Multitarget Information Fusion*, Norwood, MA, USA: Artech House, 2007, pp. 185–195.

[2] R.P.S. Mahler, "Multi-target Bayes Filtering via First-Order Multi-target Moments," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 4, Oct. 2003, pp. 1152–1178.

[3] R. Mahler, "PHD Filters of Higher Order in Target Number," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 43, no. 4, Oct. 2007, pp. 1523–1543.

[4] B.T. Vo, B.N. Vo, and A. Cantoni, "The Cardinality Balanced Multi-target Multi-bernoulli Filter and its Implementations," *IEEE Trans. Signal Process.*, vol. 57, no. 2, 2009, pp. 409–423.

[5] B.N. Vo, S. Singh, and A. Doucet, "Sequential Monte Carlo Implementation of the PHD Filter for Multi-target Tracking," *Proc. Int. Conf. Inf. Fusion*, Qeensland, Australia, July 8–11, 2003, pp. 792–799.

[6] B.N. Vo, and W.K. Ma, "The Gaussian Mixture Probability Hypothesis Density Filter," *IEEE Trans. Signal Process.*, vol. 54, no. 11, Nov. 2006, pp. 4091–4104.

[7] S. Reuter et al., "The Labeled Multi-bernoulli Filter," *IEEE Trans. Signal Process.*, vol. 62, no. 12, May 2014, pp. 3246–3260.

[8] B.N. Vo, B.T. Vo, and D. Phung, "Labeled Random Finite Sets and the Bayes Multi-target Tracking Filter," *IEEE Trans. Signal Process.*, vol. 62, no. 24, Oct. 2014, pp. 6554–6567.

[9] F. Papi et al., "Generalized Labeled Multi-bernoulli Approximation of Multi-object Densities," *IEEE Trans. Signal Process.*, vol. 63, no. 20, Oct. 2015, pp. 5487–5497.

[10] X. Tang et al., "A Multiple-Detection Probability Hypothesis Density Filter," *IEEE Trans. Signal Process.*, vol. 63, no. 8, Apr. 2015, pp. 2007–2019.

[11] H.Q. Zhang et al., "A GM-PHD Algorithm for Multiple Target Tracking Based on False Alarm Detection with Irregular Window," *Signal Process.*, vol. 120, Mar. 2016, pp. 537–552.

[12] X.L. Zhou et al., "Tracking Multiple Video Targets with an Improved GM-PHD Tracker," *Sensors*, vol. 15, no. 12, Dec. 2015, pp. 30240–30260.

[13] C.M. Shi et al., "Micro-Object Motion Tracking Based on the Probability Hypothesis Density Particle Tracker," *J. Math. Biol.*, vol. 72, no. 5, Apr. 2016, pp. 1225–1254.

[14] F. Papi and D.Y. Kim, "A Particle Multi-target Tracker for Superpositional Measurements Using Labeled Random Finite Sets," *IEEE Trans. Signal Process.*, vol. 63, no. 16, Aug. 2015, pp. 4348–4358.

[15] A.K. Gostar, R. Hoseinnezhad, and A. Bab-Hadiashar, "Multi-bernoulli Sensor-Selection for Multi-target Tracking with Unknown Clutter and Detection Profiles," *Signal Process.*, vol. 119, Feb. 2016, pp. 28–42.

[16] B. Ristic, D. Clark, and B.N. Vo, "Improved SMC Implementation of the PHD Filter," *Conf. Inf. Fusion*, Edinburgh, Scotland, July 26–29, 2010, pp. 1–8.

[17] B. Ristic et al., "Adaptive Target Birth Intensity for PHD and CPHD Filters," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 2, Apr. 2012, pp. 1656–1668.

[18] Y. Wang et al., "Detection-Guided Multi-target Bayesian Filter," *Signal Process.*, vol. 92, no. 2, Feb. 2012, pp. 564–574.

[19] H.J. Zhang et al., "A GM-PHD Filter for New Appearing Targets Tracking," *Int. Conf. Image Signal Process.*, Hangzhou, China, Dec. 16–18, 2013, pp. 1153–1159.

[20] X.L. Zhou et al., "GM-PHD-Based Multi-target Visual Tracking Using Entropy Distribution and Game Theory," *IEEE Trans. Ind. Inf.*, vol. 10, no. 2, May 2014, pp. 1064–1076.

[21] X.L. Zhou, Y.F. Li, and B.W. He, "Entropy Distribution and Coverage Rate-Based Birth Intensity Estimation in GM-PHD Filter for Multi-target Visual tracking," *Signal Process.*, vol. 94, Jan. 2014, pp. 650–660.

[22] K. Panta, B.N. Vo, and S. Singh, "Novel Data Association Schemes for the Probability Hypothesis Density Filter," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 43, no. 2, 2007, pp. 556–570.

[23] M. Mallick and B.L. Scala, "Comparison of Single-Point and Two-Point Difference Track Initiation Algorithm Using Position Measurements," *Acta Autom. Sinica*, vol. 34, no. 3, Mar. 2008, pp. 258–265.

[24] D. Musicki and T.L. Song, "Track Initialization: Prior Target Velocity and Acceleration Moments," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 49, no. 1, Jan. 2013, pp. 665–670.

[25] T.Y. Jiang et al., "An Efficient Measurement-Driven Sequential Monte Carlo Multi-bernoulli Filter for Multi-target Filtering," *J. Zhejiang Univ. Sci. C*, vol. 15, no. 6, June 2014, pp. 445–457.

[26] Y.M. Zheng et al., "An Efficient Data-Driven Particle PHD Filter for Multitarget Tracking," *IEEE Trans. Ind. Inf.*, vol. 9, no. 4, Nov. 2013, pp. 2318–2326.

[27] M. Yazdian-Dehkordi, Z. Azimifar, and M.A. Masnadi-Shirazi, "Competitive Gaussian Mixture Probability Hypothesis Density Filter for Multiple Target Tracking in the Presence of Ambiguity and Occlusion," *IET Radar, Sonar Navig.*, vol. 6, no. 4, Apr. 2012, pp. 251–262.

[28] D. Schuhmacher, B.T. Vo, and B.N. Vo, "A Consistent Metric for Performance Evaluation of Multi-object Filters," *IEEE Trans. Signal Process.*, vol. 56, no. 8, Aug. 2008, pp. 3447–3457.

Huanqing Zhang received his MS degree in computer science and engineering from Shandong Normal University, Jinan, China in 2013. He is currently pursuing a PhD from the School of Internet of Things Engineering at Jiangnan University, Jiangsu, China. His research interests include target tracking, statistical signal processing, and multi-sensor fusion.

Hongwei Ge received his MS degree in computer science from Nanjing University of Aeronautics and Astronautics, China in 1992, and his PhD in control engineering from Jiangnan University in 2008. He is currently a professor and PhD supervisor in the School of Internet of Things, Jiangnan University. His research interests include artificial intelligence, machine learning, and pattern recognition and their applications.

Jinlong Yang received his MS degree in circuits and systems from Northwest Normal University, Lanzhou, China in 2009, and his PhD in pattern recognition and intelligent systems from Xidian University, Xi'an, China in 2012. He is an associate professor in Jiangnan University. His research interests include target tracking, information fusion, and signal processing.