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Diffraction Corrections for Second Harmonic Beam Fields and Effects on the Nonlinearity Parameter Evaluation

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Abstract The nonlinearity parameter is frequently measured as a sensitive indicator in damaged material characterization or tissue harmonic imaging. Several previous studies have employed the plane wave solution, and ignored the effects of beam diffraction when measuring the non-linearity parameter β. This paper presents a multi-Gaussian beam approach to explicitly derive diffraction corrections for fundamental and second harmonics under quasilinear and paraxial approximation. Their effects on the nonlinearity parameter estimation demonstrate complicated dependence of β on the transmitter-receiver geometries, frequency, and propagation distance. The diffraction effects on the non-linearity parameter estimation are important even in the nearfield region. Experiments are performed to show that improved β values can be obtained by considering the diffraction effects.

Keywords: Diffraction Correction, Multi-Gaussian Beam, Nonlinearity Parameter

1. Introduction

When an ultrasonic transducer radiates waves into a medium, the on-axis response of amplitude and phase will deviate from that of a plane wave. This phenomenon is known as diffraction effects. One needs to adjust amplitudes or phases of actual acoustic waves to their plane wave values before they are used. This is the effect referred to as diffraction correction. In linear acoustics, it is well known that accurate corrections for diffraction effects are necessary for precise measurements ultrasonic quantities such as attenuation or velocity. It is also important to make appropriate corrections for diffraction in nonlinear acoustics, for instance, to realize a high precision measurement of the nonlinearity parameter β using the finite amplitude method [1].

The early studies on diffraction corrections associated with linear waves date back to Williams [2]. He obtained an exact integral expression for diffraction correction for a uniformly vibrating piston when both transmitting and receiving transducers are the same size. Based on this, Rogers and Van Buren [3] developed an approximate solution of a closed form, valid for $(ka)^{1/2} \gg 1$. Williams' diffraction theory was later used by Khimunin [4-6] to formulate a diffraction correction relative to plane waves. In Khimunin's [4,5] approach, the diffraction correction was defined as the ratio of the average pressure over a receiving transducer to the plane wave pressure at the same distance. Limitations of previous studies on receiver geometries have been addressed by Yamada and Fujii [7], Beissner [8] and Szabo [9], where the diffraction correction was solved for an arbitrary receiver size. All these diffraction issues apply to the corrections of fundamental wave fields. An approximate diffraction correction for the second harmonic wave was presented by Ingenito and Williams [10], and used in later studies of nonlinearity parameter measure- ments [11,12]. Diffraction effects were neglected in many previous studies and measurements were usually

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made within the near-field region of the piston transducer [13,14]. This imposed restrictions related to the size of sample being studied.

The purpose of this study is to explicitly derive diffraction corrections for the fundamental and second harmonics and to show through simulation and experiment the importance of making diffraction corrections for accurate determination of nonlinearity parameter β. In the companion paper [15], we have developed an efficient and accurate method to calculate nonlinear diffraction beam fields using the multi-Gaussian (MGB) model approach. It was also shown that the MGB model approach, due to its paraxial approximation, easily separates diffraction effects out of combined beam fields. Since diffraction corrections depend on receiver types, we present closed-form expressions of MGB model-based diffraction corrections for point receiver and area receiver. Their accuracy is tested through comparisons with integral solution-based approach. Finally it is demonstrated through simulation and experiment that improved β can be acquired with proper diffraction corrections.

2. Integral Solutions and Received Beam Fields

Two governing equations for the fundamental and second harmonic pressure components were derived in Ref. [15] using the quasilinear theory of the Westervelt equation. Solutions for these equations were then obtained by integrating over the product of the Green's function and appropriate source function to sum up the contributions from all source points. For a circular piston radiator of radius a with source pressure p_0 at $(x',y',z'=0)$ and the z axis taken as the propagation direction the integral solutions are given by

$$p_1(x,y,z) = -\frac{ikp_0}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\exp(ikr)}{r}\,dx'dy' \tag{1}$$

$$p_2(x,y,z) = \frac{\beta k^2}{2\pi\rho c^2}\int_0^z\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p_1^2(x',y',z')\,\frac{\exp(i2kR)}{R}\,dx'dy'dz' \tag{2}$$

where k is the wave number, c is the longitudinal wave velocity of fundamental wave, ρ is the density, β is the acoustic nonlinearity parameter of fluids, $r=\sqrt{(x-x')^2+(y-y')^2+z^2}$, and $R=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$.

Eq. (1) represents the transducer radiation as a superposition of spherical waves radiating from point sources distributed on the plane $z'=0$. Eq. (1) is known as the Rayleigh-Sommerfeld (RS) integral to the linear wave equation.

Harmonic generation experiments are usually performed in a through-transmission mode. Propagated beam fields can be acquired on the reception side by a point receiver or by a finite area receiver of the same shape and size as the transmitting transducer. In case of finite size receivers the received signals can be obtained by taking the weighted average of the calculated field parameter over the receiver aperture. Therefore, magnitudes of received signals will depend on the receiver types, and the diffraction corrections will change accordingly. In this study, we will consider an ideal point receiver and a circular area receiver to calculate received signals and diffraction corrections.

First consider the received pressure by a point receiver moving along the z axis. Then, the on-axis pressures of fundamental and second harmonic fields can be obtained from Eqs. (1) and (2) by setting $x=y=0$:

$$p_1(z) = p_1(0,0,z) = -\frac{ikp_0}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\exp(ikr)}{r}\,dx'dy' \tag{3}$$

$$p_2(z) = p_2(0,0,z) = \frac{\beta k^2}{2\pi\rho c^2}\int_0^z\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p_1^2(x',y',z')\,\frac{\exp(i2kR)}{R}\,dx'dy'dz' \tag{4}$$

where $r=\sqrt{x'^2+y'^2+z^2}$, $R=\sqrt{x'^2+y'^2+z^2}$, and $p_1(x',y',z')$ is given by Eq. (1).

To calculate the received pressure at a distance z by a circular transducer of radius b, the concept of average pressure will be used and defined as follows:

$$\tilde{p}_n(z) = \frac{1}{\pi b^2}\int_0^b p_n(x,y,z)2\pi r dr, \quad n=1,2 \tag{5}$$

where $p_n(x,y,z)$ is computed from Eqs. (1) and (2), and $r=\sqrt{x^2+y^2}$.

3. MGB Models and Received Beam Fields

We have derived the multi-Gaussian beam (MGB) models to efficiently compute the fundamental and second harmonic beam fields obtained by the integral solutions [15]. The MGB models provided accurate results when compared with those of integral solutions for different transmitter-receiver geometries. Moreover, since the MGB model approach, due to the paraxial approximation, easily separates diffraction effects out of combined beam fields, diffraction corrections can be obtained explicitly. Since received beam fields depend on receiver types, diffraction corrections will vary accordingly. In this section, we will derive MGB model-based diffraction corrections for two types of receivers: point receiver and area receiver.

From the MGB model approach in Ref. [15], the pressure beam fields of the fundamental and the second harmonic are respectively given by

$$p_1^{MGB}(x,y,z) = [p_0\exp(ikz)]\left[\sum_{m=1}^{25}\frac{A_m}{1+iB_m z/D_R}\exp\left(\frac{i\omega}{2}\frac{iB_m/cD_R}{1+iB_m z/D_R}(x^2+y^2)\right)\right] \tag{6}$$

$$p_2^{MGB}(x,y,z) = \left[\frac{\beta k p_0^2 z}{2\rho c^2}\exp(2ikz)\right]\times \left[\frac{1}{z}\int_0^z\sum_{m=1}^{25}\sum_{n=1}^{25}\frac{2A_mA_n}{(2+B_a z)+(B_a-2B_b z)z'}\exp\left\{ik(x^2+y^2)\left(\frac{B_a-2B_b z'}{(2+B_a z)+(B_a-2B_b z)z'}\right)\right\}dz'\right] \tag{7}$$

where $D_R = ka^2/2$ is the Rayleigh distance, $B_a = i(B_m+B_n)/D_R$, and $B_b = B_mB_n/D_R^2$. It is noticed that in Eqs. (6) and (7) the first term in the right-hand side represents the plane wave solution for the fundamental and the second harmonic, respectively,

$$p_1^{plane}(z) = p_0\exp(ikz) \tag{8}$$

$$p_2^{plane}(z) = \frac{\beta k p_0^2 z}{2\rho c^2}\exp(2ikz) \tag{9}$$

The same procedure as in the previous section can be followed to obtain the received beam fields from the MGB model equations. When a point receiver is used, the received beam fields for the fundamental and second harmonics along the z axis can be obtained by setting $x=y=0$ in Eqs. (6) and (7)

$$p_1^{MGB}(z) = p_1^{MGB}(x=0,y=0,z) = p_1^{plane}(z)D_1^{MGB}(a,f,z) \tag{10}$$

$$p_2^{MGB}(z) = p_2^{MGB}(x=0,y=0,z) = p_2^{plane}(z)D_2^{MGB}(a,f,z) \tag{11}$$

where $D_1^{MGB}(a,f,z)$ and $D_2^{MGB}(a,f,z)$ are the MGB model-based diffraction corrections of the fundamental and second harmonics for a point receiver

$$D_1^{MGB}(a,f,z) = \sum_{m=1}^{25}\frac{A_m}{1+iB_m z/D_R} \tag{12}$$

$$D_2^{MGB}(a,f,z) = \frac{1}{z}\int_0^z\sum_{m=1}^{25}\sum_{n=1}^{25}\frac{2A_mA_n}{(2+B_a z)+(B_a-2B_b z)z'}dz' \tag{13}$$

When a circular transducer of radius b is used as a receiver, the received beam fields for the fundamental and second harmonics can be obtained by applying the concept of averaging to Eqs. (6) and (7). The results can be written in the following forms:

$$\tilde{p}_1^{MGB}(z) = \frac{1}{\pi b^2}\int_0^b p_1^{MGB}(x,y,z)2\pi r dr = p_1^{plane(z)}\tilde{D}_1^{MGB}(a,b,f,z) \tag{14}$$

$$\tilde{p}_2^{MGB}(z) = \frac{1}{\pi b^2}\int_0^b p_2^{MGB}(x,y,z)2\pi r dr = p_2^{plane(z)}\tilde{D}_2^{MGB}(a,b,f,z) \tag{15}$$

where $\tilde{D}_1^{MGB}(a,b,f,z)$ and $\tilde{D}_2^{MGB}(a,b,f,z)$ are the MGB model-based diffraction corrections of the fundamental and second harmonics for a circular receiver of radius b

$$\tilde{D}_1^{MGB}(a,b,f,z)=\frac{1}{\pi b^2}\int_0^b\left[\sum_{m=1}^{25}\frac{A_m}{1+iB_m z/D_R}\exp\left(\frac{i\omega}{2}\frac{iB_m/cD_R}{1+iB_m z/D_R}(x^2+y^2)\right)\right]2\pi r dr \tag{16}$$

$$\tilde{D}_2^{MGB}(a,b,f,z)=$$
$$\frac{1}{\pi b^2}\int_0^b\left[\frac{1}{z}\int_0^z\sum_{m=1}^{25}\sum_{n=1}^{25}\frac{2A_mA_n}{(2+B_a z)+(B_a-2B_b z)z'}\exp\left\{ik(x^2+y^2)\left(\frac{B_a-2B_b z'}{(2+B_a z)+(B_a-2B_b z)z'}\right)\right\}dz'\right]2\pi r dr \tag{17}$$

The paraxial MGB approach easily separates diffraction corrections from total beam fields obtained from the quasilinear theory. As shown in Eqs. (12), (13), (16) and (17), the MGB model-based diffraction corrections for the fundamental and second harmonics are given in explicit forms and depend on the transmitter-receiver geometries, the fundamental frequency and the propagation distance. On the contrary, the integral solution approach provides combined beam fields of plane wave and diffraction effects, so that it is not easy to separate one effect from the other. In the next section, we will define the integral solution-based diffraction corrections.

It can be proved that Eq. (16) is the same as diffraction correction derived by Rogers and Van Buren[3] in linear acoustics when the circular transmitter and receiver sizes are the same, $2a=2b$.

4. Integral Solution-Based Diffraction Corrections

Now we define diffraction corrections based on the integral solutions of received fields in section 3, Eqs. (3) and (4) for point receivers, and Eq. (5) for area receivers. Following the definition by Khimunin[4,5] and neglecting medium absorption, the diffraction corrections for the fundamental and the second harmonic can be found in terms of average pressure of the fundamental and the second harmonic over the receiver area at a distance z divided by their corresponding plane wave pressures at the same distance. For a point receiver, the diffraction corrections for the fundamental and second harmonics can be defined as

$$D_1(a,f,z)=\frac{p_1(z)}{p_1^{plane}} \tag{18}$$

$$D_2(a,f,z)=\frac{p_2(z)}{p_2^{plane}} \tag{19}$$

In a similar manner, for a circular receiver of radius b the diffraction corrections are defined as

$$\tilde{D}_1(a,b,f,z)=\frac{\tilde{p}_1(z)}{p_1^{plane}} \tag{20}$$

$$\tilde{D}_2(a,b,f,z)=\frac{\tilde{p}_2(z)}{p_2^{plane}} \tag{21}$$

The diffraction corrections defined in Eqs. (16) to (19) will be referred to as the integral solution (IS)-based diffraction corrections.

When the transmitting and receiving transducers are the same, an approximate diffraction correction for the second harmonic was presented by Ingenito and Williams [10].

$$D_2(a=b,f,z)=\frac{1}{z}\int_0^z\tilde{D}_1\left(z-\frac{\psi}{2}\right)^2 d\psi \tag{22}$$

This approximate diffraction corrections were subsequently used by Cobb [11] and Hurley and Fortunko [12] in their measurements of nonlinearity parameter of fluids and solids. We will test validity of Eq. (22) by comparing with more accurate diffraction corrections, Eqs. (17) and (21).

5. Comparison of Diffraction Corrections

For simulation, consider a piston transducer of $2a$=9.5 mm diameter radiating into water at 3.5 MHz, where a denotes the radius of the transmitting transducer. The properties of water used are: c=1480 m/s, ρ=1000 kg/m^3, β=3.5.

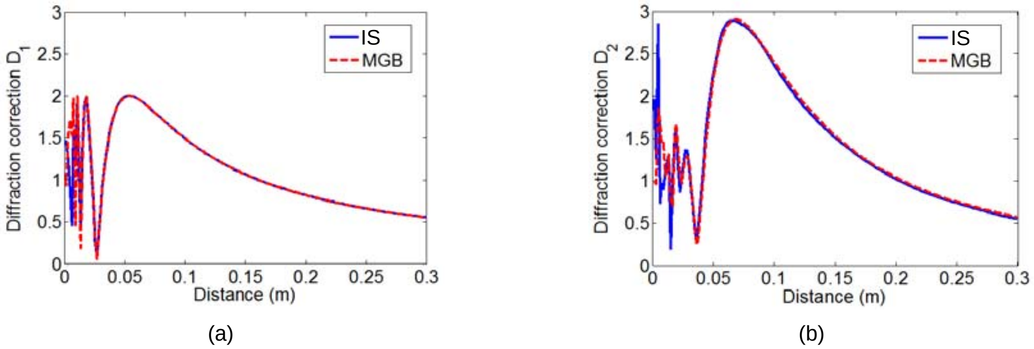


Fig. 1 Comparison of MGB model-based diffraction corrections with integral solution (IS)-based diffraction corrections for a point receiver: (a) fundamental wave, and (b) second harmonic wave

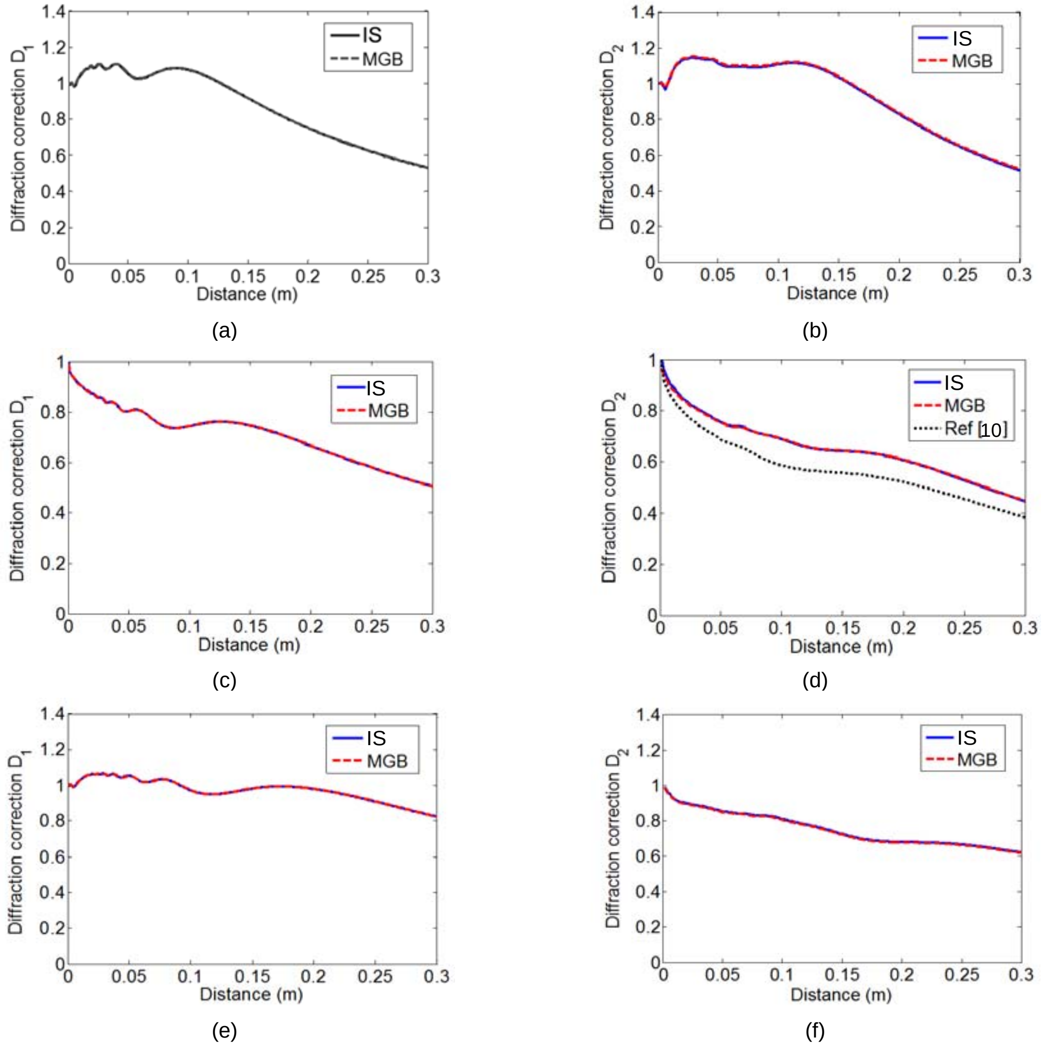


Fig. 2 Comparison of MGB model-based diffraction corrections with integral solution (IS)-based diffraction corrections for different transmitter and receiver sizes: (a), (b) 2*a*=9.5 mm, 2*b*=6.35 mm, (c), (d) 2*a*=9.5 mm, 2*b*=9.5 mm, and (e), (f) 2a=9.5 mm, 2*b*=12.7 mm. Figures in the left column represent diffraction corrections for the fundamental, while figures in the right column represent diffraction corrections for the second harmonic.

The number of expansion coefficients for A_n and B_n used in the MGB model are known to have effects on the beam field of the fundamental wave in the very near field. N=25 expansion coefficients are used here to calculate the MGB model-based diffraction corrections because a larger number of expansion coefficients provided better beam fields [15]. The expansion coefficients are listed in [16].

First, MGB model-based diffraction corrections [Eqs. (12) and (13)] are compared with integral solution-based diffraction corrections [Eqs. (18) and (19)] for a point receiver. As seen in Fig. 1, the overall agreement between two models is good except a short region close to the source transducer. This occurs because of paraxial approximation used in MGB models. The point receiver considered here is an ideal case not frequently encounter in reality. As can be seen in further simulation below, the effect of paraxial approximation is negligible in cases of finite size receivers, and the overall agreement of MGB model with the integral solution is pretty good.

Next, the accuracy of MGB model-based diffraction corrections [Eqs. (16) and (17)] was tested for area receivers by comparing with integral solution-based diffraction corrections [Eqs. (20) and (21)]. The accuracy of approximate second harmonic diffraction corrections, Eq. (22), was also tested when the receiver diameter $2b$ equals the transmitter diameter $2a$.

In case of finite size receivers, the effect of paraxial approximation is negligible, and the overall agreement of MGB model results with integral solution-based results is pretty good for the most part of the range covered as seen in Figs. 2(a) through 2(f). The approximate diffraction correction for the second harmonic, Eq. (22), shows considerable difference when compared with the integral solution- or the MGB model-based diffraction corrections, as seen in Fig. 2(d). The difference is about 15% at propagation distance z=0.3 m. Therefore, one should be careful in using this equation for diffraction corrections of the second harmonic.

6. Effects of Diffraction Correction on Nonlinearity Parameter

The plane wave solutions, Eqs. (8) and (9), provide a practical means to determine the nonlinearity parameter β. From these equations, the expression for calculating β can be obtained as

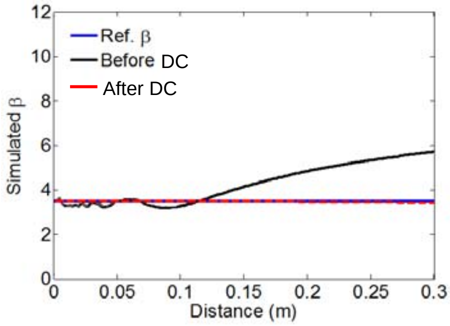
$$\beta = \frac{2\rho c^2}{kz}\frac{p_2^{plane}(z)}{[p_1^{plane}(z)]^2} \tag{23}$$

The issue with using Eq. (23) to estimate β is that it is based on the plane wave assumption, and do not fully account for the actual acoustic fields. The plane wave pressure should be replaced with actual pressure with proper diffraction correction. Eqs. (14) and (15) show the average pressure that is expressed in a quasi-plane wave form modified by diffraction corrections. Use of Eqs. (14) and (15) in Eq. (23) thus yields the β expression in terms of the average pressure corrected for diffraction as follows. It can also be expressed in terms of the average displacement using the relation $p_n = -i\rho c n\omega A_n,\ n=1,\ 2$.

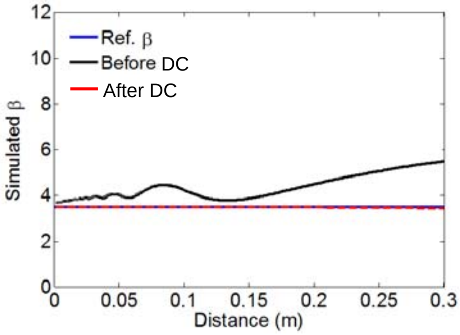
$$\beta = \frac{2\rho c^2}{kz}\frac{\tilde{p}_2(z)}{[\tilde{p}_1(z)]^2}\frac{\left|\tilde{D}_1(a,b,f,z)\right|^2}{\left|\tilde{D}_2(a,b,f,z)\right|} = \frac{4}{k^2 z}\frac{\tilde{A}_2(z)}{[\tilde{A}_1(z)]^2}\frac{\left|\tilde{D}_1(a,b,f,z)\right|^2}{\left|\tilde{D}_2(a,b,f,z)\right|} \tag{24}$$

where $\widetilde{A}_1(z)$ and $\widetilde{A}_2(z)$ are the average displacement of the fundamental and the second harmonic, respectively, at distance z. Since the diffraction corrections found from the MGB model agree well with the integral solutions, either of them can be used in Eq. (24).

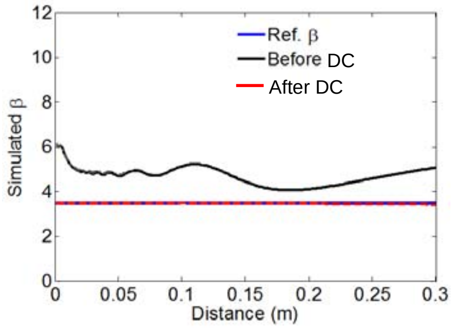
According to Eq. (24), the nonlinearity parameter β can now be determined, at a distance z from the transmitter, using the received displacement of the fundamental, $\widetilde{A}_1(z)$,



(a)



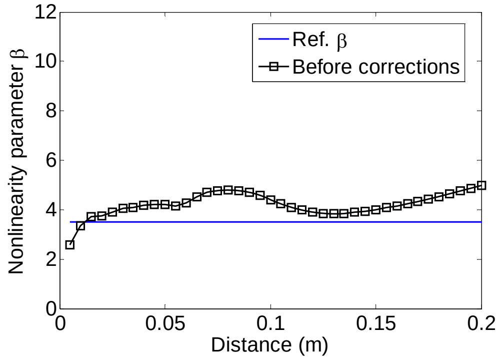
(b)



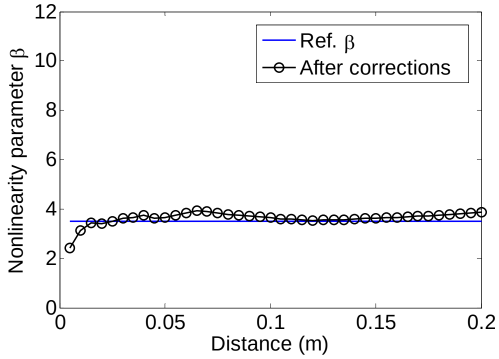
(c)

Fig. 3 Effects of diffraction correction (DC) on the determination of nonlinearity parameter β for different transmitter and receiver sizes: (a) 2*a*=9.5 mm, 2*b*=6.35 mm, (b) 2*a*=9.5 mm, 2*b*=9.5 mm, and (c) 2*a*=9.5 mm, 2*b*=12.7 mm

and that of the second harmonic, $\widetilde{A}_2(z)$, with proper corrections for diffraction. Fig. 3 shows simulation results on the effects of making diffraction corrections on β determination for different combinations of transmitter-receiver sizes. The horizontal line was included to represent the plane wave solution for water, β = 3.5 [11,17]. For given fundamental frequency



(a)



(b)

Fig. 4 Effects of diffraction corrections on β determination through harmonic generation experiments in water: (a) before diffraction corrections, and (b) after diffraction corrections

and transmitter size (diameter $2a$), diffraction corrections show complicated behavior depending on the propagation distance and receiver size (diameter $2b$). The nearfield distance in this simulation is estimated to be about 0.53 m at 3.5 MHz. When the measurements are made within the nearfield using the same size of receiver as the transmitter [Fig. 3(b)], use of uncorrected displacements will overestimate β by about 14% at maximum. When a larger size receiver is used [Fig. 3(c)], this overestimation becomes worse. On the other hand, use of a smaller receiver [Fig. 3(a)] underestimates β by about 5%. According to this observation, the receiver size can be optimized to minimize the diffraction effects. Based on these simulation results, we conclude that it is generally important to make diffraction corrections for accurate determination of β even when the measurements are performed within the nearfield region.

The effects of diffraction corrections on β estimation were further investigated through harmonic generation experiments in water. Experimental details can be found in [18] including receiver calibration for absolute measurements of fundamental and second harmonic displacements.

The nonlinearity parameter β of water was calculated according to Eq. (24) using the measured displacement of the fundamental and that of the second harmonic at each transmitter-receiver distance z. Fig. 4 shows such results on β estimation before and after diffraction corrections were made. The horizontal line represents the literature value of β = 3.5. Fig. 4(a) shows that the uncorrected β fluctuates and departs noticeably from the constant value of 3.5. After corrections were made for diffraction in Fig. 4(b), considerable improvements are observed in β values. It is found that after diffraction corrections βcan be determined with less than 10% errors except for very short transmitter-receiver distances, as can be seen in Fig. 4(b).

7. Conclusions and Future Work

The multi-Gaussian beam(MGB) model-based diffraction corrections for the fundamental and second harmonics were developed in this study. They are given in closed-forms under the paraxial and quasilinear approximation, and are functions of transmitter and receiver geometries, fundamental frequency, and propagation distance. They were found accurate and computationally efficient when compared to the integral solution-based diffraction corrections. Effects of making diffraction corrections on the estimation of nonlinearity parameter β were studied through simulation and experiment in water. Both results showed significance of making proper diffraction corrections for accurate determination of β. Attenuation corrections are another factor to be considered in nonlinearity parameter evaluation, and future study should address this issue. The diffraction corrections derived in this work can also be applied to isotropic solids if further assumptions are made in the nonlinear wave equation on which they are based.

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