

Improved Method for Determining the Height of Center of Gravity of Agricultural Tractors

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Abstract

Purpose: This study aimed to improve the method for determining the position of the center of gravity for agricultural tractors. **Methods:** The proposed method uses trigonometric functions and coordinate transformation. Data were measured according to the ISO 789-6 test procedures for the center of gravity of agricultural tractors. The height calculated using the proposed method was compared with that determined from an AutoCAD drawing. To find the center of gravity of the tractor, the algorithm for finding the intersection of the two lines was used. **Results:** The vertical height from the ground to the center of gravity is 682.06 mm. The vertical coordinates obtained from the calculation and the drawing were the same. **Conclusions:** The developed method uses trigonometric and polar coordinate transformation. The method was compared and verified with the AutoCAD drawing results. The results indicate that users can apply this developed method instead of the plotting method which is an inconvenient and time-consuming. Further, users can program Microsoft Excel to easily determine the vertical coordinate. In addition, researchers will propose this method to the ISO as a standard method for determining the center of gravity in accordance with ISO 789-6.

Keywords: Agricultural tractor, Center of gravity, Test procedure

Introduction

Agricultural tractors are basic equipment used for crop production and transportation. An increasing number of tractors are being used for agriculture. RDA (2013) reports that 26.3% of tractor accidents occur by rollover. It is important to determine the location of the center of gravity to evaluate the safety of tractors in order to prevent such rollover accidents. ISO 16231-2 specifies test procedures for determining the static stability of self-propelled agricultural machinery except for tractors to help prevent rollover accidents (ASABE, 2015).

Steinbruegge (1969) proposed a weighing method to determine the center of gravity of tractors that has been used as the ASABE standard with two conditions: (1) the

tires are stiff and (2) the center of mass remains unchanged if the front axles are raised. The JIS standard is similar to the ASABE standard but it uses a different equation for calculating the vertical coordinate. This JIS standard has been used to determine the center of gravity of tractors in Japan. ISO 789-6 has been widely used internationally since 1982. RDA (2012) reported that the ASABE and JIS standards ensure safety; however, they adversely affect tractor exports because the resulting vertical coordinates are higher than those determined by the ISO standard. In Korea, KS B ISO 789-6 was established as a national standard in accordance with ISO 789-6 in 2001.

ISO 789-6, the suspension and ground reaction method, is a simple and practical method that has been used internationally and as an OECD test code. The horizontal fore-and-aft coordinate and lateral coordinate in the horizontal plane are determined by an equation. However, this procedure is an inconvenient and time-consuming

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method for determining the vertical coordinate. Therefore, researchers generally plot the coordinates on the scribing board fixed to the tractor.

This study aimed to develop an equation to determine the vertical coordinate that is consistent with ISO 789-6.

Materials and Methods

Definitions of terms

The terms used in this study are defined as follows: (1) Vertical coordinate is the vertical distance of the center of gravity from the horizontal reference plane. (2) Horizontal reference plane is the ground level.

Test procedures

1) As shown in Figure 1, front-axle load (F_f) and rear-axle load (F_r), heights of the centers of the front axle (R_f) and rear axle (R_r), wheelbase (L), and distance between the centers of the front and rear axles (L') were measured. The following equations were derived from Figure 1.

$$\Delta r = R_r - R_f, \quad (1)$$

where

Δr = height difference between the centers of the axles,
 R_f = static load height of the centers of front axle, and
 R_r = static load height of the centers of rear axle.

$$L' = \sqrt{L^2 + \Delta r^2}, \quad (2)$$

where

L' = difference from center of front axle to center of rear axle, and

L = wheelbase.

$$\theta = \tan^{-1}\left(\frac{\Delta r}{L}\right), \quad (3)$$

where

θ = angle between the centers of the axles.

2) As shown in Figure 2, the rear axle was raised. The front-axle load (F_{f1}), heights of the centers of the front axle (R_{f1}) and rear axle (R_{r1}), horizontal distance between the centers of the front and rear axles (L_1),

and height of the support (n_1) were measured. The horizontal distance (c_1) from the center of gravity to the rear axle was calculated.

$$\Delta r_f = R_f - R_{f1}, \quad (4)$$

where

Δr_f = amount of compression of the front tires,

R_f = static load height of the centers of front axle, and

R_{f1} = height of the centers of front axle from ground when rear axle is in raised position.

$$c_1 = \frac{L_1 \times F_{f1}}{W}, \quad (5)$$

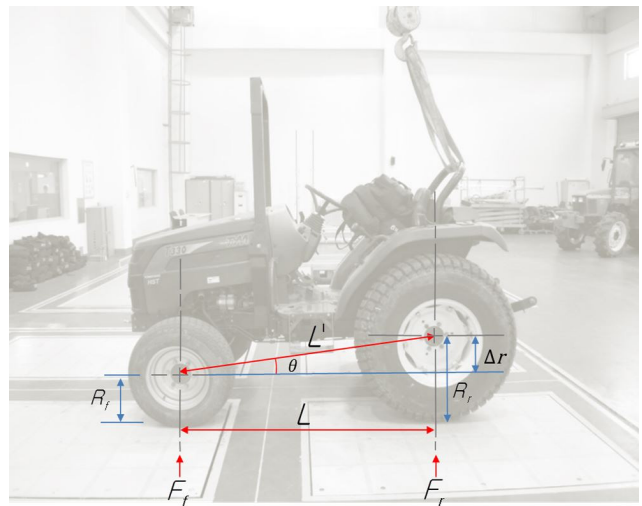


Figure 1. Determination of the location of the center of gravity on an unsuspended agricultural tractor.

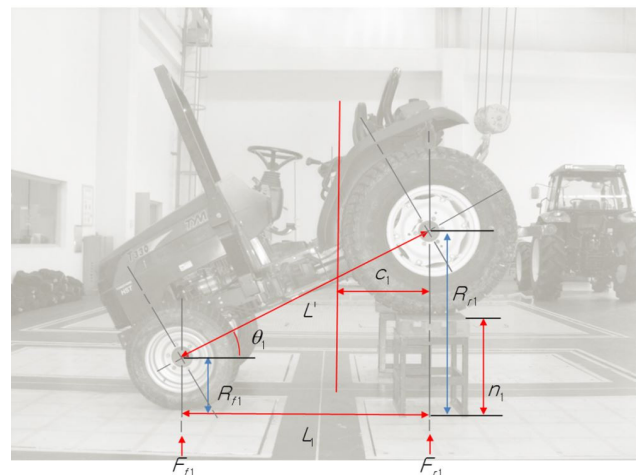


Figure 2. Determination of the location of the center of gravity on a suspended rear axle.

where

$$L_1 = L' \cos \theta_1,$$

$$\theta_1 = \sin^{-1} \left(\frac{R_{f1} - R_{r1}}{L'} \right),$$

θ_r = rotated angle ($\theta_1 - \theta$), and

W = the weight of tractor in kilograms.

3) As shown in Figure 3, the forward axle was raised. The rear-axle load (F_{r2}), heights of the centers of the front axle (R_{f2}) and rear axle (R_{r2}), horizontal distance between the centers of the front and rear axles (L_2), and height of the support (n_2) were measured. The horizontal distance (c_2) from the center of gravity to the front axle was calculated.

$$\Delta r_r = R_r - R_{r2}, \quad (6)$$

where

Δr_r = amount of compression of the rear tires,

R_r = static load height of the centers of rear axle, and

R_{r2} = height of the centers of rear axle from ground when front axle is in raised position.

$$c_2 = \frac{L_2 \times F_{r2}}{W}, \quad (7)$$

where

$$L_2 = L' \cos \theta_2,$$

$$\theta_2 = \sin^{-1} \left(\frac{R_{f2} - R_{r2}}{L'} \right),$$

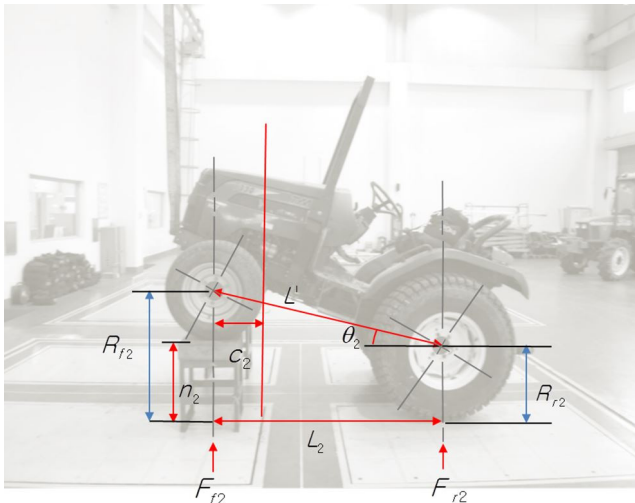


Figure 3. Determination of the location of the center of gravity on a suspended front axle.

θ_f = rotated angle ($\theta_2 + \theta$), and

W = the weight of tractor in kilograms.

Procedure for verifying the equation using AutoCAD drawing

The compression of the tires is negligible when compared with the radius of the tire. Therefore, it is not considered in this procedure. A horizontal line called the horizontal reference plane is drawn. The first circle, the radius of which is the height of the center of the front axle from the ground level, is drawn; and the second circle, the radius of which is the height of the center of the rear axle, is drawn horizontally at a wheelbase distance and vertically at $R_r - R_f$ distance from the first circle. The first and second circles describe the front and rear wheels, respectively. A circle identical to the first circle was drawn horizontally at L_1 distance and vertically at $R_{r1} + \Delta r_f$ distance from the center of the rear wheel; and a vertical line was drawn horizontally at c_1 distance from the center of the rear wheel. The vertical line was rotated based on the rear wheel until the two circles coincided. A circle identical to the second circle is drawn horizontally at L_2 distance and vertically at $R_{f2} + \Delta r_r$ distance from the center of the front wheel; and a vertical line is drawn horizontally at c_2 distance from the center of the front wheel. The vertical line is rotated based on the front wheel until two the circles coincide. The point of intersection of the two lines is found, and the distance from this point to the horizontal reference plane is taken as the vertical coordinate.

Algorithm for finding point of intersection of two lines

In general, a line has two coordinates. As shown in Figure 4, two lines have four coordinates. The method for finding the point of intersection of two lines is defined by Equation 8.

$$\begin{aligned} X &= \frac{Y_{10} - Y_{20}}{\left(\frac{Y_b - Y_a}{X_b - X_a} - \frac{Y_d - Y_c}{X_d - X_c} \right)}, \\ Y &= \frac{Y_d - Y_c}{X_d - X_c} \times \frac{Y_{10} - Y_{20}}{\left(\frac{Y_b - Y_a}{X_b - X_a} - \frac{Y_d - Y_c}{X_d - X_c} \right)} + Y_{10}, \end{aligned} \quad (8)$$

where $(X_a, Y_a), (Y_b, Y_b)$ = Selecting two points on Line A in

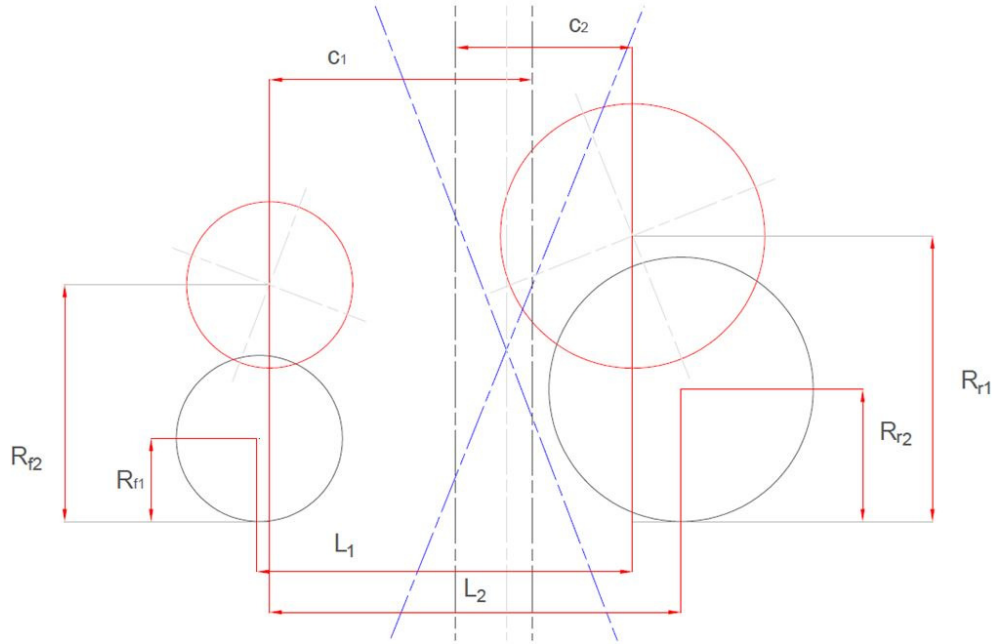


Figure 4. Verification of the location of the center of gravity using AutoCAD drawing.

raised position of front axle (rectangular coordinates), (X_c, Y_c) , (X_d, Y_d) = Selecting two points on Line B in raised position of rear axle (rectangular coordinates), $Y_{10} = Y_c - \frac{Y_d - Y_c}{X_d - X_c} X_c$ and $Y_{20} = Y_b - \frac{Y_b - Y_a}{X_b - X_a} X_b$.

Results and Discussion

Polar coordinate transformation

Two coordinates, $(L_1 - c_1, L)$ and $(L_1 - c_1, -L)$, were generated and transformed into polar coordinates when the rear axle was raised. The polar coordinates are described in terms of the radius ($r_{pr} = \sqrt{(L_1 - c_1)^2 + L^2}$) and orientation ($\alpha_r = \tan^{-1} \frac{L_1 - c_1}{L}$) from the center of the front axle. Therefore, the generated polar coordinates are $(X_d = r_{pr} \cos \alpha_r, Y_d = r_{pr} \sin \alpha_r)$ and $(X_c = r_{pr} \cos \alpha_r, Y_c = -r_{pr} \sin \alpha_r)$. The rear axle was rotated by θ_r degrees based on the center of the front axle, and the polar coordinates were transformed into $(X_d = r_{pr} \cos(\alpha_r - \theta_r), Y_d = r_{pr} \sin(\alpha_r - \theta_r))$, and $(X_c = r_{pr} \cos(\alpha_r + \theta_r), Y_c = -r_{pr} \sin(\alpha_r + \theta_r))$. The origin of the polar coordinates was moved to the center of the rear wheel when not raised, and the polar coordinates were transformed into $(X_c = r_{pr} \cos(\alpha_r + \theta_r), Y_c = -r_{pr} \sin(\alpha_r + \theta_r) + R_f)$ and $(X_d = r_{pr} \cos(\alpha_r -$

$\theta_r), Y_d = r_{pr} \sin(\alpha_r - \theta_r) + R_f)$.

Two coordinates, $(L_2 - c_2, L)$ and $(L_2 - c_2, -L)$, were generated and transformed into polar coordinates when the rear wheels were raised. The polar coordinates are described in terms of the radius ($r_{pr} = \sqrt{(L_2 - c_2)^2 + L^2}$) and orientation ($\alpha_f = \tan^{-1} \frac{L_2 - c_2}{L}$). Therefore, the generated polar coordinates are $(X_a = -r_{pf} \cos \alpha_f, Y_a = r_{pf} \sin \alpha_f)$ and $(X_b = -r_{pf} \cos \alpha_f, Y_b = -r_{pf} \sin \alpha_f)$. The front axle was rotated by θ_f degree based on the center of the rear axle, and the polar coordinates were transformed into $(X_a = r_{pf} \cos(\alpha_f - \theta_f), Y_a = r_{pf} \sin(\alpha_f - \theta_f))$ and $(X_b = -r_{pf} \cos(\alpha_f + \theta_f), Y_b = -r_{pf} \sin(\alpha_f + \theta_f))$. The origin of the polar coordinates was moved to the center of the front wheel when not raised, and the polar coordinates were transformed into $(X_a = -r_{pf} \cos(\alpha_f - \theta_f) + L, Y_a = r_{pf} \sin(\alpha_f - \theta_f) + R_r)$ and $(X_b = -r_{pf} \cos(\alpha_f + \theta_f) + L, Y_b = -r_{pf} \sin(\alpha_f + \theta_f) + R_r)$.

Verification of calculation method using AutoCAD drawing

The developed calculation method was verified using AutoCAD and test report data from FACT(The Foundation of Agri. Tech, Commercialization and Transfer) (RDA, 2012). All data are listed in the following tables to calculate the height of the center of gravity for agricultural tractors.

Table 1. Input data for calculation of the center of gravity of agricultural tractors

Data description	Symbol	Value	Unit
The mass of tractor	W	1479	kg
Wheel base	L	1675	mm
Static load radius of front axle	R_f	330	mm
Height of center of front axle from ground when rear axle is in raised position	R_{f1}	328	mm
Height of center of front axle from the ground in raised position	R_{f2}	938	mm
Static load radius of rear axle	R_r	525	mm
Height of center of rear axle from ground when rear axle is in raised position	R_{r1}	1132	mm
Height of center of rear axle from the ground when front axle is in raised position	R_{r2}	523	mm
Load on front axle	F_f	632	kg
Load on front axle in rear axle raised without support weight	F_{f1}	701	kg
Load on rear axle in front axle raised without support weight	F_{f2}	943	kg

Table 2. Calculation of the center of gravity of agricultural tractors

Data description	Symbol	Unit	Calculation	Result
Vertical distance from the center of the front axle and the center of the rear axle	Δr	mm	$R_r - R_f$	195
Distance between the center of the front axle and the center of the rear axle	L'	mm	$\sqrt{L^2 + \Delta r^2}$	1686.31
Angle between the center of the front axle and the center of the rear axle	θ	rad	$\tan^{-1} \frac{\Delta r}{L}$	0.116
Angle between the center of the front axle and the center of the rear axle	θ_1	rad	$\sin^{-1} \frac{R_{f1} - R_{r1}}{L}$	0.497
Raised angle of rear axle	θ_r	rad	$\theta_1 - \theta$	0.381
Horizontal distance from the center of rear axle to the center of front axle	L_1	mm	$L \cos \theta_1$	1482.31
Horizontal distance from the center of rear axle to the center of gravity	c_1	mm	$\frac{F_{f1} \times L_1}{W}$	702.57
Horizontal distance from the center of front axle to the center of gravity	t_1	mm	$L_1 - c_1$	779.74
Angle between the center of the front axle and the center of the rear axle	θ_2	rad	$\sin^{-1} \left(\frac{R_{f2} - R_{r2}}{L} \right)$	0.249
Raised angle of front axle	θ_f	rad	$\theta + \theta_2$	0.365
Horizontal distance from the center of rear axle to the center of front axle	L_2	mm	$L \cos \theta_2$	1634.45
Horizontal distance from the center of front axle to the center of gravity	c_2	mm	$\frac{F_{r2} \times L_2}{W}$	1042.11
Horizontal distance from the center of rear axle to the center of gravity	t_2	mm	$L_2 - c_2$	592.34
Selecting two points on Line B when rear axle is raised (rectangular coordinates)	(X_c, Y_c) (X_d, Y_d)		(t_1, L) $(t_1, -L)$	(779.74, 1675) (779.74, -1675)
Polar coordinates angle of Line B	α	rad	$\tan^{-1} \left(\frac{t_1}{L} \right)$	1.135
Two points with rotating Line B by θ_r	X'_c Y'_c X'_d Y'_d		$\sqrt{t_1^2 + L^2} \times \cos(\alpha + \theta_r),$ $-\sqrt{t_1^2 + L^2} \times \sin(\alpha + \theta_r),$ $\sqrt{t_1^2 + L^2} \times \cos(\alpha - \theta_r),$ $\sqrt{t_1^2 + L^2} \times \sin(\alpha - \theta_r)$	100.81 -1844.85 1346.79 1264.82

Table 2. Calculation of the center of gravity of agricultural tractors (Continued)

Data description	Symbol	Unit	Calculation	Result
Synchronization of coordinates	X_c		$X_c = X'_c$	100.81
	Y_c		$Y_c = Y'_c + R_f$	-1594.82
	X_d		$X_d = X'_d$	1346.79
	Y_c		$Y_d = Y'_d + R_f$	1594.82
Selecting two points on Line A in raised position of front axle (rectangular coordinates)	(X_a, Y_a)		(t_2, L)	(-592.34, 1675)
	(X_b, Y_b)		$(t_2, -L)$	(-592.34, -1675)
Polar coordinates angle of Line A	α_f	rad	$\tan^{-1}\left(\frac{t_2}{L}\right)$	1.23
Two points with rotating Line A by θ_r	X_a		$\sqrt{t_2^2 + L^2} \times \cos(\alpha_f - \theta_r)$,	-1150.6
	Y_a		$\sqrt{t_2^2 + L^2} \times \sin(\alpha_f - \theta_r)$,	1353.74
	X_b		$\sqrt{t_2^2 + L^2} \times \cos(\alpha_f + \theta_r)$,	43.7752
	Y_b		$\sqrt{t_2^2 + L^2} \times \sin(\alpha_f + \theta_r)$	-1776.11
Synchronization of coordinates	X_a		$X_a = X'_a + L$,	524.40
	Y_a		$Y_a = Y'_a + R_f$,	1878.74
	X_b		$X_b = X'_b + R_f$,	1718.78
	Y_b		$Y_b = Y'_b + R_f$	-1251.11
Finding the cross point	(X, Y)		$\left(\frac{Y_{10} - Y_{20}}{\frac{Y_b - Y_a}{X_b - X_a} - \frac{Y_d - Y_c}{X_d - X_c}}, \frac{Y_d - Y_c}{\frac{Y_b - Y_a}{X_b - X_a} - \frac{Y_d - Y_c}{X_d - X_c}} \right)$ $\times \left(\frac{Y_{10} - Y_{20}}{\frac{Y_b - Y_a}{X_b - X_a} - \frac{Y_d - Y_c}{X_d - X_c}} + Y_{10} \right)$	(981.07, 682.06)

As shown in Figure 5, the equations to find the intersection point were programmed in Microsoft Excel 2010, and the vertical height from the ground to the center of gravity was 682.06 mm.

As shown in Figure 6, the vertical height from the ground to the center of gravity obtained by plotting was 682.06 mm in the AutoCAD drawing.

The result calculated using Microsoft Excel 2010 is the same as that obtained using the AutoCAD drawing. Therefore, the calculation method can be used instead of the plotting method.

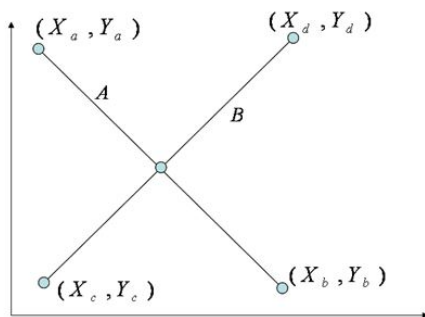


Figure 5. Finding the location of the center of gravity.

Conclusions

This study aimed to develop a calculation method for determining the vertical coordinate of an agricultural tractor's center of gravity in accordance with ISO 789-6. This method used trigonometric functions and polar coordinate transformation, and it was verified through a comparison with an AutoCAD drawing. The result showed that the developed method can be used instead of the plotting method. If the calculation is programmed using Microsoft Excel, the vertical coordinate can be determined easily. Furthermore, the rollover angle can be predicted easily using this method. Therefore, the authors will propose this method to the ISO.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Input				Calculated value							
2	Height of front axle	Rf	330	mm	Vertical distance from the center of the front axle and the center of the rear axle	$\Delta r = R_r - R_f$	195	mm		Cartesian coordinates		
3	Height of rear axle	Rr	525	mm	Distance between the center of the front axle and the center of the rear axle	L'	1686.312545	mm			Xd	779.7396
4	Wheel base	L	1675	mm	Angle between the center of the front axle and the center of the rear axle	theta	0.115896204	radian			Yd	1675
5	Front axle load	Ff	632	kg	When the rear axle was raised						Xc	779.7396
6	Tractor weight	W	1479	kg	Angle between the center of the front axle and the center of the rear axle	theta_1	0.496987834	radian			Yc	-1675
7	When the rear wheel was raised				Raised angle	theta_r	0.38109163	radian		Polar coordinates		
8	Front axle load without support weight	Ff1	701	kg	Horizontal distance from the center of rear axle to the center of front axle	L1	1482.306986	mm		Orientation of polar coordinates	alpha_r	1.135114
9	Height of rear axle	Rr1	1132	mm	Horizontal distance from the center of rear axle to the center of gravity	c1	702.5674084	mm		Radius of polar coordinates	r_pr	1847.598
10	Height of front axle	Rf1	328	mm	Horizontal distance from the center of front axle to the center of gravity	t1=L1-c1	779.7395774	mm		Transformation and rotation		
11	When the front wheel was raised				Synchronization of coordinates					$\sqrt{X_d^2 + Y_d^2} \cos(\theta_{r1} - \alpha_r)$	Xd'	1346.79
12	Rear axle load without support weight	Fr2	943	kg	$X_d = X_d$	Xd	1346.789877			$\sqrt{X_d^2 + Y_d^2} \sin(\theta_{r1} - \alpha_r)$	Yd'	1264.822
13	Height of front axle	Rf2	938	mm	$Y_d = Y_d + R_f$	Yd	1594.822452			$\sqrt{X_c^2 + Y_c^2} \cos(\theta_{r1} + \alpha_r)$	Xc'	100.811
14	Height of rear axle	Rr2	523	mm	$X_c = X_c$	Xc	100.8109553			$-\sqrt{X_c^2 + Y_c^2} \sin(\theta_{r1} + \alpha_r)$	Yc'	-1844.846
15					$Y_c = Y_c + R_f$	Yc	-1514.845782					
16					When the front axle was raised					Cartesian coordinates		
17					Angle between the center of the front axle and the center of the rear axle	theta_2	0.248653506				Xa	-592.336
18					Raised angle	theta_f	0.36454971				Ya	1675
19					Horizontal distance from the center of rear axle to the center of front axle	L2	1634.449449				Xb	-592.336
20					Horizontal distance from the center of front axle to the center of gravity	c2	1042.113475				Yb	-1675
21					Horizontal distance from the center of rear axle to the center of gravity	t2=L2-c2	592.3359733			Polar coordinates		
22					Synchronization of coordinates					Orientation of polar coordinates	alpha_f	1.230888
23										Radius of polar coordinates	r_pf	1776.65
24					$X_a = L$	Xa	524.4042848			Transformation and rotation		
25					$Y_a = R_r$	Ya	1878.741632				Xa'	-1150.596
26					$X_b = L$	Xb	1718.7752				Ya'	1353.742
27					$Y_b = R_r$	Yb	-1251.111099				Xb'	43.7752
28					Finding the cross point						Yb'	-1776.111
29					x	x	981.0651956					
30					Height from the ground	y	682.0602818					

Figure 6. Calculation of location of center of gravity by using Microsoft Excel.

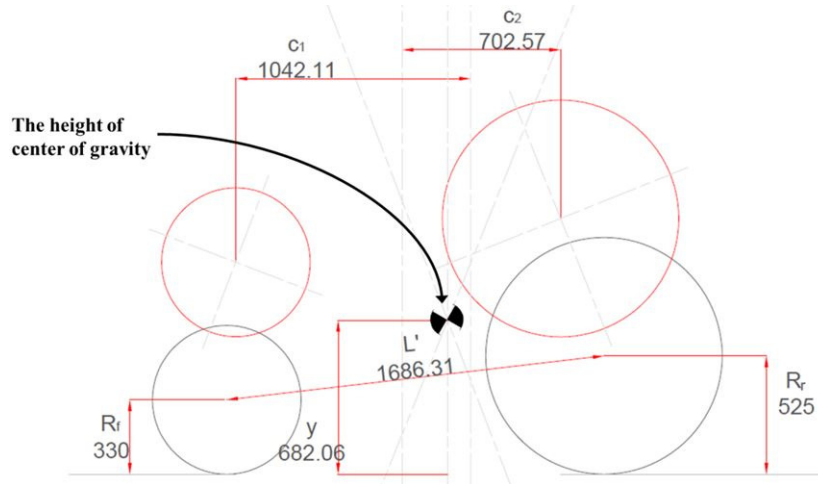


Figure 7. Calculation of location of center of gravity using AutoCAD.

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