KYUNGPOOK Math. J. 56(2016), 867-876
http://dx.doi.org/10.5666/KMJ.2016.56.3.867
pISSN 1225-6951 eISSN 0454-8124

On Certain Subclasses of Starlike p-valent Functions

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Abstract. The object of the present paper is to investigate the starlikeness of the class of functions $f(z) = z^p + \sum_{k=n}^{\infty} a_{p+k} z^{p+k}$ $(p, n \in \mathbb{N} = \{1, 2, ...\})$ which are analytic and $p-$valent in the unit disc U and satisfy the condition

$$\left|(1-\lambda)(\frac{f(z)}{z^p})^{\alpha} + \lambda \frac{zf^{'}(z)}{pf(z)}(\frac{f(z)}{z^p})^{\alpha} - 1\right| < \mu \quad (0 < \mu \leq 1, \lambda \geq 0, \alpha > 0, z \in U).$$

The starlikeness of certain integral operator are also discussed. The results obtained generalize the related works of some authors and some other new results are also obtained.

1. Introduction

Let $\mathcal{H}$ denote the class of functions analytic in unit disc $U = \{z : |z| < 1\}$ and let $A_p(n) \subset \mathcal{H}$ denote the class of functions of the form

$$f(z) = z^p + \sum_{k=n}^{\infty} a_{p+k} z^{p+k} \ (p, n \in \mathbb{N} = \{1, 2, ...\}), \tag{1.1}$$

which are analytic and $p-$valent in U. A function $f(z) \in A_p(n)$ is called $p-$ valently starlike in U if it satisfies

$$\Re\left\{\frac{zf^{'}(z)}{f(z)}\right\} > 0 \ (z \in U), \tag{2}$$

we denote by $S_p(n)$ the subclass of $A_p(n)$ consisting of functions $f(z)$ which are $p-$valently starlike in U. Also, we write $A_1(1) = A$ and $S_1(1) = S$ (the class of

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* Corresponding Author.
Received September 14, 2015; revised May 16, 2016; accepted May 30, 2016.
2010 Mathematics Subject Classification: 30C45.
Key words and phrases: Analytic; p-valently starlike functions; subordination.

univalent starlike functions). For some recent investigations on the starlikeness of analytic functions, one can refer to see ([3], [4], [6], [7], [11], [12], [14], [15], [17], [18], [19]). Various analogous classes of analytic multivalent functions were studied in many papers. For example, in the paper [16] several interesting properties of a new class of analytic and $p-$valent functions involving higher-order derivatives were investigated, in the paper [1] the authors investigate some applications of the differential subordination and the differential superordination of certain admissible classes of multivalent functions. A family of multiplier transformations and several subclasses of multivalent functions which are defined by means of convolution and several interesting results were considered in the paper [13]. For two functions f and g which are analytic in U, we say that f is subordinate to g, or g is superordinate to f, if there exists a Schwarz function $w(z)$ in U with $w(0) = 0$ and $|w(z)| < 1$ $(z \in U)$, such that $f(z) = g(w(z))$. In such case we write $f \prec g$ or $f(z) \prec g(z)$ $(z \in U)$. If $g(z)$ is univalent in U, then the following equivalence relationship holds true.

$$f(z) \prec g(z)(z \in U) \Longleftrightarrow f(0) = g(0) \text{ and } f(U) \subset g(U).$$

Definition 1.1. A function $f(z) \in A_p(n)$ is said to be in the class $B_p(n, \alpha, \lambda, \mu)$ $(\alpha > 0, \lambda \geq 0, 0 < \mu \leq 1)$ if it satisfies

$$\left|(1-\lambda)(\frac{f(z)}{z^p})^\alpha + \lambda \frac{zf^{'}(z)}{pf(z)}(\frac{f(z)}{z^p})^\alpha - 1\right| < \mu \ (z \in U). \tag{3}$$

Remark 1.2.

(i) For $\lambda = 1$, the class $B_p(n, \alpha, 1, \mu) = B_p(n, \alpha, \mu)$ $(\alpha, \mu > 0)$, which introduced and studied by Yang [20],

(ii) The subclass $B_1(1, \alpha, 1, \mu)$ $(\alpha > 0)$ has been studied by Ponnusamy [8].

(iii) The subclass $B_1(n, \alpha, \lambda, \mu)$ $(\alpha > 0)$ has been studied by Ponnusamy and Rajasekaran [9].

The object of the present paper is to investigate the conditions of starlikeness for functions in the class $B_p(n, \alpha, \lambda, \mu)$. The starlikeness of certain integral operator are also obtained. Relevant connections of the results presented here with those obtained in earlier works are pointed out.

We shall use the following Lemmas to prove our results.

Lemma 1.3.([5]) *Let $h(z)$ be a convex function in U (i.e. $h(z)$ is analytic and univalent in U and $h(U)$ is a convex domain), $h(0) = 1$, and let $g(z) = 1 + b_n z^n + b_{n+1} z^{n+1} + \ldots$ be analytic in U. If*

$$g(z) + \frac{1}{c} z g'(z) \prec h(z),$$

then

$$g(z) \prec \frac{c}{n} z^{\frac{-c}{n}} \int_0^z t^{\left(\frac{c}{n}\right)-1} h(t)dt,$$

where $c \neq 0$ *and* $Rec \geq 0$.

Lemma 1.4.([10]) *Let* $(0 < \mu_1 < \mu < 1)$ *and let* g *be analytic in* U, *satisfying*

$$g(z) \prec 1 + \mu_1 z, \ g(0) = 1,$$

(a) *if* p *is analytic in* U, $p(0) = 1$, *and satisfies*

$$g(z)\left[\gamma + (1-\gamma)p(z)\right] \prec 1 + \mu z,$$

where

$$\gamma = \begin{cases} \frac{1-\mu}{1+\mu_1}, & 0 < \mu + \mu_1 \leq 1 \\ \frac{1-(\mu^2+\mu_1^2)}{2(1-\mu_1^2)}, & \mu^2 + \mu_1^2 \leq 1 \leq \mu + \mu_1 \end{cases} \tag{4}$$

then $Re\{p(z)\} > 0$ $(z \in U)$.

(b) *if* w *is analytic in* U, *with* $w(0) = 0$, *and*

$$g(z)\left[1 + w(z)\right] \prec 1 + \mu z,$$

then

$$|w(z)| \leq \frac{\mu + \mu_1}{1 - \mu_1} = r \leq 1, \quad \mu + 2\mu_1 \leq 1. \tag{5}$$

The value of γ *given by (4) and the bounds (5) are best possible.*

2. Main results

Theorem 2.1. *Let* $f(z) \in B_p(n, \alpha, \lambda, \mu)$, *for some* $\lambda, \ \lambda > 0, \ \alpha > 0$,

(a) *If*

$$\mu = \begin{cases} \frac{\lambda(\lambda n + p\alpha)}{p\alpha(2-\lambda)+\lambda n}, & 0 < \lambda \leq \sqrt{\frac{2p\alpha}{n} + \frac{(3p\alpha-n)}{4n^2}^2} - \frac{3p\alpha-n}{2n} \\ \frac{(p\alpha+\lambda n)\sqrt{2\lambda-1}}{\sqrt{\lambda^2 n^2+2p\alpha\lambda(n+p\alpha)}}, & \sqrt{\frac{2p\alpha}{n} + \frac{(3p\alpha-n)}{4n^2}^2} - \frac{3p\alpha-n}{2n} \leq \lambda \leq 1 \end{cases} \tag{2.1}$$

then $f(z) \in S_p(n)$.

(b) *If* $\left|\frac{zf'(z)}{pf(z)} - 1\right| < r, \quad z \in U$, *where*

$$r = \frac{\mu\left[2p\alpha + \lambda n\right]}{\lambda\left[p\alpha\left(1-\mu\right) + \lambda n\right]}, \quad 0 < \mu \leq \frac{p\alpha + \lambda n}{3p\alpha + \lambda n}. \tag{2.2}$$

Then $f(z) \in S_p(n)$.

Proof. Since $f(z) \in A_p(n)$ satisfies (3) we can write

$$(1-\lambda)(\frac{f(z)}{z^p})^{\alpha} + \lambda \frac{zf^{'}(z)}{pf(z)}(\frac{f(z)}{z^p})^{\alpha} \prec 1 + \mu z \ (z \in U). \tag{2.3}$$

Let $g(z) = (\frac{f(z)}{z^p})^{\alpha}$, then $g(z) = 1 + b_n z^n + b_{n+1} z^{n+1} + ...$ is analytic in U and

$$(1-\lambda)(\frac{f(z)}{z^p})^{\alpha} + \lambda \frac{zf^{'}(z)}{pf(z)}(\frac{f(z)}{z^p})^{\alpha} = g(z) + \frac{\lambda}{p\alpha} z g'(z).$$

Therefore, it follows from (2.3) that

$$g(z) + \frac{\lambda}{p\alpha} z g'(z) \prec 1 + \mu z \ ,$$

and an application of Lemma 1.3 with $h(z) = 1 + \mu z$ yields

$$g(z) \prec 1 + \frac{p\alpha\mu}{p\alpha + \lambda n} z \ := 1 + \mu_1 z, \ 0 \leq \mu_1 = \frac{p\alpha\mu}{p\alpha + \lambda n} < \mu < 1, \tag{2.4}$$

since the condition (2.3) is equivalent to

$$(\frac{f(z)}{z^p})^{\alpha} \left[(1-\lambda) + [1-(1-\lambda)] \frac{zf^{'}(z)}{pf(z)} \right] \prec 1 + \mu z \ (z \in U). \tag{2.5}$$

Putting $g(z) = (\frac{f(z)}{z^p})^{\alpha}, \ p(z) = \frac{zf^{'}(z)}{pf(z)}$ and $\gamma = 1 - \lambda$ where $\lambda, \ \mu, \ \mu_1$ and $g(z)$ satisfy the relation (2.1) and (2.4), then we can easily check that the conditions in Lemma (1.4)(a) are satisfied which implies that

$$\Re \{p(z)\} > 0 \ (z \in U), \quad i.\ e., f(z) \in S_p(n).$$

(b) In this case we write the condition (3) in the form

$$(\frac{f(z)}{z^p})^{\alpha} \left[1 + \lambda \left(\frac{zf^{'}(z)}{pf(z)} - 1 \right) \right] \prec 1 + \mu z, \tag{2.6}$$

and if we put $g(z)$ as in (a), $w(z) = \lambda \left(\frac{zf^{'}(z)}{pf(z)} - 1 \right)$, μ and μ_1 as mentioned above, then by Lemma 1.4 (b) we have $\left| \frac{zf^{'}(z)}{pf(z)} - 1 \right| < r$, where r is given by (2.2). □

Remark 2.2. Let $\lambda = 1$ in Theorem 2.1, we get the following corollary which given by Yang [20].

Corollary 2.3. *If $f(z) \in A_p(n)$ satisfy*

$$\left| \frac{zf^{'}(z)}{pf(z)} (\frac{f(z)}{z^p})^{\alpha} - 1 \right| < \frac{p\alpha + n}{\sqrt{(p\alpha)^2 + (p\alpha+n)^2}},$$

then $f(z) \in S_p(n)$.

For $\alpha = 1$, our theorem gives

Corollary 2.4. *Let $f(z) \in B_p(n, 1, \lambda, \mu)$ satisfy the condition*

$$(1-\lambda)(\frac{f(z)}{z^p}) + \lambda \frac{f^{'}(z)}{pz^{p-1}} \prec 1 + \mu z \ (z \in U), \tag{2.7}$$

for some $\lambda,\ \lambda > 0$,

(a) *If*

$$\mu = \begin{cases} \frac{\lambda(\lambda n + p)}{p(2-\lambda)+\lambda n}, & 0 < \lambda \le \sqrt{\frac{2p}{n} + \frac{(3p-n)}{4n^2}^2} - \frac{3p-n}{2n} \\ \frac{(p+\lambda n)\sqrt{2\lambda - 1}}{\sqrt{\lambda^2 n^2 + 2p\lambda(n+p)}}, & \sqrt{\frac{2p}{n} + \frac{(3p-n)}{4n^2}^2} - \frac{3p-n}{2n} \le \lambda \le 1 \end{cases}$$

then $f(z) \in S_p(n)$.

(b) *If* $\left| \frac{zf^{'}(z)}{pf(z)} - 1 \right| < r, \quad z \in U$, *where*

$$r = \frac{\mu\left[2p + \lambda n\right]}{\lambda\left[p\left(1-\mu\right) + \lambda n\right]}, \qquad 0 < \mu \le \frac{p + \lambda n}{3p + \lambda n} \le 1.$$

Then $f(z) \in S_p(n)$.

Remark 2.5. Setting $p = n = \alpha = 1$ in Theorem 2.1, we get the result obtained by Daghreery [2].

Theorem 2.6. *If $c > -p\alpha$. Let $f(z) \in B_p(n, \alpha, \lambda, \mu)$, for some $\lambda,\ \lambda > 0,\ \alpha > 0$, then*

(a) *The function $F(z)$ defined by*

$$F(z) = \left[\frac{c + p\alpha}{z^c} \int_0^z t^{c-1} \left(f(t)\right)^{\alpha} dt\right]^{\frac{1}{\alpha}},$$

belongs to $S_p(n)$, where

$$\mu = \begin{cases} \frac{\lambda(c+p\alpha+n)(\lambda n + p\alpha)}{(c+p\alpha)[p\alpha(2-\lambda)+\lambda n]}, & 0 < \lambda \le \sqrt{\frac{2p\alpha}{n} + \frac{(3p\alpha-n)}{4n^2}^2} - \frac{3p\alpha-n}{2n} \\ \frac{(c+p\alpha+n)(p\alpha+\lambda n)\sqrt{2\lambda-1}}{(c+p\alpha)\sqrt{\lambda^2 n^2 + 2p\alpha\lambda(n+p\alpha)}}, & \sqrt{\frac{2p\alpha}{n} + \frac{(3p\alpha-n)}{4n^2}^2} - \frac{3p\alpha-n}{2n} \le \lambda \le 1 \end{cases} \tag{2.8}$$

(b) *If* $\left|\frac{zF'(z)}{pF(z)} - 1\right| < r, \quad z \in U,$ *where*

$$r = \frac{\mu\,(c+p\alpha)\,[2p\alpha+\lambda n]}{\lambda\,[(c+p\alpha)\,(p\alpha+\lambda n-\mu)+n\,(p\alpha+\lambda n)]}, \qquad 0 < \mu \le \frac{(c+p\alpha+n)\,(p\alpha+\lambda n)}{(c+p\alpha)\,(3p\alpha+\lambda n)} \le 1.$$

Then $F(z) \in S_p(n)$.

Proof. It is clear that the function $F(z)$ is in $A_p(n)$. Differentiating both sides of the equality

$$z^c\,(F(z))^\alpha = (c+p\alpha)\int_0^z t^{c-1}\,(f(t))^\alpha\,dt,$$

we have

$$c\,(F(z))^{\alpha-1}\,F'(z) + \left(z\,(F(z))^{\alpha-1}\,F'(z)\right)' = (c+p\alpha)\,(f(z))^{\alpha-1}\,f'(z). \tag{2.9}$$

Letting

$$G(z) = (1-\lambda)(\frac{F(z)}{z^p})^\alpha + \lambda\frac{zF'(z)}{pF(z)}(\frac{F(z)}{z^p})^\alpha = 1 + b_n z^n + \ldots,$$

then (2.9) becomes

$$G(z) + \frac{zG'(z)}{(c+p\alpha)} = (1-\lambda)(\frac{f(z)}{z^p})^\alpha + \lambda\frac{zf'(z)}{pf(z)}(\frac{f(z)}{z^p})^\alpha. \tag{2.10}$$

It follows from (3) and (2.10) that

$$G(z) + \frac{zG'(z)}{(c+p\alpha)} \prec 1 + \mu z\ ,$$

and an application of Lemma 1.3 with $h(z) = 1 + \mu z$ yields

$$G(z) \prec 1 + \frac{\mu\,(c+p\alpha)}{(c+p\alpha+n)}z, \tag{2.11}$$

and hence

$$\left|(1-\lambda)(\frac{F(z)}{z^p})^\alpha + \lambda\frac{zF'(z)}{pF(z)}(\frac{F(z)}{z^p})^\alpha - 1\right| < \frac{\mu\,(c+p\alpha)}{(c+p\alpha+n)} := \delta \tag{2.12}$$

for $z \in U$. The conditions of Theorem 2.6 follows immediately from (2.12), by replacing $f(z)$ by $F(z)$ and μ by $\delta := \frac{\mu(c+p\alpha)}{(c+p\alpha+n)}$ in Theorem 2.1. □

Remark 2.7. Setting $\lambda = 1$ in Theorem 2.6, we get the result obtained by Yang [20]. Taking $\alpha = 1$ and in Theorem 2.6, we obtain

Corollary 2.8. *If* $c > -p$. *Let*

$$(1-\lambda)(\frac{f(z)}{z^p}) + \lambda\frac{f'(z)}{pz^{p-1}} \prec 1 + \mu z\ (z \in U),\ \textit{for some}\ \lambda > 0\ \textit{and}\ 0 < \mu \le 1,$$

then,

(a) *The function* $F(z)$ *defined by*

$$F(z)=\frac{c+p}{z^c}\int_0^z t^{c-1}f(t)dt,$$

belongs to $S_p(n)$, *where*

$$\mu=\begin{cases}\frac{\lambda(c+p+n)(\lambda n+p)}{(c+p)[p(2-\lambda)+\lambda n]}, & 0<\lambda\leq\sqrt{\frac{2p}{n}+\frac{(3p-n)}{4n^2}^2}-\frac{3p-n}{2n}\\ \frac{(c+p+n)(p+\lambda n)\sqrt{2\lambda-1}}{(c+p)\sqrt{\lambda^2n^2+2p\lambda(n+p)}}, & \sqrt{\frac{2p}{n}+\frac{(3p-n)}{4n^2}^2}-\frac{3p-n}{2n}\leq\lambda\leq 1\end{cases}$$

(b) *If* $\left|\frac{zF'(z)}{pF(z)}-1\right|<r, \quad z\in U,$ *where*

$$r=\frac{\mu\,(c+p)\,[2p+\lambda n]}{\lambda\,[(c+p)\,(p+\lambda n-\mu)+n\,(p+\lambda n)]},\qquad 0<\mu\leq\frac{(c+p+n)\,(p+\lambda n)}{(c+p)\,(3p+\lambda n)}\leq 1.$$

Then $F(z)\in S_p(n)$.

Theorem 2.9. *Let* $Re\beta\geq 0,\ \beta\neq 0,$ *and* $0<\mu<\frac{|p\alpha+n\beta|}{p\alpha}$. *If* $f(z)$ *in* $A_p(n)$, *satisfies*

$$\left|(1-\beta)(\frac{f(z)}{z^p})^\alpha+\beta\frac{zf^{'}(z)}{pf(z)}(\frac{f(z)}{z^p})^\alpha-1\right|<\mu\ (z\in U),\tag{2.13}$$

then

$$\left|\frac{zf^{'}(z)}{pf(z)}-1\right|<\frac{\mu\,[p\alpha+|p\alpha+n\beta|]}{|\beta|\,[|p\alpha+n\beta|-p\alpha\mu]}\ (z\in U).\tag{2.14}$$

Proof. As in Theorem 2.1 the function $g(z)=(\frac{f(z)}{z^p})^\alpha=1+b_nz^n+\ldots$ is analytic in U and it follows from (2.13) that

$$(1-\beta)(\frac{f(z)}{z^p})^\alpha+\beta\frac{zf^{'}(z)}{pf(z)}(\frac{f(z)}{z^p})^\alpha=g(z)+\frac{\beta}{p\alpha}zg^{'}(z)\prec 1+\mu z.$$

By Lemma 1.3, we have

$$(\frac{f(z)}{z^p})^\alpha\prec 1+\frac{\mu p\alpha}{p\alpha+n\beta}z,\tag{2.15}$$

which implies that

$$\left|(\frac{f(z)}{z^p})^\alpha-1\right|<\frac{\mu p\alpha}{|p\alpha+n\beta|},\qquad\left|(\frac{f(z)}{z^p})^\alpha\right|>1-\frac{\mu p\alpha}{|p\alpha+n\beta|}>0\text{ for }z\in U.\tag{2.16}$$

Making use of (2.13) and (2.16), we deduce that

$$|\beta|\left|\frac{zf^{'}(z)}{pf(z)}(\frac{f(z)}{z^{p}})^{\alpha}-(\frac{f(z)}{z^{p}})^{\alpha}\right|$$

$$\leq\left|(\frac{f(z)}{z^{p}})^{\alpha}-1+\beta\left[\frac{zf^{'}(z)}{pf(z)}(\frac{f(z)}{z^{p}})^{\alpha}-(\frac{f(z)}{z^{p}})^{\alpha}\right]\right|+\left|(\frac{f(z)}{z^{p}})^{\alpha}-1\right|$$

$$<\frac{\mu\left[p\alpha+|p\alpha+n\beta|\right]}{|p\alpha+n\beta|-p\alpha\mu}\left[1-\frac{\mu p\alpha}{|p\alpha+n\beta|}\right]$$

$$<\frac{\mu\left[p\alpha+|p\alpha+n\beta|\right]}{|p\alpha+n\beta|-p\alpha\mu}\left|(\frac{f(z)}{z^{p}})^{\alpha}\right| \quad (z\in U),$$

which yields (2.14) and the proof is complete. □

From Theorem 2.9 we easily have

Corollary 2.10. *If $Re\beta\geq 0,\ \beta\neq 0,$ and $f(z)$ in $A_{p}(n)$, satisfies*

$$\left|(1-\beta)(\frac{f(z)}{z^{p}})^{\alpha}+\beta\frac{zf^{'}(z)}{pf(z)}(\frac{f(z)}{z^{p}})^{\alpha}-1\right|<\frac{|\beta\,(p\alpha+n\beta)|}{p\alpha\,(1+|\beta|)+|p\alpha+n\beta|}\ (z\in U),$$

then $f(z)\in S_{p}(n)$ and

$$\left|\frac{zf^{'}(z)}{pf(z)}-1\right|<1\ (z\in U).$$

Corollary 2.11. *For $\alpha=1$ in Theorem 2.9, we get the result obtained by Yang [20].*

For $p=n=\beta=\alpha=1$, Theorem 2.9 reduces to

Corollary 2.12. *If $0<\mu<2$ and $f(z)$ in A satisfies $\left|f^{'}(z)-1\right|<\mu\ (z\in U)$, then*

$$\left|\frac{zf^{'}(z)}{f(z)}-1\right|<\frac{3\mu}{2-\mu}\ (z\in U).$$

Acknowledgements. The authors would like to thank the referee for his careful reading and making some valuable comments which have essentially improved the presentation of this paper.

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