

Prediction of Possible Intercept Time by Considering Flight Trajectory of Nodong Missile

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Abstract

This paper presents research on predicting the possible intercept time for a Nodong missile based on its flight trajectory. North Korea possesses ballistic missiles of various ranges, and nuclear warhead miniaturization tests and ballistic missile launch tests conducted last year and in previous years have made these missiles into a serious security threat for the international community. With North Korea's current miniaturization skills, the range of the nuclear capable Nodong missiles can be adjusted according to their use goals and operating environment by using a variety of adjustment methods such as payload, fuel mass, Isp, loft angle, cut-off, etc., and therefore precise flight trajectory prediction is difficult. In this regards, this research performs model simulations of the flight trajectory of North Korea's domestically developed Nodong missiles and uses these as a basis for predicting the possible intercept times for major ballistic missile defense systems such as PAC-3, THAAD, and SM-3.

Key Words : Nodong Missile, Flight trajectory, Nuclear Warhead Miniaturization, Ballistic Missile Defense

1. Introduction

Ballistic missiles can have a variety of flight trajectories according to changes in input parameters. The trajectory of a ballistic missile is determined by burn-out time, thrust, and loft angle [10]. Input parameters such as payload, fuel mass, Isp, loft angle, and cut-off time can be adjusted according to use goals and operating environment in order to strike targets within a maximum range [2]. However, existing research has focused only on cut-off time adjustment methods or loft angle adjustment angles, and overall flight trajectory research has been inadequate [1-5]. In Japan, where the Nodong

missile is considered to be a major security threat, 3 methods have been used in their analysis, unlike in South Korea, and these include loft angle, payload, and Isp (Specific Impulse)[6-8]. With a payload of 1,000kg, the Nodong missile is a nuclear capable missile according to North Korea's current level of nuclear miniaturization [9]. Also, the Nodong missile has a range of 300~1300km, which puts the entire Korean peninsula within its range. With this in mind, this research has modeled the Nodong missile's flight characteristics for each flight phase and simulated its flight trajectory. Based on these simulations, it has produced possible intercept times for PAC-3, THAAD, and SM-3.

2. Derivation of simulation input parameters

Typical flight stage of a ballistic missile consist of boost (propulsion flight stage, including vertical

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launch and programmed rotation), mid-flight, and reentry stages. Nodong generates propulsion via burning of the propellant and once the boost stage is completed, propulsion is terminated and it stays in flight via kinetic energy only. Meanwhile, just like regular ballistic missiles, Nodong losses its mass as the propellant is burned up, and since gravitational acceleration and atmospheric density change according to changes in altitude, these factors must be reflected in the equation of motion. This type of Nodong motion can generally be expressed as a from of a differential equation, as shown in equation(1). Here, Earth was considered as having a completely spherical from that does not rotate and thrust was not considered, assuming the angle of attack was small. Moreover, the effective exhaust velocity of the propellant was assumed to be constant.

$$\vec{F} = m(t)\vec{g} + \vec{T} + \vec{F}_{drag}(\vec{v}) \quad (1)$$

Here, $m(t)\vec{g}$, \vec{T} , $\vec{F}_{drag}(\vec{v})$ represent gravity, thrust, and drag, respectively. Meanwhile, drag, $\vec{F}_{drag}(\vec{v})$, is a function of velocity, which can be expressed as shown in equation (2) [10].

$$\vec{F}_{drag} = \frac{1}{2} C_D \rho(h) A V^2 \quad (2)$$

Here, C_D , $\rho(h)$, A , and V represent drag coefficient, atmospheric density based on altitude, frontal cross-sectional area of the ballistic missile, and velocity of missile, respectively.

$$\rho(h) = \rho_0 \exp^{-P_y/7254} \quad (3)$$

In order to simulate the equation of motion for this type of ballistic missile, the ballistic missile was considered the mass point, and expressing its flight on xy planes, the position vector (\vec{P}) and velocity vector (\vec{v}) of the ballistic missile was expressed as follows:

$$\dot{v}_x = c \frac{dm}{m(t)} \cos\theta - \frac{\rho(P_x) C_D A V^2}{2m} \cos\theta \quad (4)$$

$$\dot{v}_y = c \frac{dm}{m(t)} \sin\theta - \frac{\rho(P_y) C_D A V^2}{2m} \sin\theta - g_0 \left(\frac{R_e}{R_e + P_y} \right)^2 \quad (5)$$

Here, V is equivalent to $\sqrt{v_x^2 + v_y^2}$. Moreover, the first term on the right side of equations (4) and (5) represents acceleration by thrust; second term, acceleration by drag; and third term, gravitational acceleration. Meanwhile, because a ballistic missile

does not generate thrust after the boost stage, in the equation of motion for the ballistic missile after the boost stage, the term for acceleration by thrust is deleted from equations (4) and (5). In addition to minimum energy, Nodong can be operated by controlling the various ranges according to mission and operational environments, as shown in Table 1. Various methods of control, such as controlling loft angle at burn-out, specific impulse(Isp), Payload, cutting off(fuel mass) are being applied [11].

Table 1 The way of adjusting the missile range

Categories	Methods
Loft Angle	Minimum Energy, Over-Lofted, depressed
Isp	Specific Impulse, Fuel & Oxidizing Agent
Payload	Reduce Warhead-Mass
Cutting-off (Fuel-Mass)	Control Burn-Out Time, Fuel-Mass

The flight trajectory of a ballistic missile is determined by its velocity, energy, and loft at burn-out [12]. In general, the method most often used for maximum range is achieved by minimum energy when controlling the loft angle, and increasing or decreasing the launch angle from this results in decreased range.

3. Flight Trajectory Simulation Results

A ballistic missile uses a rocket to achieve its initial velocity, and after burning its rocket motor, it goes into free flight. There are ballistic missiles which are guided after reentering the atmosphere, but the guidance path is a small part of the overall flight path [13]. For ballistic missile flight trajectory characteristics, we used a verified model which we developed using MATLAB [2, 14] and completed it by adding an algorithm for each flight phase, as shown in Fig. 1. In this model, the earth's curvature was assumed to be a perfect sphere which was fixed and did not rotate, and it was also assumed that there was no boost phase force and the loaded propellant was completely spent at burnt-out time. The input parameters in Table 2 were set considering the characteristics of the normal Nodong missile's separable warhead [6-8, 14-16].

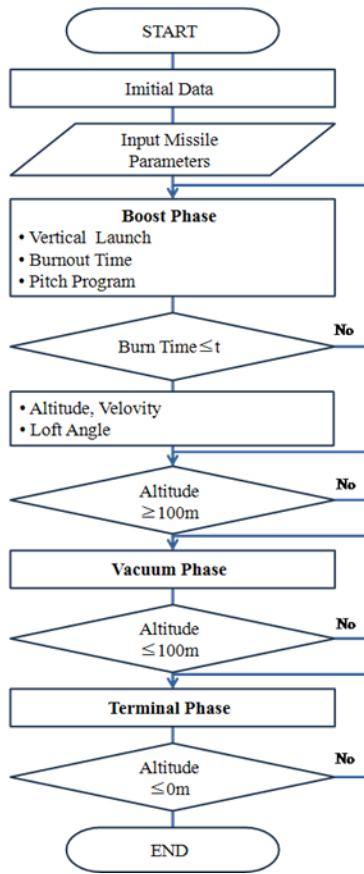


Fig. 1 Model Algorithm

Table 2 Nodong Input Parameter

Category		Parameter
1 st stage	stage diameter(m)	1.3
	max warhead diameter(m)	1.3
	payload mass(kg)	1,000
	fueled weight(kg)	16,000
	stage dry(kg)	4,000
	Isp(sec)	240
	burn time(sec)	70
2 nd stage	stage diameter(m)	1.3
	max warhead diameter(m)	1.3
	payload mass(kg)	1,000

Table 3 Defense System Input Parameter

Category	Intercept Altitude(km)	Maximum Range(km)
PAC-3	10-30	170
THAAD	40-150	400
SM-3	70-500	1,200

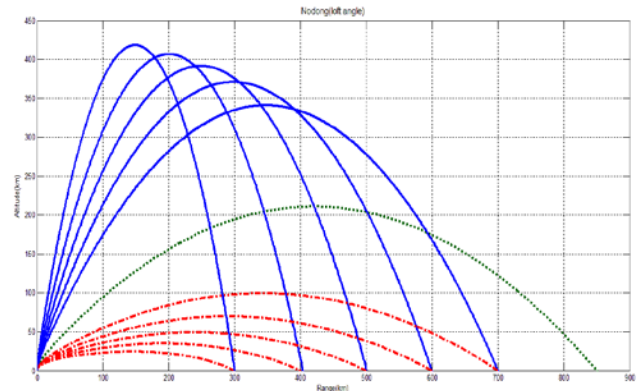


Fig. 2 Range-Altitude According to Loft Angle

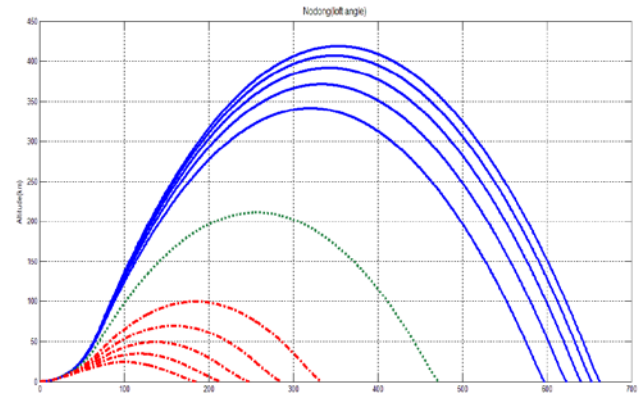


Fig. 3 Time-Altitude According to Loft Angle

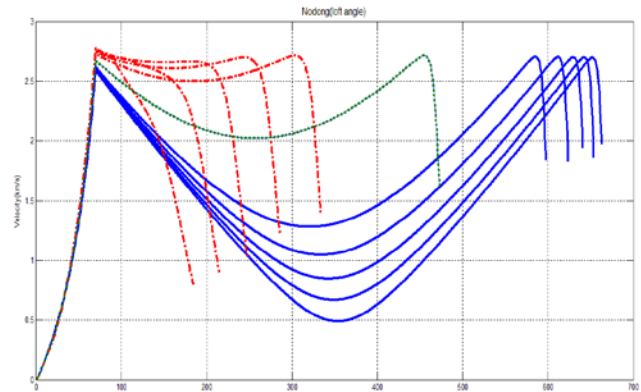


Fig. 4 Time-Velocity According to Loft Angle

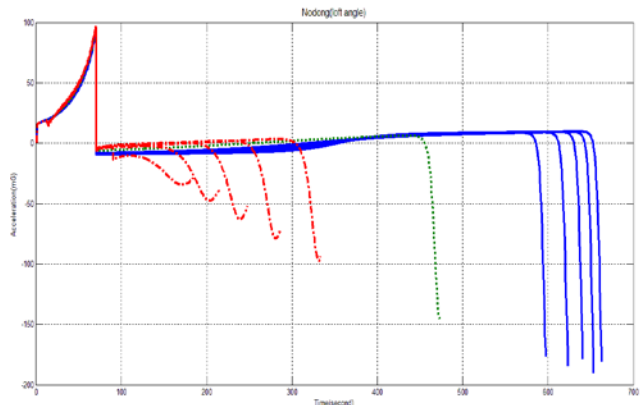


Fig. 5 Time-Acceleration According to Loft Angle

Table 4 Intercept Time According to Loft Angle

Category		Cutting Off(km)					M.E
		300	400	500	600	700	852
Intercept Time(s)	PAC-3	32	23	19	16	13	10
	THAAD	50	88	112	134	104	69
	SM-3	-	77	170	230	288	339

We selected PAC-3, THAAD, and SM-3 as the intercept missiles which respond to the ballistic missile, as they are currently the main intercept missiles used in regional defense. We used publically available data for this, as shown in Table 3.

Figures 2, 3, 4, and 5 show the ballistic missile flight characteristics which result from changes in loft angle. As shown in Fig. 2, if the maximum payload was assumed to be 1,000 kg, and the Isp (Specific Impulse) of TM-185 (20% gasoline, 80% kerosene) using AK271 oxidizer was assumed to be 240 seconds, the minimum energy trajectory range was 852km, and the apogee was 211km. This is within the maximum range to apogee ratio of 20 to 25%, an observed value for normal ballistic missiles [17]. If the missile was over-lofted, the apogee increased, but the range decreased, and if it was depressed, the apogee and range both decreased. In Fig. 2, the apogee for a 500km range, which can strike Seoul, is 49km for depressed and 391km for over-lofted, and as the distance is reduced, the difference in the two launch methods becomes more distinct. Table 4 shows the defense system intercept times based on flight trajectory simulation results. Here, the possible intercept time is defined as the time during which the ballistic missile flies within the intercept missile's possible intercept altitude. In Table 4, only PAC-3 could intercept a depressed trajectory under 400km, and as the range increased, the possible intercept time of the PAC-3 was reduced, but the possible intercept times of the THAAD and SM-3 increased. On the other hand, in the case of an over-lofted trajectory, the possible intercept times of low level defense systems such as PAC-3 and THAAD were reduced as the range was reduced, while the times for the SM-3 increased. Figs. 4 and 5 show velocity and acceleration characteristics. From launch until burn-out time, velocity and acceleration were affected by thrust and increased greatly, then in the ascent phase after burn-out time until apogee, velocity and acceleration were reduced due to the

effects of gravity. From the apogee to the impact, the missile descended, and in the terminal phase, the velocity increased again until it achieved maximum velocity. After this, the velocity was rapidly reduced by atmospheric resistance, and the size of the acceleration increased again. In Fig. 4, the Time-Velocity graph shows a typical M shape, and the missile's maximum velocity was 2.7km/s (mach 8) and did not show much difference according to its course, but in an over-lofted trajectory, its minimum velocity for the mid-course phase was 0.5km/s (mach 1.5), which is an advantageous environment for interception.

The acceleration characteristics in Fig. 5 show that the acceleration and variation in the terminal phase were fairly large, and atmospheric instability created a high possibility of unstable behavior in the reentry vehicle, which makes it difficult to precisely predict the flight trajectory. The cutting off method of range adjustment is used in Nodong missiles, which are used in a cluster as the 1st phase propellant of the Unha-3. As shown in Figs. 6 and 7, the altitude becomes lower as the range is reduced, similar to the depressed method. In the loft angle control method, the maximum velocity shows almost uniform characteristics regardless of range, but in the cutting off method, as the distance becomes shorter, the velocity and acceleration are reduced, as shown in Figs. 8 and 9. Table 5 shows that the possible intercept time for ranges under 500km was large for ground-based defense systems such as PAC-3 and THAAD, while above 500km, the intercept time of the sea-based SM-3 defense system increased greatly as the range grew longer.

The Isp range adjustment method is based on using fuels with different thrust performances. For 200 seconds, SCUD-B fuel TM-185 (20% gasoline, 80% kerosene) and AK-271 (27% N₂O₄ and 73% HNO₃) oxidizer are used. For 230 seconds, SCUD-C fuel is used, which is SCUD-B fuel expanded by 25%. The Isp of the Nodong missile, which uses the improved

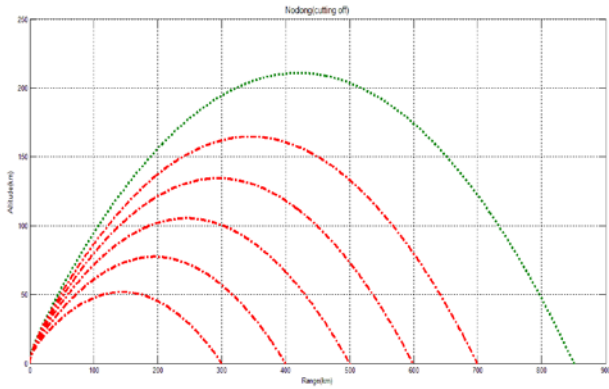


Fig. 6 Range-Altitude According to Cutting Off

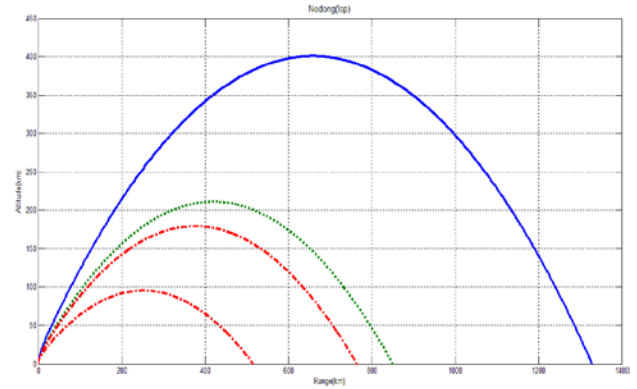


Fig. 10 Range-Altitude According to Isp

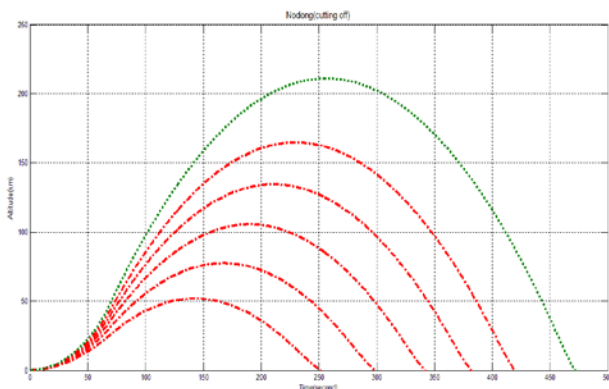


Fig. 7 Time-Altitude According to Cutting Off

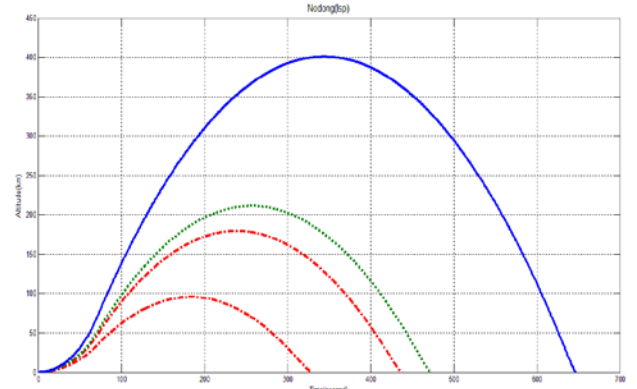


Fig. 11 Time-Altitude According to Isp

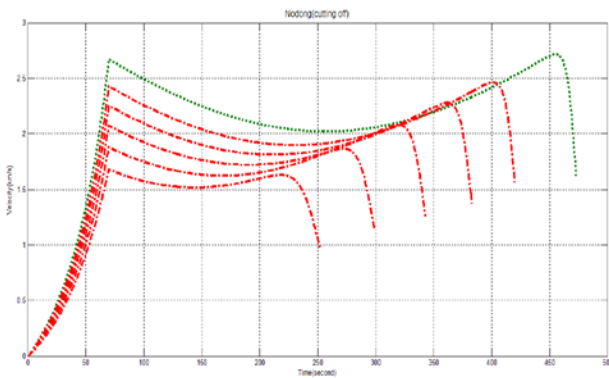


Fig. 8 Time-Velocity According to Cutting Off

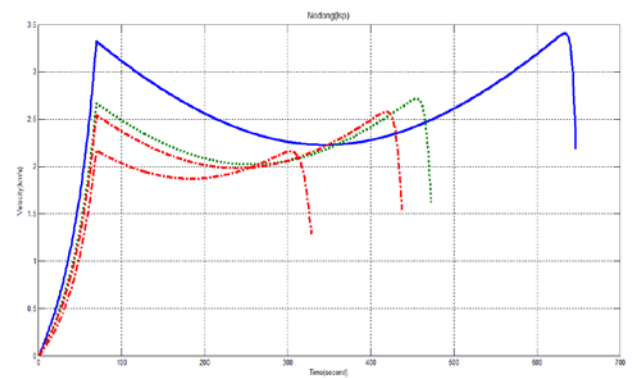


Fig. 12 Time-Velocity According to Isp

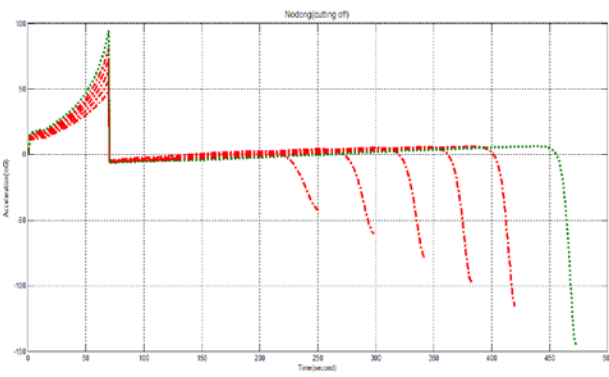


Fig. 9 Time-Acceleration According to Cutting Off

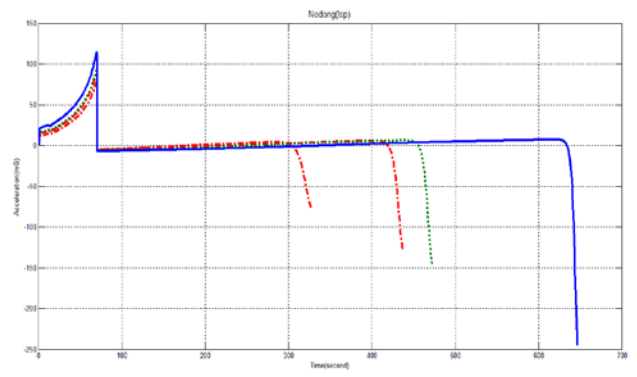


Fig. 13 Time-Acceleration According to Isp

Table 5 Intercept Time According to Isp

Category		Isp(sec)			
		200	230	240(M.E)	290
Intercept Time(s)	PAC-3	24	18	10	7
	THAAD	105	85	69	43
	SM-3	130	284	339	522

Table 6 Intercept Time According to Payload

Category		Payload(kg)				
		1,000(M.E)	800	600	400	200
Intercept Time(s)	PAC-3	10	9	9	8	8
	THAAD	69	66	64	62	60
	SM-3	339	362	382	404	420

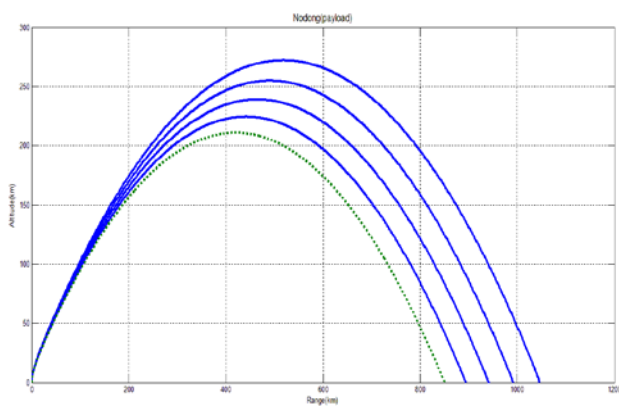


Fig. 14 Range-Altitude According to Payload

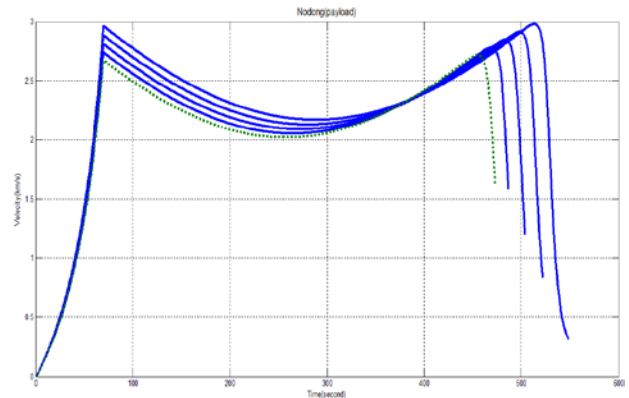


Fig. 16 Time-Velocity According to Payload

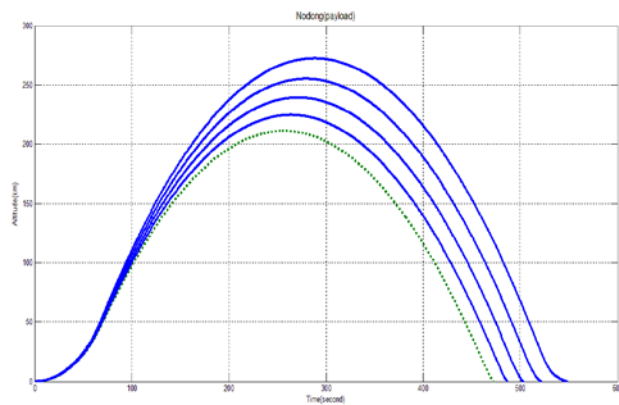


Fig. 15 Time-Altitude According to Payload

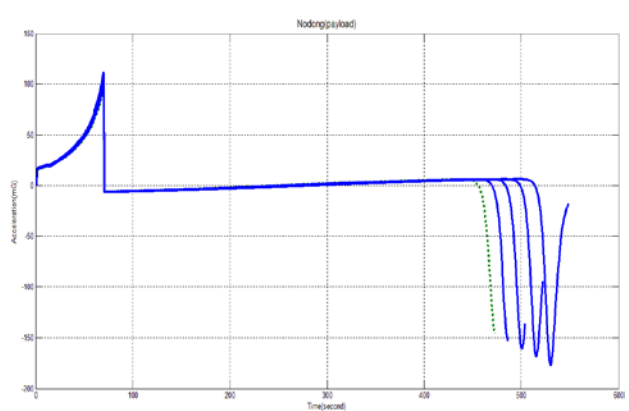


Fig. 17 Time-Acceleration According to Payload

AK-27P oxidizer, is 240 seconds. The Isp of the Musudan missile, which uses UDMH(Unsymmetrical Demethylhydrazine) fuel and IRFNA(Inhibited Red Fuming Nitric Acid), is 290 seconds. Figs. 10, 11, 12, and 13 show that as the Isp increased, the range increased, and the maximum velocity and acceleration also increased. As shown in Table 6, the possible intercept times of PAC-3 and THAAD were reduced as fuel with good performance was used, but the possible intercept time of the SM-3 increased greatly.

The payload range adjustment method is a method of adjusting range by adjusting the weight of the payload, and in reality, when North Korea developed the SCUD-C missile, it reduced the weight of the SCUD-B warhead to increase the range. Figs. 14, 15, 16, and 17 show that if the payload is reduced, the range is increased, and the maximum velocity and acceleration are also increased. As shown in Table 7, the possible intercept times of PAC-3 and THAAD were reduced as the payload was reduced, but the possible intercept time of the SM-3 increased greatly.

4. Conclusions

The Nodong missile has a variety of flight characteristics which change during the overall flight time according to the launch method, and during the terminal phase the changes are more rapid. In a minimum energy launch, the simulation showed a maximum of 852km, which is a maximum range shorter than the generally known range of 1,300 to 1,500km. Based on the simulation results, the Nodong missile's payload must be reduced and its Isp increased to make a 1,300 to 1,500km flight. As the loft angle was increased to make it over-lofted, the range was reduced, and the apogee and flight time were increased so that it had flight characteristics in which terminal phase acceleration was greatly increased. In a depressed scenario, as the loft angle was decreased, the range, apogee, and flight time were decreased, and it showed flight characteristics in which terminal phase acceleration was unstable. The significance of this research is that it shows the objective threat posed by the nuclear capable Nodong missile via simulation, which stands as the Korean peninsula's greatest threat. The limitation of this research is that it considered possible intercept time as the only mission effectiveness metric for defense systems that counter the Nodong missile's

various range adjustment methods. Future research must identify the capabilities needed by detection sensors with regards to the Nodong missile's various range adjustment methods.

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