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A Note on Continued Fractions and Mock Theta Functions

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Abstract. Mock theta functions are the most interesting topic mentioned in Ramanujan's Lost Notebook, due to its emerging application in the field of Number theory, Quantum invariants theory and etc. In the present research articles we have made an attempt to develop continued fractions representation of all the existing Mock theta functions.

1. Introduction

Continued fraction is a link between numbers and functions and it has been the subject of interest of mathematicians for centuries. In the beginning of the 20^{th} century, S. Ramanujan [36, 37] gave a remarkable contribution to continued fraction expansion of an analytic function, chapter 12 and 16 of second notebook of Ramanujan contain large number of results of continued fraction representation of hypergeometric function (ordinary and basic). Generalized hypergeometric function (ordinary and basic) drawn the attention to several mathematicians W. N. Bailey [38], L. J. Slater [18], H. M. Srivastava [12, 13, 14] have been a very significant tool in the derivation of continued fraction representation. G. E. Andrews [10], B. C. Berndt [5, 6], S. Bhargava and C. S. Adiga [32], S. Bhargava, C. S. Adiga and D. D. Somashekara [33], K. G. Ramanathan [15], R. P. Agrawal [26, 27], R. Y. Denis [29], R. Y. Denis, S. N. Singh and S. P. Singh [30, 31], M. Pathak and Pankaj

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Srivastava [20] Pankaj Srivastava and R. V. G. K. Mohan [24] and their co workers gives remarkable results on continued fractions. Recently R. Y. Denis, S. N. Singh and S. P. Singh [31] developed beautiful results involving polybasic hypergeometric series and continued fractions. In recent communication Pankaj Srivastava [21, 22] has developed continued fractions for the ratio of the basic bilateral hypergeometric series ${}_3\psi_3/{}_3\psi_3$ and ${}_2\psi_2/{}_2\psi_2$.

Mock theta functions are the last gift to the mathematical world given by Ramanujan. He named and defined Mock theta function as a function $f(q)$ defined by a q-series which converges for $|q| < 1$ and which satisfies the following two conditions:

(1) For every root of unity ζ, there is a theta function $\theta_\zeta(q)$ such that the difference $f(q) - \theta_\zeta(q)$ is bounded as $q \to \zeta$ radially.

(2) There is no single theta function which work for all ζ; i.e., for every theta function $\theta(q)$ there is some root of unity ζ for which $f(q) - \theta(q)$ is unbounded as $q \to \zeta$ radially.

Ramanujan [35] defined four third-order Mock theta functions, ten fifth-order Mock theta functions in two groups each having five functions and three seventh-order Mock theta functions. G. N. Watson [11] in his presidential address to London Mathematical Society in 1936 introduced the mathematical world about a new class of functions that Ramanujan developed and named as Mock theta functions, added three more Mock theta function of order three. Y. S. Choi [39] given a list of four functions found in the Lost Notebook and designated them as Mock theta function of order-ten. G. E. Andrews and D. Hickerson [9] introduced seven Mock theta functions of order six and three Mock theta functions of order two. Some new development appeared during the end of the 20^{th} century, B. Gordon and R. J. McIntosh [8] published their research article on 'Some Mock theta function of order eight '. In 2006 B. C. Berndt, S. H. Chan [7] gave two new Mock theta functions of sixth-order. Recently K. Hikami [16, 17] introduced Mock theta function of order two, order four, order eight respectively and he was silent about their interrelationship. In order to investigate interrelationship of these newly developed Mock theta functions Pankaj Srivastava and A. J. Wahidi [23] developed their generalized form and integral representations. Some of the continued fraction representation for the Mock theta function and ratio of Mock theta function are found in the literatures and it is evident that R. P. Agrawal [28], S. N. Singh [34], A. K. Srivastava [1, 2], B. Srivastava [3, 4], M. Pathak and Pankaj Srivastava [20], explored continued fractions representation approach for Mock theta function. In the present research work we made an attempt to develop continued fraction representations of all the existing Mock theta functions available in literatures.

2. Definitions and Notations

A continued fraction is a ratio of the type,

$$\frac{a_1}{a_2+}\;\frac{a_3}{a_4+}\;\frac{a_5}{a_6+}\;\frac{a_7}{a_8+}\;\frac{a_9}{a_{10}+}\cdots$$

where $a_1, a_2, a_3, a_4 \dots$ are real or complex numbers.

For real or complex number a and q ($|q| < 1$), let $(a;q)_n$ be defined by,

$$(a;q)_n = \begin{cases} 1, & \text{if } n = 0, \\ (1-a)(1-aq)\dots(1-aq^{n-1}), & \text{if } n \in N \end{cases}$$

and $(a;q)_n = \dfrac{(a;q)_\infty}{(aq^n;q)_\infty}$. For arbitrary parameter a and integer n, we define the generalized basic hypergeometric function as follows,

$${}_r\Phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r \\ b_1, b_2, \dots, b_s; q^\lambda \end{matrix} ;q, z \right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, \dots, a_r; q)_n}{(q, b_1, b_2, \dots, b_s; q)_n} q^{\lambda n(n-1)/2} z^n,$$

when $\lambda > 0$ series converges for $|z| < \infty$, when $\lambda = 0$ then series converges for $r = s+1$, max.$(|q|, |z|) < 1$ and when $r \leq s$ series converges for any z and $|q| < 1$ provided that no zero appear in the denominator.

3. Main Result

In this section we establish our main result for developing continued fractions representation of mock theta functions,

$$F(a,b,c,d,e,f:z) = -E_m(a,b,c,d,e,f) - \frac{C(a,b,c,d,e,f)E_m(a,b,c,d,e,f)}{-C(a,b,c,d,e,f) + A_m(a,b,c,d,e,f)\;-}$$

$$\frac{B_m(a,b,c,d,e,f)}{A_m(aq,bq,cq,dq,eq,fq)\;-}\;\frac{B_m(aq,bq,cq,dq,eq,fq)}{A_m(aq^2,bq^2,cq^2,dq^2,eq^2,fq^2)\;-}$$

$$\frac{B_m(aq^2,bq^2,cq^2,dq^2,eq^2,fq^2)}{A_m(aq^3,bq^3,cq^3,dq^3,eq^3,fq^3)\;-}\;\frac{B_m(aq^3,bq^3,cq^3,dq^3,eq^3,fq^3)}{A_m(aq^4,bq^4,cq^4,dq^4,eq^4,fq^4)\;-}\cdots \tag{3.1}$$

where a, b, c, d, e, f, z, q, are complex numbers and $|q| < 1$ and $|z| < 1$.

$$F(a,b,c,d,e,f:z) = \sum_{n=0}^{m} \frac{(a,b,c;q)_n}{(d,e,f;q)_n} z^n,$$

$$C(a,b,c,d,e,f) = \frac{(1-a)(1-b)(1-c)}{(1-d)(1-e)(1-f)} z,$$

$$E_m(a,b,c,d,e,f)=\frac{(a,b,c;q)_{m+1}}{(d,e,f;q)_{m+1}}z^{m+1}-1,$$

$$A_m(a,b,c,d,e,f)=C(a,b,c,d,e,f)+\frac{E_m(a,b,c,d,e,f)}{E_m(aq,bq,cq,dq,eq,fq)}$$

and

$$B_m(a,b,c,d,e,f)=\frac{C(aq,bq,cq,dq,eq,fq)E_m(a,b,c,d,e,f)}{E_m(aq,bq,cq,dq,eq,fq)}.$$

4. Proof of Main Result

In this section, we establish our main result eq.(3.1) as,

$$(4.1)\qquad F(a,b,c,d,e,f:z)$$
$$=C(a,b,c,d,e,f)F(aq,bq,cq,dq,eq,fq:z)-E_m(a,b,c,d,e,f).$$

In order to prove eq.(4.1) we consider,

$$F(a,b,c,d,e,f:z)-C(a,b,c,d,e,f)F(aq,bq,cq,dq,eq,fq:z)$$
$$=\sum_{n=0}^{m}\frac{(a,b,c;q)_n}{(d,e,f;q)_n}z^n-\frac{(1-a)(1-b)(1-c)}{(1-d)(1-e)(1-f)}z\sum_{n=0}^{m}\frac{(aq,bq,cq;q)_n}{(dq,eq,fq;q)_n}z^n$$
$$=1-\frac{(a,b,c;q)_{m+1}}{(d,e,f;q)_{m+1}}z^{m+1}$$

this proves eq.(4.1).
From eq.(4.1), we can write,

$$(4.2)\qquad \frac{F(a,b,c,d,e,f:z)}{F(aq,bq,cq,dq,eq,fq:z)}=C(a,b,c,d,e,f)-\frac{E_m(a,b,c,d,e,f)}{F(aq,bq,cq,dq,eq,fq:z)}$$

after some simplification eq.(4.2) can be written as,

$$(4.3)\qquad F(aq,bq,cq,dq,eq,fq:z)=-\frac{E_m(a,b,c,d,e,f)}{-C(a,b,c,d,e,f)+\frac{F(a,b,c,d,e,f:z)}{F(aq,bq,cq,dq,eq,fq:z)}}$$

from eq.(4.1) and eq.(4.3) we obtain,
(4.4)

$$F(a,b,c,d,e,f:z)=-E_m(a,b,c,d,e,f)-\frac{C(a,b,c,d,e,f)E_m(a,b,c,d,e,f)}{-C(a,b,c,d,e,f)+\frac{F(a,b,c,d,e,f:z)}{F(aq,bq,cq,dq,eq,fq:z)}}.$$

Now replacing a,b,c,d,e,f by aq,bq,cq,dq,eq,fq respectively in eq.(4.1), we obtain

$$F(aq,bq,cq,dq,eq,fq:z)$$

(4.5)
$= C(aq, bq, cq, dq, eq, fq)F(aq^2, bq^2, cq^2, dq^2, eq^2, fq^2 : z) - E_m(aq, bq, cq, dq, eq, fq)$

multiply eq.(4.1) by $E_m(aq, bq, cq, dq, eq, fq)$ and eq.(4.5) by $E_m(a, b, c, d, e, f)$ after subtracting and further simplifying, we obtain

$$F(a, b, c, d, e, f : z)E_m(aq, bq, cq, dq, eq, fq)$$

$$= [E_m(a, b, c, d, e, f) + C(a, b, c, d, e, f)E_m(aq, bq, cq, dq, eq, fq)]$$
$$F(aq, bq, cq, dq, eq, fq : z) - C(aq, bq, cq, dq, eq, fq)$$

$$F(aq^2, bq^2, cq^2, dq^2, eq^2, fq^2 : z)E_m(a, b, c, d, e, f) \tag{4.6}$$

now dividing eq.(4.6) by $F(aq, bq, cq, dq, eq, fq : z)E_m(aq, bq, cq, dq, eq, fq)$, and after simplification we get

$$\frac{F(a, b, c, d, e, f : z)}{F(aq, bq, cq, dq, eq, fq : z)} = C(a, b, c, d, e, f) + \frac{E_m(a, b, c, d, e, f)}{E_m(aq, bq, cq, dq, eq, fq)}$$

$$-\frac{C(aq, bq, cq, dq, eq, fq)E_m(a, b, c, d, e, f)/E_m(aq, bq, cq, dq, eq, fq)}{\frac{F(aq,bq,cq,dq,eq,fq:z)}{F(aq^2,bq^2,cq^2,dq^2,eq^2,fq^2:z)}} \tag{4.7}$$

further eq.(4.7), can be written as

$$\frac{F(a, b, c, d, e, f : z)}{F(aq, bq, cq, dq, eq, fq : z)} = A_m(a, b, c, d, e, f) - \frac{B_m(a, b, c, d, e, f)}{\frac{F(aq,bq,cq,dq,eq,fq:z)}{F(aq^2,bq^2,cq^2,dq^2,eq^2,fq^2:z)}} \tag{4.8}$$

further eq.(4.8), can be written as

$$\frac{F(a, b, c, d, e, f : z)}{F(aq, bq, cq, dq, eq, fq : z)}$$

$$= A_m(a, b, c, d, e, f) - \frac{B_m(a, b, c, d, e, f)}{A_m(aq, bq, cq, dq, eq, fq) - \frac{B_m(aq,bq,cq,dq,eq,fq)}{\frac{F(aq^2,bq^2,cq^2,dq^2,eq^2,fq^2:z)}{F(aq^3,bq^3,cq^3,dq^3,eq^3,fq^3:z)}}} \tag{4.9}$$

continuing the process of eq.(4.9), we obtain

$$\frac{F(a, b, c, d, e, f : z)}{F(aq, bq, cq, dq, eq, fq : z)} = A_m(a, b, c, d, e, f) - \frac{B_m(a, b, c, d, e, f)}{A_m(aq, bq, cq, dq, eq, fq)} -$$

$$\frac{B_m(aq, bq, cq, dq, eq, fq)}{A_m(aq^2, bq^2, cq^2, dq^2, eq^2, fq^2)} - \frac{B_m(aq^2, bq^2, cq^2, dq^2, eq^2, fq^2)}{A_m(aq^3, bq^3, cq^3, dq^3, eq^3, fq^3)} - \cdots \tag{4.10}$$

Finally combining eq.(4.4) and eq.(4.10), we get the main result eq.(3.1) after simplification.

5. Special Cases

Continued fraction representation of mock theta function's of order three,

(5.1)
$$f(q) = 1 + \frac{q}{(1+q)^2} - \frac{q^3(1+q)^2}{q^3+(1+q^2)^2} - \frac{q^5(1+q^2)^2}{q^5+(1+q^3)^2} - \frac{q^7(1+q^3)^2}{q^7+(1+q^5)^2} - \cdots$$

(5.2)
$$\varphi(q) = 1 + \frac{q}{1+q^2} - \frac{q^3(1+q^2)}{1+q^3+q^4} - \frac{q^5(1+q^4)}{1+q^5+q^6} - \frac{q^7(1+q^6)}{1+q^7+q^8} - \cdots$$

(5.3)
$$\psi(q) = \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} - \frac{q^5(1-q)(1-q^3)}{1} - \frac{q^7(1-q^5)}{1} - \cdots$$

(5.4)
$$\chi(q) = 1 + \frac{q}{1+\omega q} + \frac{q^2+\omega^2 q}{1-\omega^2 q+\omega q^2} + \frac{q^3+\omega^2 q}{1-\omega^2 q+\omega q^3} + \frac{q^4+\omega^2 q}{1-\omega^2 q+\omega q^4} + \cdots$$

(5.5)
$$\omega(q) = \frac{1}{(1-q)^2} + \frac{q^4}{(1-q)^2(1-q^3)^2} - \frac{q^8(1-q)^2(1-q^3)^2}{q^8+(1-q^5)^2} - \frac{q^{12}(1-q^5)^2}{q^{12}+(1-q^7)^2} - \cdots$$

(5.6)
$$\upsilon(q) = \frac{1}{1+q} + \frac{q^2}{(1+q)(1+q^3)} - \frac{q^4(1+q)(1+q^3)}{1+q^4+q^5} - \frac{q^6(1+q^5)}{1+q^6+q^7} - \cdots$$

(5.7)
$$\rho(q) = \frac{1}{1-\omega q} + \frac{\omega^2 q}{(1-\omega q)(1-\omega q^3)} - \frac{(1-\omega q)(\omega^2 q-q^4)}{1+\omega^2 q-\omega q^5} - \frac{\omega^2 q-q^6}{1+\omega^2 q-\omega q^7} - \cdots$$

Continued fraction representation of mock theta function's of order five,

(5.8)
$$f_0(q) = 1 + \frac{q}{1+q} - \frac{q^3(1+q)}{1+q^2+q^3} - \frac{q^5(1+q^2)}{1+q^3+q^5} - \frac{q^7(1+q^3)}{1+q^4+q^7} - \cdots$$

(5.9)
$$\varphi_0(q) = 1 + \frac{q(1+q)}{1} - \frac{q^3(1+q^3)}{1+q^3+q^6} - \frac{q^5(1+q^5)}{1+q^5+q^{10}} - \frac{q^7(1+q^7)}{1+q^7+q^{14}} - \cdots$$

(5.10)
$$\psi_0(q) = q + \frac{q^3(1+q)}{1} - \frac{q^3(1+q^2)}{1+q^3+q^5} - \frac{q^4(1+q^3)}{1+q^4+q^7} - \frac{q^5(1+q^4)}{1+q^5+q^9} - \cdots$$

(5.11) $F_0(q) = 1 + \frac{q^2}{1-q} - \frac{q^6(1-q)}{1-q^3+q^6} - \frac{q^{10}(1-q^3)}{1-q^5+q^{10}} - \frac{q^{14}(1-q^5)}{1-q^7+q^{14}} - \cdots$

(5.12)
$$\chi_0(q) = 1 + \frac{q}{1-q^2} - \frac{q(1-q^2)}{1+q+q^2-q^3-q^5} - \frac{q(1+q^2)(1-q^3)}{1+q+q^3-q^5-q^8} - \cdots$$

(5.13) $f_1(q) = 1 + \frac{q^2}{1+q} - \frac{q^4(1+q)}{1+q^2+q^4} - \frac{q^6(1+q^2)}{1+q^3+q^6} - \frac{q^8(1+q^3)}{1+q^4+q^8} - \cdots$

(5.14)
$$\phi_1(q) = q + \frac{q^4(1+q)}{1} - \frac{q^5(1+q^3)}{1+q^5+q^8} - \frac{q^7(1+q^5)}{1+q^7+q^{12}} - \frac{q^9(1+q^7)}{1+q^9+q^{16}} - \cdots$$

(5.15) $\psi_1(q) = 1 + \frac{q(1+q)}{1} - \frac{q^2(1+q^2)}{1+q^2+q^4} - \frac{q^3(1+q^3)}{1+q^3+q^6} - \frac{q^4(1+q^4)}{1+q^4+q^8} - \cdots$

(5.16)
$$F_1(q) = \frac{1}{1-q} + \frac{q^4}{(1-q)(1-q^3)} - \frac{q^8(1-q)(1-q^3)}{1-q^5+q^8} - \frac{q^{12}(1-q^5)}{1-q^7+q^{12}} - \cdots$$

(5.17)
$$\chi_1(q) = \frac{1}{1-q} + \frac{q}{(1-q)(1-q^2)} - \frac{q(1-q)(1-q^2)}{1+q+q^2-q^3-q^5} - \frac{q(1+q^2)(1-q^3)}{1+q+q^3-q^5-q^8} - \cdots$$

Continued fraction representation of mock theta function's of order seven,

(5.18)
$$F_0(q) = 1 + \frac{q}{1-q^2} - \frac{q^3(1-q^2)}{1+q^2-q^5} - \frac{q^5(1+q^2)(1-q^3)}{1+q^3-q^8} - \frac{q^7(1+q^3)(1-q^5)}{1+q^4-q^{11}} - \cdots$$

(5.19)
$$F_1(q) = \frac{q}{1-q} + \frac{q^4}{(1-q^2)(1-q^3)} - \frac{q^5(1-q^2)(1-q^3)}{1+q^2-q^7} - \frac{q^7(1+q^2)(1-q^5)}{1+q^5-q^{10}} - \cdots$$

(5.20)
$$F_2(q) = \frac{1}{1-q} + \frac{q^2}{(1-q^2)(1-q^3)} - \frac{q^4(1-q^2)(1-q^3)}{q^4+(1+q^2)(1-q^5)} - \frac{q^6(1+q^2)(1-q^5)}{q^6+(1+q^3)(1-q^7)} - \cdots$$

Continued fraction representation of mock theta function's of order six,

(5.21)
$$\phi_L(q) = 1 - \frac{q(1-q)}{1+q+q^2+q^3} + \frac{q^3(1+q)(1+q^2)(1-q^3)}{1+q^4+q^6+q^7} + \frac{q^5(1+q^3)(1+q^4)(1-q^5)}{1+q^6+q^{10}+q^{11}} + \cdots$$

(5.22)
$$\psi_L(q) = \frac{q}{1+q} - \frac{q^4(1-q)}{(1+q)(1+q^2+q^3+q^5)} + \frac{q^5(1+q)(1+q^2)(1-q^6)}{1+q^4+q^8+q^9} + \cdots$$

(5.23)
$$\rho_L(q) = \frac{1}{1-q} + \frac{q(1+q)}{(1-q)(1-q^3)} - \frac{q^2(1-q)(1+q^2)(1-q^3)}{1+q^2+q^4-q^5} - \frac{q^3(1+q^3)(1-q^5)}{1+q^3+q^6-q^7} - \cdots$$

(5.24)
$$\sigma_L(q) = \frac{q}{1-q} + \frac{q^3(1+q)}{(1-q)(1-q^3)} - \frac{q^3(1-q)(1+q^2)(1-q^3)}{1+q^3} - \frac{q^4(1+q^3)(1-q^5)}{1+q^4} - \cdots$$

(5.25) $\lambda_L(q) = 1 + fracq(q-1)1 + q_- \frac{q(q^3-1)(1+q)}{1-q+q^2+q^4} - \frac{q(q^5-1)(1+q^2)}{1-q+q^3+q^6} - \cdots$

(5.26)
$$\phi_-(q) = \frac{q(1+q)}{1-q} + \frac{q^2(1+q)(1+q^2)(1+q^3)}{(1-q)(1-q^3)} - \frac{q(1-q)(1-q^3)(1+q^4)(1+q^5)}{1+q(1+q^4)(1+q^5)-q^5} - \cdots$$

(5.27) $\psi_-(q) = \frac{q}{2(1-q)} + \frac{q^2(1+q)}{(1-q)(1-q^3)} - \frac{2q(1-q)(1+q^2)(1-q^6)}{1+q(1+q^2)(1+q^3)-q^5} - \cdots$

Continued fraction representation of mock theta function's of order eight,

(5.28) $S_0(q) = 1 + \frac{q(1+q)}{1+q^2} - \frac{q^3(1+q^2)(1+q^3)}{1+q^3+q^4+q^6} - \frac{q^5(1+q^4)(1+q^5)}{1+q^5+q^6+q^{10}} - \cdots$

(5.29) $S_1(q) = 1 + \frac{q^3(1+q)}{1+q^2} - \frac{q^5(1+q^2)(1+q^3)}{1+q^4+q^5+q^8} - \frac{q^7(1+q^4)(1+q^5)}{1+q^6+q^7+q^{12}} - \cdots$

(5.30)
$$T_0(q) = \frac{q^2}{1+q} + \frac{q^6(1+q^2)}{(1+q^3)(1+q)} - \frac{q^6(1+q)(1+q^3)(1+q^4)}{1+q^5+q^6+q^{10}} - \frac{q^8(1+q^5)(1+q^6)}{1+q^8+q^7+q^{14}} - \cdots$$

(5.31)
$$T_1(q) = \frac{1}{1+q} + \frac{q^2(1+q^2)}{(1+q^3)(1+q)} - \frac{q^4(1+q)(1+q^3)(1+q^4)}{1+q^4+q^5+q^8} - \frac{q^6(1+q^5)(1+q^6)}{1+q^6+q^7+q^{12}} - \cdots$$

(5.32) $U_0(q) = 1 + \frac{q(1+q)}{1+q^4} - \frac{q^3(1+q^3)(1+q^4)}{1+q^3+q^6+q^8} - \frac{q^5(1+q^5)(1+q^8)}{1+q^5+q^{10}+q^{12}} - \cdots$

(5.33)
$$U_1(q) = \frac{q}{1+q^2} + \frac{q^4(1+q)}{(1+q^2)(1+q^6)} - \frac{q^5(1+q^2)(1+q^3)(1+q^6)}{1+q^5+q^8+q^{10}} - \frac{q^7(1+q^5)(1+q^{10})}{1+q^7+q^{12}+q^{14}} - \cdots$$

(5.34) $V_0(q) = 1 + \frac{2q(1+q)}{1-q} - \frac{q^3(1-q)(1+q^3)}{1+q^6} - \frac{q^5(1-q^3)(1+q^5)}{1+q^{10}} - \cdots$

(5.35)
$$V_1(q) = \frac{q}{1-q} + \frac{q^4(1+q)}{(1-q)(1-q^3)} - \frac{q^5(1-q)(1-q^6)}{1+q^8} - \frac{q^7(1-q^{10})}{1+q^{12}} - \cdots$$

(5.36) $$I_{12}(q) = \frac{1}{1-q} + \frac{q^2(1+q^2)}{(1-q^2)(1-q^3)} - \frac{q^2(1-q^2)(1-q^3)(1+q^4)}{q^2(1+q^4)+(1+q^2)(1-q^5)} - \cdots$$

(5.37) $$I_{13}(q) = \frac{1}{1-q} + \frac{q(1+q^2)}{(1-q^2)(1-q^3)} - \frac{q(1-q^2)(1-q^3)(1+q^4)}{1+q+q^2-q^7} - \cdots$$

Continued fraction representation of mock theta function's of order ten,
(5.38)
$$\phi_{LC}(q) = \frac{1}{1-q} + \frac{q}{(1-q)(1-q^3)} - \frac{q^2(1-q)(1-q^3)}{1+q^2-q^5} - \frac{q^3(1-q^5)}{1+q^3-q^7} - \cdots$$

(5.39)
$$\psi_{LC}(q) = \frac{q}{1-q} + \frac{q^3}{(1-q)(1-q^3)} - \frac{q^3(1-q)(1-q^3)}{1+q^3-q^5} - \frac{q^4(1-q^5)}{1+q^4-q^7} - \cdots$$

(5.40)
$$\chi_{LC}(q) = 1 - \frac{q}{1+q+q^2+q^3} + \frac{q^3(1+q)(1+q^2)}{1+q^4+q^7} + \frac{q^5(1+q^3)(1+q^4)}{1+q^6+q^{11}} + \cdots$$

(5.41)
$$X_{LC}(q) = \frac{q}{1+q} - \frac{q^4}{(1+q)(1+q^3)(1+q^4)} + \frac{q^5(1+q)(1+q^3)(1+q^4)}{1+q^6+q^{11}} + \cdots$$

Continued fraction representation of mock theta function's of order two,
(5.42)
$$A(q) = \frac{q}{(1-q)^2} + \frac{q^4(1+q)}{(1-q)^2(1-q^3)^2} - \frac{q^5(1-q)^2(1+q^3)(1-q^3)^2}{1-q^5+q^8+q^{10}} - \cdots$$

(5.43)
$$B(q) = \frac{1}{(1-q)^2} + \frac{q^2(1+q^2)}{(1-q)^2(1-q^3)^2} - \frac{q^4(1-q)^2(1-q^3)^2(1+q^4)}{1+q^4-2q^5+q^8+q^{10}} - \cdots$$

(5.44)
$$\mu(q) = 1 - \frac{q(1-q)}{(1+q)^2} + \frac{q^3(1+q^2)^2(1-q^3)}{1-q^3+2q^4+q^6+q^8} + \frac{q^5(1+q^4)^2(1-q^5)}{1-q^5+2q^6+q^{10}+q^{12}} + \cdots$$

(5.45)
$$D_5(q) = \frac{1}{1-q} + \frac{q(1+q)}{(1-q)(1-q^3)} - \frac{q(1-q)(1+q^2)(1-q^3)}{1+q+q^3-q^5} - \frac{q(1+q^3)(1-q^5)}{1+q+q^4-q^7} - \cdots$$

Continued fraction representation of mock theta function's of order four,

$$D_6(q) = \frac{1}{1-q} \;+\; \frac{q(1+q^2)}{(1-q^2)(1-q^3)} \;-\; \frac{q(1-q^2)(1-q^3)(1+q^4)}{1+q+q^2-q^7} \;-\cdots \tag{5.46}$$

Proof of Special Cases

In order to develop continued fraction, we use eq.(3.1) and replacing a by $\frac{a}{z}$, b by $\frac{b}{z}$, z by z^2 and let $z \to 0$ and $m \to \infty$ and after that put $a = 1$, $b = q$, $c = 0$, $d = e = -q$ and $f = 0$, we get the continued fraction representation for $f(q)$. Similarly for suitable selection of a, b, c, d, e and f we obtain the remaining results.

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