Control of a Segway with unknown control coefficient and input constraint

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Abstract

This paper proposes a control method of the Segway with unknown control coefficient and input saturation. To design a simple controller for the Segway with the model uncertainty, the prescribed performance function is used. Furthermore, an auxiliary variable is introduced to deal with unknown time-varying control coefficient and input saturation problem. Due to the auxiliary variable, function approximators are not used in this paper although all model uncertainties are unknown. Thus, the controller can be simple. From the Lyapunov stability theory, it is proved that all errors of the proposed control system remain within the prescribed performance bounds. Finally, the simulation results are presented to demonstrate the performance of the proposed scheme.

Keywords: Segway, prescribed performance function, input saturation, unknown control coefficient, model uncertainty.

1.   Introduction

Various forms of the Segway have recently attracted attention as a means of personal transportation. The reason is that it is easy to carry. Since the Segway is a personal vehicle, the safety of the passenger is very important and thus, it requires the design of a reliable controller. For this, it is necessary to consider the model uncertainty and input constraint. The weight and center of gravity of the passenger are different, and the control coefficient is time-varying because it is dependent on the angle of the inclination. These lead the model uncertainty. Furthermore, the Segway is driven by DC motor and it has a limit in the input voltage physically. Thus, we need to consider the input saturation problem for real applications.

To deal with the model uncertainty, previous researches used various methods such as neural network [1, 2], fuzzy logic [3, 4], and adaptive control technique [5, 6]. Aforementioned papers are useful to compensate the model uncertainty, but it has the disadvantage due to the complexity of the controller. For the unknown control direction, the Nussbaum gain technique [7, 8] was generally used. However, it can be unstable because it is constructed by the means of so-called amplitude elongation. Therefore, designing the controller based on the Nussbaum gain technique causes many difficulties in the physical point of view. The prescribed performance function based controller [9, 10] was recently proposed to solve the disadvantage of various methods dealing with the model uncertainty. In this approach, the error signals remain within the prescribed performance bounds without estimating the model uncertainty. Thus, it makes the controller simple. However, most of the researches using the prescribed performance function did not consider the time-varying control coefficient.
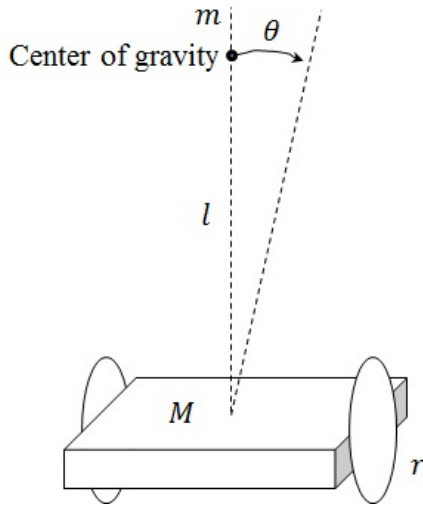
Figure 1. Segway.

Motivated by above analysis, a simple controller for the Segway with model uncertainties and input constraint is proposed to solve above problems. First, the prescribed performance function is used to deal with the model uncertainty. This means that function approximators are not used in this paper although all model uncertainties are unknown. Therefore, the controller is simple in the presence of the model uncertainty. Second, an auxiliary variable is introduced to deal with the input saturation problem and unknown time-varying control coefficient. Since the error signals of the proposed control scheme remain within the prescribed performance bounds, the safety of the passenger is guaranteed. Furthermore, the transient and steady-state responses can be adjusted by selecting the prescribed performance functions properly. Finally, the controller is designed using the Lyapunov stability theory and the simulation results are presented to verify the performance of the proposed control scheme.

2. Segway Model

In this paper, the Segway shown in Figure 1 is considered. The dynamic model of the Segway is as follows [11].

$$\ddot{\theta} = (mgl \sin \theta)/I - (mFl \cos \theta)/(M+m)I, \quad (1)$$

where m is the mass of the passenger, M is the mass of the Segway base, I is the inertia of the Segway with the passenger, θ is the angle of the inclination, g is the gravitational acceleration, l is the length between the center of gravity of the passenger and Segway base, and F is the force applied to the Segway.

Since the Segway is driven by DC motor, it is necessary to consider the motor dynamics. Neglecting the motor inductance, we have

$$F = \frac{2k_t nu}{Rr} - \frac{2k_t ne_b}{Rr}, \quad (2)$$

where k_t is the motor torque constant, e_b is the back-emf voltage, R is the resistance, r is the radius of the wheel, n is the gear reduction, $\dot{x}$ is the angular velocity of the wheel, and u is the motor input voltage. Let's define $x_1 = \theta$ and $x_2 = \dot{\theta}$. Then, substituting (2) into (1) yields

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x_1) + b(x_1)\bar{u} + d(t), \quad (3)$$

where

$$f(x_1) = \frac{mgl \sin x_1}{I},$$
$$b(x_1) = -\frac{2k_t nml \cos x_1}{(M+m)IRr},$$
$$d(t) = \frac{2k_t ne_b ml \cos x_1}{(M+m)IRr}.$$

In (3), m and l are dependent on the passenger. Thus, these are model uncertainties and they lead the unknown time-varying control coefficient $b(x_1)$. In these expressions, $\bar{u}$ is the saturated input voltage as follows.

$$\bar{u} = \begin{cases} u_M, & \text{if } u \geq u_M, \\ u, & \text{if } -u_M < u < u_M, \\ -u_M, & \text{if } u \leq u_M, \end{cases} \quad (4)$$

where u_M is the limit of the motor input voltage.

Assumption 1. *In (3), the model parameters are unknown constants, but bounded. Moreover, the back-emf voltage e_b is the unknown time-varying parameter, but bounded.*

Remark 1. To deal with the unknown time-varying coefficient $b(x_1)$, the auxiliary variable is used. Thus, the assumption on sign of $b(x_1)$ is not required in this paper.

The *control objective* is to design the controller so that the angle of the inclination tracks the desired angle(i.e., $x_d = 0°$) in the presence of the model uncertainty and input constraint.

3. Controller Design

In this section, the controller is designed using the prescribed performance functions. Define the error signals

$$e_1 = \Gamma(z_1), \quad e_2 = \Gamma(z_2),$$
$$z_1 = \frac{x_1}{\rho_1}, \quad z_2 = \frac{x_2 - \alpha - \gamma \tanh \beta}{\rho_2}, \tag{5}$$

where $\Gamma(z) = \ln \frac{1+z}{1-z}$, α is the virtual control, γ is a positive constant, and β is an auxiliary variable; ρ_1 and ρ_2 are prescribed performance functions defined by

$$\rho_1 = (\rho_{1,0} - \rho_{1,\infty})e^{-a_1 t} + \rho_{1,\infty},$$
$$\rho_2 = (\rho_{2,0} - \rho_{2,\infty})e^{-a_2 t} + \rho_{2,\infty}, \tag{6}$$

where $\rho_{1,0}$ and $\rho_{2,0}$ are initial values satisfying $\rho_{1,0} > |x_1(0)|$ and $\rho_{2,0} > |x_2(0) - \alpha(0) - \gamma \tanh \beta(0)|$, respectively, $\rho_{1,\infty}$ and $\rho_{2,\infty}$ are final values that can adjust the region of the steady-state errors; a_1 and a_2 are positive constants that can adjust the convergence rate of the errors.

Remark 2. If e_1 and e_2 are bounded, z_1 and z_2 satisfy $|z_1| < 1$ and $|z_2| < 1$, respectively. This means that $|x_1| < \rho_1$. Since ρ_1 is selected properly, the angle x_1 can be stabilized. Therefore, the controller is designed to guarantee the boundedness of e_1 and e_2.

Step 1: Using (5), the time derivative of e_1 is

$$\dot{e}_1 = \frac{2}{\rho_1} \cosh^2 \frac{e_1}{2} (z_2 \rho_2 + \alpha - z_1 \dot{\rho}_1 + \gamma \tanh \beta). \tag{7}$$

By the definition of $\Gamma(\cdot)$, $z_i = \tanh \frac{e_i}{2}$ where $i = 1, 2$. Thus, z_1 and z_2 are bounded. Furthermore, ρ_i and its time derivative are bounded by the definition of (6). This implies that there exists a positive constant satisfying

$$|z_2 \rho_2 - z_1 \dot{\rho}_1 + \gamma \tanh \beta| \le \Phi_1. \tag{8}$$

To stabilize e_1, choose the virtual control α as follows:

$$\alpha = -k_1 e_1, \tag{9}$$

where k_1 is a positive constant.

Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}e_1^2. \tag{10}$$

The time derivative of (10) along with (7) and (9) is

$$\dot{V}_1 = \frac{2}{\rho_1} \cosh^2 \frac{e_1}{2} (z_2 \rho_2 + \alpha - z_1 \dot{\rho}_1 + \gamma \tanh \beta) e_1$$
$$\le \frac{2}{\rho_1} \cosh^2 \frac{e_1}{2} (-k_1 |e_1|^2 + \Phi_1 |e_1|). \tag{11}$$

If $|e_1| > \frac{\Phi_1}{k_1}$, $\dot{V}_1$ is negative. Thus, e_1 is bounded. This implies that α and $\dot{\alpha}$ are bounded from (7) and (9).

Step 2: Using (5), the time derivative of e_2 is

$$\dot{e}_2 = \frac{2}{\rho_2} \cosh^2 \frac{e_2}{2} (f(x_1) + b(x_1)\bar{u} + d(t) - \dot{\alpha}$$
$$- z_2 \dot{\rho}_2 - \gamma \frac{\dot{\beta}}{\cosh^2 \beta}). \tag{12}$$

For the update of the auxiliary variable β, choose $\dot{\beta}$ as follows.

$$\dot{\beta} = \frac{\cosh^2 \beta}{\gamma} (-k_\beta \tanh \beta + u - \bar{u}), \tag{13}$$

where k_β is a positive constant.

Remark 3. In (13), if the actual control input u is larger than its limit u_M, it will affect the update of the auxiliary variable β to compensate the exceeded input. Otherwise, β is not working. Therefore, the auxiliary variable β can solve the input saturation problem.

Substituting (13) into (12) yields

$$\dot{e}_2 = \frac{2}{\rho_1} \cosh^2 \frac{e_2}{2} (f(x_1) + (b(x_1) + 1)\bar{u} + d(t) - \dot{\alpha}$$
$$- z_2 \dot{\rho}_2 + k_\beta \tanh \beta - u). \tag{14}$$

By Assumption 1, $f(x_1)$, $b(x_1)$, and $d(t)$ are bounded. Moreover, the boundedness of $\dot{\alpha}$, z_2, and $\dot{\rho}_2$ is proved in *Step 1*. Thus, there exists a positive constant Φ_2 satisfying

$$|f(x_1) + (b(x_1) + 1)\bar{u} + d(t)$$
$$- \dot{\alpha} - z_2 \dot{\rho}_2 + k_\beta \tanh \beta| \le \Phi_2. \tag{15}$$

To stabilize e_2, choose the actual control u as follows.

$$u = k_2 e_2, \tag{16}$$

where k_2 is a positive constant.

Consider the Lyapunov function candidate

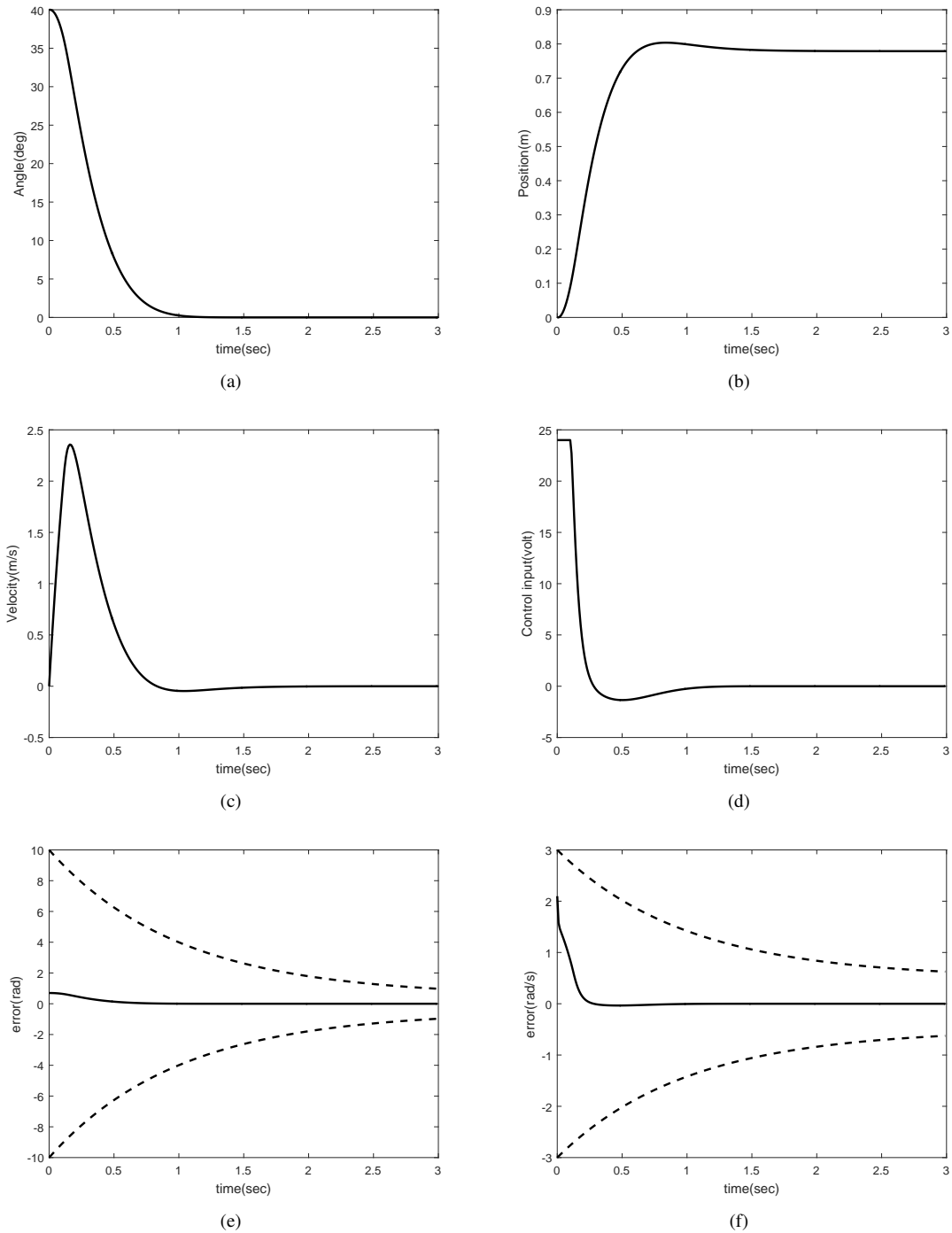$$V_2 = \frac{1}{2}e_2^2. \tag{17}$$

(a)

(b)

(c)

(d)

(e)

(f)

Figure 2. Response of the Segway with initial condition $\theta(0) = 40°$: (a) angle of the inclination; (b) position (c) linear velocity (d) motor input voltage (e) solid : x_1, dashed : ρ_1 (f) solid : $x_2 - \alpha - \gamma \tanh \beta$, dashed : ρ_1

The time derivative of (17) along with (14) and (16) is

$$\dot{V}_2 = \frac{2}{\rho_2} \cosh^2 \frac{e_2}{2} (f(x_1) + (b(x_1) + 1)\bar{u} + d(t) - \dot{\alpha}$$
$$- z_2 \dot{\rho}_2 + k_\beta \tanh \beta - k_2 e_2) e_2$$
$$\leq \frac{2}{\rho_2} \cosh^2 \frac{e_2}{2} (-k_2 |e_2|^2 + \Phi_2 |e_2|) \qquad (18)$$

If $|e_2| > \frac{\Phi_2}{k_2}$, $\dot{V}_2$ is negative. Thus, e_2 is bounded.

Theorem 1. *Consider the Segway model in (3) with the model uncertainty and input constraint. If the motor input voltage (16) is applied to the Segway, the angle x_1 remains within the prescribed performance bound. Furthermore, if $\rho_{1,\infty}$ is selected*
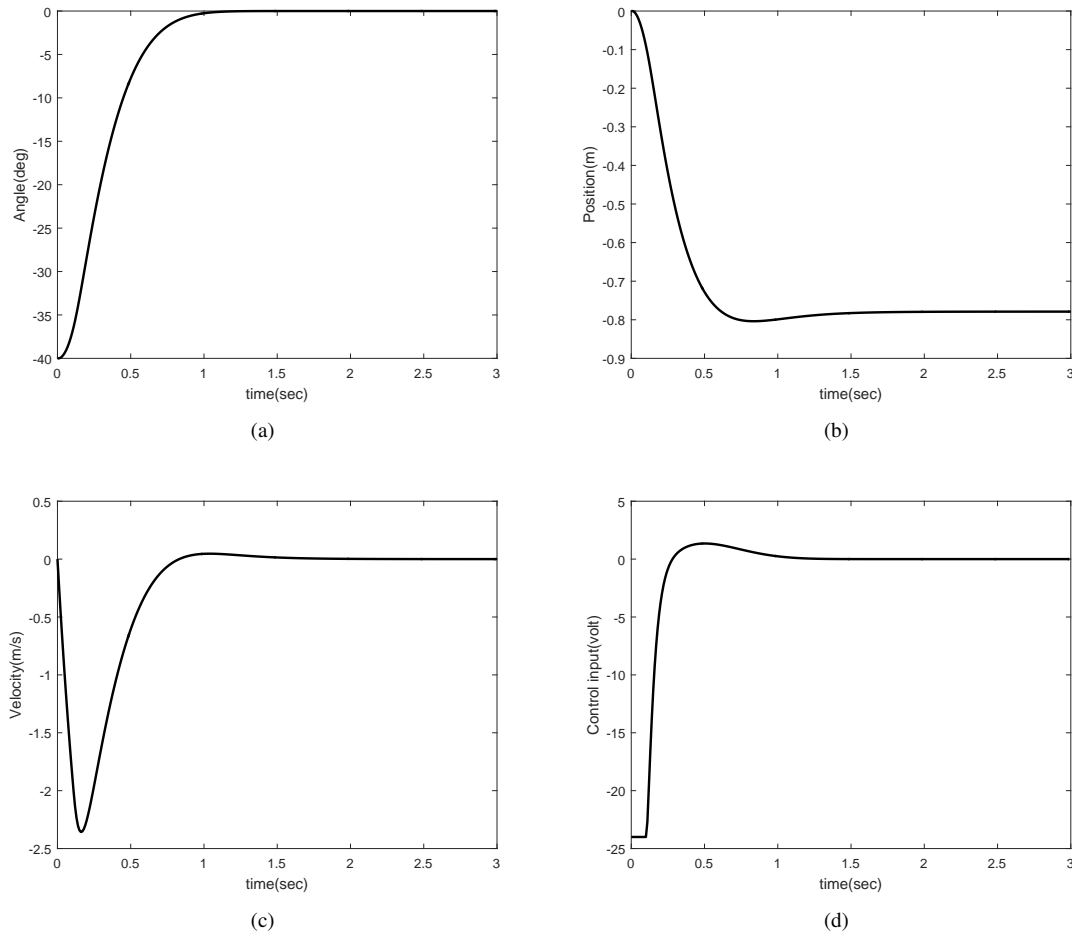
Figure 3. Response of the Segway with initial condition $\theta(0) = -40°$: (a) angle of the inclination; (b) position (c) linear velocity (d) motor input voltage

near zero, it is moved in the neighborhood of zero.

Proof. From (11) and (18), e_1 and e_2 are bounded. This leads $|x_1| < \rho_1$ as stated in Remark 1. Thus, x_1 remains within $\rho_{1,\infty}$ as $t \to \infty$. This means that if $\rho_{1,\infty}$ is selected near zero, the angle of the inclination is moved in the neighborhood of zero. $\square$

4. Simulation Results

For real applications, the model parameters in [12] are employed. These variables are shown in Table 1. The control parameters are chosen as $k_1 = 15$, $k_2 = 40$, $k_\beta = 50$, $\gamma = 1$, $\rho_{1,0} = 10$, $\rho_{2,0} = 3$, and $\rho_{1,\infty} = \rho_{2,\infty} = 0.5$. The limit of the motor input voltage is set to $u_M = 24V$. To obtain the linear velocity and position of the Segway, Newton's second law is used.

Two different cases, i.e., $\theta(0) = 40°$ and $\theta(0) = -40°$, are considered in the simulation. The simulation results are shown in Figures 2 and 3. In both cases, the angle of the inclination converges to zero even though the motor control input is saturated. This paper focuses on controlling of the angle of the inclination. Thus, the velocity control is ignored. However, from Figures 2(c) and 3(c), one can know that the linear velocity is also converged to zero as the angle of the inclination goes to zero. This means the stop of Segway if the angle of the inclination is zero. Figures 2(e) and (f) show that the error signals remain within the prescribed performance bounds.

To show the robustness of the proposed scheme against model uncertainties, the simulation is performed with various mass and length. As stated in Section 1, these values are different according to the passenger. The simulation results are shown in Figure 4. One can see that the performance difference for the various model parameters is not significant.
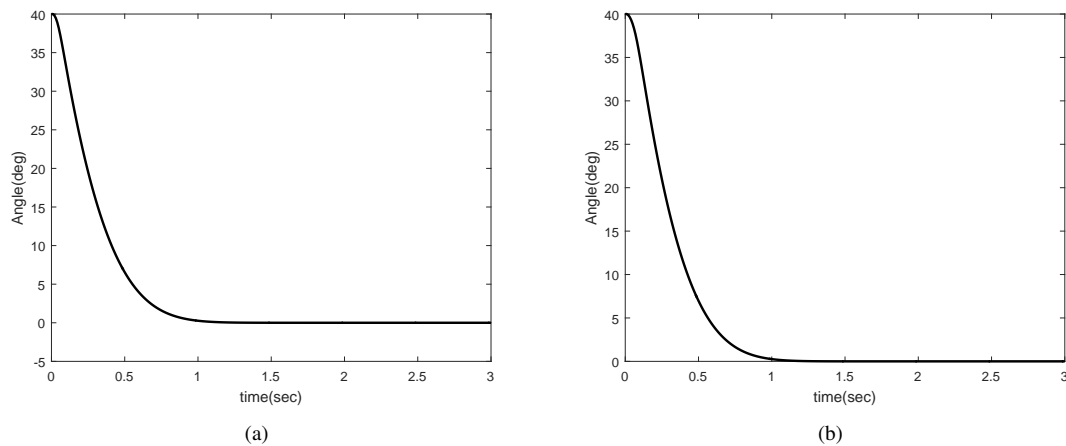
Figure 4. Response of the Segway: (a) $m = 60kg$, $l = 0.85m$; (b) $m = 40kg$, $l = 0.8m$

Table 1. Definitions of parameters of the Segway

Parameter	Definition	Value
θ	Angle of the inclination	rad
k_t	Constant of the motor torque	0.869 N/A
l	Length	0.85 m
r	Wheel radius	0.2 m
R	Resistance of the motor	1 Ω
m	Mass of the passenger	85 kg
M	Mass of the Segway	10 kg
I	Inertia of the Segway	68.98 kg·m^2
g	Acceleration of gravity	9.8 m/s^2
n	Gear reduction	10

5. Conclusions

In this paper, the tracking control problem of Segway has been investigated. The Segway is a nonlinear system with the model uncertainty and time-varying control coefficient due to various passengers. This requires the robust controller to deal with the model uncertainty and time-varying control coefficient. For this, the controller is designed using the prescribed performance functions and auxiliary variable. With the help of the auxiliary variable, the controller can solve the input saturation problem and does not require the Nussbaum gain technique which is generally used to deal with time-varying control coefficient. This leads the simple controller unlike other nonlinear controllers for Segway. The performance of the proposed scheme is verified by numerical simulations.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

Acknowledgment

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