J. Appl. Math. & Informatics Vol. **34***(2016), No. 3 - 4, pp. 331 - 340*
http://dx.doi.org/10.14317/jami.2016.331

GENERALIZED INT-SOFT SUBALGEBRAS OF BE-ALGEBRAS†

KEUM SOOK SO AND YOUNG HEE KIM*

ABSTRACT. The notion of θ-generalized int-soft subalgebras of BE-algebras is introduced, and related properties are investigated. Relations between int-soft subalgebras and θ-generalized int-soft subalgebras are discussed, and characterizations of θ-generalized int-soft subalgebras are considered.

AMS Mathematics Subject Classification : 06F35, 03G25, 06D72.
Key word and phrases : BE-algebra, subalgebra, (generalized) int-soft subalgebra.

1. Introduction

In 1966, Imai and Iséki [4] and Iséki [5] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. As a generalization of a BCK-algebra, Kim and Kim [7] introduced the notion of a BE-algebra, and investigated several properties. In [1], Ahn and So introduced the notion of ideals in BE-algebras. They gave several descriptions of ideals in BE-algebras.

Various problems in system identification involve characteristics which are essentially non-probabilistic in nature [12]. In response to this situation Zadeh [13] introduced *fuzzy set theory* as an alternative to probability theory. Uncertainty is an attribute of information. In order to suggest a more general framework, the approach to uncertainty is outlined by Zadeh [14].

Uncertainties can't be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [10]. Maji et al. [9] and Molodtsov [10] suggested that one reason for these difficulties may be due to the inadequacy of

---

Received November 18, 2015. Revised January 14, 2016. Accepted January 20, 2016.
*Corresponding author. †This work is supported by Chungbuk National University fund 2015.

the parametrization tool of the theory. To overcome these difficulties, Molodtsov [10] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [9] described the application of soft set theory to a decision making problem. Maji et al. [8] also studied several operations on the theory of soft sets. Chen et al. [3] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors.

Jun and Ahn [6] introduced the notion of int-soft subalgebras of a BE-algebra, and investigated their properties. They considered characterization of an int-soft subalgebra, and solved the problem of classifying int-soft subalgebras by their inclusive subalgebras.

In this paper, we consider a generalization of the paper [6]. We introduce the notion of θ-generalized int-soft subalgebras of BE-algebras, and investigate related properties. We discuss relations between int-soft subalgebras and θ-generalized int-soft subalgebras. We consider characterizations of θ-generalized int-soft subalgebras.

2. Preliminaries

Let $K(\tau)$ be the class of all algebras of type $\tau = (2, 0)$. By a *BE-algebra* we mean a system $(X; *, 1) \in K(\tau)$ in which the following axioms hold (see [7]):

$$(\forall x \in X)\,(x * x = 1), \tag{2.1}$$

$$(\forall x \in X)\,(x * 1 = 1), \tag{2.2}$$

$$(\forall x \in X)\,(1 * x = x), \tag{2.3}$$

$$(\forall x, y, z \in X)\,(x * (y * z) = y * (x * z)). \quad \text{(exchange)} \tag{2.4}$$

A relation "$\leq$" on a BE-algebra X is defined by

$$(\forall x, y \in X)\,(x \leq y \iff x * y = 1). \tag{2.5}$$

A nonempty subset S of a BE-algebra X is called a *subalgebra* of X if $x*y \in S$ for all $x, y \in S$.

Molodtsov [10] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathscr{P}(U)$ denotes the power set of U and $A \subset E$.

A pair $\left(\tilde{f}, A\right)$ is called a *soft set* (see [10]) over U, where $\tilde{f}$ is a mapping given by

$$\tilde{f} : A \to \mathscr{P}(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $\tilde{f}(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set $\left(\tilde{f}, A\right)$. Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [10].

For a soft set $\left(\tilde{f}, X\right)$ over U and a subset γ of U, the *γ-inclusive set* of $\left(\tilde{f}, X\right)$, denoted by $(\tilde{f};\gamma)^{\supseteq}$, is defined to be the set

$$(\tilde{f};\gamma)^{\supseteq} := \left\{x \in X \mid \gamma \subseteq \tilde{f}(x)\right\}.$$

3. θ-generalized int-soft subalgebras

In what follows, we take a BE-algebra X, as a set of parameters, and let $\mathscr{P}^*(U) = \mathscr{P}(U) \setminus \{\emptyset\}$ unless otherwise specified.

Definition 3.1 ([6])**.** A soft set $\left(\tilde{f}, X\right)$ over U is called an *int-soft subalgebra* of X if it satisfies:

$$(\forall x, y \in X)\left(\tilde{f}(x * y) \supseteq \tilde{f}(x) \cap \tilde{f}(y)\right). \tag{3.1}$$

Definition 3.2. If a soft set $\left(\tilde{f}, X\right)$ over U satisfies the following assertion:

$$(\forall x, y \in X)\,(\exists \theta \in \mathscr{P}^*(U))\left(\tilde{f}(x * y) \supseteq \theta \cap \tilde{f}(x) \cap \tilde{f}(y)\right), \tag{3.2}$$

then we say that $\left(\tilde{f}, X\right)$ is a *θ-generalized int-soft subalgebra* of X.

Obviously, every int-soft subalgebra is a θ-generalized int-soft subalgebra for all $\theta \in \mathscr{P}^*(U)$. Also, if $\theta = U$ then every θ-generalized int-soft subalgebra is an int-soft subalgebra. For every soft set $\left(\tilde{f}, X\right)$ over U, it is cleat that if $\theta \subseteq \mathrm{Im}(\tilde{f})$ then $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra of X.

For a soft set $\left(\tilde{f}, X\right)$ over U, we know that there exists nonempty subset θ of U such that $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra, but not an int-soft subalgebra as seen in the following example.

Example 3.3. Let $X = \{1, a, b\}$ be a BE-algebra with the following Cayley table:

$*$	1	a	b
1	1	a	b
a	1	1	b
b	1	a	1

Let $\left(\tilde{f}, X\right)$ be a soft set over $U = \mathbb{Z}$(; the set of integers) defined as follows:

$$\tilde{f} : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} 5\mathbb{Z} & \text{if } x = 1, \\ 4\mathbb{Z} & \text{if } x \in \{a, b\} \end{cases}$$

Then $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra of X with $\theta = 10\mathbb{Z}$, but it is not an int-soft subalgebra of X since $\tilde{f}(a * a) = \tilde{f}(1) = 5\mathbb{Z} \not\supseteq 4\mathbb{Z} = \tilde{f}(a) = \tilde{f}(a) \cap \tilde{f}(a)$. Also, $\left(\tilde{f}, X\right)$ is not a θ-generalized int-soft subalgebra of X with $\theta = 4\mathbb{Z}$ or $\theta = 8\mathbb{Z}$.

Example 3.4. Let $X = \{1, a, b, c, d, 0\}$ be a BE-algebra ([1]) with the following Cayley table:

$*$	1	a	b	c	d	0
1	1	a	b	c	d	0
a	1	1	a	c	c	d
b	1	1	1	c	c	c
c	1	a	b	1	a	b
d	1	1	a	1	1	a
0	1	1	1	1	1	1

Let $\left(\tilde{f}, X\right)$ be a soft set over $U = \mathbb{Z}$(; the set of integers) defined as follows:

$$\tilde{f} : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \mathbb{Z} & \text{if } x = 1, \\ 2\mathbb{Z} & \text{if } x = a, \\ 9\mathbb{Z} & \text{if } x = b, \\ 6\mathbb{Z} & \text{if } x = c, \\ 4\mathbb{Z} & \text{if } x = d \end{cases}$$

Then $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra of X with $\theta = 12\mathbb{Z}$, but it is not an int-soft subalgebra of X since $\tilde{f}(a*d) = \tilde{f}(c) = 6\mathbb{Z} \not\supseteq 4\mathbb{Z} = \tilde{f}(a) \cap \tilde{f}(d)$.

Theorem 3.5. *A soft set $\left(\tilde{f}, X\right)$ over U is a θ-generalized int-soft subalgebra of X if and only if $(\tilde{f};\gamma)^{\supseteq}$ is a subalgebra of X for all $\gamma \in \mathscr{P}(U)$ with $\gamma \subseteq \theta$.*

Proof. Assume that $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra of X. Let $x, y \in (\tilde{f};\gamma)^{\supseteq}$ where $\gamma \in \mathscr{P}(U)$ with $\gamma \subseteq \theta$. Then $\tilde{f}(x) \supseteq \gamma$ and $\tilde{f}(y) \supseteq \gamma$. It follows from (3.2) that

$$\tilde{f}(x * y) \supseteq \theta \cap \tilde{f}(x) \cap \tilde{f}(y) \supseteq \gamma$$

and so that $x * y \in (\tilde{f};\gamma)^{\supseteq}$. Therefore $(\tilde{f};\gamma)^{\supseteq}$ is a subalgebra of X for all $\gamma \in \mathscr{P}(U)$ with $\gamma \subseteq \theta$.

Conversely, suppose that $(\tilde{f};\gamma)^{\supseteq}$ is a subalgebra of X for all $\gamma \in \mathscr{P}(U)$ with $\gamma \subseteq \theta$. Let $x, y \in X$ be such that $\tilde{f}(x) = \gamma_x$ and $\tilde{f}(y) = \gamma_y$. Take $\gamma = \theta \cap \gamma_x \cap \gamma_y$. Then $x, y \in (\tilde{f};\gamma)^{\supseteq}$, and so $x * y \in (\tilde{f};\gamma)^{\supseteq}$. Thus

$$\tilde{f}(x * y) \supseteq \gamma = \theta \cap \gamma_x \cap \gamma_y = \theta \cap \tilde{f}(x) \cap \tilde{f}(y)$$

which shows that $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra of X. □

Lemma 3.6. *Every θ-generalized int-soft subalgebra $\left(\tilde{f}, X\right)$ over U satisfies the following inclusion:*

$$(\forall x \in X)\left(\tilde{f}(1) \supseteq \theta \cap \tilde{f}(x)\right). \tag{3.3}$$

Proof. Using (2.1) and (3.2), we have

$$\tilde{f}(1) = \tilde{f}(x * x) \supseteq \theta \cap \tilde{f}(x) \cap \tilde{f}(x) = \theta \cap \tilde{f}(x)$$

for all $x \in X$. □

Proposition 3.7. *For any θ-generalized int-soft subalgebra $\left(\tilde{f}, X\right)$ over U, if a fixed element $x \in X$ satisfies $\tilde{f}(x) = \tilde{f}(1)$, then*

$$(\forall y \in X)\left(\theta \cap \tilde{f}(y) \subseteq \tilde{f}(x * y)\right). \tag{3.4}$$

Proof. Assume that a fixed element $x \in X$ satisfies $\tilde{f}(x) = \tilde{f}(1)$. Then

$$\theta \cap \tilde{f}(y) = \theta \cap \tilde{f}(1) \cap \tilde{f}(y) = \theta \cap \tilde{f}(x) \cap \tilde{f}(y) \subseteq \tilde{f}(x * y)$$

for all $y \in X$. □

Proposition 3.8. *Let $\left(\tilde{f}, X\right)$ be a θ-generalized int-soft subalgebra over U. If a fixed element $x \in X$ satisfies the following condition:*

$$(\forall y \in X)\left(\tilde{f}(y) \subseteq \theta \cap \tilde{f}(y * x)\right), \tag{3.5}$$

then $\tilde{f}(1) = \theta \cap \tilde{f}(x)$.

Proof. Taking $y = 1$ in (3.5) implies that $\tilde{f}(1) \subseteq \theta \cap \tilde{f}(1 * x) = \theta \cap \tilde{f}(x)$ by (2.3). It follows from Lemma 3.6 that $\tilde{f}(1) = \theta \cap \tilde{f}(x)$. □

Theorem 3.9. *For every $\vartheta \in \mathscr{P}^*(U)$ with $\vartheta \subseteq \theta$, every θ-generalized int-soft subalgebra is a ϑ-generalized int-soft subalgebra.*

Proof. Let $\left(\tilde{f}, X\right)$ be a θ-generalized int-soft subalgebra over U and let $\vartheta \in \mathscr{P}^*(U)$ with $\vartheta \subseteq \theta$. For any $x, y \in X$, we have

$$\tilde{f}(x * y) \supseteq \theta \cap \tilde{f}(x) \cap \tilde{f}(y) \supseteq \vartheta \cap \tilde{f}(x) \cap \tilde{f}(y).$$

Therefore $\left(\tilde{f}, X\right)$ is a ϑ-generalized int-soft subalgebra over U for all $\vartheta \in \mathscr{P}(U)$ with $\vartheta \subseteq \theta$. □

The following example shows that the converse of Theorem 3.9 is not true in general.

Example 3.10. Consider the soft set $\left(\tilde{f}, X\right)$ which is given in Example 3.4. Note that it is a ϑ-generalized int-soft subalgebra of X with $\vartheta = 12\mathbb{Z}$. If we take $\theta = 6\mathbb{Z}$, then $\vartheta \subseteq \theta$ and $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra of X. But if we take $\theta = 4\mathbb{Z}$, then $\vartheta \subseteq \theta$ and

$$\tilde{f}(a * d) = \tilde{f}(c) = 6\mathbb{Z} \not\supseteq 4\mathbb{Z} = 4\mathbb{Z} \cap \tilde{f}(a) \cap \tilde{f}(d).$$

Hence $\left(\tilde{f}, X\right)$ is not a θ-generalized int-soft subalgebra of X with $\theta = 4\mathbb{Z}$.

Theorem 3.11. *If $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra over U, then the set*

$$X_{\tilde{f}} := \left\{x \in X \mid \tilde{f}(x) \supseteq \tilde{f}(1) \cap \theta\right\}$$

is a subalgebra of X.

Proof. Let $x, y \in X_{\tilde{f}}$. Then $\tilde{f}(x) \supseteq \tilde{f}(1) \cap \theta$ and $\tilde{f}(y) \supseteq \tilde{f}(1) \cap \theta$. It follows from (3.2) that

$$\tilde{f}(x * y) \supseteq \theta \cap \tilde{f}(x) \cap \tilde{f}(y) \supseteq \theta \cap \left(\tilde{f}(1) \cap \theta\right) = \tilde{f}(1) \cap \theta$$

and so that $x * y \in X_{\tilde{f}}$. Thus $X_{\tilde{f}}$ is a subalgebra of X. □

Theorem 3.12. *For a subset S of X, define a soft set $\left(\tilde{f}, X\right)$ over U as follows:*

$$\tilde{f} : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \gamma \cap \theta & \text{if } x \in S, \\ \tau & \text{otherwise} \end{cases}$$

where $\gamma, \tau \in \mathscr{P}(U)$ with $\tau \subsetneq \gamma \cap \theta$. Then $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra over U if and only if S is a subalgebra of X. Moreover, $X_{\tilde{f}} = S$.

Proof. Assume that $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra over U. Let $x, y \in S$. Then $\tilde{f}(x * y) \supseteq \theta \cap \tilde{f}(x) \cap \tilde{f}(y) = \theta \cap (\gamma \cap \theta) = \gamma \cap \theta$, and so $x * y \in S$. Thus S is a subalgebra of X.

Conversely, suppose that S is a subalgebra of X. Let $x, y \in X$. If $x, y \in S$, then $x * y \in S$. Hence $\tilde{f}(x * y) = \gamma \cap \theta = \tilde{f}(x) \cap \tilde{f}(y)$. If $x \notin S$ or $y \notin S$, then $\tilde{f}(x) = \tau$ or $\tilde{f}(y) = \tau$. Hence $\tilde{f}(x * y) \supseteq \tau = \tilde{f}(x) \cap \tilde{f}(y)$. Therefore $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra over U. Moreover, we have

$$\begin{aligned} X_{\tilde{f}} &= \left\{x \in X \mid \tilde{f}(x) \supseteq \tilde{f}(1) \cap \theta\right\} \\ &= \left\{x \in X \mid \tilde{f}(x) \supseteq (\gamma \cap \theta) \cap \theta\right\} \\ &= \left\{x \in X \mid \tilde{f}(x) \supseteq \gamma \cap \theta\right\} \\ &= \left\{x \in X \mid \tilde{f}(x) = \gamma \cap \theta\right\} \\ &= S. \end{aligned}$$

This completes the proof. □

For any BE-algebras X and Y, let $\mu : X \to Y$ be a function and $\left(\tilde{f}, X\right)$ and $(\tilde{g}, Y)$ be soft sets over U.

(1) The soft set

$$\mu^{-1}(\tilde{g}, Y) = \left\{\left(x, \mu^{-1}(\tilde{g})(x)\right) : x \in X,\ \mu^{-1}(\tilde{g})(x) \in \mathscr{P}(U)\right\},$$

where $\mu^{-1}(\tilde{g})(x) = \tilde{g}(\mu(x))$, is called the *soft pre-image* of $(\tilde{g}, Y)$ under μ (see [6]).

(2) The soft set

$$\mu\left(\tilde{f}, X\right) = \left\{\left(y, \mu(\tilde{f})(y)\right) : y \in Y,\ \mu(\tilde{f})(y) \in \mathscr{P}(U)\right\}$$

where

$$\mu(\tilde{f})(y) = \begin{cases} \bigcup\limits_{x \in \mu^{-1}(y)} \tilde{f}(x) & \text{if } \mu^{-1}(y) \neq \emptyset, \\ \emptyset & \text{otherwise,} \end{cases}$$

is called the *soft image* of $\left(\tilde{f}, X\right)$ under μ (see [6]).

Theorem 3.13. *Let $\mu : X \to Y$ be a homomorphism of BE-algebras and $(\tilde{g}, Y)$ a soft set over U. If $(\tilde{g}, Y)$ is a θ-generalized int-soft subalgebra over U, then the soft pre-image $\mu^{-1}(\tilde{g}, Y)$ of $(\tilde{g}, Y)$ under μ is also a θ-generalized int-soft subalgebra over U.*

Proof. For any $x_1, x_2 \in X$, we have

$$\begin{aligned} \mu^{-1}(\tilde{g})(x_1 * x_2) &= \tilde{g}\left(\mu(x_1 * x_2)\right) \\ &= \tilde{g}\left(\mu(x_1) * \mu(x_2)\right) \\ &\supseteq \theta \cap \tilde{g}(\mu(x_1)) \cap \tilde{g}(\mu(x_2)) \\ &= \theta \cap \mu^{-1}(\tilde{g})(x_1) \cap \mu^{-1}(\tilde{g})(x_2) \end{aligned}$$

Hence $\mu^{-1}(\tilde{g}, Y)$ is also a θ-generalized int-soft subalgebra over U. □

Theorem 3.14. *Let $\mu : X \to Y$ be a homomorphism of BE-algebras and $\left(\tilde{f}, X\right)$ a soft set over U. If $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra over U and μ is injective, then the soft image $\mu\left(\tilde{f}, X\right)$ of $\left(\tilde{f}, X\right)$ under μ is also a θ-generalized int-soft subalgebra over U.*

Proof. Let $y_1, y_2 \in Y$. If at least one of $\mu^{-1}(y_1)$ and $\mu^{-1}(y_1)$ is empty, then the inclusion

$$\theta \cap \mu(\tilde{f})(y_1) \cap \mu(\tilde{f})(y_1) \subseteq \mu(\tilde{f})(y_1 * y_2)$$

is clear. Assume that $\mu^{-1}(y_1) \neq \emptyset$ and $\mu^{-1}(y_2) \neq \emptyset$. Then

$$\begin{aligned}
\mu(\tilde{f})(y_1 * y_2) &= \bigcup_{x \in \mu^{-1}(y_1 * y_2)} \tilde{f}(x) \\
&= \bigcup_{\substack{x_1 \in \mu^{-1}(y_1) \\ x_2 \in \mu^{-1}(y_2)}} \left(\tilde{f}(x_1 * x_2)\right) \\
&\supseteq \bigcup_{\substack{x_1 \in \mu^{-1}(y_1) \\ x_2 \in \mu^{-1}(y_2)}} \left(\theta \cap \tilde{f}(x_1) \cap \tilde{f}(x_2)\right) \\
&= \theta \cap \left(\bigcup_{x_1 \in \mu^{-1}(y_1)} \tilde{f}(x_1)\right) \cap \left(\bigcup_{x_2 \in \mu^{-1}(y_2)} \tilde{f}(x_2)\right) \\
&= \theta \cap \mu(\tilde{f})(y_1) \cap \mu(\tilde{f})(y_2).
\end{aligned}$$

Therefore $\mu\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra over U. □

For any soft set $\left(\tilde{f}, X\right)$ over U and $\delta \in \mathscr{P}^*(U)$, let $\left(\tilde{f}_\delta, X\right)$ be a soft set over U where

$$\tilde{f}_\delta : X \to \mathscr{P}(U),\ x \mapsto \tilde{f}(x) \cap \delta.$$

Theorem 3.15. *If $\left(\tilde{f}, X\right)$ is an int-soft subalgebra over U, then so is $\left(\tilde{f}_\delta, X\right)$ for all $\delta \in \mathscr{P}^*(U)$.*

Proof. For any $x, y \in X$ and $\delta \in \mathscr{P}(U)$, we have

$$\begin{aligned}
\tilde{f}_\delta(x * y) &= \tilde{f}(x * y) \cap \delta \supseteq \tilde{f}(x) \cap \tilde{f}(y) \cap \delta \\
&= \left(\tilde{f}(x) \cap \delta\right) \cap \left(\tilde{f}(y) \cap \delta\right) \\
&= \tilde{f}_\delta(x) \cap \tilde{f}_\delta(y).
\end{aligned}$$

Hence $\left(\tilde{f}_\delta, X\right)$ is an int-soft subalgebra over U for all $\delta \in \mathscr{P}(U)$. □

We pose a question as follows.

Question. *Let $\left(\tilde{f}, X\right)$ be a soft set over U such that $\left(\tilde{f}_\delta, X\right)$ is an int-soft subalgebra over U for some $\delta \in \mathscr{P}^*(U)$. Is $\left(\tilde{f}, X\right)$ an int-soft subalgebra over U?*

The answer to the question above is false. In fact, let $\left(\tilde{f}, X\right)$ be a soft set over U which is not an int-soft subalgebra over U. If we take $\delta = \bigcap_{x \in X} \tilde{f}(x)$, then $\left(\tilde{f}_\delta, X\right)$ is an int-soft subalgebra over U.

Let $\widetilde{\mathscr{P}(U)}$ be a subclass of $\mathscr{P}(U)$ such that

$$(\forall A, B, C \in \mathscr{P}^*(U))\,(A \cap B \subseteq A \cap C \ \Rightarrow B \subseteq C)\,. \tag{3.6}$$

Theorem 3.16. *Let $\left(\tilde{f}, X\right)$ be a soft set over U such that $\left(\tilde{f}_\delta, X\right)$ is an int-soft subalgebra over U for some $\delta \in \mathscr{P}(U)$. If $\operatorname{Im}(\tilde{f}) \cup \{\delta\} \subseteq \widetilde{\mathscr{P}(U)}$, then $\left(\tilde{f}, X\right)$ is an int-soft subalgebra over U.*

Proof. For any $x, y \in X$, we have

$$\begin{aligned}\tilde{f}(x * y) \cap \delta &= \tilde{f}_\delta(x * y) \supseteq \tilde{f}_\delta(x) \cap \tilde{f}_\delta(y) \\ &= \left(\tilde{f}(x) \cap \delta\right) \cap \left(\tilde{f}(y) \cap \delta\right) \\ &= \tilde{f}(x) \cap \tilde{f}(y) \cap \delta\end{aligned}$$

and so $\tilde{f}(x*y) \supseteq \tilde{f}(x) \cap \tilde{f}(y)$ by (3.6). Therefore $\left(\tilde{f}, X\right)$ is an int-soft subalgebra over U. □

The following example shows that if $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra over U, then Theorem 3.15 is false.

Example 3.17. Consider the $\theta(= 12\mathbb{Z})$-generalized int-soft subalgebra $\left(\tilde{f}, X\right)$ over X which is given in Example 3.4. For $\delta = \mathbb{N}$, the soft set $\left(\tilde{f}_\delta, X\right)$ over $U(= \mathbb{Z})$ is described as follows:

$$\tilde{f}_\delta : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \mathbb{N} & \text{if } x = 1, \\ 2\mathbb{Z} \cap \mathbb{N} & \text{if } x = a, \\ 9\mathbb{Z} \cap \mathbb{N} & \text{if } x = b, \\ 6\mathbb{Z} \cap \mathbb{N} & \text{if } x = c, \\ 4\mathbb{Z} \cap \mathbb{N} & \text{if } x = d \end{cases}$$

Since $\tilde{f}_\delta(a*d) = \tilde{f}_\delta(c) = 6\mathbb{Z} \cap \mathbb{N} \not\supseteq 4\mathbb{Z} \cap \mathbb{N} = \tilde{f}_\delta(a) \cap \tilde{f}_\delta(d)$, we know that $\left(\tilde{f}_\delta, X\right)$ is not an int-soft subalgebra over U.

Theorem 3.18. *If $\left(\tilde{f}, X\right)$ is a θ-generalized int-soft subalgebra over U, then $\left(\tilde{f}_\delta, X\right)$ is a $(\theta \cap \delta)$-generalized int-soft subalgebra over U for all $\delta \in \mathscr{P}^*(U)$.*

Proof. Let $x, y \in X$. Then

$$\begin{aligned}\tilde{f}_\delta(x * y) &= \tilde{f}(x * y) \cap \delta \\ &\supseteq \theta \cap \tilde{f}(x) \cap \tilde{f}(y) \cap \delta \\ &= (\theta \cap \delta) \cap \left(\tilde{f}(x) \cap \delta\right) \cap \left(\tilde{f}(y) \cap \delta\right) \\ &= (\theta \cap \delta) \cap \tilde{f}_\delta(x) \cap \tilde{f}_\delta(y)\end{aligned}$$

and thus $\left(\tilde{f}_\delta, X\right)$ is a $(\theta \cap \delta)$-generalized int-soft subalgebra over U for all $\delta \in \mathscr{P}^*(U)$. □

References

1. S.S. Ahn and K.S. So, *On ideals and upper sets in BE-algerbas,* Sci. Math. Jpn. **68** (2008), 279–285.
2. A.O. Atagün and A. Sezgin, *Soft substructures of rings, fields and modules,* Comput. Math. Appl. **61** (2011), 592–601.
3. D. Chen, E.C.C. Tsang, D.S. Yeung and X. Wang, *The parametrization reduction of soft sets and its applications,* Comput. Math. Appl. **49** (2005), 757–763.
4. Y. Imai and K. iséki, *On axiom systems of propositional calculi XIV,* Proc. Japan Academy **42** (1966), 19–22.
5. K. Iséki, *An algebra related with a propositional calculus,* Proc. Japan Academy **42** (1966), 26–29.
6. Y.B. Jun and S.S. Ahn, *Applications of soft sets in BE-algebras,* Algebra **2013** (2013), Article ID 368962, 8 pages.
7. H.S. Kim and Y.H. Kim, *On BE-algerbas,* Sci. Math. Jpn. **66** (2007), 113–116.
8. P.K. Maji, R. Biswas and A.R. Roy, *Soft set theory,* Comput. Math. Appl. **45** (2003), 555–562.
9. P.K. Maji, A.R. Roy and R. Biswas, *An application of soft sets in a decision making problem,* Comput. Math. Appl. **44** (2002), 1077–1083.
10. D. Molodtsov, *Soft set theory - First results,* Comput. Math. Appl. **37** (1999), 19–31.
11. J.H. Park and Y.H. Kim, *Int-soft positive implicative filters in BE-algebras,* J. Appl. Math. & Informatics **33** (2015), 459–467.
12. L.A. Zadeh, *From circuit theory to system theory*, Proc. Inst. Radio Eng. **50** (1962), 856–865.
13. L.A. Zadeh, *Fuzzy sets,* Inform. Control **8** (1965), 338–353.
14. L.A. Zadeh, *Toward a generalized theory of uncertainty (GTU) - an outline*, Inform. Sci. **172** (2005), 1–40.

Keum Sook So

Department of Mathematics, Hallym University, Chuncheon 24252, Korea.
e-mail: ksso@hallym.ac.kr

Young Hee Kim received M.Sc. and Ph.D in Mathematics from Yonsei University. She is currently a professor at Chungbuk National University since 1983. Her research interests are BE-algebra, BCI-algebra, Fuzzy algebra and their application.

Department of Mathematics, Chungbuk National University, Cheongju 28644, Korea.
e-mail: yhkim@chungbuk.ac.kr