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Comparison of the Efficiencies of Variable Sampling Intervals Charts for Simultaneous Monitoring the means of multivariate Quality Variables

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Abstract

When the linear correlation of the quality variables are considerably high, multivariate control charts may be a more effective way than univariate charts which operate quality variables and process parameters individually. Performances and efficiencies of the multivariate control charts under multivariate normal process has been considered. Some numerical results are presented under small scale of the shifts in the process to see the improvement of the efficiency of EWMA chart and CUSUM chart, which use past quality information, comparing to Shewart chart, which do not use quality information. We can know that the decision of the optimum value of the smoothing constant in EWMA structure or the reference value in CUSUM structure are very important whether we adopt combine-accumulate technique or accumulate-combine technique under the given condition of process.

Keywords: LRT, ATS, ANSW, VSI technique, Switching behavior

1. Introduction

In many quality control, the quality of a product is usually represented by several correlated quality variables. When the quality variables are highly correlated, multivariate control procedures give better sensitivity than univariate procedures which use only one quality variable or process parameter at a time. Quality control procedure in which several correlated quality variables are of interest are called multivarite quality control procedure.

Assume that the production process of interest has $p(p \geq 2)$ related quality variables represented by the random vector $\underline{X} = (X_1, X_2, ..., X_p)'$. Under multivariate normal process $N_p(\underline{\mu}, \Sigma)$, there are many parameters to control the process. In this case, there are $(p^2 + p + 2)/2$ parameters for monitoring p related quality characteristics. Therefore, it may be very bothersome or difficult to use all the univariate control charts for monitoring p dimensional multivariate normal process.

Various types of multivariate control procedures have been proposed to take advantages of the relationships among the quality variables of interest. Alt[1] and Jackson[2] reviewed much of the literatures on multivariate quality control procedure. A Shewhart type chart for simultaneous control of standard deviation of two characteristics with a bivariate normal distribution was studied by Tuprah and Woodall[3].

A major weakness of Shewhart control chart is that it only uses the information about the process contained in the present sample, and it ignores any information given by the entire sequence of samples. Hence the Shewhart control chart is slow to signal for small or moderate shifts in the process. Two effective alternatives techniques to the Shewhart chart, the cumulative sum (CUSUM) control chart and the exponentially weighted moving average (EWMA) control chart, can be applied to improve the efficiency of the basic Shewhart chart when small or moderate shifts in the process has occurred.

A nomogram that is useful in the design of CUSUM charts was proposed by Goel and Wu[4]. Ewan and Kemp[5] evaluated integral equation method for the approximations of the run length (RL) distribution and Van Dobben de Bryun[6] recommended using Monte Carlo simulation method to estimate the RL distribution when average run length (ARL) is not large.

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Lucas and Saccucci[7] presented a Markov chain method to estimate the RL properties of EWMA chart for monitoring the univariate mean of normally distributed process and they showed that the performances of EWMA chart are close to those of CUSUM chart. Im and Cho[8] studied multiparameter CUSUM charts with variable sampling intervals (VSI) and they showed that a combined VSI CUSUM chart is comparatively more efficient than any other chart if the changes in both mean and variance are small.

In applying multivariate EWMA and CUSUM charts there are two basic approaches, combine-accumulate technique and accumulate-combine technique, and these techniques use past sample information. Crosier[9] proposed accumulate-combine technique in multivariate CUSUM chart and Lowry et al[10] proposed accumulate-combine technique in multivariate EWMA chart for monitoring mean vector of quality variables.

In this paper the performances and efficiencies of the multivariate Shewhart, EWMA and CUSUM charts are evaluated and compared when the linear correlation of all the related quality variables are very high. Process engineers may choose between combine-accumlate and accumulate-combine according to their process circumstances. As a result, it is recommended for them to operate multivariate control procedures, because it uses smoothing constant or reference values to response sensitively to each process shift by giving prompt signal even though the amount of shifts are very small, and process engineers may want to detect them as soon as possible.

2. Accumulate-Combine and Combine-Accumulate Techniques

Shewhart chart has a good property to detect large shift quickly and is easy to implement in the process. However, Shewhart chart is slow to signal for small or moderate shift. CUSUM chart and EWMA chart can be adopted to overcome this Shewhart chart's shortcoming.

A multivariate EWMA control chart for monitoring mean vector of a multivariate normal process using accumulate-combine approach was presented by Lowry *et al*.[10]. Through simulation, they showed that the performance of the multivariate EWMA procedure performs better than the multivariate CUSUM procedures Pignatiello and Runger[11].

Suppose that the production process has $p(=2, 3,...)$ quality variables represented by the random vector $\underline{X} = (X_1, X_2,..., X_p)$ and $\underline{X}$ has a multivariate normal distribution $N_p(\underline{\mu}_0, \Sigma_0)$ where the target mean vector $\underline{\mu}_0$ is specified and Σ_0 are known. And we also assume that sequential observation vectors between and within samples are independent and identically distributed. Let the sample of n observations taken at each sampling time i can be represented by $np \times 1$ vector $\underline{X}_i' = (\underline{X}_{i1}', \underline{X}_{i2}',..., \underline{X}_{in}')$ where $\underline{X}_{ij}' = (X_{ij1}', X_{ij2}',..., X_{ijp}')$.

In this study we assume that $\underline{\mu}_0 = \underline{0}$, all diagonal elements of Σ_0 are 1 and all off-diagonal elements of Σ_0 are 0.1 or 0.7, for simplicity of evaluation and comparison of the considered control charts's performances.

In the process the target mean vector $\underline{\mu}_0$ is set in advance, any deterioration in quality is generally reflected by a change in mean vector. To obtain the control statistic for mean vector $\underline{\mu}$ of $N_p(\underline{\mu}, \Sigma)$, we use likelihood ratio test (LRT) statistic for testing $H_0 : \underline{\mu} = \underline{\mu}_0$ vs $H_1 : \underline{\mu} \neq \underline{\mu}_0$ under multivriate normal process where Σ is known as Σ_0. By simple calculation, the LRT statistic can be obtained as follows:

$$Z_i^2 = n(\overline{\underline{X}}_i - \underline{\mu})' \Sigma_0^{-1} (\overline{\underline{X}}_i - \underline{\mu}) \tag{2.1}$$

Thus, the statistic Z_i^2 in (2.1) can be used as the control statistic for monitoring mean vector of p correlated quality variables. For arbitrary $\underline{\mu}$, control statistic Z_i^2 has a non-central chi-squared distribution with p degrees of freedom with noncentrality parameter $\tau^2 = n(\underline{\mu} - \underline{\mu}_0)' \Sigma_0^{-1} (\underline{\mu} - \underline{\mu}_0)$.

The null hypothesis $H_0 : \underline{\mu} = \underline{\mu}_0$ will be rejected whenever $Z_i^2 > \chi_\alpha(p)$, where $\chi_\alpha(p)$ is the upper 100αth percentile of a chi-square distribution with p degrees of freedom. Thus the LRT statistic Z_i^2 can be used as Shewhart control statistic for $\underline{\mu}$ and the control limits be set as $\{0, \chi_\alpha(p)\}$. If the statistic Z_i^2 in (2.1) plots above the control limit $\chi_\alpha(p)$, then the production process is deemed to be out of control state and an assignable cause of variation has occurred.

2.1. Combine-Accumulate Technique

The most intuitive method of replacing the multivariate Shewhart chart stataistic in (2.1) by a multivariate CUSUM chart statistic is to form a cumulative sum of the scalars $Z_j^2 (j = 1, 2,...)$. In this case, multivariate CUSUM statistic for $\underline{\mu}$ at the jth sample can be stated as

$$Y_j = \max\{Y_{j-1}, 0\} + (Z_j - k) \quad (2.2)$$

where $Y_0 = \omega \cdot I_{(\omega \geq 0)}$ is a constant. This multivariate CUSUM chart signals whenever $Y_j \geq h$.

Similarly, multivariate EWMA chart based on LRT statistic in (2.1) can be stated as

$$Y_{Z^2,i} = (1-\lambda)Y_{Z^2,-1} + \lambda Z_i^2 \quad (2.3)$$

where $Y_{Z^2,0} = \omega(\omega \geq 0)$ and $\lambda(0 < \lambda \leq 1)$ is a smoothing constant. This multivariate EWMA chart signals whenever $Y_{Z^2,i} \geq h$. Control limits *h's* in (2.2) and (2.3) can be obtained by using Markov chain method or integral equations.

2.2. Accumulate-Combine Technique

Crosier[9] and Lowry *et al.*[10] proposed new accumulate-combine techniques applying past sample information to multivariate CUSUM cart and multivariate EWMA cart respectively. They proposed charts which first accumulate past sample information for each parameter of interest and then form a univariate chart statistic from the sequence of multivariate statistic.

Pignatiello and Runger[10] also proposed a multivariate CUSUM chart based on accumulate-combine technique for $\underline{\mu}$ of $N_p(\underline{\mu}, \Sigma)$ process as Crosier[9] and the multivariate CUSUM chart by them is based on the following statistics. Let the chart statistic

$$\underline{D}_i = \sum_{j=i-1_i+1}^{i} (\underline{\overline{X}}_j - \underline{\mu}_0)$$

and

$$MC1_i = \max\{0, (n\underline{D}_i' \Sigma_0^{-1} \underline{D}_i)^{1/2} - kl_i\} \quad (2.4)$$

where reference value $k > 0$,

$$l_i = \begin{cases} l_{i-1}+1 & \text{if } MC1_{i-1} > 0 \\ 1 & otherwise \end{cases}$$

and $i = 1, 2, \cdots$. Multivariate CUSUM chart based on (2.4) signals whenever $MC1_i > h(h > 0)$.

Lowry *et al.*[10] proposed multivariate EWMA chart with accumulate-combine technique for $\underline{\mu}$, called MEWMA chart, and they constructed chart statistic by forming a univariate EWMA statistic from a multivariate EWMA statistic. The vectors of EWMA's are defined as

$$\underline{Y}_i = \sum_{k=1}^{i} \Lambda(I-\Lambda)^{i-k} \underline{\overline{X}}_i + (I-\Lambda)^i \underline{\mu}_0 \quad (2.5)$$

$i = 1, 2, 3, \cdots$, where $\underline{Y}_0 = \underline{\mu}_0$ and $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_p)$ $0 < \lambda_j \leq 1 (j = 1, 2, \ldots, p)$. This MEWMA chart signals whenever $T_i^2 = (\underline{Y}_i - \underline{\mu}_0)' \Sigma_{\underline{Y}_i}^{-1} (\underline{Y}_i - \underline{\mu}_0) > h$. They showed that the distribution of T_i^2 depends on $\underline{\mu}$ and Σ only through the noncentrality parameter $\tau^2 = n(\underline{\mu} - \underline{\mu}_0)' \Sigma_{\underline{Y}_i}^{-1} (\underline{\mu} - \underline{\mu}_0)$. Control limits *h's* for the sample statistic in (2.4) and (2.5) can be obtained to achieve a specified in control ARL by using simulation.

3. Variable Sampling Intervals Structure

The usual practice in using a control chart is to take samples from the process with fixed sampling intervals (FSI). The ability of a control chart is measured by the length of time required for the chart signal when the process has shifted. In FSI chart, the RL is defined as the random number of samples required for the chart to signal and average run length (ARL) is the expected value of the RL.

The idea of VSI control chart is intuitively reasonable. If a chart statistic falls close to a control limit. then one would naturally wonder whether subsequent samples actually be outside the control limit. In this case, the natural inclination would be to take next sample quickly and the sampling time interval $t_{i+1} - t_i$ should be short. On the other hands, if the current chart statistic close to the target value then the sampling time interval $t_{i+1} - t_i$ should be long on VSI structure.

Reynolds *et al.*[12] stated that the efficiency of VSI chart can also be interpreted as the value of the average number of samples to signal (ANSS) and the average time to signal (ATS). The number of samples to signal (NSS) and ANSS in VSI chart is the same definition of RL and ARL in FSI chart,

One disadvantage of VSI scheme is that frequent switching between different sampling intervals requires more cost and effort to administer the process than corresponding FSI scheme. Therefore for measuring the efficiency of a considered VSI control scheme, it is necessary to obtain the average number of switches (ANSW) made from the start of the process until the chart signals, or the probability of switches $P(\text{switch}) = \Sigma_{i=1}^{\eta} P(d_i) \cdot P(d_j|d_i)$ where $P(d_i)$ is the probability of using sampling interval d_i, and $P(d_j|d_i)$ is the conditional probability of using sampling interval d_i in the current sample given that the sampling interval $d_j(d_i \neq d_j)$ was used in the previous sample.

To apply η sampling intervals multivariate VSI structure, the interval of chart statistic in (2.1), (2.2), (2.3), (2.4) and T_i^2 are devided into in-control region C and out-of-control region C'. And the in control region C must be partitioned into η disjoint regions $I_1, I_2,..., I_\eta$ where I_j is the region in which the sampling interval d_j is used. Thus, the sampling interval used between $\underline{X}_i$ and $\underline{X}_{i+1}$ is $d(\underline{X}_i)$. If we use a finite number of interval lengths $d_1, d_1,..., d_\eta$ where $d_1 < d_2 < d_\eta (d_1 > 0)$, these possible interval lengths must be chosen to satisfy $l_1 < l_2 < l_2$ $(l_1 > 0)$. In this paper, the sample statistics (2.1), (2.2), (2.3), (2.4) and (2.5) used in FSI multivariate chart are applied to VSI multivariate charts, and then the performances and efficiencies of the VSI charts with two sampling intervals are evaluated and compared.

4. Comparison and Efficiencies of the Considered Charts

Upper control limit h value in FSI chart and g value in two sampling intervals VSI chart of the considered multivariate charts were obtained from the percentage points of chi-square distribution or simulation with 10,000 iterations. In our numerical computation, the ARL and ATS of the considered charts were fixed to 400.0 for the process in-control and the sample size n for each characteristic was 5 for $p = 2, 3$. Numerical performances of MEWMA chart has obtained under the condition $\lambda_1 = \lambda_2 = ... = \lambda_p = \lambda$.

To calculate the performances and compare the efficiencies of the charts, we employed $d_1 = 0.1$ and $d_2 = 1.9$ for the two sampling intervals VSI charts. Chang[13] presented and compared that ARL performances of two sampling intervals VSI procedure and three sampling interval VSI procedure. The numerical results shows that ARL performances are similar regardless the amount of shifts for monitoring both $\underline{\mu}$ and Σ. However, the values of switching behaviors including ANSW are less efficient in three sampling intervals VSI procedures than in two sampling intervals VSI procedures.

In Table 1 to Table 9, the values of the measures ARL, ATS and ANSW are presented. Using these measures we can evaluate and compare the efficiency of the multivarite control charts considered in this study.

The tables present the numerical results with the various amount of shifts τ^2, the difference according to whether the technique is combine-accumulate technique or accumulate-combine technique, and various values of design parameters, smoothing constant λ_i's or reference values k_i's. Through the presented numerical results, the performances and efficiencies can be examined.

In the combine-accumulate approach, it shows that when the amount of process shift τ^2 is large the Shewart chart, not adopting past quality (sample) information, is

Table 1. Performances of EWMA and Shewhart chart with C-A approach ($p = 2$, $\rho_0 = 0.7$)

	EWMA ($\lambda = 0.1$)			EWMA ($\lambda = 0.2$)			EWMA ($\lambda = 0.3$)			Shewhart chart ($\lambda = 1.0$)		
	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW
In control	400.00	400.00	53.16	400.00	400.00	76.57	400.00	400.00	95.70	400.00	400.00	199.84
$\tau = 0.5$	147.22	114.59	18.52	164.25	130.22	30.80	177.93	145.72	41.55	216.89	201.08	107.81
$\tau = 1.0$	35.98	25.50	3.83	37.48	20.02	5.72	42.58	21.77	8.10	71.45	53.23	33.29
$\tau = 1.5$	15.16	13.39	2.32	13.01	7.87	2.35	13.48	6.35	2.49	24.52	13.28	8.86
$\tau = 2.0$	8.68	8.74	2.04	6.69	5.07	1.87	6.32	3.87	1.67	9.79	3.97	2.68
τ=2.5	5.76	6.26	1.93	4.24	3.74	1.59	3.81	2.92	1.33	4.65	1.80	1.13
τ=3.0	4.17	4.78	1.79	3.02	2.96	1.31	2.65	2.42	1.03	2.63	1.25	0.67
$\tau = 3.5$	3.20	3.83	1.60	2.32	2.48	1.03	2.01	2.14	0.78	1.75	1.09	0.44
$\tau = 4.0$	2.57	3.18	1.34	1.87	2.18	0.77	1.62	1.99	0.55	1.33	1.04	0.25
τ=4.5	2.15	2.70	1.07	1.56	2.01	0.54	1.36	1.93	0.35	1.14	1.01	0.12
$\tau = 5.0$	1.84	2.34	0.82	1.34	1.94	0.33	1.18	1.91	0.18	1.05	1.01	0.05
$\tau = 5.5$	1.59	2.11	0.59	1.17	1.91	0.18	1.08	1.90	0.08	1.02	1.00	0.02
$\tau = 6.0$	1.38	1.98	0.38	1.08	1.90	0.08	1.03	1.90	0.03	1.00	1.00	0.00

Table 2. Performances of CUSUM chart with C-A approach ($p = 2$, $\rho_0 = 0.7$)

	CUSUM ($k = 2.1$)			CUSUM ($k = 2.2$)			CUSUM ($k = 2.3$)			CUSUM ($k = 2.4$)		
	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW
In control	400.00	400.00	32.26	400.00	400.00	43.57	400.00	400.00	55.72	400.00	400.00	67.26
$\tau = 0.5$	121.10	90.01	9.54	122.75	91.69	12.52	127.17	96.18	16.31	132.52	102.50	20.76
$\tau = 1.0$	32.29	19.93	3.18	29.54	16.96	3.37	28.37	15.33	3.61	28.08	14.84	3.95
$\tau = 1.5$	14.51	9.03	2.17	12.76	7.39	2.11	11.75	6.37	2.04	11.14	5.83	2.02
$\tau = 2.0$	8.34	5.43	1.81	7.25	4.48	1.65	6.59	3.90	1.55	6.16	3.59	1.49
$\tau =2.5$	5.49	3.81	1.56	4.77	3.22	1.40	4.32	2.88	1.27	4.02	2.70	1.21
$\tau =3.0$	3.95	2.95	1.34	3.44	2.59	1.19	3.11	2.39	1.08	2.89	2.29	1.02
$\tau = 3.5$	3.03	2.47	1.15	2.64	2.26	1.02	2.40	2.15	0.92	2.23	2.10	0.86
$\tau = 4.0$	2.43	2.20	0.99	2.13	2.08	0.87	1.94	2.03	0.76	1.81	2.00	0.67
$\tau =4.5$	2.02	2.06	0.85	1.78	2.00	0.69	1.62	1.97	0.57	1.51	1.96	0.48
$\tau = 5.0$	1.73	1.99	0.67	1.52	1.96	0.49	1.38	1.94	0.37	1.30	1.93	0.29
$\tau = 5.5$	1.49	1.95	0.48	1.31	1.93	0.30	1.21	1.92	0.21	1.15	1.92	0.15
$\tau = 6.0$	1.29	1.93	0.29	1.16	1.92	0.16	1.09	1.91	0.09	1.06	1.91	0.06

Table 3. Performances of EWMA chart with A-C approach ($p = 2$, $\rho_0 = 0.7$)

	EWMA ($\lambda = 0.1$)			EWMA ($\lambda = 0.$)			EWMA ($\lambda = 0.3$)			EWMA ($\lambda = 0.9$)		
	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW
In control	399.97	399.96	86.93	399.97	400.01	119.47	399.96	399.19	143.08	400.01	399.97	198.94
$\tau = 0.5$	33.66	17.38	5.77	49.30	30.26	11.55	67.92	47.29	20.15	194.46	177.43	94.91
$\tau = 1.0$	9.32	3.92	1.92	11.21	4.54	2.29	13.68	5.76	3.03	57.50	40.35	24.88
$\tau = 1.5$	4.67	2.04	1.27	5.17	2.14	1.34	5.71	2.28	1.42	18.40	9.09	6.11
$\tau = 2.0$	2.95	1.45	0.98	3.19	1.49	1.02	3.37	1.52	1.05	7.57	2.92	1.95
$\tau =2.5$	2.11	1.20	0.74	2.26	1.22	0.79	2.34	1.23	0.81	3.79	1.53	0.97
$\tau =3.0$	1.65	1.10	0.53	1.74	1.11	0.59	1.79	1.11	0.61	2.31	1.19	0.65
$\tau = 3.5$	1.36	1.04	0.33	1.43	1.05	0.38	1.46	1.06	0.40	1.64	1.07	0.43
$\tau = 4.0$	1.18	1.02	0.18	1.23	1.02	0.22	1.24	1.03	0.23	1.31	1.03	0.25
$\tau =4.5$	1.08	1.01	0.08	1.10	1.01	0.10	1.11	1.01	0.11	1.13	1.01	0.12
$\tau = 5.0$	1.03	1.00	0.03	1.04	1.00	0.04	1.04	1.00	0.04	1.05	1.01	0.05
$\tau = 5.5$	1.01	1.00	0.01	1.01	1.00	0.01	1.01	1.00	0.01	1.02	1.00	0.02
$\tau = 6.0$	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00

Table 4. Performances of CUSUM chart with A-C approach ($p = 2$, $\rho_0 = 0.7$)

	CUSUM ($k = 0.4$)			CUSUM ($k = 0.5$)			CUSUM ($k = 0.6$)			CUSUM ($k = 0.7$)		
	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW
In control	400.08	400.15	100.32	399.97	399.54	120.88	400.12	400.03	140.42	400.12	399.99	159.01
$\tau = 0.5$	34.81	21.15	6.54	40.07	26.10	9.16	47.45	33.17	12.97	56.02	41.57	18.02
$\tau = 1.0$	10.94	5.11	2.17	10.81	4.92	2.25	11.12	5.08	2.48	11.75	5.54	2.87
$\tau = 1.5$	6.39	2.74	1.53	5.98	2.51	1.47	5.77	2.40	1.46	5.69	2.33	1.47
$\tau = 2.0$	4.57	1.89	1.27	4.18	1.71	1.21	3.93	1.63	1.18	3.77	1.58	1.17
$\tau =2.5$	3.60	1.49	1.12	3.26	1.39	1.09	3.03	1.34	1.07	2.86	1.30	1.05
$\tau =3.0$	3.00	1.30	1.05	2.70	1.23	1.03	2.50	1.20	1.01	2.36	1.17	0.98
$\tau = 3.5$	2.58	1.19	1.02	2.33	1.15	1.00	2.17	1.13	0.97	2.05	1.11	0.91
$\tau = 4.0$	2.28	1.13	1.00	2.10	1.11	0.97	1.97	1.10	0.90	1.82	1.08	0.79
$\tau =4.5$	2.09	1.11	0.99	1.95	1.10	0.92	1.79	1.08	0.78	1.62	1.06	0.62
$\tau = 5.0$	1.99	1.10	0.97	1.83	1.08	0.82	1.61	1.06	0.61	1.42	1.04	0.42
$\tau = 5.5$	1.91	1.09	0.91	1.66	1.07	0.66	1.41	1.04	0.41	1.25	1.03	0.25
$\tau = 6.0$	1.80	1.08	0.80	1.47	1.05	0.47	1.24	1.02	0.24	1.12	1.01	0.12

Table 5. Performances of EWMA chart with A-C approach ($p = 2$, $\rho_0 = 0.7$)

	EWMA ($\lambda = 0.1$)			EWMA ($\lambda = 0.2$)			EWMA ($\lambda = 0.3$)			EWMA ($\lambda = 0.9$)		
	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW
In control	399.97	399.91	86.93	399.97	399.94	119.48	399.96	400.02	143.02	400.01	400.03	198.94
$\tau = 0.5$	33.37	17.31	5.77	49.38	30.30	11.58	67.21	47.03	19.94	194.53	177.55	94.98
$\tau = 1.0$	9.30	3.92	1.92	11.08	4.54	2.29	13.65	5.77	3.03	56.27	39.60	24.36
$\tau = 1.5$	4.67	2.05	1.28	5.17	2.16	1.35	5.68	2.29	1.43	18.22	9.05	6.08
$\tau = 2.0$	2.96	1.46	0.98	3.21	1.50	1.02	3.37	1.53	1.04	7.48	2.90	1.93
$\tau =2.5$	2.12	1.20	0.75	2.26	1.22	0.80	2.35	1.23	0.81	3.76	1.53	0.97
$\tau =3.0$	1.64	1.10	0.52	1.74	1.11	0.58	1.79	1.11	0.61	2.31	1.19	0.65
$\tau = 3.5$	1.36	1.04	0.33	1.42	1.05	0.38	1.45	1.05	0.40	1.64	1.07	0.42
$\tau = 4.0$	1.18	1.02	0.18	1.23	1.02	0.22	1.24	1.03	0.23	1.30	1.03	0.25
$\tau =4.5$	1.08	1.01	0.08	1.10	1.01	0.10	1.11	1.01	0.11	1.13	1.01	0.12
$\tau = 5.0$	1.03	1.00	0.03	1.04	1.00	0.04	1.04	1.00	0.04	1.05	1.01	0.05
$\tau = 5.5$	1.01	1.00	0.01	1.01	1.00	0.01	1.01	1.00	0.01	1.02	1.00	0.02
$\tau = 6.0$	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00	1.00	0.00

Table 6. Performances of EWMA and Shewhart chart with C-A approach ($p = 3$, $\rho_0 = 0.7$)

	EWMA ($\lambda = 0.1$)			EWMA ($\lambda = 0.2$)			EWMA ($\lambda = 0.3$)			Shewhart chart ($\lambda = 1.0$)		
	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW
In control	400.00	400.00	54.96	400.00	400.00	78.42	400.00	400.00	97.49	400.00	400.00	200.18
$\tau = 0.5$	172.41	139.92	22.09	190.29	157.21	36.67	204.51	173.57	49.17	245.90	230.38	123.83
$\tau = 1.0$	45.57	32.88	4.66	48.11	27.42	7.66	54.77	30.63	11.27	91.32	70.66	43.29
$\tau = 1.5$	19.19	17.27	2.47	16.49	10.07	2.72	17.26	8.27	3.11	32.51	18.78	12.74
$\tau = 2.0$	11.02	11.47	2.11	8.32	6.40	2.05	7.84	4.74	1.91	12.85	5.48	3.75
$\tau =2.5$	7.33	8.27	2.00	5.23	4.69	1.82	4.62	3.49	1.55	5.90	2.20	1.42
$\tau =3.0$	5.31	6.31	1.93	3.69	3.67	1.59	3.17	2.80	1.27	3.18	1.37	0.79
$\tau = 3.5$	4.06	5.03	1.84	2.81	3.00	1.34	2.38	2.38	1.01	2.01	1.13	0.53
$\tau = 4.0$	3.25	4.15	1.68	2.26	2.54	1.06	1.90	2.12	0.76	1.46	1.05	0.32
$\tau =4.5$	2.68	3.52	1.47	1.88	2.23	0.81	1.57	1.99	0.53	1.20	1.02	0.17
$\tau = 5.0$	2.28	3.04	1.21	1.60	2.05	0.58	1.34	1.93	0.34	1.08	1.01	0.08
$\tau = 5.5$	1.99	2.63	0.96	1.38	1.95	0.38	1.18	1.91	0.18	1.03	1.00	0.03
$\tau = 6.0$	1.76	2.30	0.75	1.21	1.92	0.21	1.08	1.90	0.08	1.01	1.00	0.01

Table 7. Performances of CUSUM chart with C-A approach ($p = 3$, $\rho_0 = 0.7$)

	CUSUM ($k = 3.1$)			CUSUM ($k = 3.2$)			CUSUM ($k = 3.3$)			CUSUM ($k = 3.4$)		
	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW
In control	400.00	400.00	31.71	400.00	400.00	41.24	400.00	400.00	51.04	400.00	400.00	60.32
$\tau = 0.5$	142.58	108.86	10.91	143.80	110.31	13.91	147.58	115.14	17.63	152.38	121.18	21.50
$\tau = 1.0$	40.14	24.79	3.53	37.07	21.41	3.79	35.61	19.83	4.08	35.11	19.23	4.46
$\tau = 1.5$	18.08	11.06	2.35	16.06	9.12	2.30	14.80	8.00	2.26	13.99	7.31	2.26
$\tau = 2.0$	10.35	6.54	1.94	9.08	5.39	1.81	8.26	4.72	1.72	7.69	4.31	1.64
$\tau =2.5$	6.78	4.49	1.70	5.93	3.76	1.53	5.36	3.35	1.42	4.97	3.10	1.35
$\tau =3.0$	4.84	3.40	1.48	4.23	2.92	1.32	3.83	2.67	1.22	3.55	2.52	1.15
$\tau = 3.5$	3.68	2.76	1.28	3.23	2.46	1.14	2.92	2.30	1.06	2.71	2.22	1.01
$\tau = 4.0$	2.93	2.38	1.13	2.58	2.20	1.02	2.34	2.12	0.94	2.18	2.07	0.87
$\tau =4.5$	2.42	2.16	1.01	2.14	2.06	0.89	1.95	2.02	0.79	1.81	2.00	0.71
$\tau = 5.0$	2.05	2.05	0.88	1.82	2.00	0.74	1.66	1.97	0.61	1.54	1.96	0.51
$\tau = 5.5$	1.79	1.99	0.73	1.57	1.96	0.55	1.43	1.94	0.42	1.33	1.93	0.33
$\tau = 6.0$	1.56	1.96	0.55	1.36	1.94	0.37	1.24	1.93	0.24	1.17	1.92	0.17

Table 8. Performances of EWMA chart with A-C approach ($p = 3$, $\rho_0 = 0.7$)

	EWMA ($\lambda = 0.1$)			EWMA ($\lambda = 0.2$)			EWMA ($\lambda = 0.3$)			EWMA ($\lambda = 0.9$)		
	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW
In control	400.02	400.01	84.97	400.08	400.00	116.83	400.01	399.70	140.18	399.91	399.73	199.04
$\tau = 0.5$	38.67	20.65	6.72	59.84	38.31	14.40	83.30	60.37	25.29	227.23	209.66	111.44
$\tau = 1.0$	10.39	4.36	2.07	12.74	5.22	2.58	16.07	6.94	3.61	74.37	54.51	33.16
$\tau = 1.5$	5.18	2.26	1.37	5.79	2.38	1.44	6.48	2.56	1.56	24.57	12.89	8.64
$\tau = 2.0$	3.25	1.54	1.04	3.52	1.59	1.09	3.72	1.64	1.11	9.62	3.81	2.57
τ=2.5	2.32	1.26	0.83	2.47	1.28	0.87	2.57	1.29	0.88	4.65	1.78	1.15
τ=3.0	1.79	1.13	0.62	1.89	1.14	0.67	1.94	1.14	0.69	2.69	1.27	0.74
$\tau = 3.5$	1.46	1.06	0.42	1.52	1.07	0.46	1.56	1.07	0.48	1.84	1.11	0.52
$\tau = 4.0$	1.25	1.03	0.24	1.30	1.03	0.28	1.32	1.04	0.30	1.41	1.05	0.32
τ=4.5	1.12	1.01	0.12	1.14	1.02	0.14	1.16	1.02	0.15	1.19	1.02	0.17
$\tau = 5.0$	1.05	1.01	0.05	1.06	1.01	0.06	1.07	1.01	0.07	1.08	1.01	0.08
$\tau = 5.5$	1.02	1.00	0.02	1.02	1.00	0.02	1.03	1.00	0.03	1.03	1.00	0.03
$\tau = 6.0$	1.00	1.00	0.00	1.01	1.00	0.01	1.01	1.00	0.01	1.01	1.00	0.01

Table 9. Performances of CUSUM chart with A-C approach ($p = 3$, $\rho_0 = 0.7$)

	CUSUM ($k = 0.4$)			CUSUM ($k = 0.5$)			CUSUM ($k = 0.6$)			CUSUM ($k = 0.7$)		
	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW	ARL	ATS	ANSW
In control	399.98	399.99	85.44	400.02	399.97	103.76	400.08	399.98	121.42	399.90	400.00	138.55
$\tau = 0.5$	37.71	22.92	6.30	44.39	29.00	9.02	52.92	37.14	12.87	63.38	47.57	18.18
$\tau = 1.0$	11.78	5.58	2.23	11.65	5.32	2.29	11.94	5.41	2.47	12.62	5.92	2.83
$\tau = 1.5$	6.98	3.13	1.64	6.51	2.78	1.56	6.24	2.57	1.50	6.11	2.47	1.50
$\tau = 2.0$	5.01	2.19	1.39	4.57	1.91	1.28	4.28	1.76	1.23	4.08	1.68	1.20
τ=2.5	3.97	1.72	1.22	3.59	1.51	1.14	3.31	1.41	1.10	3.11	1.36	1.08
τ=3.0	3.31	1.41	1.10	2.98	1.30	1.06	2.73	1.25	1.04	2.54	1.21	1.02
$\tau = 3.5$	2.86	1.26	1.04	2.55	1.19	1.02	2.35	1.16	1.01	2.19	1.14	0.97
$\tau = 4.0$	2.52	1.18	1.01	2.25	1.14	1.00	2.10	1.12	0.97	1.98	1.10	0.91
τ=4.5	2.24	1.13	1.00	2.07	1.11	0.99	1.96	1.10	0.92	1.81	1.08	0.80
$\tau = 5.0$	2.08	1.11	1.00	1.97	1.10	0.95	1.82	1.08	0.82	1.64	1.06	0.63
$\tau = 5.5$	2.00	1.10	0.98	1.88	1.09	0.87	1.66	1.07	0.66	1.44	1.04	0.44
$\tau = 6.0$	1.96	1.10	0.96	1.75	1.08	0.75	1.47	1.05	0.47	1.27	1.03	0.27

more efficient than EWMA chart or CUSUM chart, adopting past quality (sample) information in control chart, in terms of ARL, ATS, ANSW. However, the efficiency of most control charts is focused on the small or moderate process shifts, and in the cases EWMA chart or CUSUM chart are more efficient than Shewart chart.

Table 1 and Table 5 show performances for the linear correlation coefficient of quality variables ρ_0 being 0.7 which means high correlation and 0.1 which means almost independency. The values of ARL, ATS, ANSW in the tables shows that in in-control state when the quality variables's mean vector μ are changed but their linear correlation coefficient ρ_0 are the same, the efficiency only depends on the amount of process shift τ^2.

In Tables 1, 3, 6, and 8, when $p = 2$, 3 the performance and efficiency of the combine-accumulate procedure in EWMA chart can be compared with Shewart chart. In Tables 2, 4, 7, and 9, when $p = 2$, 3 the performance and efficiency of combine-accumulate procedure and accumulate-combine procedure in CUSUM chart can be compared. The results recommend that if possible, EWMA chart or CUSUM chart rather than Shewhart chart and accumulate-combine technique rather than combine-accumulate technique are more efficient and so better to apply.

In industrial production process, as a strategy for improving efficiency in control chart, it is recommended that first search the optimum value of smoothing constants or reference values according to the shift in the present process and then design multivariate control chart using past quality information to decide whether to use EWMA chart or CUSUM chart. According to the process situations we may decide whether to use combine-accumulate technique or accumulate-combine technique. However, the decision of the optimum value of smoothing constant or reference values based on the process shift expected is also very important.

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