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Using DEA and AHP for Hierarchical Structures of Data

Mohammad Sadegh Pakkar*
Faculty of Management, Laurentian University, Sudbury, Canada

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ABSTRACT

In this paper, we propose an integrated data envelopment analysis (DEA) and analytic hierarchy process (AHP) methodology in which the information about the hierarchical structures of input-output data can be reflected in the performance assessment of decision making units (DMUs). Firstly, this can be implemented by extending a traditional DEA model to a three-level DEA model. Secondly, weight bounds, using AHP, can be incorporated in the three-level DEA model. Finally, the effects of incorporating weight bounds can be analyzed by developing a parametric distance model. Increasing the value of a parameter in a domain of efficiency loss, we explore the various systems of weights. This may lead to various ranking positions for each DMU in comparison to the other DMUs. An illustrative example of road safety performance for a set of 19 European countries highlights the usefulness of the proposed approach.

Keywords: Data Envelopment Analysis, Analytic Hierarchy Process, Hierarchical Structures

* Corresponding Author, E-mail: ms_pakkar@laurentian.ca

1. INTRODUCTION

Data envelopment analysis (DEA) is an objective data-oriented approach for assessing the relative efficiency of decision making units (DMUs) with multiple inputs and outputs. In classical DEA models, each DMU is allowed to choose its own favorable system of weights to maximize its relative efficiency. This freedom from choosing weights is equivalent to keeping the preferences of the decision-maker (DM) out of the decision process.

On the other hand, the analytic hierarchy process (AHP) is a multi-attribute decision- making method that can reflect a priori information about the relative priority of inputs, outputs or even DMUs in the efficiency assessment. AHP can be combined with DEA models in different ways. The most common way is the imposition of weight restrictions in DEA models. Referring to the literature, AHP can estimate the bounds of the following restrictions in DEA:

- Absolute weight restrictions. These restrictions directly impose upper and (or) lower bounds on the weights of inputs (outputs) using AHP (Pakkar, 2014a; Entani *et al*., 2004).
- Relative weight restrictions. These restrictions limit the relationship between the weights of inputs (outputs) using AHP (Lee *et al*., 2012; Liu *et al*., 2005; Takamura and Tone, 2003; Tseng *et al*., 2009; Kong and Fu, 2012).
- Virtual weight restrictions. A single virtual input (output) is defined as the weighted sum of all inputs (outputs). We refer to the proportion of each component of such sum as the "*virtual weight*" of an input (output). These restrictions limit virtual weights using AHP (Premachandra, 2001; Shang and Sueoshi, 1995).
- Restrictions on input (output) units. These restrictions impose bounds on changes of inputs (outputs) while the relative importance of such changes is computed using AHP (Lozano and Villa, 2009).

There are a number of other methods that do not necessarily apply additional restrictions to a DEA model. Such as converting the qualitative data in DEA to the quantitative data using AHP (Azadeh *et al*., 2008; Ertay *et al*., 2006; Jyoti *et al*., 2008; Korpela *et al*., 2007; Lin *et al*., 2011; Ramanathan, 2007; Yang and Kuo, 2003; Raut, 2011), ranking the efficient/inefficient units in DEA models using AHP in a two stage process (Ho and Oh , 2010; Jablonsky, 2007; Sinuany-Stern *et al*., 2000), weighting the efficiency scores obtained from DEA using AHP (Chen, 2002), weighting the inputs and outputs in the DEA structure (Pakkar, 2014b; Cai and Wu, 2001; Feng *et al*., 2004; Kim, 2000), constructing a convex combination of weights using AHP and DEA (Liu and Chen, 2004) and estimating missing data in DEA using AHP (Saen *et al*., 2005). Nevertheless, the above- mentioned literature is limited to the applications with one-level DEA models, which may not entirely satisfy the need for increasingly complex assessment problems.

In a recent paper, Pakkar (2015) proposes an integrated DEA and AHP approach to assess the performance of DMUs. The core logic of the proposed approach is to reflect the relative priority of inputs and outputs in performance assessment under hierarchical structures of data. This approach can be organized into the following steps:

1. The classical CCR (one-level) DEA model is used to compute the efficiency of each DMU after normalizing the original data. The computed efficiencies in this step will be part of the data used in the next step.
2. A two-level DEA model is used to obtain a set of weights of inputs and outputs for each DMU under the hierarchical structures of data (the minimum efficiency loss).
3. The two level-DEA model is bounded by AHP weights to reflect the priority weights of inputs and outputs in the performance assessment (the maximum efficiency loss).
4. A parametric-distance model is developed to explore various sets of weights within the defined domain of efficiency losses.

In two-level DEA models which originally developed by Meng *et al*. (2008), the inputs and outputs of similar characteristics are grouped into their own categories using a weighted sum approach. Nonetheless, these inputs and outputs might also belong to different sub-categories and further be linked to one another constituting a three-level hierarchical structure. To overcome this limitation, we similarly integrate AHP to a three-level DEA model. A three-level DEA model reflects the characteristics of the generalized multi-level DEA model developed by Shen *et al*. (2011). Theoretically, the approach proposed in this paper may also be considered as an extension to the three-level DEA model without explicit inputs, using AHP, to constructing composite indicators proposed by Pakkar (2016).

2. Methodology

2.1 A Basic DEA Model

In a classical DEA model, the optimal values of the variables (weights) are highly sensitive to the scales used for each input and output (Cooper *et al*., 2004). It seems logical and desirable to have scale independent weights that can be interpreted in some meaningful way. This may be achieved by using a unified-scale or normalized data. For this purpose, the distance to a reference approach is adopted as follows (OECD, 2008):

$$x_{ij} = \frac{\hat{x}_{ij}}{\hat{x}_{i(\max)}}, \quad \hat{x}_{i(\max)} = \max\{\hat{x}_{i1}, \hat{x}_{i2}, \cdots, \hat{x}_{in}\} \tag{1}$$

for inputs.

$$y_{rj} = \frac{\hat{y}_{rj}}{\hat{y}_{r(\max)}}, \quad \hat{y}_{r(\max)} = \max\{\hat{y}_{r1}, \hat{y}_{r2}, \cdots, \hat{y}_{rn}\} \tag{2}$$

for outputs.

Where $\hat{x}_{ij}$ and $\hat{y}_{rj}$ are the raw values of input $i(i, 2, \cdots, m)$ and output $r(r=1, 2, \cdots, s)$ for DMU $j(j=1, 2, \cdots, n)$. x_{ij} and y_{rj} are the corresponding normalized values of input i and output r for DMU j. Then the fractional CCR-DEA model can be developed as follows (Charnes *et al*., 1978):

$$\text{Max} \quad E_k = \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \tag{3}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \;\; \forall j, \tag{4}$$

$$u_r, v_i > 0 \quad \forall r, \; i, \tag{5}$$

where E_k is the relative efficiency of DMU under assessment. k is the index for the DMU under assessment where k ranges over 1, 2, $\cdots$, n. v_i and u_r are the weights of input $i(i=1, 2, \cdots, m)$ and output $r(r=1, 2, \cdots, s)$. The first set of constraints (4) assures that if the computed weights are applied to a group of n DMUs, $(j=1, 2, \cdots, n)$, they do not attain an efficiency value of larger than 1. The second set of constraints (5) indicates the non-negative conditions for the model variables.

2.2 Three-Level DEA Model

We develop our formulation based on the generalized distance model (Kao and Hung, 2005) in such a way that the hierarchical structures of data, using a weighted-average approach, are taken into consideration (Shen *et al*., 2011). Let $x_{hh'ij}$ be the value of input $i(i=1, 2, \cdots, m)$ of sub-category $h'(h'=1, 2, \cdots, M')$ of category $h\,(h=1, 2, \cdots, M)$, and $y_{ll'rj}$ be the value of output $r\,(r=1, 2, \cdots, s)$ of sub-category $l'(l'=1, 2, \cdots, S')$ of category $(l = 1, 2, \cdots, S)$ for DMU $j\,(j=1, 2, \cdots, n)$ after normalizing the original data. Let $v_{hh'i}$ be the internal weight of input i of sub-category h' of category h and $u_{ll'r}$ be the internal weight of output r of sub-category l' of category l, while $\sum_{i=1}^{m} v_{hh'i} = 1$ and $\sum_{r=1}^{s} u_{ll'r} = 1$. Then the values of sub-category h' of category h and sub-category l' of category l for the DMU j are defined as $x_{hh'j} = \sum_{i=1}^{m} v_{hh'i} x_{hh'ij}$ and $y_{ll'j} = \sum_{r=1}^{s} u_{ll'r} y_{ll'rj}$, respectively. Let $q_{hh'}$ be the internal weight of sub-category h' of category h and $p_{ll'}$ be the internal weight of sub-category l' of category l, while $\sum_{h'=1}^{M'} q_{hh'} = 1$ and $\sum_{l'=1}^{S'} p_{ll'} = 1$. Then the values of categories h and l are defined as $x_{hj} = \sum_{h'=1}^{M'} q_{hh'} x_{hh'j}$ and $y_{lj} = \sum_{l'=1}^{S'} p_{ll'} y_{ll'j}$, respectively. Let q_h and p_l be the weights of categories h and l, respectively. Then, the new multipliers of input i of sub-category h' of category h and output r of sub-category l' of category l are defined as: $v'_{hh'i} = q_h q_{hh'} v_{hh'i}$ and $u'_{ll'r} = p_l p_{ll'} u_{ll'r}$, respectively. Similarly, the new multiplier of sub-category l' of category l is defined as: $p'_{ll'} = p_l p_{ll'}$.

Let E_k^* $(k = 1, 2, \cdots, n)$ be the best attainable efficiency value for the DMU under assessment, calculated from the CCR-DEA model. We want the efficiency value $E_k(u'_{ll'r}, v'_{hh'i})$, calculated from the set of weights $u'_{ll'r}$ and $v'_{hh'i}$ to be *closest* to E_k^*. Our definition of *closest* is that the largest distance is at its minimum. Hence we choose the form of the minimax model: $\min_{u'_{ll'r}, v'_{hh'i}} \max_k \left\{ E_k^* - E_k(u'_{ll'r}, v'_{hh'i}) \right\}$ to minimize a single deviation which is equivalent to the following nonlinear model:

$$\text{Min} \quad \theta \tag{6}$$

s.t.

$$E_k^* - \frac{\sum_{l=1}^{S}\sum_{l'=1}^{S'}\sum_{r=1}^{s} u'_{ll'r} y_{ll'rk}}{\sum_{h=1}^{M}\sum_{h'=1}^{M'}\sum_{i=1}^{m} v'_{hh'i} x_{hh'ik}} \le \theta, \tag{7}$$

$$\frac{\sum_{l=1}^{S}\sum_{l'=1}^{S'}\sum_{r=1}^{s} u'_{ll'r} y_{ll'rj}}{\sum_{h=1}^{M}\sum_{h'=1}^{M'}\sum_{i=1}^{m} v'_{hh'i} x_{hh'ij}} \le E_j^* \quad \forall j, \tag{8}$$

$$\sum_{l'=1}^{S'}\sum_{r=1}^{s} u'_{ll'r} = p_l \quad \forall l, \tag{9}$$

$$\sum_{h'=1}^{M'}\sum_{i=1}^{m} v'_{hh'i} = q_h \quad \forall h, \tag{10}$$

$$\sum_{r=1}^{s} u'_{ll'r} = p'_{ll'} \quad \forall l, l', \tag{11}$$

$$\sum_{i=1}^{m} v'_{hh'i} = q'_{hh'} \quad \forall h, h', \tag{12}$$

$$u'_{ll'r}, p'_{ll'}, p_l, v'_{hh'i}, q'_{hh'}, q_h > 0 \quad \forall l, l', r, h, h', i, \tag{13}$$

$$0 \le \theta \le 1. \tag{14}$$

The combination of (6)-(14) forms a three-level DEA model that identifies the minimum efficiency loss, $\theta = \theta_{\min}$, needed to arrive at an optimal set of weights. Constraint (7) ensures that each DMU loses no more than θ of its best attainable efficiency value, E_k^*. The second set of constraints (8) satisfies that the efficiency values of all DMUs are less than or equal to their upper bound of E_j^*. The sets of constraints (9) to (12) imply that the sum of weights under each (sub-) sub-category equals to the weight of that (sub-) sub-category. It should be noted that the original (or internal) weights of outputs and inputs used for calculating the weighted averages are obtained as $u_{ll'r} = u'_{ll'r} / p'_{ll'}$ and $v_{hh'i} = v'_{hh'r} / q'_{hh'}$ while $p_{ll'} = p'_{ll'} / p_l$ and $q_{hh'} = q'_{hh'} / q_l$, respec-tively.

2.3 Prioritizing Weights Using AHP

The three-level DEA model identifies the minimum efficiency loss $\theta_{\min}$ needed to arrive at a set of weights of inputs and outputs by the internal mechanism of DEA. On the other hand, the priority weights of inputs and outputs, and the corresponding (sub-) categories are defined out of the internal mechanism of DEA by AHP.

In order to more clearly demonstrate how AHP is integrated into the three- level DEA model, this research presents an analytical process in which output weights are bounded by the AHP method. All the description on imposing weight bounds for output weights may be easily extendable to input weights. The AHP procedure for imposing weight bounds may be broken down into the following steps:

Step 1: A decision maker makes a pairwise comparison matrix of different criteria, denoted by A, with the entries of a_{lo} $(l = o = 1, 2, \cdots, S)$. The comparative importance of criteria is provided by the decision maker using a rating scale. Saaty (1980) recommends using a 1-9 scale.

Step 2: The AHP method obtains the priority weights of criteria by computing the eigenvector of matrix A (Eq. 15), $w = (w_1, w_2, \cdots, w_S)^T$, which is related to the largest eigenvalue, $\lambda_{\max}$.

$$Aw = \lambda_{\max} w. \tag{15}$$

To determine whether or not the inconsistency in a comparison matrix is reasonable the random consistency ratio, *C.R.*, can be computed by the following equation:

$$C.R. = \frac{\lambda_{\max} - N}{(N-1)R.I.} \tag{16}$$

where *R.I.* is the average random consistency index and N is the size of a comparison matrix. In a similar way, the priority weights of (sub-) sub-criteria under each (sub-) criterion can be computed. To obtain the weight bounds for output weights in the three-level DEA model, this study aggregates the priority weights of three different levels in AHP as follows:

$$\bar{u}_{ll'r} = w_l e_{ll'} f_{ll'r}, \quad \sum_{l=1}^{S} w_l = 1, \quad \sum_{l'=1}^{S'} e_{ll'} = 1 \text{ and } \sum_{r=1}^{s} f_{ll'r} = 1 \tag{17}$$

where w_l is the priority weight of criterion $l\,(l=1,\cdots,S)$ in AHP, $e_{ll'}$ is the priority weight of sub-criterion $l'(l'=1,2,\cdots,S')$ under criterion l and $f_{ll'r}$ is sub-sub-criterion $r\,(r=1,\cdots,s)$ under sub-criterion l'. Similarly, the weight bounds for input weights in the three-level DEA model, using AHP, is obtained as:

$$\bar{v}_{hh'i}=w'_h\,e'_{hh'}\,f'_{hh'i},\ \sum_{h=1}^{M}w'_h=1,\ \sum_{h'=1}^{M'}e'_{hh'}=1 \text{ and } \sum_{i=1}^{m}f'_{hh'i}=1 \quad (18)$$

where w'_h is the priority weight of criterion $h\,(h=1,2,\cdots,M)$ in AHP, $e'_{hh'}$ is the priority weight of sub-criterion $h'(h'=1,2,\cdots,M')$ under criterion h and $f'_{hh'i}$ is sub-sub-criterion $i\,(i=1,2,\cdots,m)$ under sub-criterion h'.

In order to estimate the maximum efficiency loss $\theta_{\max}$ necessary to achieve the priority weights of inputs and outputs for each DMU the following sets of constraints are added to the three-level DEA model:

$$u'_{ll'r}=\alpha\bar{u}_{ll'r}\ \ \forall l,\ l',\ r,\ \text{while } \alpha>0. \quad (19)$$

$$v'_{hh'i}=\beta\bar{v}_{hh'i}\ \ \forall h,h',i,\ \text{while } \beta>0. \quad (20)$$

The two sets of constraints (19)and (20) change the AHP computed weights to weights for the new system by means of two scaling factors α and β. The scaling factors α and β are added to avoid the possibility of contradicting constraints leading to infeasibility or underestimating the relative efficiencies of DMUs (Podinovski, 2004).

2.4 A Parametric Distance Model

We can now develop a parametric distance model for various discrete values of parameter θ such that $\theta_{\min} \le \theta \le \theta_{\max}$. Let $u'_{ll'r}(\theta)$ and $v'_{hh'i}(\theta)$ be the weights of outputs and inputs for a given value of parameter θ, where outputs are under sub-category $l'(l'=1,2,\cdots,S')$ of category $l\,(l=1,2,\cdots,S)$ and inputs are under sub-category $h'(h'=1,2,\cdots,M')$ of category $h\,(h=1,2,\cdots,M)$. Let $u'^{*}_{ll'r}$ and $v'^{*}_{hh'i}$ be the priority weights of outputs and inputs obtained from the three-level DEA model after adding (19) and (20). Our objective is to minimize the total deviations of $u'_{ll'r}(\theta)$ and $v'_{hh'i}(\theta)$ from their priority weights, $u'^{*}_{ll'r}$ and $v'^{*}_{hh'i}$, with the shortest Euclidian distance measure subject to the constraints (7) to (13):

Min

$$Z_k(\theta)=\left(\sum_{l=1}^{S}\sum_{l'=1}^{S'}\sum_{r=1}^{s}(u'_{ll'r}-u'^{*}_{ll'r})^2+\sum_{h=1}^{M}\sum_{h'=1}^{M'}\sum_{i=1}^{m}(v'_{hh'i}-v'^{*}_{hh'i})^2\right)^{1/2} \quad (21)$$

s.t. Constraints (7) to (13).

Because the range of deviations computed by the objective function is different for each DMU, it is necessary to normalize it by using relative deviations rather than absolute ones. Hence, the normalized deviations can be computed by:

$$\Delta_k(\theta)=\frac{Z_k^*(\theta_{\min})-Z_k^*(\theta)}{Z_k^*(\theta_{\min})}, \quad (22)$$

where $Z_k^*(\theta)$ is the optimal value of the objective function for $\theta_{\min} \le \theta \le \theta_{\max}$. We define $\Delta_k(\theta)$ as a *measure of closeness* which represents the relative closeness of each DMU to its priority weight in the range [0, 1]. Increasing the parameter (θ), we improve the deviations between the two systems of weights obtained from the three-level DEA model before and after adding the two sets of constraints (19) and (20). This may lead to different ranking positions for each DMU in comparison to the other DMUs. It should be noted that in a special case where the parameter $\theta=\theta_{\max}=0$, we assume $\Delta_k(\theta)=1$.

3. A NUMERICAL EXAMPLE: ROAD SAFETY PERFORMANCE

In this section we present the application of the proposed approach to assess the road safety performance of a set of European countries (or DMUs).The data on 13 road safety performance indicators (SPIs) in terms of road user behavior (inputs) and 4 safety outcomes (outputs) for 19 European countries have been adopted from Shen *et al*. (2011). The resulting normalized data based on (1) and (2) are presented in Table 1.The notations in Table 1 are as follows: AT = Austria, BE = Belgium, CZ = Czech Republic, DK = Denmark, FR = France, DE = Germany, EL = Greece, HU = Hungary, IE = Ireland, LV = Latvia, LU = Luxembourg, NL = Netherlands, PL = Poland, PT = Portugal, SI = Slovenia, ES = Spain, SE = Sweden, CH = Switzerland, UK = United Kingdom.

Since in DEA-based road safety models, the most efficient countries are those with minimum output levels given input levels, we treat all the outputs (inputs) as inputs (outputs) (Shen *et al*., 2012). Tables 2 and 3 depict the priority weights of safety outcomes and SPIs as constructed by the author in Expert Choice software. One can argue that the priority weights of SPIs must be judged by road safety experts. However, since the aim of this section is just to show the application of the proposed approach on numerical data, we see no problem to use our judgment alone.

Solving the three-level DEA model for the country under assessment, we obtain an optimal set of weights with minimum efficiency loss ($\theta_{\min}$). It should be noted that the efficiency value of all countries calculated from the three-level DEA model is identical to that calculated from the CCR-DEA model. Therefore, the minimum efficiency loss for the country under assessment is $\theta_{\min}=0$ (Table 4). This implies that the measure of relative closeness to the AHP weights for the country under assessment is $\Delta_k(\theta_{\min})=0$. On the other hand, solving the three-level DEA model for the country under assessment after adding the two sets of constraints (19) and (20), we adjust the priority weights of SPIs (outputs) and safety

Table 1. Normalized data on SPIs (outputs) and safety outcomes (inputs) for 19 european countries

Countries*	Outputs								
	Alcohol	Speed						Protective systems	
	% of Fatalities	Mean speed			% of Speed limit violation			Non-use of seat belt	
		On urban roads	On rural roads	On motor-ways	On urban roads	On rural roads	On motor-ways	In front seats	In rear seats
AT	0.107	0.965	0.810	0.911	0.913	0.387	0.446	0.275	0.680
BE	0.181	0.948	0.888	0.931	0.816	0.649	0.604	0.550	0.800
CZ	0.059	0.835	0.681	0.822	0.495	0.346	0.696	0.275	0.787
DK	0.276	0.857	0.760	0.927	0.643	0.486	0.602	0.375	0.493
FR	0.472	0.835	0.793	0.941	0.585	0.435	0.733	0.050	0.240
DE	0.197	0.849	0.872	0.935	0.689	0.448	0.575	0.125	0.160
EL	0.150	0.803	0.838	0.949	0.668	0.511	0.585	1.000	1.000
HU	0.150	0.983	0.698	0.954	0.934	0.188	0.588	0.825	0.880
IE	0.309	0.904	0.845	0.901	0.709	0.493	0.268	0.350	0.720
LV	0.375	0.783	0.828	0.887	0.690	0.457	0.611	0.575	0.907
LU	0.179	0.892	0.795	0.911	0.675	0.487	0.089	0.500	0.533
NL	0.432	0.870	0.857	0.950	0.770	0.494	0.723	0.250	0.480
PL	0.149	1.000	0.819	0.826	1.000	0.672	0.608	0.650	0.733
PT	0.176	0.783	1.000	1.000	0.522	1.000	0.964	0.350	0.733
SI	1.000	0.913	0.809	0.949	0.654	0.532	0.543	0.450	0.680
ES	0.354	0.866	0.831	0.976	0.702	0.479	0.561	0.275	0.413
SE	0.278	0.943	0.853	0.990	0.706	0.487	0.607	0.100	0.267
CH	0.333	0.748	0.810	0.921	0.247	0.333	0.518	0.450	0.627
UK	0.294	0.826	0.776	1.000	0.515	0.378	1.000	0.225	0.213

Table 1. Continued

Country	Outputs				Inputs			
	Protective systems				Casualties			Crashes
	% of non-use of Child restraint	% of Non-use of helmet			No of fatalities	No of serious injuries	No of soft injuries	No of crashes
		Cyclists	Moped riders	Motor-cyclists				
AT	0.310	0.849	0.450	0.522	0.451	1.000	0.651	0.551
BE	0.362	0.880	0.419	0.390	0.554	0.382	0.641	0.454
CZ	1.000	0.871	0.328	0.496	0.641	0.246	0.327	0.249
DK	0.429	1.000	0.308	0.593	0.402	0.377	0.086	0.113
FR	0.190	0.872	0.038	0.800	0.408	0.410	0.140	0.147
DE	0.276	0.978	0.654	1.000	0.326	0.601	0.578	0.455
EL	0.398	0.851	0.373	0.618	0.783	0.107	0.214	0.155
HU	0.351	0.877	0.337	0.382	0.668	0.531	0.256	0.229
IE	0.413	0.873	0.392	0.488	0.457	0.136	0.228	0.154
LV	0.370	0.888	1.000	0.600	1.000	0.183	0.320	0.234
LU	0.741	0.883	0.416	0.446	0.522	0.391	0.289	0.222
NL	0.483	0.874	0.269	0.411	0.234	0.387	0.169	0.176
PL	0.241	0.883	0.320	0.000	0.793	0.276	0.165	0.145
PT	0.380	0.887	0.355	0.307	0.500	0.185	0.544	0.371
SI	0.367	0.870	0.433	0.601	0.788	0.421	1.000	0.644
ES	0.347	0.870	0.269	0.200	0.462	0.282	0.367	0.250
SE	0.086	0.785	0.385	0.200	0.277	0.274	0.335	0.226
CH	0.259	0.710	0.231	0.200	0.370	0.480	0.411	1.000
UK	0.121	0.875	0.412	0.450	0.272	0.310	0.493	0.344

Table 2. The AHP hierarchical model for road safety outcomes (inputs)

Objective level	Criteria level	Sub-criteria level	Sub sub-criteria level
Prioritizing road safety outcomes (inputs)	Injuries $w'_1 = 0.75$	Fatalities $e'_{11} = 0.667$	Fatalities $f'_{111} = 1.00$
		Serious injuries $e'_{12} = 0.222$	Serious injuries $f'_{121} = 1.00$
		Slight injuries $e'_{13} = 0.111$	Slight injuries $f'_{131} = 1.00$
	Crashes $w'_2 = 0.25$	Crashes $e'_{21} = 1.00$	Crashes $f'_{211} = 1.00$

Table 3. The AHP hierarchical model for SPIs (outputs)

Objective level	Criteria level	Sub-criteria level	Sub sub-criteria level
Prioritizing road user behavior (outputs)	Alcohol $w_1 = 0.2667$	Fatalities involving at least one driver impaired by alcohol $e_{11} = 1.000$	Fatalities involving at least one driver impaired by alcohol $f_{111} = 1.000$
	Speed $w_2 = 0.40$	Mean speed $e_{21} = 0.40$	Mean speed of vehicles on urban roads $f_{211} = 0.333$
			Mean speed of vehicles on rural roads, $f_{212} = 0.267$
			Mean speed of vehicles on motorways $f_{213} = 0.40$
		Speed limit violations $e_{22} = 0.60$	% of vehicles exceeding the speed limit on urban roads $f_{221} = 0.267$
			% of vehicles exceeding the speed limit on rural roads $f_{222} = 0.333$
			% of vehicles exceeding the speed limit on motorways $f_{223} = 0.40$
	Protective systems $w_3 = 0.333$	Seat belt $e_{31} = 0.333$	Seatbelt in front seats $f_{311} = 0.60$
			Seatbelt in rear seats $f_{312} = 0.40$
		Child restraints $e_{32} = 0.267$	Child restraints $f_{321} = 1.000$
		Helmet $e_{33} = 0.40$	Helmet by cyclists $f_{331} = 0.40$
			Helmet by moped riders $f_{332} = 0.333$
			Helmet by motorcyclists $f_{333} = 0.267$

outcomes (inputs) obtained from AHP in such a way that they become compatible with the weights' structure in the three level DEA model. This results in the maximum efficiency loss, $\theta_{\max}$, for the country under assessment (Table 4). In addition, this implies that the measure of relative closeness to the AHP weights for the country under assessment is $\Delta_k(\theta_{\max}) = 1$.

Table 5 presents the optimal weights of SPIs and safety outcomes as well as the scaling factors for the best performing country, the Netherlands. It should be

Table 4. Minimum and maximum efficiency losses for each country

Countries	E_k^*	$\theta_{\min}$	$\theta_{\max}$
AT	0.8022	0.000	0.4684
BE	0.9270	0.000	0.4792
CZ	1	0.000	0.5455
DK	1	0.000	0.2686
FR	1	0.000	0.3272
DE	1	0.000	0.5198
EL	1	0.000	0.5057
HU	1	0.000	0.5690
IE	1	0.000	0.3098
LV	1	0.000	0.5930
LU	1	0.000	0.5014
NL	1	0.000	0.0000
PL	1	0.000	0.5612
PT	1	0.000	0.4346
SI	1	0.000	0.5697
ES	0.8584	0.000	0.2838
SE	1	0.000	0.2836
CH	1	0.000	0.6561
UK	1	0.000	0.3405

Table 5. Optimal weights of SPIs and safety outcomes for the Netherlands obtained from the three-level DEA model bounded by AHP

Weights of categories	Weights of sub-categories	Weights of sub sub-categories
$q_1 = 3.1307$	$q'_{11} = 2.0882$	$v'_{111} = 2.0882$
	$q'_{12} = 0.6950$	$v'_{121} = 0.6950$
	$q'_{13} = 0.3475$	$v'_{131} = 0.3475$
$q_2 = 1.0436$	$q'_{21} = 1.0436$	$v'_{211} = 1.0436$
$p_1 = 0.4669$	$p'_{11} = 0.4669$	$u'_{111} = 0.4669$
$p_2 = 0.7003$	$p'_{21} = 0.2801$	$u'_{211} = 0.0933$
		$u'_{212} = 0.0748$
		$u'_{213} = 0.1120$
	$p'_{22} = 0.4202$	$u'_{221} = 0.1122$
		$u'_{222} = 0.1399$
		$u'_{223} = 0.1681$
$p_3 = 0.5830$	$p'_{31} = 0.1941$	$u'_{311} = 0.1165$
		$u'_{312} = 0.0777$
	$p'_{32} = 0.1557$	$u'_{321} = 0.1557$
	$p'_{33} = 0.2332$	$u'_{331} = 0.0933$
		$u'_{332} = 0.0777$
		$u'_{333} = 0.0623$
$\alpha = 1.7507$ and $\beta = 4.1743$		

noted that the priority weights of AHP used for incorporating weight bounds on the weights of safety outcomes and SPIs are obtained as $\bar{v}_{hh'i} = \frac{v'_{hh'i}}{\beta}$ and $\bar{u}_{ll'r} = \frac{u'_{ll'r}}{\alpha}$, respectively. In addition, the priority weights of safety outcomes in the AHP model can be obtained as follows:

$w'_h = \frac{q_h}{\beta}$ while $\sum_{h'=1}^{M'}\sum_{i=1}^{m} v'_{hh'i} = q_h$ and $\sum_{i=1}^{m} v'_{hh'i} = q'_{hh'}$ for criteria level,

$e'_{hh'} = q'_{hh'}/q_h$ for sub-criteria level,

$f'_{hh'i} = v'_{hh'i}/q'_{hh'}$ for sub-sub-criteria levels.

Similarly, the priority weights of SPIs in the AHP model can be obtained as follows:

$w_l = \frac{p_l}{\alpha}$ while $\sum_{l'=1}^{S'}\sum_{r=1}^{s} u'_{ll'r} = p_l$ and $\sum_{r=1}^{s} u'_{ll'r} = p'_{ll'}$ for criteria level,

$e_{ll'} = p'_{ll'}/p_l$ for sub-criteria level,

$f_{ll'r} = u'_{ll'r}/p'_{ll'}$ for sub-sub-criteria levels.

Going one step further to the solution process of the parametric distance model (21), we proceed to the estimation of total deviations from the AHP weights for each country while the parameter θ is $0 \le \theta \le \theta_{\max}$. Table 6 represents the ranking position of each country based on the minimum deviation from the priority weights of SPIs and safety outcomes for $\theta = 0$. It should be noted that in a special case where the parameter $\theta = \theta_{\max} = 0$ we assume $\Delta_k(\theta) = 1$.

Table 6 shows that the Netherlands is the best performer in terms of the efficiency value and its relative closeness to the priority weights of SPIs and safety outcomes. Nevertheless, increasing the value of θ from 0 to $\theta_{\max}$ has two main effects on the performance of the other countries: improving the degree of deviations and reducing the value of efficiency. This, of course, is a phenomenon, one expects to observe frequently. The

Table 6. The ranking position of each country based on the minimum distance to priority weights of SPIs and safety outcomes

Countries	$Z^*(\eta)$	Rank
AT	0.6811	13
BE	0.7685	16
CZ	0.6024	10
DK	0.6663	12
FR	0.5613	9
DE	0.5239	7
EL	0.4294	2
HU	0.8174	18
IE	0.5366	8
LV	0.5048	4
LU	0.7871	17
NL	0.0000	1
PL	0.6998	14
PT	0.4907	3
SI	0.5225	6
ES	1.3648	19
SE	0.6111	11
CH	0.7225	15
UK	0.5156	5

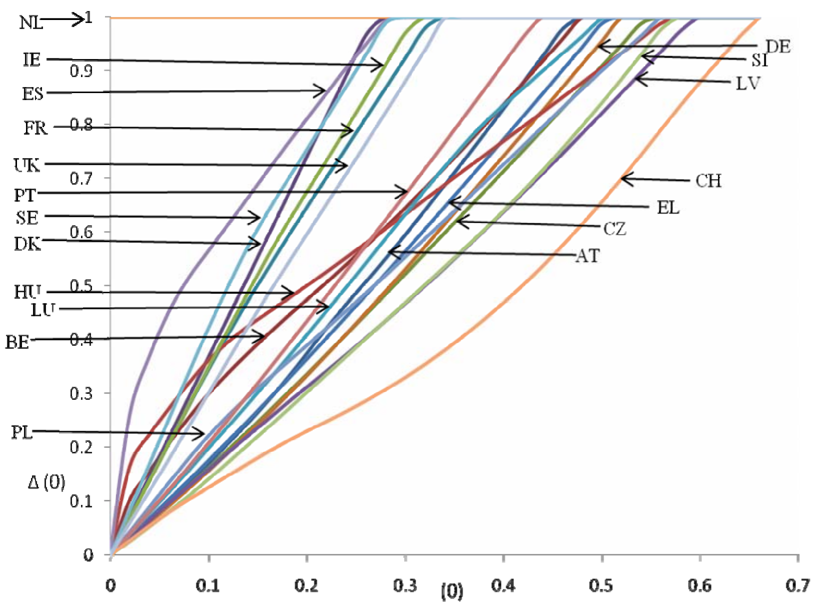

Figure 1. The relative closeness to the priority weights of SPIs and safety outcomes [Δ (θ)], versus efficiency loss (θ) for each country.

graph of $\Delta(\theta)$ versus θ, as shown in Figure 1, is used to describe the relation between the relative closeness to the priority weights of SPIs and safety outcomes, versus efficiency loss for each country. This may result in different ranking positions for each country in comparison to the other countries (Appendix A).

4. CONCLUSION

We develop an integrated approach based on DEA and AHP methodologies for hierarchical structures of inputs and outputs. We define two sets of weights of inputs and outputs in a three-level DEA framework. The first set represents the weights of inputs and outputs with minimum efficiency loss. The second set represents the corresponding priority weights of hierarchical inputs and outputs, using AHP, with maximum efficiency loss. We assess the performance of each DMU in comparison to the other DMUs based on the relative closeness of the first set of weights to the second set of weights. Improving the measure of relative closeness in a defined range of efficiency loss, we explore the various ranking positions for the DMU under assessment in comparison to the other DMUs. To demonstrate the effectiveness of the proposed approach, an illustrative example of road safety performance of a set of 19 European countries is carried out.

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Appendix A. The measure of relative closeness to the priority weights of hierarchical SPIs and safety outcomes $[\Delta_k(\theta)]$ vs. composite loss [θ] for each country*

θ	AT	BE	CZ	DK	FR	DE	EL	HU	IE	LV
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Rank	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.02	0.0341	0.1029	0.0315	0.0727	0.0692	0.0302	0.0341	0.1654	0.0686	0.0323
Rank	14	4	16	6	7	17	13	3	8	15
0.04	0.0987	0.2085	0.0957	0.2202	0.2084	0.0920	0.1038	0.2731	0.2082	0.0967
Rank	14	6	16	5	7	17	13	3	8	15
0.06	0.1335	0.2562	0.1286	0.2941	0.2774	0.1240	0.1395	0.3198	0.2787	0.1283
Rank	14	8	15	5	7	17	13	4	6	16
0.08	0.1701	0.3000	0.1621	0.3684	0.3451	0.1569	0.1758	0.3621	0.3492	0.1595
Rank	14	9	15	4	7	17	13	5	6	16
0.1	0.2079	0.3392	0.1962	0.4427	0.4104	0.1907	0.2125	0.3961	0.4193	0.1906
Rank	14	9	15	4	6	16	13	7	5	17
0.12	0.2461	0.3752	0.2307	0.5168	0.4727	0.2254	0.2497	0.4249	0.4880	0.2216
Rank	14	9	15	4	6	16	13	7	5	17
0.14	0.2861	0.4100	0.2656	0.5914	0.5319	0.2610	0.2875	0.4501	0.5545	0.2523
Rank	14	9	15	4	6	16	13	8	5	17
0.16	0.3292	0.4432	0.3011	0.6665	0.5898	0.2973	0.3257	0.4753	0.6167	0.2827
Rank	13	9	15	4	6	16	14	8	5	17
0.18	0.3753	0.4751	0.3371	0.7418	0.6468	0.3345	0.3644	0.5007	0.6753	0.3128
Rank	13	9	15	4	6	16	14	8	5	17
0.2	0.4224	0.5073	0.3736	0.8172	0.7024	0.3725	0.4035	0.5265	0.7341	0.3424
Rank	12	9	15	3	6	16	14	8	5	17
0.22	0.4668	0.5403	0.4105	0.8926	0.7577	0.4113	0.4431	0.5527	0.7931	0.3727
Rank	12	9	16	3	6	15	14	8	5	17
0.24	0.5089	0.5743	0.4478	0.9677	0.8133	0.4509	0.4831	0.5793	0.8523	0.4036
Rank	12	10	16	2	6	15	14	8	5	17
0.26	0.5514	0.6094	0.4855	1.0000	0.8688	0.4912	0.5235	0.6061	0.9116	0.4352
Rank	12	9	16	2	6	15	13	10	5	18
0.28	0.5956	0.6454	0.5233	1.0000	0.9244	0.5322	0.5643	0.6332	0.9710	0.4675
Rank	12	9	16	1	6	15	13	11	5	18
0.3	0.6414	0.6824	0.5613	1.0000	0.9800	0.5738	0.6054	0.6605	1.0000	0.5004
Rank	12	9	16	1	6	15	13	11	1	18
0.32	0.6881	0.7203	0.5994	1.0000	1.0000	0.6158	0.6469	0.6879	1.0000	0.5339
Rank	11	10	16	1	6	15	13	12	1	18
0.34	0.7355	0.7589	0.6375	1.0000	1.0000	0.6582	0.6886	0.7152	1.0000	0.5680
Rank	11	10	16	1	1	14	13	12	1	18
0.36	0.7828	0.7983	0.6759	1.0000	1.0000	0.7008	0.7307	0.7424	1.0000	0.6028
Rank	11	10	16	1	1	14	13	12	1	18

Appendix A. Continued

θ	AT	BE	CZ	DK	FR	DE	EL	HU	IE	LV
0.38	0.8308	0.8382	0.7145	1.0000	1.0000	0.7433	0.7730	0.7694	1.0000	0.6380
Rank	11	9	16	1	1	14	12	13	1	18
0.4	0.8798	0.8786	0.7533	1.0000	1.0000	0.7856	0.8156	0.7964	1.0000	0.6738
Rank	9	10	16	1	1	14	12	13	1	18
0.42	0.9293	0.9195	0.7924	1.0000	1.0000	0.8281	0.8583	0.8236	1.0000	0.7101
Rank	9	10	16	1	1	13	12	14	1	18
0.44	0.9792	0.9605	0.8316	1.0000	1.0000	0.8710	0.9013	0.8508	1.0000	0.7469
Rank	9	10	16	1	1	13	12	14	1	18
0.46	1.0000	1.0000	0.8709	1.0000	1.0000	0.9140	0.9444	0.8782	1.0000	0.7841
Rank	9	10	15	1	1	13	12	14	1	18
0.48	1.0000	1.0000	0.9103	1.0000	1.0000	0.9572	0.9876	0.9056	1.0000	0.8216
Rank	1	1	14	1	1	13	12	15	1	18
0.5	1.0000	1.0000	0.9497	1.0000	1.0000	1.0000	1.0000	0.9330	1.0000	0.8595
Rank	1	1	14	1	1	13	1	15	1	18
0.52	1.0000	1.0000	0.9891	1.0000	1.0000	1.0000	1.0000	0.9604	1.0000	0.8977
Rank	1	1	14	1	1	1	1	16	1	18
0.54	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9877	1.0000	0.9362
Rank	1	1	1	1	1	1	1	16	1	18
0.56	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9748
Rank	1	1	1	1	1	1	1	1	1	18
0.58	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Rank	1	1	1	1	1	1	1	1	1	1
0.6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Rank	1	1	1	1	1	1	1	1	1	1
0.62	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Rank	1	1	1	1	1	1	1	1	1	1
0.64	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Rank	1	1	1	1	1	1	1	1	1	1
0.66	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Rank	1	1	1	1	1	1	1	1	1	1

Appendix A. Continued

θ	LU	NL	PL	PT	SI	ES	SE	CH	UK
0	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Rank	N/A	1	N/A	N/A	N/A	N/A	N/A	N/A	N/A
0.02	0.0391	1.0000	0.0443	0.0400	0.0261	0.2616	0.0840	0.0282	0.0605
Rank	12	1	10	11	19	2	5	18	9
0.04	0.1179	1.0000	0.1352	0.1223	0.0817	0.4534	0.2546	0.0802	0.1790
Rank	12	1	10	11	18	2	4	19	9
0.06	0.1581	1.0000	0.1797	0.1648	0.1103	0.5163	0.3394	0.1037	0.2398
Rank	12	1	10	11	18	2	3	19	9
0.08	0.1988	1.0000	0.2218	0.2081	0.1415	0.5658	0.4220	0.1261	0.3013
Rank	12	1	10	11	18	2	3	19	8
0.1	0.2399	1.0000	0.2600	0.2522	0.1727	0.6146	0.5002	0.1484	0.3630
Rank	12	1	10	11	18	2	3	19	8
0.12	0.2813	1.0000	0.2943	0.2971	0.2045	0.6641	0.5732	0.1703	0.4241
Rank	12	1	11	10	18	2	3	19	8
0.14	0.3234	1.0000	0.3266	0.3425	0.2369	0.7142	0.6377	0.1915	0.4835
Rank	12	1	11	10	18	2	3	19	7
0.16	0.3663	1.0000	0.3581	0.3885	0.2696	0.7637	0.6963	0.2118	0.5404
Rank	11	1	12	10	18	2	3	19	7
0.18	0.4099	1.0000	0.3896	0.4348	0.3028	0.8112	0.7547	0.2309	0.5966
Rank	11	1	12	10	18	2	3	19	7
0.2	0.4540	1.0000	0.4215	0.4816	0.3361	0.8587	0.8132	0.2492	0.6528
Rank	11	1	13	10	18	2	4	19	7
0.22	0.4985	1.0000	0.4539	0.5286	0.3692	0.9058	0.8719	0.2676	0.7091
Rank	11	1	13	10	18	2	4	19	7
0.24	0.5435	1.0000	0.4868	0.5758	0.4023	0.9501	0.9307	0.2872	0.7659
Rank	11	1	13	9	18	3	4	19	7
0.26	0.5887	1.0000	0.5202	0.6229	0.4355	0.9921	0.9895	0.3082	0.8234
Rank	11	1	14	8	17	3	4	19	7
0.28	0.6339	1.0000	0.5539	0.6703	0.4689	1.0000	1.0000	0.3308	0.8813
Rank	10	1	14	8	17	1	1	19	7
0.3	0.6787	1.0000	0.5879	0.7182	0.5024	1.0000	1.0000	0.3550	0.9397
Rank	10	1	14	8	17	1	1	19	7
0.32	0.7226	1.0000	0.6221	0.7666	0.5360	1.0000	1.0000	0.3810	0.9984
Rank	9	1	14	8	17	1	1	19	7
0.34	0.7646	1.0000	0.6565	0.8154	0.5693	1.0000	1.0000	0.4087	1.0000
Rank	9	1	15	8	17	1	1	19	7
0.36	0.8035	1.0000	0.6911	0.8645	0.6038	1.0000	1.0000	0.4383	1.0000
Rank	9	1	15	8	17	1	1	19	1

Appendix A. Continued

θ	LU	NL	PL	PT	SI	ES	SE	CH	UK
0.38	0.8371	1.0000	0.7260	0.9139	0.6397	1.0000	1.0000	0.4698	1.0000
Rank	10	1	15	8	17	1	1	19	1
0.4	0.8692	1.0000	0.7611	0.9636	0.6770	1.0000	1.0000	0.5032	1.0000
Rank	11	1	15	8	17	1	1	19	1
0.42	0.9014	1.0000	0.7964	1.0000	0.7158	1.0000	1.0000	0.5387	1.0000
Rank	11	1	15	1	17	1	1	19	1
0.44	0.9336	1.0000	0.8316	1.0000	0.7561	1.0000	1.0000	0.5763	1.0000
Rank	11	1	15	1	17	1	1	19	1
0.46	0.9657	1.0000	0.8665	1.0000	0.7977	1.0000	1.0000	0.6160	1.0000
Rank	11	1	16	1	17	1	1	19	1
0.48	0.9977	1.0000	0.9004	1.0000	0.8407	1.0000	1.0000	0.6575	1.0000
Rank	11	1	16	1	17	1	1	19	1
0.5	1.0000	1.0000	0.9329	1.0000	0.8850	1.0000	1.0000	0.7008	1.0000
Rank	1	1	16	1	17	1	1	19	1
0.52	1.0000	1.0000	0.9655	1.0000	0.9305	1.0000	1.0000	0.7457	1.0000
Rank	1	1	15	1	17	1	1	19	1
0.54	1.0000	1.0000	0.9981	1.0000	0.9770	1.0000	1.0000	0.7917	1.0000
Rank	1	1	15	1	17	1	1	19	1
0.56	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8375	1.0000
Rank	1	1	1	1	1	1	1	19	1
0.58	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8810	1.0000
Rank	1	1	1	1	1	1	1	19	1
0.6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9228	1.0000
Rank	1	1	1	1	1	1	1	19	1
0.62	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9653	1.0000
Rank	1	1	1	1	1	1	1	19	1
0.64	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000
Rank	1	1	1	1	1	1	1	19	1
0.66	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Rank	1	1	1	1	1	1	1	1	1

* AT = Austria, BE = Belgium, CZ = Czech Republic, DK = Denmark, FR = France, DE = Germany, EL = Greece, HU = Hungary, IE = Ireland, LV = Latvia, LU = Luxembourg, NL = Netherlands, PL = Poland, PT = Portugal, SI = Slovenia, ES = Spain, SE = Sweden, CH = Switzerland, UK = United Kingdom.