International Journal of Reliability and Applications
Vol. 17, No. 2, pp. 107-119, 2016

On NBU_{mgf} class at specific age

A. M. Gadallah*
Department of Basic Sciences, Thebes Higher Institute for Engineering
Thebes Academy, Cairo, Egypt

Received 13 February 2016; revised 06 May 2016; accepted 30 August 2016

Abstract: A new concept of aging classes namely new better (worse) than used at age t_0 in moment generating function order, $NBU_{mgf} - t_0$ ($NWU_{mgf} - t_0$) is introduced. For the classes $NBU_{mgf} - t_0$ ($NWU_{mgf} - t_0$), preservation under convolution, mixture, mixing and the homogeneous Poisson shock model are studied. In the sequel, nonparametric test is proposed, the asymptotic normality of the class is established and the asymptotic null variance is estimated. The percentiles and powers of this test are tabulated. The asymptotic efficiencies for some alternatives distributions are derived. Finally sets of real data are used as examples to elucidate the use of the proposed test in practical application.

Key Words: *convolution, hypothesis test,* m*ixture, Poisson shock model, U-Statistic*

1. INTRODUCTION

The new better than used aging classes play an important role in modeling situations which the lifetime of a new unit is better than the lifetime of a used one. These classes can be defined using the concept of stochastic comparisons between the residual live of a new unit with the residual live of a used one. The stochastic and the increasing concave comparisons are used by Muller and Stoyan (2002) and Shaked and Shanthikumar (1994). Formally, if X and Y are two non-negative random variables with distributions F and G (survival functions $\overline{F}$ and $\overline{G}$), respectively, then we say X is smaller than Y in the:

(a) Stochastic order (denoted by $X \leq st\ Y$) if

$$E\left[\phi(X)\right] \leq E\left[\phi(Y)\right] \text{ for all increasing functions } \phi.$$

(b) Increasing concave order (denoted by $X \leq icv\ Y$) if

$$E\left[\phi(X)\right] \leq E\left[\phi(Y)\right] \text{ for all increasing concave functions } \phi.$$

*Corresponding Author.
E-mail address: *alaadean_ mag@yahoo.com*

(c) Moment generating function order (denoted by X ≤mgf Y) if

$$\int_0^\infty e^{\lambda x}\bar{F}(x)dx \le \int_0^\infty e^{\lambda x}\bar{G}(x)dx \quad \forall\lambda \ge 0.$$

In the context of lifetime distributions, some of the above o rderings of distributions have been used to give characterizations and new definitions of aging classes. We say X is

(a) New better than used (denoted by X $\in$ NBU) if $\bar{F}(x+t) \le \bar{F}(t)\bar{F}(x)$, $\forall x,t \ge 0$.
(b) New better than used in the increasing concave order (denoted by X $\in$ NBU(2)) If

$$\int_0^x \bar{F}(u+t)dx \le \bar{F}(t)\int_0^x \bar{F}(u)du, \quad \forall x,t \ge 0.$$

(c) New better than used in the moment generating function order (denoted by X $\in$ NBU_{mgf}) if

$$\int_0^\infty e^{\lambda x}\bar{F}(x+t)dx \le \bar{F}(t)\int_0^\infty e^{\lambda x}\bar{F}(x)dx, \quad \forall\lambda,t \ge 0.$$

For more details about the aging notions in (a),(b) and (c), one may refer to Bryson and Siddiqui (1969), Barlow and Proschan (1981), Deshpande et al. (1986)and Wang (1996). Some properties of the NBU_{mgf} class including some preservations properties have been discussed by Li (2004), while Zhang and Li (2004) showed that the NBU_{mgf} class is preserved under both the non-homogeneous Poisson shock model and the general shock model. Some new results are given including some closure properties, characterizations and testing exponentially against the NBU_{mgf} class is addressed by Ahmad and Kayid (2004) and Mahmoud and Gadallah (2012).
There are situation in real life where the components of the system gradually deteriorate up time t_0 which is warranty guarantee time provided by most manufactures, then maintenance through repairs or spare parts replacement take place after time t_0. Here maintenance is expected to improve the performance of the system but can not bring it back to a better situation than it was at age t_0.
Hollander et al. (1986) introduced the concept of the class of life distributions called new better than used of specified age t_0. Ebrahimi and Habbibullah (1990), Ahmad (1998) and Pandit and Anuradha (2007) investigated the testing of new better than used of specified age t_0 ($NBU-t_0$) alternative. The classes new better than used in expectation at specific age t_0 ($NBUE-t_0$), harmonic new better than used in expectation at specific age t_0 ($HNBUE-t_0$) and new better than used of age t_0 in Laplace transform order ($NBUL-t_0$) are investigated by Mahmoud et al. (2013, 2014).
A generalization of NBU_{mgf} aging class is defined in the following definition.

Definition 1. X is new better (worse) than used at age t_0 in moment generating function order (denoted by X $\in$ $NBU_{mgf}-t_0$) if

$$\int_0^\infty e^{\lambda x}\bar{F}(x+t_0)dx \le \bar{F}(t_0)\int_0^\infty e^{\lambda x}\bar{F}(x)dx, \quad \forall\lambda, t_0 \ge 0.$$

One can note that

$$NBU \Rightarrow NBU-t_0 \Rightarrow NBU_{mgf}-t_0 \Rightarrow NBUE-t_0 \Rightarrow HNBUE-t_0 .$$

The paper is organized as follows. In Section 2 we discus preservation under convolution, mixture, mixing and the homogeneous Poisson shock model of the $NBU_{mgf} - t_0$ ($NWU_{mgf}-t_0$) classes. In Section 3, based on U statistic we study a procedure to test that X is exponential versus that it is $NBU_{mgf}-t_0$ and not exponential.

2. SOME PPROPERTIES OF THE $NBU_{mgf}-t_0$ CLASS

In this section we discuss preservation and nonpreservation properties of the $NBU_{mgf}-t_0$ and $NWU_{mgf}-t_0$ classes.

2.1 Convolution, mixture and mixing properties

Our aim in this subsection is to discuss preservation under convolution, mixture and mixing properties of $NBU_{mgf}-t_0$ and $NWU_{mgf}-t_0$ classes.

Theorem 1. The $NBU_{mgf}-t_0$ class is preserved under convolution.

Proof. Suppose that F_1 and F_2 are two independent $NBU_{mgf}-t_0$ lifetime distributions then their convolution is given by:

$$\bar{F}(z) = \int_0^\infty \bar{F}_1(z-y)dF_2(y).$$

And therefore:

$$\int_0^\infty e^{\lambda x}\bar{F}(x+t_0)dx = \int_0^\infty\int_0^\infty e^{\lambda x}\bar{F}_1(x+t_0-u)dF_2(u)dx$$
$$= \int_0^\infty\int_0^\infty e^{\lambda x}\bar{F}_1(x+t_0-u)dxdF_2(u),$$

since F_1 is $NBU_{mgf}-t_0$ then

$$\int_0^\infty e^{\lambda x}\bar{F}(x+t_0)dx \le \int_0^\infty\int_0^\infty e^{\lambda x}\bar{F}_1(t_0)\bar{F}_1(x-u)dxdF_2(u)$$
$$= \bar{F}_1(t_0)\int_0^\infty e^{\lambda x}\bar{F}(x)dx,$$

making use of $\bar{F}_i(z) \le \bar{F}(z)$ for i = 1, 2, gives

$$\int_0^\infty e^{\lambda x}\bar{F}(x+t_0)dx \le \bar{F}(t_0)\int_0^\infty e^{\lambda x}\bar{F}(x)dx,$$

which complete the proof.
The following example is presented to show that $NWU_{mgf} - t_0$ class is not preserved under convolution.

Example 1. The convolution of the exponential distribution F(x) = $1-e^{x}$ with itself yields the gamma distribution of order 2: G(x) = $1-\left(1\ +\ x\right)e^{x}$, with strictly increasing failure rate. Thus G(x) is not $NWU_{mgf} - t_0$.

The following example shows that the $NBU_{mgf} - t_0$ class is not preserved under mixtures.

Example 2. Let $\overline{F\alpha}(x) = e^{-\alpha x}$ and $\overline{G}(x) = \int_0^\infty \overline{F\alpha}(x)e^{-\alpha}$ dα = $(x+1)^{-1}$. Then the failure rate function is $r_g(x) = (x+1)^{-1}$, which is strictly decreasing thus $\overline{G}(x)$ is not $NBU_{mgf} - t_0$. The following theorem is stated and proved to show that the $NWU_{mgf} - t_0$ class is preserved under mixture.

Theorem 2. The $NWU_{mgf} - t_0$ class is preserved under mixture.

Proof. Suppose F(x) is the mixture of F_α, where each F_α is $NWU_{mgf} - t_0$ then,

$$\begin{aligned}\int_0^\infty e^{\lambda x}\bar{F}(x+t_0)dx &= \int_0^\infty\int_0^\infty e^{\lambda x}\bar{F}_\alpha(x+t_0)dG(\alpha)dx \\ &= \int_0^\infty\int_0^\infty e^{\lambda x}\bar{F}_\alpha(x+t_0)dxdG(\alpha).\end{aligned} \tag{1}$$

Since Fα is $NWU_{mgf} - t_0$ then

$$\int_0^\infty\int_0^\infty e^{\lambda x}\bar{F}_\alpha(x+t_0)dxdG(\alpha) \ge \int_0^\infty\int_0^\infty e^{\lambda x}\bar{F}_\alpha(t_0)\bar{F}_\alpha(x)dxdG(\alpha). \tag{2}$$

Upon using Chebyschev inequality for similarity ordered functions we get

$$\begin{aligned}&\int_0^\infty\int_0^\infty e^{\lambda x}\bar{F}_\alpha(t_0)\bar{F}_\alpha(x)dG(\alpha)dx \ge \\ &\int_0^\infty e^{\lambda x}\left\{\int_0^\infty \bar{F}_\alpha(t_0)dG(\alpha).\int_0^\infty \bar{F}_\alpha(x)dG(\alpha)\right\}dx.\end{aligned} \tag{3}$$

Using (1), (2) and (3) the proof is completed.

The following example illustrates that the $NBU_{mgf} - t_0$ class is not preserved under mixing.

Example 3. Let $\overline{F}_1 = e^{-\delta x}$ and $\overline{F}_2 = e^{-\gamma x}$. Let $\overline{F} = \frac{1}{2}\overline{F}_1 + \frac{1}{2}\overline{F}_2$. It follows that both $\overline{F}_1$ and $\overline{F}_2$ are $NBU_{mgf} - t_0$ but $\overline{F}$ is not $NBU_{mgf} - t_0$.

2.2 Homogeneous Poisson shock model

Here our purpose is to check closure of the homogeneous Poisson shock model for $NBU_{mgf} - t_0$. Suppose that a device is subjected to sequence shocks occurring randomly in the time according to a Poisson process with constant intensity s. Suppose further that the device has probability $\overline{p}_k$ of surviving the first k shocks, where $1 = \overline{p}_0 \geq \overline{p}_1 \geq \cdots$. Then the survival function of the device is given by

$$\bar{H}(t) = \sum_{k=0}^{\infty} \bar{p}_k \frac{(st)^k}{k!} e^{-st}. \tag{4}$$

This shock model has been studied by Esary et al. (1969) for IFR, IFRA, DMRL, NBU and $NBUE$ classes. Klefsjo (1981) for $HNBUE$, Mahmoud et al. (2013) for $NBUE - t_0$, $HNBUE - t_0$ and Mahmoud et al. (2014) for $NBUL - t_0$. To discuss this property we need the following definition.

Definition 2. A discrete distribution $p_k, k = 0,1,\cdots,\infty$ or its survival probabilities $\overline{p}_k, k = 0,1,\cdots,\infty$ is said to have discrete new better (worse) than used at age t0 in moment generating function order ($NBU_{mgf} - t_0$)($NWU_{mgf} - t_0$) if

$$\sum_{r=0}^{\infty} \bar{p}_{r+j} z^{-r} \leq (\geq) \bar{p}_j \sum_{r=0}^{\infty} \bar{p}_r z^{-r}, 0 \leq z \leq 1, j = 0, 1, \cdots. \tag{5}$$

Now, let us state and prove the following theorem.

Theorem 3. If p_k is discrete $NBU_{mgf} - t_0$, then $\overline{H}(t)$ given by (4) is $NBU_{mgf} - t_0$.

Proof. It must be shown that

$$\int_0^{\infty} e^{\lambda x} \bar{H}(x + t_0) dx \leq \bar{H}(t_0) \int_0^{\infty} e^{\lambda x} \bar{H}(x) dx.$$

Upon using (4), we get

$$\int_0^{\infty} e^{\lambda x} \bar{H}(x + t_0) dx = \int_0^{\infty} e^{\lambda x} \sum_{k=0}^{\infty} \bar{p}_k \frac{[s(t_0 + x)]^k}{k!} e^{-s(x+t_0)} dx$$

$$= e^{-st_0} \sum_{k=0}^{\infty} \bar{p}_k \sum_{r=0}^{k} \binom{k}{r} \frac{(st_0)^{k-r}}{k!} \int_0^{\infty} (sx)^r e^{-x(s-\lambda)} dx.$$

Integrating by parts yields

$$\int_0^\infty e^{\lambda x}\bar{H}(x+t_0)dx = \frac{e^{-st_0}}{(s-\lambda)}\sum_{r=0}^{\infty}\sum_{k=r}^{\infty}\bar{p}_k\frac{(st_0)^{(k-r)}}{(k-r)!}(\frac{s}{s-\lambda})^r$$

$$= \frac{e^{-st_0}}{(s-\lambda)}\sum_{r=0}^{\infty}\sum_{j=0}^{\infty}\bar{p}_{j+r}\frac{(st_0)^j}{j!}(\frac{s-\lambda}{s})^{-r}$$

$$\leq \frac{e^{-st_0}}{(s-\lambda)}\sum_{j=0}^{\infty}\bar{p}_j\sum_{r=0}^{\infty}\bar{p}_r\frac{(st_0)^j}{j!}(\frac{s-\lambda}{s})^{-r}$$

$$= \frac{1}{(s-\lambda)}\sum_{j=0}^{\infty}\frac{(st_0)^j}{j!}\bar{p}_je^{-st_0}\sum_{r=0}^{\infty}\bar{p}_r(\frac{s}{s-\lambda})^r$$

$$= \sum_{j=0}^{\infty}\frac{(st_0)^j}{j!}\bar{p}_je^{-st_0}\sum_{r=0}^{\infty}\bar{p}_r\frac{s^r}{r!}\int_0^\infty x^re^{-x(s-\lambda)}dx$$

$$= \bar{H}(t_0)\int_0^\infty e^{\lambda x}\sum_{r=0}^{\infty}\frac{(sx)^r}{r!}\bar{p}_re^{-sx}dx$$

$$= \bar{H}(t_0)\int_0^\infty e^{\lambda x}\bar{H}(x)dx.$$

The proof for the $NWU_{mgf}-t_0$ class is obtained by reversing the inequality.

3. TESTING EXPONENTIALITY VERSUS THE $NBU_{mgf}-t_0$ CLASS

In this section we propose a test statistic for testing H_0 : F is exponential versus H_1 : F belongs to $NBU_{mgf}-t_0$ class of life distributions and not exponential. Let

$$\triangle = \int_0^\infty e^{\lambda x}\bar{F}(t_0)\bar{F}(x)dx - \int_0^\infty e^{\lambda x}\bar{F}(x+t_0)dx. \quad (6)$$

It is clear that

$$\int_0^\infty e^{\lambda x}\bar{F}(x+t_0)dx = \frac{1}{\lambda}E[e^{\lambda(X-t_0)}-1], \quad (7)$$

and

$$\int_0^\infty e^{\lambda x}\bar{F}(x)dx = \frac{1}{\lambda}E[e^{\lambda X}-1]. \quad (8)$$

Upon using (7) and (8) in (6) we get

$$\triangle = \frac{1}{\lambda}E[1-\bar{F}(t_0)+e^{\lambda X}\bar{F}(t_0)-e^{\lambda(X-t_0)}]. \quad (9)$$

One can notice that the value of $\triangle$ under H_0 equals ζ, where

$$\zeta = 1-\bar{F}(t_0)-\frac{e^{-\lambda t_0}-\bar{F}(t_0)}{(1-\lambda)}.$$

Setting the measure of departure from H0 is $\delta = \triangle - \zeta$, then

$$\delta = \frac{1}{\lambda}[e^{-\lambda t_0}-\bar{F}(t_0)]E[\frac{1}{1-\lambda}-e^{\lambda X}]. \quad (10)$$

Note that under $H_0 : \delta = 0$, while under $H_1 : \delta > 0$.

To estimate δ, let $X_1, X_2, \cdots, X_n$ be a random sample from F, so the empirical form of δ in(10) is

$$\hat{\delta}_n = \frac{[e^{-\lambda t_0} - \bar{F}(t_0)]}{n\lambda} \sum_{i=1}^{n} [\frac{1}{1-\lambda} - e^{\lambda X_i}]. \tag{11}$$

It easily to show that $\hat{\delta}_n$ is an unbiased estimator for δ.

To find the limiting distribution of $\hat{\delta}_n$ we resort to the U-statistic theory. Let

$$\phi(X) = \frac{1}{1-\lambda} - e^{\lambda X},$$

and define the symmetric kernel

$$\psi(X) = \sum_{R} \phi(X_i),$$

where the sum is over all arrangements of Xi, this leads that $\hat{\delta}_n$ in (11) is equivalent to U-statistic given by

$$U_n = \frac{1}{n} \sum_{R} \psi(X_i).$$

The next results summarizes the asymptotic normality of $\hat{\delta}_n$.

Theorem 4. (i) As n $\to \infty$, $\sqrt{n}(\hat{\delta} n - \delta)$ is asymptotically normal with mean 0 and varianceis

$$\sigma^2 = Var\left\{ \frac{(e^{-\lambda t_0} - \bar{F}(t_0))}{\lambda} [\frac{1}{1-\lambda} - e^{\lambda X}] \right\}. \tag{12}$$

(ii) Under H_0, the variance is reduced to

$$\sigma_0^2 = \frac{(e^{-\lambda t_0} - \bar{F}(t_0))^2}{(1-2\lambda)(1-\lambda)^2}. \tag{13}$$

Hence we reject H_0 if $\sqrt{n}\ \hat{\delta}_n / \sigma_0 > z_\alpha$, where z_α the standard normal variate.

3.1 The Pitman Asymptotic Efficiency (PAE) of δ

To asses how good this procedure is relative to others in the literature we employ the concept of Pittman asymptotic efficiency (PAE) for the following two alternatives these are:

1. Linear failure rate family (LFR): $\overline{F}\theta(x) = exp(-x - \frac{\theta}{2}x^2)$, $x > 0$, $\theta \geq 0$.

2. Makeham family: $\overline{F}\theta = exp(-x + \theta(x + e^{-1} - 1))$, $x > 0$, $\theta \geq 0$.

The PAE of δ is defined by

$$PAE(\delta) = \frac{1}{\sigma_0} \left| \frac{d\delta}{d\theta} \right|_{\theta \to \theta_0}.$$

The following is PAE of δ for any alternative F,

$$PAE(\hat{\delta},F)=\frac{1}{\sigma_0}\left|e^{-t_0}\int_0^{\infty}e^{\lambda x}\bar{F}_\theta'(x)dx+\frac{\bar{F}_\theta'(t_0)}{(1-\lambda)}-e^{-\lambda t_0}\int_{t_0}^{\infty}e^{\lambda x}\bar{F}_\theta'(x)dx\right|.$$

So

$$PAE(\hat{\delta},LFR)=\frac{1}{\sigma_0}\left|\frac{t_0e^{-t_0}}{(1-\lambda)^2}\right|,$$

and

$$PAE(\hat{\delta},Makeham)=\frac{1}{\sigma_0}\left|\frac{e^{-t_0}(1-e^{-t_0})}{(1-\lambda)(2-\lambda)}\right|.$$

After Searching, we find λ = 0.34 that maximizes the PAE($\hat{\delta}$,LFR) and the PAE($\hat{\delta}$, Makeham).
Figure 1 shows the relation between efficiencies and t_0 < 3.

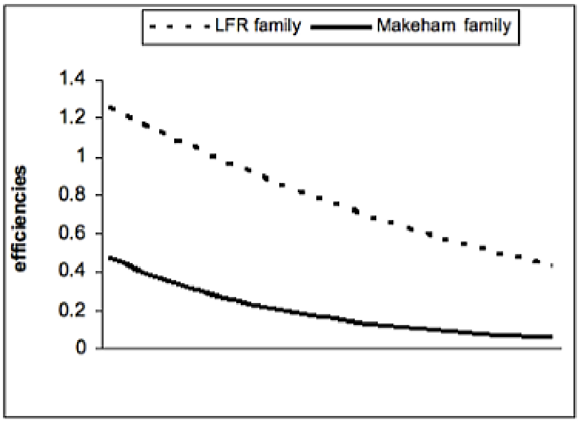

Figure 1. The relation between efficiencies and t_0 < 3 at λ = 0.34

In view of Figure 1, it is noticed that the PAE's of δ are decreasing as t_0 increasing and the PAE's for the LFR alternative is greater than the PAE's for Makeham alternative.
We compare the above PAE's at t_0 = 0.1 and λ = 0.34 with that of Hollander and Proschan (1972) and Mahmoud and Gadallah (2012) and the results are shown in Table 1.

Table 1. The PAE's for LFR and Makeham families

Test	LFR	Makeham
Hollander-Proschan	0.8660	0.2886
Mahmoud-Gadallah	0.6974	0.395
δ	1.25625	0.4753

Table 1 shows that our class $NBU_{mgf}-t_0$, which is larger than NBU and NBU_{mgf}, is more efficient for all used alternatives.

Table 2. The percentiles of statistic $\hat{\delta}_n$ at t0 = 0.1& λ = 0.34

n	0.01	0.05	0.10	0.90	0.95	0.99
2	-0.458	-0.165	-0.091	0.076	0.083	0.089
3	-0.383	-0.148	-0.086	0.068	0.074	0.082
4	-0.317	-0.131	-0.080	0.062	0.070	0.081
5	-0.248	-0.126	-0.080	0.059	0.065	0.076
6	-0.228	-0.115	-0.076	0.054	0.061	0.072
7	-0.190	-0.110	-0.070	0.050	0.058	0.068
8	-0.182	-0.098	-0.064	0.049	0.057	0.066
9	-0.183	-0.092	-0.063	0.047	0.054	0.065
10	-0.158	-0.095	-0.057	0.045	0.051	0.063
11	-0.148	-0.084	-0.056	0.042	0.049	0.061
12	-0.140	-0.083	-0.058	0.042	0.048	0.059
13	-0.146	-0.085	-0.055	0.038	0.047	0.057
14	-0.130	-0.081	-0.057	0.040	0.047	0.056
15	-0.128	-0.077	-0.050	0.038	0.044	0.056
16	-0.127	-0.074	-0.048	0.037	0.045	0.053
17	-0.122	-0.068	-0.049	0.036	0.044	0.054
18	-0.123	-0.066	-0.044	0.035	0.042	0.053
19	-0.116	-0.063	-0.048	0.034	0.039	0.052
20	-0.105	-0.058	-0.045	0.034	0.042	0.050
21	-0.100	-0.062	-0.045	0.033	0.039	0.050
22	-0.105	-0.060	-0.042	0.033	0.040	0.050
23	-0.098	-0.058	-0.041	0.033	0.038	0.051
24	-0.105	-0.063	-0.041	0.033	0.038	0.050
25	-0.101	-0.057	-0.042	0.032	0.039	0.048
26	-0.108	-0.055	-0.039	0.032	0.039	0.048
27	-0.100	-0.053	-0.037	0.031	0.038	0.048
28	-0.092	-0.053	-0.038	0.031	0.037	0.048
29	-0.093	-0.053	-0.036	0.031	0.037	0.045
30	-0.086	-0.055	-0.035	0.028	0.036	0.046
31	-0.085	-0.051	-0.036	0.029	0.036	0.045
32	-0.098	-0.049	-0.036	0.029	0.036	0.044
33	-0.083	-0.048	-0.035	0.028	0.035	0.048
34	-0.094	-0.051	-0.032	0.028	0.035	0.044
35	-0.073	-0.047	-0.034	0.027	0.034	0.045
36	-0.083	-0.047	-0.034	0.027	0.034	0.043
37	-0.086	-0.048	-0.034	0.028	0.033	0.044
38	-0.081	-0.047	-0.034	0.027	0.033	0.044
39	-0.081	-0.044	-0.034	0.027	0.032	0.043
40	-0.072	-0.047	-0.030	0.026	0.032	0.041
41	-0.083	-0.046	-0.033	0.026	0.031	0.043
42	-0.073	-0.044	-0.033	0.026	0.032	0.043
43	-0.068	-0.043	-0.030	0.025	0.030	0.041
44	-0.076	-0.040	-0.030	0.025	0.031	0.041
45	-0.065	-0.043	-0.031	0.026	0.031	0.038
46	-0.068	-0.041	-0.029	0.025	0.030	0.037
47	-0.070	-0.041	-0.028	0.025	0.030	0.038
48	-0.069	-0.041	-0.028	0.024	0.030	0.040
49	-0.068	-0.040	-0.028	0.024	0.029	0.037
50	-0.065	-0.040	-0.029	0.024	0.029	0.039

3.2 Monte carlo null distribution critical points

Many practitioners, such as applied statisticians, and reliability analysts are interested in simulated percentiles. Table 2 gives these percentile points of the statistic $\hat{\delta}_n$ given in (11) at t_0 = 0.1, λ = 0.34 and the calculations are based on 10000 simulated samples of sizes n = 2(1)50. It is noticed from Table 2 that the critical values are increasing as the confidence level increasing and almost decreasing as the sample size increasing.

3.3 The power estimates

Table 3 shows the estimated power of the test statistic $\hat{\delta}_n$ given in (13) at the significant level 0.05 using LFR and Makeham distributions. The estimate are based on 10000 simulated samples for sizes n = 10,20 and 30.

Table 3. Power estimates using α = 0.05 at t_0 = 0.1

distribution	n	$\theta = 1$ Powers	$\theta = 2$ Powers	$\theta = 3$ Powers	$\theta = 4$ Powers
LFR	10	0.439	0.839	0.975	0.999
	20	0.797	0.993	1.000	1.000
	30	0.910	1.000	1.000	1.000
Makeham	10	1.000	1.000	1.000	1.000
	20	1.000	1.000	1.000	1.000
	30	1.000	1.000	1.000	1.000

It is clear from Table 3 the more increasing the sample size the more increasing the powers and the powers will increase due to the θ increasing for LFR distribution and the test has perfect power for Makeham distribution.

3.4 Applications

Example 4. The following data represent 39 liver cancers patients taken from Elminia cancer center Ministry of Health - Egypt, which entered in (1999) (see Attia et al. (2005)). The ordered life times (in years) are:

0.027	0.038	0.038	0.038	0.038	0.038	0.041	0.047	0.049	0.055
0.055	0.055	0.055	0.055	0.063	0.063	0.066	0.071	0.082	0.082
0.085	0.110	0.314	0.140	0.143	0.164	0.167	0.184	0.195	0.203
0.206	0.238	0.263	0.288	0.293	0.293	0.293	0.318	0.411	

It was found that $\hat{\delta}_n$ = 0.740 which is greater than the tabulated critical value in Table 2. There is enough evidence to accept H1 which states that the data set has $NBU_{mgf} - t_0$ property.

Example 5. The following data set consists of 16 intervals in operating days between successive failures of air conditioning equipment in a Boeing 720 aircraft (see Edgeman et al. (1988)):

4.25	8.708	0.583	2.375	2.25	1.333	2.792	2.458
5.583	6.333	1.125	0.583	9.583	2.75	2.542	1.417

It was found that $\hat{\delta}_n$ = 0.38 and this value exceeds the tabulated critical value in Table 3.2. Then we conclude that this data set has $NBU_{mgf} - t_0$ property.

Example 6. Let us consider the following data, which represent failure times in hours, for a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress (Lawless (1982, p.138):

0.205	0.363	0.407	0.770	0.720	0.782	1.178	1.255
1.592	1.635	2.310					

It was found that the value of test statistic for the data set by formula (11) $\hat{\delta}_n = -0.007$, which is smaller than the critical value in Table 2. Then we accept the null hypothesis which states that the data set has exponential proper

ACKNOWLEDGMENT

The authors are thankful to the referees and the editor in chief for valuable comments which enhanced the presentation of the paper.

REFERENCES

Ahmad, I. A. (1998). Testing whether a survival distribution is new better than used of an unknown specified age, *Biometrka*, **85**, 451-456.

Ahmad, I. A.and Kayid, M. (2004). Some results about NBU_{mgf} class of life distribution, *International Journal of Reliability and Applications*, **5**, 155-162.

Attia, A. F., Mahmoud, M. A. W. and Tiab, I. B. (2005). On testing exponential better than used in average based on the total time on test transform, *The 7th Annual Conference on Statistics and Modeling in Human and Social Science*, 76-83.

Barlow, R. E. and proschan, F. (1981). *Statistical Theory of Reliability and Life Testing Probability Models*, To begin with, Silver Spring, MD.

Bryson, M. C. and Siddiqui, M. M. (1969). Some criteria for aging, *Journal of the American Statistical Association*, **64**, 1472-1483.

Deshpande, J. V., Kochar, S. C. and Singh, H. (1986). Aspects of positive aging, *Journal of Applied Probability*, **288**, 773-779.

Ebrahimi, N. and Habbibullah, M. (1990). Testing whether the survival distribution is new better than used of specified age, *Biometrika,* **77**, 212-215.

Edgman R. L., Scott R. C. and Pavur R. J. (1988). A modified kolmogorov- smirnov test for the inverse Gaussian density with unknown parameters, Communications in Statistics-simulation and Computations, **17**, 1203-1212.

Esary, J. D., Marshal, A. W. and Proschan, F. (1969). Shock models and wear process, *Annals of Probability*, **1**, 627-649.

Hollander, R. M., Park, D. H. and Proschan, F. (1986). A class of life distributions for aging, *Journal of the American Statistical Association*, **81**, 91-95.

Hollander, R. M. and Proschan, F. (1972). Testing whether new is better than used, *Annals of Mathematical Statistics*, **43**, 1136-1146.

Klefsjo, B. (1981). HNBUE survival under some shock models, *Scandinavian Journal of Statistics*, **8**, 39-47.

Lawless, J. F. (1982). *Statistical Models & Methods for Lifetime Data*, Wiley & sons, New York.

Li, X. (2004). Some properties of aging notions based upon the moment generating function order, *Journal of Applied Probability*, **41**, 927-934.

Mahmoud, M. A. W. and Gadallah, A. M. (2012). Moment inequalites of NBUmgf with testing hypotheses application, *International Journal of Reliability and Applications*, **13,** 57-69.

12 Mahmoud, M. A. W., Moshref, M. E., Gadallah, A. M. and Shawky, A. I. (2013). New classes at specific age: properties and testing hypotheses, *Journal of Statistical Theory and Applications*, **12,** 106-119.

Mahmoud, M. A. W., Moshref, M. E. and Gadallah, A. M. (2014). On NBUL class at specific age, *International Journal of Reliability and Applications*, **15,** 11-22.

Muller, A. and Stoyan D. (2002). *Coparison Methods for Queues and Other Stochastic Models*, Wiley & sons, New York.

Parameshwar, V. Pandit and Anuradha, M. P. (2007). On testing exponentiality against new better than used of specified age, *Statistical Methodology*, **4**, 13-21.

Shaked, M. and Shanthikumar, J.G. (1994). *Stochastic Orders and their Applications*, Academic press, New York.

Wang, W. Y. (1996). Life distribution classes and two unit standby redundant system, *Ph.D. dissertation, Chinese Academy of Science, Beijing*.

Zhang, S. and Li, X. (2004). Preservation property of NBU_{mgf} under shock models, *International Journal of Reliability and Applications*, **4**, 71-77.