

# A Signaling Theory of Education under the Presence of Career Concerns

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*A person's life consists of two important stages: the first stage as a student and the second stage as a worker. In an integrated model of education and career concerns, I analyze the welfare effects of education. In Spence's job market signaling model, education as a sorting device improves efficiency by mitigating the lemon market problem. In contrast, in the integrated model, education as a sorting device can be detrimental to social welfare, as it eliminates work incentives generated by career concerns.*

Key Word: education, signaling, career concerns

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## I. Introduction

Life consists of two important stages. In the first stage, as students, people decide how much education to obtain. In the second stage, as workers, they choose jobs and exert effort in the workplace (and then they retire). Therefore, education and careers are two important choices people make in life. However, people do not choose education and careers independently. They choose education while anticipating its impact on their future careers. However, the existing literature rarely considers possible interactions between education and careers. The standard model of (stand-alone) education is the job market signaling model of Spence (1973), in which education is used to signal workers' hidden productivity to the labor market. The standard model of (stand-alone) careers is the career concerns model of Holmstrom (1999), in which workers exert efforts to achieve good performance, which signals to the labor market that they are talented workers. Although the education and career concerns literatures starting from the two aforementioned seminal papers are both vast, little work has been done on the interactions between education and career concerns.

This paper examines an integrated model of education and career concerns. By

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explicitly considering the interactions between education and career concerns, I find new welfare implications of education: if society uses education to sort agents according to productivity, then education could reduce the total surplus. This is inconsistent with Spence's standard job market signaling model, which implies that education as a sorting device enhances welfare (even if it has no human capital value). Suppose that there are high type and low type agents. The labor market treats them equally, as they are indistinguishable. In such a case, high type agents leave and only low type agents prevail in the labor market (with an appropriate assumption on reservation payoffs). This loss of top talent (i.e., the lemon market problem) can be resolved if education reveals hidden types.

This story of the lemon market problem, however, may be overturned if not merely hidden productivity (type) but also hidden effort can contribute to output. Suppose that there exists a post-education work stage, in which an agent exerts effort to produce output. If the labor market does not fully know the agent's hidden productivity, she works diligently to demonstrate her talent. However, if education reveals the hidden talent, she is demotivated, exerts little effort, and hence output decreases. Therefore, education as a sorting device could be detrimental to welfare.

This paper is organized as follows. Section II presents an integrated model of education and career concerns. Section III analyzes the welfare implications of education. Section IV discusses an important technical issue in the integrated model. Spence's standard model has a unique separating equilibrium and no pooling equilibrium if one uses the Intuitive Criterion suggested by Cho and Kreps (1987), a standard equilibrium refinement. If the integrated model also has no pooling equilibrium, this paper's main welfare implication has of little value. In Section IV, I argue that pooling equilibria could survive the Intuitive Criterion in the integrated model. Section 5 presents the conclusion.

## II. The Model

An agent's hidden productivity  $\theta$  is either  $h$  (high) or  $l$  (low) where  $0 = l < h < 1$ . There are many agents whose total measure is normalized to one. One half is of high type and the other half is of low type.

The timeline is as follows. There are three periods  $t = 0, 1, 2$ . At  $t = 0$ , each agent as a student chooses a publicly-observable education level  $e \in [0, \infty)$  at the cost of education  $C(\theta, e)$  such that  $C(\theta, 0) = 0$ ,  $C_e > 0$ ,  $C_\theta < 0$ , and  $C_{e\theta} < 0$ , where the lower subscript denotes the (cross-) partial derivatives. At  $t = 1, 2$ , the competitive labor market pays wage  $w_t$ ; Given  $w_t$ , each agent as a worker chooses privately-observable effort level  $a_t \in [0, \bar{a}]$  at the cost of effort  $c(a_t)$  such that  $c(0) = c'(0) = 0$ ,  $c' > 0$ , and  $c'' > 0$ ; Given  $a_t$ , a publicly-observable output  $y_t \in \{0, 1\}$  is realized.

Effort and productivity contribute to output in the following manner: let  $f(\theta, a_t)$  be the conditional probability of success

$$(1) \quad f(\theta, a_t) \equiv \mathbb{P}(y_t = I | \theta, a_t) = \theta + ka_t, \quad k > 0$$

Note that the probability of success increases with effort and productivity. (The following analysis holds even if  $f(\theta, a_t)$  has a general functional form such that  $f_a > 0$  and  $f_\theta > 0$ ). To ensure that  $f(\theta, a_t) \leq 1$  for any  $a_t \in [0, \bar{a}]$ , it is necessary to assume that  $\bar{a} \leq 1 - h$ . Given that  $y_t \in \{0, I\}$ , one can interpret  $f(\theta, a_t)$  as the expected output conditional on  $(\theta, a_t)$ .

I assume that the labor market is competitive, and hence wage  $w_t$  equals the market's expectation of output. I assume that output  $y_t$  is not contractible.

Given a wage structure, a type  $\theta$  agent's preference is represented by

$$(2) \quad -C(\theta, e) + \mathbb{E}^\theta[w_1 | \cdot] - c(a_1) + \delta \{ \mathbb{E}^\theta[w_2 | \cdot] - c(a_2) \}$$

where  $\delta \in (0, 1)$  is the discount factor and  $\mathbb{E}^\theta[\cdot | \cdot]$  denotes the type  $\theta$  agent's expectation with respect to the relevant information structure.

Below, I consider two cases. First, high type agents choose a higher level of education than low type agents—separating equilibrium. Second, both types choose the same level of education—pooling equilibrium.

### A. Separating Equilibrium

Note that education has no human capital value. It can only signal hidden productivity. Thus, the most reasonable case is one in which low type agents choose zero education while high type agents choose the minimum level of education, which the low type agents cannot mimic because of higher cost of education. This separating equilibrium is known as the Riley equilibrium. I confine my attention to the Riley equilibrium, which is later shown to be the only separating equilibrium that satisfies the Intuitive Criterion suggested by Cho and Kreps (1987).

In the Riley equilibrium, types are revealed, as is the expected output. Suppose the market expects that  $a_t^e(\theta)$  will be chosen in equilibrium. A type  $\theta$  agent's wage is then, for  $t = 1, 2$ ,

$$w_t = f(\theta, a_t^e(\theta)) = \theta + ka_t^e(\theta)$$

That is, the wage is fixed and independent of effort  $a_t$  (though it depends on  $a_t^e(\cdot)$ ). Let  $a_t^*(\theta)$  denote the equilibrium effort type  $\theta$  agent actually chooses. Because effort is costly and has no effect on wages, it follows that  $a_t^*(\theta) = a_t^e(\theta) = 0$  in rational expectations equilibria.

Let  $e(\theta)$  be the equilibrium level of education, which the type  $\theta$  agent

chooses. Thus,  $e(l) = 0$  and  $e(h) > 0$ . Note that  $e(h)$  represents the minimum education level low type agents cannot profitably mimic. That is,  $e(h)$  is characterized by

$$(3) \quad h(1 + \delta) - C(l, e(h)) = l(1 + \delta) - C(l, e(l)) = 0$$

Let  $u^\theta$  denote the type  $\theta$  agent's reservation payoff, which she can obtain by leaving the labor market. This is perhaps the payoff of self-production or of participating in an alternative labor market. I assume that  $u^l = 0$  for simplicity and that

**Assumption 1**  $\frac{l}{2}h < u^h < h$

Assumption 1 implies that the lemon market problem exists: high type agents will not participate unless the types are revealed, despite the fact that their participation is socially efficient. I also assume that education is not prohibitively costly for the high type (though it is highly costly for the low type):

**Assumption 2**  $(1 + \delta)[h - u^h] > C(h, e(h))$

(3) implies that Assumption 2 holds if the education cost decreases sufficiently with productivity. Assumption 2 implies that the high type willingly participates in the labor market by undertaking costly education, as the hidden type is revealed, and therefore, a higher wage is promised.

The total surplus in this Riley separating equilibrium is given by

$$(4) \quad \frac{l}{2} \{ (1 + \delta)h - C(h, e(h)) \}$$

as the low type agent's utility equals zero regardless of whether or not they participate.

As for a benchmark case, suppose that education cannot be used as a sorting device. Due to the lemon market problem, the total surplus is then given by

$$(5) \quad \frac{l}{2} \{ (1 + \delta)u^h \}$$

Assumption 2 and equations (4) and (5) imply that education as a sorting device mitigates the lemon market problem by inducing high type agents' participation in the labor market. Furthermore, education as a sorting device improves efficiency in the Paretian sense, as low type agents' payoffs are unchanged while high type agents' payoffs increase.

## B. Pooling Equilibrium

In a pooling equilibrium, education has no sorting effect, and hence, the labor market cannot distinguish types. Given that education has no role but is only costly, I confine my attention to the most efficient pooling equilibrium, in which both type agents choose zero education. In Spence's standard job market signaling model, where interactions between education and career concerns are ignored, this pooling equilibrium fails to satisfy the Intuitive Criterion. Later, I shall discuss that this is not the case if one considers the interactions between education and career concerns.

Suppose that the market expects that  $a_i^e(\theta)$  is chosen in equilibrium. Let  $\mathbf{a}_i^e$  denote  $(a_i^e(h), a_i^e(l))$  and  $\mathbb{E}^m$  denote the market's expectation conditional on  $\mathbf{a}_i^e$ . Then, wages are given by

$$(6) \quad w_1 = \mathbb{E}^m [y_1]$$

$$(7) \quad w_2(y_1) = \mathbb{E}^m [y_2 | y_1]$$

Importantly, the date-2 wage (i.e., the expected date-2 output) depends on the date-1 output  $y_1$ . This is because market observes the date-1 performance  $y_1$ , which is informative of an agent's hidden productivity  $\theta$ , which determines the date-2 performance  $y_2$ . (In a separating equilibrium, in contrast, the date-1 performance has no value of information, as the hidden productivity is already unraveled). This observation is crucial in this paper. Even if there are no explicit incentive contracts, agents face date-1 incentives because the date-1 effort (stochastically) determines the date-1 output, which determines the date-2 wage. However, the date-1 wage is fixed and independent of output, as no output is realized in the beginning of date-1.

To observe this formally, note that at date-2, a type  $\theta$  agent maximizes  $w_2(y_1) - c(a_2)$ . Since  $w_2(y_1)$  is independent of  $a_2$  (while it depends on  $a_2^e(\theta)$ ), the agent chooses  $a_2^*(\theta) = 0$  for  $\theta = h, l$ . Then, in rational expectations equilibria, it follows that  $a_2^e(\theta) = a_2^*(\theta) = 0$ . At date-1, the agent chooses  $a_1 = a_1^*(\theta)$ , which maximizes

$$(8) \quad w_1 - c(a_1) + \delta \left\{ \mathbb{E}^\theta [w_2(y_1) | a_1] - c(a_2) \right\}$$

where  $\mathbb{E}^\theta$  is the type  $\theta$  agent's expectation conditional on  $\theta$  and  $\mathbf{a}_i^e$ . Since  $a_1$  affects the probability of  $y_1$ ,  $a_1 = a_1^*(\theta)$ , is characterized by the following first-order condition

$$\begin{aligned}
 c'(a_1) &= \delta \frac{\partial}{\partial a_1} \mathbb{E}^\theta [w_2(y_1) | a_1] \\
 (9) \quad &= \delta \frac{\partial}{\partial a_1} [f(\theta, a_1)w_2(1) + \{1 - f(\theta, a_1)\}w_2(0)] \\
 &= \delta f_a(\theta, a_1)[w_2(1) - w_2(0)]: \text{ the marginal incentive}
 \end{aligned}$$

In general,  $a_1^*(\theta)$  depends on  $\theta$ , as  $f_a(\theta, a_1)$  depends on  $\theta$ . Given the simplification that  $f(\theta, a_1) = \theta + ka_1$ , however,  $a_1^*(\theta)$  is independent of  $\theta$ , though the main result of this paper is robust to the functional form of  $f(\theta, a_1)$  under certain regularity conditions.<sup>1</sup> Thus, I let  $a_1^*(\theta) = a_1^*$  and  $a_1^e(\theta) = a_1^e$ . One may expect that the wage wedge  $[w_2(1) - w_2(0)]$  is positive. This is true because better date-1 performance implies that hidden productivity is greater, indicating that the expected date-2 performance would be greater. The following lemma provides a formal account.

**Lemma 1** *The marginal incentive in (9) is positive, independent of  $\theta$ , and equal to*

$$(10) \quad \delta k \frac{VAR^m(\theta)}{VAR^m(y_1)}$$

where  $VAR^m$  is the market's assessed variance conditional on  $a_1^e$ .

*Proof:* First of all, the marginal incentive is positive since the variances are always nonnegative and  $\theta$  is a nondegenerate random variable from the market's perspective. Independence with respect to  $\theta$  is obvious. Next, I shall prove that  $[w_2(1) - w_2(0)]$  equals the ratio of variances. Initially, the wage is given by

$$\begin{aligned}
 w_2(y_1) &= \mathbb{E}^m [y_2 | y_1] = \mathbb{E}^m [\mathbb{E}^m [y_2 | \theta, y_1] | y_1] \quad (\text{the iterated expectation}) \\
 &= \mathbb{E}^m [1 \cdot f(\theta, a_1^e) + 0 \cdot \{1 - f(\theta, a_1^e)\} | y_1] \\
 (11) \quad &= \underbrace{f(h, a_1^e)}_{=h} \mathbb{P}^m(h | y_1) + \underbrace{f(l, a_1^e)}_{=0} \{1 - \mathbb{P}^m(h | y_1)\}
 \end{aligned}$$

<sup>1</sup>Assume a set of regularity conditions:  $f_a > 0$ ,  $f_\theta > 0$ , and  $f_{a\theta} \geq 0$  for any  $(\theta, a)$ , where the subscripts denote (cross) partial derivatives. In this case, it can be shown that  $a_1^*(h) \geq a_1^*(l) > 0$ . Although the equilibrium effort depends on type, it is unobservable by the market. Thus, under the absence of education as a sorting device, the market cannot identify the high type from the low type, and hence, the agent has work incentives. I thank an anonymous referee for having pointed out to me the necessity of certain regularity conditions.

where  $\mathbb{P}^m(h|y_i)$  is the market's posterior of  $\theta = h$  given  $y_i$  and  $a_i^e$ . This posterior is given by

$$\mathbb{P}^m(h|y_i = I) = \frac{\frac{I}{2}f(h, a_i^e)}{\frac{I}{2}f(h, a_i^e) + \frac{I}{2}f(l, a_i^e)}$$

$$\mathbb{P}^m(h|y_i = 0) = \frac{\frac{I}{2}[1 - f(h, a_i^e)]}{\frac{I}{2}[1 - f(h, a_i^e)] + \frac{I}{2}[1 - f(l, a_i^e)]}$$

Then, it follows

$$(12) \quad w_2(I) - w_2(0) = \frac{\frac{I}{4}f(h, a_i^e)[f(h, a_i^e) - f(l, a_i^e)]}{\frac{I}{2}[f(h, a_i^e) + f(l, a_i^e)] \left[ 1 - \frac{I}{2}\{f(h, a_i^e) + f(l, a_i^e)\} \right]}$$

Note that the numerator equals  $\frac{I}{4}h^2$ , which equals  $VAR^m(\theta)$  since

$$VAR^m(\theta) = \mathbb{E}^m[\theta^2] - \mathbb{E}^m[\theta]^2 = \frac{I}{2}h^2 - \frac{I}{4}h^2 = \frac{I}{4}h^2$$

The denominator equals  $VAR^m(y_i)$  since

$$VAR^m(y_i) = \mathbb{E}^m[y_i^2] - \mathbb{E}^m[y_i]^2$$

$$= \mathbb{E}^m[y_i](I - \mathbb{E}^m[y_i]) \text{ since } y_i \in \{0, I\} \text{ implies } \mathbb{E}^m[y_i^2] = \mathbb{E}^m[y_i]$$

where  $\mathbb{E}^m[y_i] = \mathbb{E}^m[\mathbb{E}^m[y_i|\theta]] = \mathbb{E}^m[f(\theta, a_i^e)] = \frac{I}{2}\{f(h, a_i^e) + f(l, a_i^e)\}$ . ■

Lemma 1 implies that the date-1 marginal incentive is determined by the signal-to-noise ratio—the extent to which the data  $y_i$  conveys information about a hidden variable  $\theta$ . Holmstrom (1999) finds the same result under a simpler model in which neither the market nor the agent knows the hidden type. Thus, making  $\theta$  private information does not affect the size of the career concerns motive of work incentives under the current specification, in which the expected output is linear in

$\theta$  and  $a_t$  (i.e.  $f(\theta, a_t) = \theta + ka_t$ ). In a more general specification, work incentives depend on  $\theta$ .

A key observation from (10) is as follows. If an agent's hidden ability is not revealed to the labor market, the agent faces date-1 work incentives in order to convince the market that she is of high ability.

The next lemma demonstrates the existence of a rational expectations equilibrium ( $a_t^* = a_t^e$ ). Note that  $w_2(y_t)$  depends on  $a_t^e$ . To highlight this dependence, I write  $w_2(y_t) = w_2(y_t)(a_t^e)$ .

**Lemma 2** Suppose  $c'\left(\frac{1-h}{k}\right) > h\delta k$ . Then, there exists a rational expectations equilibrium  $a_t^* \in \left(0, \frac{1-h}{k}\right)$  such that

$$\delta k [w_2(1)(\hat{a}_t) - w_2(0)(\hat{a}_t)] = c'(\hat{a}_t) \quad \text{at } \hat{a}_t = a_t^*$$

*Proof:* Let  $v(a_t^e) \equiv w_2(1)(a_t^e) - w_2(0)(a_t^e) > 0$ . In addition, let  $g(\cdot) \equiv \frac{c'(\cdot)}{\delta k}$ .

Then, (\*) there exists a unique  $a_t \in \left(0, \frac{1-h}{k}\right)$  for each given  $a_t^e$  such that

$g(a_t) = v(a_t^e)$  since  $c'(0) = 0$ ,  $c'\left(\frac{1-h}{k}\right) > h\delta k$ , and  $h > v(a_t^e)$ . Then,

$a_t = g^{-1}(v(a_t^e))$ . Let  $\Psi(a_t^e) \equiv g^{-1}(v(a_t^e))$ .  $\Psi(a_t^e)$  is a continuous function on

the closed interval  $\left[0, \frac{1-h}{k}\right]$  to the same interval. Then, the Brouwer fixed point

theorem and (\*) imply that there exists an  $a_t^* \in \left(0, \frac{1-h}{k}\right)$  such that  $a_t^* = \Psi(a_t^*)$ .

That is, there exists a rational expectations equilibrium  $a_t^*$ . ■

Given the equilibrium effort  $a_t^*$ , the date-1 wage equals

$$w_t = \mathbb{E}^m [y_t] = \mathbb{E}^m [\mathbb{E}^m [y_t | \theta]] = \mathbb{E}^m [f(\theta, a_t^*)] = \frac{1}{2}h + ka_t^*$$

Additionally, according to (11), a type  $\theta$  agent's expectation of the date-2 wage equals

$$\mathbb{E}^\theta [w_2(y_t)] = h\mathbb{E}^\theta [\mathbb{P}^m(h | y_t)]$$



Subsequently, a type  $\theta$  agent's payoff equals

$$(13) \quad U_p(\theta) \equiv \frac{I}{2}h + ka_1^* - c(a_1^*) + \delta h \mathbb{E}^\theta [\mathbb{P}^m(h|y_1)]$$

Note that this payoff increases with the level of equilibrium effort  $a_1^*$  since (9) implies that  $a_1^*$  is lower than the first-best effort, which maximizes  $ka - c(a)$ . Thus, if there are two equilibrium efforts,  $a'$  and  $a''$  such that  $a' < a''$ , then every agent is strictly better off with  $a''$  than  $a'$ . Therefore, the remaining part of this paper focuses on the maximum (or the supremum) of equilibrium efforts based on the notion of the Pareto optimality.

Note that low type agents will always participate in the labor market, as their reservation payoff  $u^l$  is zero. However, high type agents will participate if and only if

$$\textbf{Assumption 3} \quad \frac{I}{2}h + ka_1^* - c(a_1^*) + \delta h \mathbb{E}^h [\mathbb{P}^m(h|y_1)] \geq (I + \delta)u^h$$

To observe when high type agents profitably participate, suppose for the moment  $k = 0$ , and hence,  $a_1^* = 0$  by (10). Note that  $\mathbb{E}^\theta [\mathbb{P}^m(h|y_1)] \in (0, I)$  is the type  $\theta$  agent's expectation of the market's posterior given  $y_1$ . High type agents are more optimistic about the future than low type agents, as they know their superior ability:  $\mathbb{E}^l [\mathbb{P}^m(h|y_1)] < \frac{I}{2} < \mathbb{E}^h [\mathbb{P}^m(h|y_1)]$ . This is related to the market's learning effect. The labor market updates its expectation of date-2 output by observing the date-1 output. High type agents then expect more income, as the date-1 output is (stochastically) higher. If the learning effect is slight (i.e.,  $|\mathbb{E}^h [\mathbb{P}^m(h|y_1)] - \frac{I}{2}|$  has a small value), Assumption 1 implies that Assumption 3 is violated. If instead the learning effect is large enough, Assumption 3 is satisfied.

Even if the learning effect is slight, if  $k > 0$  and the surplus  $ka_1^* - c(a_1^*)$  generated by career concerns is large enough, then Assumption 3 is satisfied.

As long as high type agents find it optimal to participate (i.e. Assumption 3 holds), the total surplus in this pooling equilibrium is  $\frac{I}{2}[U_p(h) + U_p(l)]$ , which equals

$$(14) \quad \frac{I}{2}(I + \delta)h + ka_1^* - c(a_1^*)$$

according to the martingale property, that is, the expectation of the posterior equals its prior.

### III. Welfare Implications

By comparing (14) to (4), it is apparent that the total surplus in the pooling equilibrium is greater than that in the Riley separating equilibrium. That is, the use of education as a sorting device reduces the total surplus. There are two reasons. First, education as a signaling device reveals hidden productivity. Thus, agents need not persuade their ability to potential employers in the labor market, which means that their post-education work incentives (motivated by career concerns) are eliminated. In consequence, the work stage surplus  $ka_1^* - c(a_1^*)$  is not realized.

Second, education is wasteful and incurs cost  $C(h, e(h))$ .

The detrimental effect of education as a sorting device hinges on Assumption 3. If this assumption is not satisfied, only low type agents participate in the labor market. Expecting this fact rationally, the market then pays zero to the participants. Thus, the total surplus in the pooling equilibrium equals

$$\frac{1}{2}(1 + \delta)u^h$$

Thus, we return to the standard result, where the use of education as a signaling device improves the total surplus. One might wonder when Assumption 3 is more likely to be satisfied. This depends on  $h$  and  $u^h$ . The degree of the lemon market problem can be measured by  $|h - u^h|$  since adverse selection discourages the participation of high type agents in the labor market. Note that Assumption 3 holds if  $h$  is large or  $u^h$  is small, as in this case high type agents will participate even if the work stage surplus is relatively low. Thus, Assumption 3 is satisfied if the degree of the lemon market problem is severe, which is of high interest, whereas it is violated if the lemon market problem is not very important, which is of little interest. In the standard Spence model in which the post-education work stage is ignored, education as a sorting device is more beneficial if the lemon market problem is more severe. However, in this very case, education as a sorting device is more detrimental if the post-education work stage is explicitly considered.

Summarizing the analysis brings the following main result:

**Proposition 1** *Suppose that the lemon market problem exists (i.e., Assumption 1) and that education is not prohibitively costly (i.e., Assumption 2). Then,*

(i) *If there is no post-education work stage, the use of education as a sorting device increases the total surplus.*

*Suppose for the following that there is a post-education work stage.*

(ii) *If high type agents profitably participate in the labor market in the most efficient pooling equilibrium (i.e. Assumption 3), which is the case when the lemon market problem is severe, then education as a sorting device decreases the total surplus.*

(iii) *If high type agents cannot profitably participate in the most efficient pooling equilibrium (i.e. Assumption 3 is violated), which is the case when the lemon*

*market problem is unimportant, then education as a sorting device increases the total surplus.*

In the following, I consider three extensions. In the first extension, I consider multiple types. Productivity is determined by various hidden factors. For instance, intelligence and fitness-to-work are two important factors that contribute to overall productivity. Let  $\theta_1$  denote intelligence, which agents observe but the market does not. The marginal cost of education  $C_e(\theta_1, e)$  decreases in  $\theta_1$ . Let  $\theta_2$  denote fitness-to-work, which is unobservable to agents and the market. Let  $\theta \equiv g(\theta_1, \theta_2)$  be productivity, where  $g$  increases in each argument. In this case, education can reveal only  $\theta_1$ . Lemma 1 then implies that an agent's post-education marginal incentive under separating equilibria equals

$$\delta k \frac{VAR^m(\theta | \theta_1)}{VAR^m(y_1)}$$

while the marginal incentive under pooling equilibria equals

$$\delta k \frac{VAR^m(\theta)}{VAR^m(y_1)}$$

That is, the marginal incentive is greater under pooling equilibria. This result holds irrespective of whether or not  $\theta_1$  and  $\theta_2$  are independent. That is, the more information education reveals, the more likely it will be detrimental to welfare.

In the second extension, I assume that education has both a human capital-enhancing effect and a sorting effect. This situation can be modeled in the following way: the overall productivity  $\theta'$  equals  $\theta + \alpha e$ , where  $\alpha > 0$ . In this case, the welfare implication of education is ambiguous. On the one hand, it improves efficiency by raising productivity. On the other hand, it reduces efficiency by discouraging post-education work incentives.

In the third extension, explicit incentive contracts are considered. Thus far, I have assumed that workers face only the implicit incentive generated by career concerns but not an explicit incentive provided by a performance-based contract. Suppose instead that output  $y_t$  is contractible at no cost. Then, the linear contract  $w_t(y_t) = y_t - K$ , where  $K$  is a constant, induces the first-best effort since the agent becomes a residual claimant. However, such a 'perfect' contractibility is not satisfied in many real-life principal-agent relationships. Performance measures of workers in administration offices are often nonverifiable. Subjective and hence non-contractible performance measures (such as quality) often contain more information than objective performance measures (such as quantity). For these reasons, suppose that  $y_t$  is non-contractible. Let  $p_t$  be a contractible but imperfect performance measure. Workers then face an imperfect explicit incentive

generated by a contract  $w_i(p_i)$ . The implicit incentive generated by career concerns could then complement the imperfect explicit incentive. Therefore, education as a sorting device could reduce the welfare to an extent, which increases in the imperfectness of  $p_i$ .

Proposition 1 implies that uncertainty in types is beneficial for social welfare. This seems at odd at a first glance, as one might expect that the uncertainty, as a source of market failure, reduces social welfare. However, this is consistent with the general theory of second-best suggested by Lipsey and Lancaster (1956): if there is an existing source of market failure (so that the economy is in a second-best outcome), an additional source of market failure could either increase or decrease social welfare (see Milgrom and Roberts (1982) and Kim (2004) for other examples of the general theory.) However, Proposition 1 is not a simple corollary of the general theory. In fact, the general theory is too general to explain why an additional source of market failure is socially beneficial; the general theory only raises the possibility that an additional market failure is beneficial.

Proposition 1 is also related to the information disclosure literature. In a dynamic tournament setting, Ederer (2010) shows that disclosing interim performance can reduce incentives due to the trade-off between evaluation and motivation effects. In the presence of career concerns and relational contracting, Mukherjee (2008) shows that if the current employer discloses workers' performance levels to the labor market, the career concerns motive of incentives increases, whereas the effectiveness of relational contracting is reduced. Proposition 1 is, however, different from these papers in that (1) it examines the effect of disclosing types rather than performance on incentives and (2) these papers consider only post-education workplace behaviors, while the present study considers both education and post-education behaviors.

#### IV. Intuitive Criterion

There are infinitely many separating and pooling equilibria in the standard job market signaling model (where there is no post-education work stage). However, Cho and Kreps (1987) show that only the Riley separating equilibrium (and no pooling equilibria) satisfies their Intuitive Criterion, which currently is the standard equilibrium refinement criterion, and is hence reasonable. Recently, Alos-Ferrer and Prat (2012) show that certain pooling equilibria can satisfy the Intuitive Criterion if there is learning by the market (or employers). If there is a post-education work stage, as in this paper, pooling equilibria are more likely consistent with the Intuitive Criterion. These points are elaborated below.

##### A. Separating Equilibrium

Consider initially the standard job market signaling model. Let  $(e^l, e^h)$  be a non-Riley separating equilibrium such that  $e^l = 0$  and  $e^h > e(h)$ , where  $e(h)$

is given by (3). (Note that  $e^l$  cannot be positive since otherwise low type agents profitably deviate to zero education). Then, if an agent chooses  $e'$  such that,  $e(h) < e' < e^h$ , the market should *not* believe that the agent is high type since otherwise high type agents profitably deviate to  $e'$ . However, such a belief is not *intuitive* for the following reason. If low type agents deviate to  $e'$ , they are worse off even in the best scenario in which the market believes them as high type and hence pays  $h$  rather than  $l$ . If high type agents deviate to  $e'$ , they are better off in the best scenario. That is, low type agents never deviate while high type agents may deviate. Thus, the market should believe that those who deviate to  $e'$  are high type. Therefore, if only the *intuitive* belief is allowed, as required by the Intuitive Criterion,  $(e^l, e^h)$  is no longer a separating equilibrium. Nothing changes in this argument even if one introduces a post-education work stage, as there is no uncertainty in types.

### B. Pooling Equilibrium

Consider the standard job market signaling model. Let  $e_p$  be a pooling equilibrium. Then, there is  $e_p^\theta$  such that

$$\mathbb{E}^m[\theta](I + \delta) - C(\theta, e_p) = h(I + \delta) - C(\theta, e_p^\theta)$$

It follows then  $e_p < e_p^l < e_p^h$ . Suppose that  $e_p$  is a pooling equilibrium. Then, the market should *not* believe that the agent who deviates to  $e' \in (e_p^l, e_p^h)$  is high type, since otherwise high type agents profitably deviate to  $e'$ . But this belief is not *intuitive*. By deviating to  $e'$ , low type agents are worse off even under the best scenario (i.e., the market believes them as high type), while high type agents are better off under the best scenario. Thus, only high type agents may deviate to  $e'$ , and hence, the market should believe that those who choose  $e'$  are high type. If only this type of *intuitive* belief is allowed,  $e_p$  is no longer a pooling equilibrium.

However, this is not the case if one adds post-education learning by the market (or employers). After the education choice, the output signal  $y_t$  is realized. (To focus on the learning effect, I assume for now that  $k = 0$  so that agents do not exert any work effort). By observing  $y_t$ , the labor market learns (partially) about the hidden type and then adjusts its expectation of the date-2 output  $\mathbb{E}^m[\theta | y_t]$ . High type agents know that their date-1 output will be (stochastically) greater than that of low type agents. Thus, even if types are not revealed in pooling equilibria, high type agents expect larger incomes (i.e.,  $\mathbb{E}^h[\mathbb{E}^m[\theta | y_t]] > \mathbb{E}^l[\mathbb{E}^m[\theta | y_t]]$ ). Then, a high type agent's gain from separating himself (by gaining more education than  $e_p$ ) from low type agents is lower than it is when there is no post-education

learning. To see this, note that  $e_p^\theta$  is determined via

$$\mathbb{E}^m [\theta] + \delta \mathbb{E}^\theta [\mathbb{E}^m [\theta | y_l]] - C(\theta, e_p) = h(1 + \delta) - C(\theta, e_p^\theta)$$

If this learning effect ( $\mathbb{E}^h [\mathbb{E}^m [\theta | y_l]] - \mathbb{E}^l [\mathbb{E}^m [\theta | y_l]]$ ) is large enough, it follows that  $e_p^l > e_p^h$ . Then, there is no education level  $e'$  such that low type agents are worse off even under the best scenario while high type agents are better off under the best scenario. Thus, this pooling equilibrium satisfies the Intuitive Criterion. (See Alos-Ferrer and Prat (2012) for more details).

If one also considers post-education working (i.e.,  $k > 0$ ), then the pooling equilibria become more robust with respect to the Intuitive Criterion. By separating himself from low type agents, high type agents lose the work stage surplus  $ka_l^* - c(a_l^*)$ , which is realized only in pooling equilibria. If this surplus is large enough, high type agents are worse off by separating himself from low type agents (by taking more education than  $e_p$ ) even in the best scenario. That is, for any  $e > e_p$ , it follows that

$$\mathbb{E}^m [\theta] + \delta \mathbb{E}^h [\mathbb{E}^m [\theta | y_l]] + ka_l^* - c(a_l^*) - C(h, e_p) > h(1 + \delta) - C(h, e)$$

The pooling equilibrium  $e_p$  then satisfies the Intuitive Criterion.

## V. Conclusion

I revisit Spence's signaling theory of education by providing an integrated model of education and career concerns. Under the absence of post-education career concerns, education as a sorting device improves welfare by mitigating the lemon market problem. Under the presence of post-education career concerns, sorting by education could be detrimental to social welfare, as it eliminates work incentives generated by career concerns.

Although this paper examines the welfare effects of education, one should be careful in drawing educational policy implications directly from the current paper, as the main result is based on several assumptions. Rather, the merit of this paper is that it provides a theoretic framework with which researchers may conduct thought experiments regarding possible dynamics of education, careers, incentives, and policies.

## REFERENCES

- Alos-Ferrer, Carlos and Julien Prat. 2012. "Job Market Signaling and Employer Learning." *Journal of Economic Theory* 147: 1787–1817.
- Cho, In-Koo and David M Kreps. 1987. "Signaling Games and Stable Equilibria." *Quarterly*

*Journal of Economics*: 179–221.

- Ederer, Florian.** 2010. “Feedback and Motivation in Dynamic Tournaments.” *Journal of Economics and Management Strategy* 19: 733–769.
- Holmstrom, Bengt.** 1999. “Managerial Incentive Problems: A Dynamic Perspective.” *Review of Economic Studies* 66: 169–182.
- Kim, Jaehong.** 2004. “Efficiency of Entry Regulation under Incomplete Information.” mimeo.
- Lipsey, R. and Kelvin Lancaster.** 1956. “The General Theory of Second Best.” *Review of Economic Studies* 24: 11–32.
- Milgrom, Paul and John Roberts.** 1982. “Limit Pricing and Entry under Incomplete Information: An Equilibrium Analysis.” *Econometrica* 50: 443–459.
- Mukherjee, Arijit.** 2008. “Sustaining Implicit Contracts when Agents Have Career Concerns: the Role of Information Disclosure.” *RAND Journal of Economics* 39: 469–490.
- Spence, Michael.** 1973. “Job Market Signaling.” *Quarterly Journal of Economics* 87: 355–374.