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A NEW SUBCLASS OF MEROMORPHIC FUNCTIONS DEFINED BY HILBERT SPACE OPERATOR

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Abstract. In this paper, we introduce and investigate a new subclass of meromorphic functions associated with a certain integral operator on Hilbert space. For this class,we obtain several properties like the coefficient inequality, extreme points, radii of close-to-convexity, starlikeness and meromorphically convexity and integral transformation. Further, it is shown that this class is closed under convex linear combination.

1. Introduction

Let Σ denote the class of analytic functions in the punctured unit disc

$$\mathbb{U}^* = \{z \in \mathbb{C} : 0 < |z| < 1\} = \mathbb{U} \backslash \{0\},$$

with a simple pole at the origin of the form

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n. \tag{1}$$

Let $g \in \Sigma$ given by

$$g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n. \tag{2}$$

Then the Hadamard product(or convolution) [6] of the functions f and g , denoted by $f * g$, is given by

$$(f * g)(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n b_n z^n. \tag{3}$$

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There are many papers about some subclasses of meromorphic functions associated with several families of integral operators and derivate operators (see for example [2], [9], [10], [11], [12], [13]).

Lashin [10] defined an integral operator $J_\mu : \Sigma \to \Sigma$:

$$J_\mu = J_\mu f(z) = \frac{\mu}{z^{\mu+1}} \int_0^z t^\mu f(t)\, dt \qquad (\mu > 0;\ z \in \mathbb{U}^*), \tag{4}$$

integrating (4) ,we obtain

$$J_\mu f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{\mu}{n+\mu+1} a_n z^n = \frac{1}{z} + \sum_{n=1}^{\infty} L(n,\mu) a_n z^n, \tag{5}$$

where

$$L(n,\mu) = \frac{\mu}{n+\mu+1}. \tag{6}$$

Let H be a Hilbert space on the complex field and $L(H)$ denote the algebra of all bounded linear operators on H. For a complex-valued function f analytic in a domain E of the complex plain containing the spectrum $\sigma(T)$ of the bounded linear operator T, let $f(T)$ denote the operator on H defined by the Riesz-Dunford integral [5]

$$f(T) = \frac{1}{2\pi i} \int_C (zI - T)^{-1} f(z) dz,$$

where I is the identity operator on H and C is a positively oriented simple closed rectifiable closed contour containing the spectrum $\sigma(T)$ in the interior domain [8]. The operator $f(T)$ can also be defined by the following series:

$$f(T) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} T^n.$$

which converges in the norm topology.

The class of all functions $f \in \Sigma$ with $a_n \geq 0$ is denoted by Σ_p. The object of the present paper is to investigate the following subclass of Σ_p associated with the integral operator $J_\mu f(z)$.

Definition 1.1. *For $0 \leq \beta < 1$ and $0 \leq \alpha < 1$, a function $f \in \Sigma_p$ given by (1) is in the class $M_p(\alpha, \beta, T)$ if*

$$\begin{aligned} & \left\| T(J_\mu f(T))' - \{(\beta-1) J_\mu f(T) + \beta T (J_\mu f(T))'\} \right\| \\ < \ & \left\| T(J_\mu f(T))' + (1-2\alpha)\{(\beta-1) J_\mu f(T) + \beta T (J_\mu f(T))'\} \right\|. \end{aligned}$$

for all operators T with $\|T\| < 1$ and $T \neq \Theta$ (Θ is the zero operator on H).

Akgul and Bulut [1] defined new subclass for meromoprphic functions associated with a certain integral operator on Hilbert spaces and investigated some properties of this class. In this study, we obtain coefficient estimates, radii of starlikeness, and convexity for the functions in the class $M_p(\alpha, \beta, T)$. We employ the technique adopted by [1], [2], [3] and [4].

2. Coefficient Bounds

We first give a characterization of the class $M_p(\alpha, \beta, T)$ by finding necessary and sufficient condition for a function in this class. This characterization implies coefficient estimates.

Theorem 2.1. *A function $f \in \Sigma_p$ given by (1) is in the class $M_p(\alpha, \beta, T)$ for all proper contraction T with $T \neq \Theta$ if and only if*

$$\sum_{n=1}^{\infty}[n+\alpha-\alpha\beta(n+1)]L(n,\mu)a_n \leq 1-\alpha. \tag{7}$$

The result is sharp for the function

$$f(z) = \frac{1}{z} + \frac{1-\alpha}{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}z^n \quad (n \geq 1)\,. \tag{8}$$

Proof. Suppose that (7) is true for $0 \le \beta < 1$ and $0 \le \alpha < 1$. Then

$$\begin{aligned}
& \left\| T(J_\mu f(T))' - \{(\beta-1)J_\mu f(T) + \beta T(J_\mu f(T))'\} \right\| \\
& - \left\| T(J_\mu f(T))' + (1-2\alpha)\{(\beta-1)J_\mu f(T) + \beta T(J_\mu f(T))'\} \right\| \\
= & \left\| \sum_{n=1}^{\infty} (n+1)(1-\beta)L(n,\mu)a_n T^n \right\| \\
& - \left\| 2(1-\alpha)T^{-1} - \sum_{n=1}^{\infty} [n + (1-2\alpha)(\beta - 1 + \beta n)]L(n,\mu)a_n T^n \right\| \\
\le & \sum_{n=1}^{\infty} (n+1)(1-\beta)L(n,\mu)a_n \|T\|^n - 2(1-\alpha)\left\|T^{-1}\right\| \\
& + \sum_{n=1}^{\infty} [n + (1-2\alpha)(\beta - 1 + \beta n)]L(n,\mu)a_n \|T\|^n \\
= & \; 2\sum_{n=1}^{\infty} [n + \alpha - \alpha\beta(n+1)]L(n,\mu)a_n \|T\|^n - 2(1-\alpha)\left\|T^{-1}\right\| \\
\le & \; 2(1-\alpha) - 2(1-\alpha) = 0, \qquad \text{(by using (7))}
\end{aligned}$$

and so $f \in \Sigma_p$ is in the class $M_p(\alpha, \beta, T)$.
Conversely, let $f \in M_p(\alpha, \beta, T)$. We need only show that each function f of the class $M_p(\alpha, \beta, T)$ satisfies the coefficient inequality (7). Since $f \in M_p(\alpha, \beta, T)$, then

$$\begin{aligned}
& \left\| T(J_\mu f(T))' - \{(\beta-1)J_\mu f(T) + \beta T(J_\mu f(T))'\} \right\| \\
< & \left\| T(J_\mu f(T))' + (1-2\alpha)\{(\beta-1)J_\mu f(T) + \beta T(J_\mu f(T))'\} \right\|.
\end{aligned}$$

From this inequality, it is obtained that

$$\begin{aligned}
& \left\| \sum_{n=1}^{\infty} (n+1)(1-\beta)L(n,\mu)a_n T^{n+1} \right\| \\
< & \left\| 2(1-\alpha) - \sum_{n=1}^{\infty} [n + (1-2\alpha)(\beta - 1 + \beta n)]L(n,\mu)a_n T^{n+1} \right\|.
\end{aligned}$$

By choosing $T = rI$ $(0 < r < 1)$ in above inequality, we get

$$\frac{\sum\limits_{n=1}^{\infty} (n+1)(1-\beta)L(n,\mu)a_n r^{n+1}}{2(1-\alpha) - \sum\limits_{n=1}^{\infty} [n + (1-2\alpha)(\beta - 1 + \beta n)]L(n,\mu)a_n r^{n+1}} < 1.$$

Letting $r \to 1^-$ in the above inequality, we obtain the assertion (7). This completes the proof of our theorem. $\square$

From Theorem 2.1 we have the following result.

Corollary 2.2. *If a function $f \in \Sigma_p$ given by* (1) *is in the class $M_p(\alpha, \beta, T)$, then*

$$a_n \leq \frac{1-\alpha}{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)} \quad (n \geq 1).$$

The result is sharp for the function f of the form (8).

3. Extreme points

Theorem 3.1. *Let*

$$f_0(z) = \frac{1}{z}$$

and

$$f_n(z) = \frac{1}{z} + \frac{1-\alpha}{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)} z^n \quad (n = 1, 2, \ldots). \tag{9}$$

Then $f \in M_p(\alpha, \beta, T)$ if and only if it can be represented in the form

$$f(z) = \sum_{n=0}^{\infty} \tau_n f_n(z) \quad \left(\tau_n \geq 0,\ \sum_{n=0}^{\infty} \tau_n = 1\right).$$

Proof. Assume that $f(z) = \sum_{n=0}^{\infty} \tau_n f_n(z)$, ($\tau_n \geq 0$, $n = 0, 1, 2, \ldots$; $\sum_{n=0}^{\infty} \tau_n = 1$). Then we have

$$\begin{aligned} f(z) &= \sum_{n=0}^{\infty} \tau_n f_n(z) \\ &= \tau_0 f_0(z) + \sum_{n=1}^{\infty} \tau_n f_n(z) \\ &= \frac{1}{z} + \sum_{n=1}^{\infty} \tau_n \frac{1-\alpha}{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)} z^n. \end{aligned}$$

Therefore

$$\begin{aligned}
&\sum_{n=1}^{\infty}[n+\alpha-\alpha\beta(n+1)]L(n,\mu)\tau_n\frac{1-\alpha}{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}\\
=\ &(1-\alpha)\sum_{n=1}^{\infty}\tau_n\\
=\ &(1-\alpha)(1-\tau_0)\\
\leq\ &(1-\alpha).
\end{aligned}$$

Hence by Theorem 2.1, $f\in M_p(\alpha,\beta,T)$. Conversely, suppose that $f\in M_p(\alpha,\beta,T)$. Since, by Corollary 2.2,

$$a_n\leq\frac{1-\alpha}{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}\quad(n\geq 1)\,,$$

setting

$$\tau_n=\frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha}a_n\qquad(n\geq 1)$$

and $\tau_0=1-\sum_{n=1}^{\infty}\tau_n$, we obtain

$$f(z)=\tau_0 f_0(z)+\sum_{n=1}^{\infty}\tau_n f_n(z).$$

This completes the proof of the theorem. □

4. Radii of Close-to-Convexity, Starlikeness and Convexity

We concentrate upon getting the radii of meromorphically close-to-convexity, starlikeness and convexity for functions f in the class $M_p(\alpha,\beta,T)$.

Theorem 4.1. *Let $f\in M_p(\alpha,\beta,T)$. Then f is meromorphically close-to-convex of order γ $(0\leq\gamma<1)$ in the disk $|z|<r_1$, where*

$$r_1=\inf_{n\in\mathbb{N}}\left[\frac{(1-\gamma)\,[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{n\,(1-\alpha)}\right]^{\frac{1}{n+1}}\quad.$$

The result is sharp for the extremal function given by (8).

Proof. It sufficies to show that

$$\left\|\ f^{'}(T)T^2\ +1\right\|<1-\gamma. \tag{10}$$

By Theorem 2.1, we have

$$\sum_{n=1}^{\infty} \frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha} a_n \leq 1.$$

So the inequality

$$\left\| f^{'}(T)T^2 \ +1 \right\| = \left\| \sum_{n=1}^{\infty} n a_n T^{n+1} \right\| \leq \sum_{n=1}^{\infty} n a_n \left\| T \right\|^{n+1} < 1-\gamma$$

holds true if

$$\frac{n \left\| T \right\|^{n+1}}{1-\gamma} \leq \frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha}.$$

Then, (10) holds true if

$$\left\| T \right\|^{n+1} \leq \frac{(1-\gamma)\,[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{n\,(1-\alpha)} \qquad (n \geq 1)\,,$$

which yields the close-to-convexity of the function and completes the proof. □

Theorem 4.2. *Let $f \in M_p(\alpha, \beta, T)$. Then f is meromorphically starlike of order γ $(0 \leq \gamma < 1)$ in the disk $|z| < r_2$, where*

$$r_2 = \inf_{n\in\mathbb{N}} \left[\left(\frac{1-\gamma}{n+2-\gamma} \right) \frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha} \right]^{\frac{1}{n+1}} .$$

The result is sharp for the extremal function given by (8) *.*

Proof. By using the technique employed in the proof of Theorem 4.1, we can show that

$$\left\| \frac{Tf^{'}(T)}{f(T)} + 1 \right\| < 1-\gamma,$$

for $|z| < r_2$, and prove that the assertion of the theorem is true. □

Theorem 4.3. *Let $f \in M_p(\alpha, \beta, T)$. Then f is meromorphically convex of order γ $(0 \leq \gamma < 1)$ in the disk $|z| < r_3$ where*

$$r_3 = \inf_{n\in\mathbb{N}} \left[\left(\frac{1-\gamma}{n+2-\gamma} \right) \frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{n\,(1-\alpha)} \right]^{\frac{1}{n+1}} .$$

The result is sharp for the extremal function given by

$$f_n(z) = \frac{1}{z} + \frac{n\,(1-\alpha)}{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)} z^n .$$

Proof. By using the technique employed in the proof of Theorem 4.1 we can show that

$$\left\| \frac{Tf''(T)}{f'(T)} + 2 \right\| < 1 - \gamma,$$

for $|z| < r_3$ and prove that the assertion of the theorem is true. $\square$

5. Hadamard Product

Theorem 5.1. *For functions $f, g \in \Sigma_p$ defined by* (1) *and* (2) *, respectively, let $f, g \in M_p(\alpha, \beta, T)$. Then the Hadamard product $f * g \in M_p(\rho, \beta, T)$ where*

$$\rho \leq 1 - \frac{(1-\alpha)^2(n+1)(1-\beta)}{(1-\alpha)^2(1-\beta(n+1)) + [n+\alpha-\alpha\beta(n+1)]^2 L(n,\mu)}$$

Proof. We need to find the largest ρ such that

$$\sum_{n=1}^{\infty} \frac{[n+\rho-\rho\beta(n+1)]L(n,\mu)}{1-\rho} a_n b_n \leq 1.$$

Since $f, g \in M_p(\alpha, \beta, T)$ by Theorem 2.1, we have

$$\sum_{n=1}^{\infty} \frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha} a_n \leq 1, \tag{11}$$

and

$$\sum_{n=1}^{\infty} \frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha} b_n \leq 1. \tag{12}$$

From (11) and (12) we find, by means of the Cauchy-Schwartz inequality, that

$$\sum_{n=1}^{\infty} \frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha} \sqrt{a_n b_n} \leq 1. \tag{13}$$

We want only to show that

$$\begin{aligned} &\frac{[n+\rho-\rho\beta(n+1)]L(n,\mu)}{1-\rho} a_n b_n \\ &\leq \frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha} \sqrt{a_n b_n}, \end{aligned}$$

that is,

$$\sqrt{a_n b_n} \leq \frac{(1-\rho)\,[n+\alpha-\alpha\beta(n+1)]}{(1-\alpha)\,[n+\rho-\rho\beta(n+1)]}. \tag{14}$$

On the other hand, from (13) we have

$$\sqrt{a_n b_n} \leq \frac{1-\alpha}{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}. \tag{15}$$

Therefore in view of (14) and (15) it is enough to find the largest ρ that

$$\frac{1-\alpha}{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)} \leq \frac{(1-\rho)\,[n+\alpha-\alpha\beta(n+1)]}{(1-\alpha)\,[n+\rho-\rho\beta(n+1)]}$$

which yields

$$\rho \leq \frac{[n+\alpha-\alpha\beta(n+1)]^2\,L(n,\mu)-n(1-\alpha)^2}{[n+\alpha-\alpha\beta(n+1)]^2\,L(n,\mu)+(1-\alpha)^2\,[1-\beta(n+1)]}$$

that is,

$$\rho \leq 1-\frac{(1-\alpha)^2(n+1)(1-\beta)}{(1-\alpha)^2(1-\beta(n+1))+[n+\alpha-\alpha\beta(n+1)]^2\,L(n,\mu)}.$$

□

Theorem 5.2. *For functions $f,g \in \Sigma_p$ defined by (1) and (2), respectively, let $f,g \in M_p(\alpha,\beta,T)$. Then the function $k(z)=\frac{1}{z}+\sum\limits_{n=1}^{\infty}(a_n^2+b_n^2)z^n$ is in the class $M_p(\rho,\beta,T)$ where*

$$\rho \leq 1-\frac{2(1-\alpha)^2L(n,\mu)[1-\beta(n+1)+n]}{\{[n+\alpha-\alpha\beta(n+1)]\,L(n,\mu)\}^2+2(1-\alpha)^2L(n,\mu)[1-\beta(n+1)]}$$

Proof. Since $f,g \in M_p(\alpha,\beta,T)$ we have

$$\sum_{n=1}^{\infty}\left\{\frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)a_n}{1-\alpha}\right\}^2 \leq 1 \tag{16}$$

and

$$\sum_{n=1}^{\infty}\left\{\frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)b_n}{1-\alpha}\right\}^2 \leq 1 \tag{17}$$

Combining the last two inequalities, we get

$$\sum_{n=1}^{\infty}\frac{1}{2}\left\{\frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha}\right\}^{2}(a_n^2+b_n^2)\leq 1 \tag{18}$$

But we need to find the largest ρ such that

$$\sum_{n=1}^{\infty}\frac{[n+\rho-\rho\beta(n+1)]L(n,\mu)(a_n^2+b_n^2)}{1-\rho}\leq 1 \tag{19}$$

The inequality(19) would hold if

$$\frac{[n+\rho-\rho\beta(n+1)]L(n,\mu)}{1-\rho}\leq\frac{1}{2}\left\{\frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha}\right\}^{2}$$

Then we have

$$\begin{aligned}\rho &\leq \frac{\{[n+\alpha-\alpha\beta(n+1)]\,L(n,\mu)\}^{2}-2n(1-\alpha)^2L(n,\mu)}{\{[n+\alpha-\alpha\beta(n+1)]\,L(n,\mu)\}^{2}+2(1-\alpha)^2L(n,\mu)[1-\beta(n+1)]}\\ &= 1-\frac{2(1-\alpha)^2L(n,\mu)[1-\beta(n+1)+n]}{\{[n+\alpha-\alpha\beta(n+1)]\,L(n,\mu)\}^{2}+2(1-\alpha)^2L(n,\mu)[1-\beta(n+1)]}.\end{aligned}$$

□

6. Integral Operators

In this section, we consider integral transforms of functions in the class $M_p(\alpha,\beta,T)$ of the type considered by Goel and Sohi [7] .

Theorem 6.1. *Let the function $f\in\Sigma_p$ given by (1) is in the class $M_p(\alpha,\beta,T)$.Then the integral operator*

$$F(z)=c\int_0^1 u^c f(uz)\,du,\ldots\ldots..(0<u\leq 1,0<c<\infty) \tag{20}$$

is in $M_p(\rho,\beta,T)$, where

$$\rho=1-\frac{(1-\alpha)(1+2\beta)+c}{(1+\alpha-2\alpha\beta)(c+2)+(1-\alpha)(1-2\beta)}$$

The result is sharp for the function

$$f(z)=\frac{1}{z}+\frac{(1-\alpha)(\mu+2)}{(1+\alpha-2\alpha\beta)\mu}z$$

Proof. Let $f \in \Sigma_p$ given by (1) is in the class $M_p(\alpha, \beta, T)$.Then

$$F(z) = c\int_0^1 u^c f(uz)\,du$$

$$= \frac{1}{z} + \sum_{n=1}^{\infty} \frac{c}{c+n+1} a_n z^n \tag{21}$$

We have to show that

$$\sum_{n=1}^{\infty} \frac{c\,[n+\rho-\rho\beta(n+1)]\,L(n,\mu)}{(1-\rho)(c+n+1)} a_n \leq 1. \tag{22}$$

Since $f \in M_p(\alpha, \beta, T)$,we have

$$\sum_{n=1}^{\infty} \frac{[n+\alpha-\alpha\beta(n+1)]L(n,\mu)}{1-\alpha} a_n \leq 1,$$

The inequality (22) satisfied if

$$\frac{c\,[n+\rho-\rho\beta(n+1)]}{(1-\rho)(c+n+1)} \leq \frac{[n+\alpha-\alpha\beta(n+1)]}{1-\alpha}$$

Then we get

$$\rho \leq \frac{[n+\alpha-\alpha\beta(n+1)](n+c+1)-(1-\alpha)cn}{[n+\alpha-\alpha\beta(n+1)](n+c+1)+c(1-\alpha)(1-\beta(n+1))}$$

$$= 1 - \frac{(1-\alpha)[1+\beta(n+1)]+cn}{[n+\alpha-\alpha\beta(n+1)](n+c+1)+c(1-\alpha)[1-\beta(n+1)]}$$

Since

$$\phi(n) = 1 - \frac{(1-\alpha)[1+\beta(n+1)]+cn}{[n+\alpha-\alpha\beta(n+1)](n+c+1)+c(1-\alpha)[1-\beta(n+1)]}$$

is an increasing function of $n (n \geq 1)$ we obtain the desired result. $\square$

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