J. Korean Math. Soc. **53** (2016), No. 2, pp. 305–313
http://dx.doi.org/10.4134/JKMS.2016.53.2.305

SYMMETRY OF COMPONENTS FOR RADIAL SOLUTIONS OF γ-LAPLACIAN SYSTEMS

Yun Wang

Abstract. In this paper, we give several sufficient conditions ensuring that any positive radial solution (u, v) of the following γ-Laplacian systems in the whole space $\mathbb{R}^n$ has the components symmetry property $u \equiv v$

$$\begin{cases} -\mathrm{div}(|\nabla u|^{\gamma-2}\nabla u) = f(u, v) & \text{in } \mathbb{R}^n, \\ -\mathrm{div}(|\nabla v|^{\gamma-2}\nabla v) = g(u, v) & \text{in } \mathbb{R}^n. \end{cases}$$

Here $n > \gamma$, $\gamma > 1$.

Thus, the systems will be reduced to a single γ-Laplacian equation:

$$-\mathrm{div}(|\nabla u|^{\gamma-2}\nabla u) = f(u) \quad \text{in } \mathbb{R}^n.$$

Our proofs are based on suitable comparation principle arguments, combined with properties of radial solutions.

1. Introduction

In 2008, Li and Ma [10] studied the stationary Schrödinger system

$$\begin{cases} -\Delta u = u^p v^q & \text{in } \mathbb{R}^n, \\ -\Delta v = u^q v^p & \text{in } \mathbb{R}^n, \end{cases} \tag{1.1}$$

and obtained a components symmetry result:

Proposition 1.1. *Assume $n > \gamma$, $1 \le p, q \le \frac{n+2}{n-2}$ and $p + q = \frac{n+2}{n-2}$. Then any $(L^{\frac{2n}{n-2}}(\mathbb{R}^n))^2$-positive solution pair (u, v) to (1.1) is radial symmetric, and hence $u \equiv v = a(b^2 + |x - x_0|^2)^{(2-n)/2}$ with $a, b > 0$ and $x_0 \in \mathbb{R}^n$.*

The proof was achieved by the classification result in [4] and the method of moving planes based on the conformal invariant property. Afterwards, Lei and Li ([8]) studied the asymptotic radial symmetry and decay estimates of positive integrable solutions of

$$\begin{cases} -\mathrm{div}(|\nabla u|^{\gamma-2}\nabla u) = u^p v^q & \text{in } \mathbb{R}^n, \\ -\mathrm{div}(|\nabla v|^{\gamma-2}\nabla v) = v^p u^q & \text{in } \mathbb{R}^n, \end{cases} \tag{1.2}$$

Received November 30, 2014; Revised July 12, 2015.
2010 *Mathematics Subject Classification.* 35B08, 35J47, 35J6.
Key words and phrases. γ-Laplacian system, components symmetry property, radial solution.

where $n > \gamma$, $\gamma > 1$, and $p, q > 0$ and $p + q = \frac{n\gamma}{n-\gamma} - 1$.

In 2012, Quittner and Souplet studied the more general Laplacian systems (cf. [12])

$$\begin{cases} -\Delta u = f(u,v) \quad \text{in } \mathbb{R}^n, \\ -\Delta v = g(u,v) \quad \text{in } \mathbb{R}^n, \end{cases} \tag{1.3}$$

under some 'monotonicity' assumption

$$(X - Y)[f(X,Y) - g(X,Y)] \leq 0, \quad X, Y \geq 0, \tag{1.4}$$

and also obtained further interesting components symmetry results.

Such class of systems appears in the modeling of Bose-Einstein condensates which is described by the static Schrödinger equations [11]. The physical and mathematic background can be see in [1] and [3] and other related references.

In this paper, we expect to generalize those components symmetry property in [12] to the γ-Laplacian systems

$$\begin{cases} -\text{div}(|\nabla u|^{\gamma-2}\nabla u) = f(u,v) \quad \text{in } \mathbb{R}^n, \\ -\text{div}(|\nabla v|^{\gamma-2}\nabla v) = g(u,v) \quad \text{in } \mathbb{R}^n. \end{cases} \tag{1.5}$$

Here $n > \gamma$, $\gamma > 1$, and f, g satisfy (1.4) and other suitable growth assumptions on f, g. In what follows, we assume that $f, g : [0, \infty) \to \mathbb{R}$ are continuous.

For the γ-Laplacian equations with $\gamma \neq 2$, it seems difficult to handle the general classical solutions in view of its nonlinearity and degeneration. As a try for it, we only consider the radial classical solutions in this paper.

We say that a couple of nonnegative functions (u, v) is semitrivial if one component is equal to 0 and the other is not (with the convention $0^0 = 1$).

Theorem 1.2. *Let $n > \gamma$, $\gamma > 1$, and $0 \leq p, t \leq \frac{(n-1)\gamma}{n-\gamma} - 1$. Assume that f, g satisfy (1.4), and for each $\eta > 0$, there exists $c = c(\eta) > 0$ such that*

$$f(u,v) \geq cu^p \quad \textit{for all} \;\; v \geq \eta, \; u \geq 0, \tag{1.6}$$

and

$$g(u,v) \geq cv^t \quad \textit{for all} \;\; u \geq \eta, \; u \geq 0. \tag{1.7}$$

Then any nonnegative radial solution (u, v) of (1.5) is either semitrivial or satisfies $u \equiv v$.

The following corollary is a special case of Theorem 1.2 concerning the system

$$\begin{cases} -\text{div}(|\nabla u|^{\gamma-2}\nabla u) = u^p v^q \quad \text{in } \mathbb{R}^n, \\ -\text{div}(|\nabla v|^{\gamma-2}\nabla v) = v^t u^s \quad \text{in } \mathbb{R}^n. \end{cases} \tag{1.8}$$

Here $n > \gamma$, $\gamma > 1$, and $p, q, t, s \geq 0$.

Corollary 1.3. *Let $n > \gamma$, $\gamma > 1$,*

$$q - t = s - p \geq 0, \tag{1.9}$$

and

$$0 \le p, t \le \frac{(n-1)\gamma}{n-\gamma} - 1. \tag{1.10}$$

Then any nonnegative radial solution (u, v) *of* (1.8) *is either semitrivial or satisfies* $u \equiv v$.

The following corollary is also a special case of Corollary 1.3 concerning the system (1.2).

Corollary 1.4. *Assume* $n > \gamma$, $\gamma > 1$, *and* $p+q = \frac{n\gamma}{n-\gamma} - 1$. *Then any positive radial solution* (u, v) *of* (1.2) *satisfies* $u \equiv v$.

Remark 1.1. (i) Comparing with the works of [5] and [10], the positive solutions of (1.2) with $\gamma \neq 2$ may have not the radial symmetry property even if the exponent $p + q$ satisfies the critical condition for Sobolev embedding. In fact, the γ-Laplacian equations have not the conformal invariant property except for some energy minimal solutions (ground states) (cf. [2]). Therefore, the classification result is hardly obtained.

(ii) When $u \equiv v$, (1.2) is reduced to a single equation. According to [8] and [9], the integrable solutions of this single equation decay with the fast rate when $|x| \to \infty$.

According to the conclusion pointed out in [12], the condition $q-t = s-p$ in (1.9) is necessary. In addition, the following theorem implies that the conditions (1.9) and (1.10) are not purely technical.

Theorem 1.5. *Let* $n > \gamma$, $\gamma > 1$ *and* $p, q, t, s \ge 0$.

(i) *Assume* $q - t = s - p \ge 0$. *Then any nonnegative solution* (u, v) *of* (1.8) *satisfies* $u \ge v$ *or* $v \ge u$. *Furthermore, if* $p + q \le \frac{(n-1)\gamma}{n-\gamma} - 1$, *the nonnegative radial solution is semitrivial.*

(ii) *Let* $q = t \ge \frac{n\gamma}{n-\gamma} - 1$ *and* $p = s \ge 0$. *Then there exists a positive solution* (u, v) *of* (1.8), *such that* $u > v$ *in* $\mathbb{R}^n$. *More precisely, we have* $\lim_{|x|\to\infty} v(x) = 0$ *and* $u \equiv v + 1$. *Moreover, if* $q = s$, *then the couple* (v, u) *is also a solution.*

(iii) *Let* $p = t \ge \frac{n\gamma}{n-\gamma} - 1 - \frac{\gamma^2}{2(n-\gamma)}$ *and* $q = s = p - (\gamma - 1)$. *Then there exists a positive function* w *such that the couple* $(u, v) = (cw, w/c)$ *solves* (1.8) *for any* $c > 0$.

2. Proof of Theorem 1.2

The properties of our mainly study of the radial solution $U(x) = u(r)$ for

$$-\Delta_\gamma U := -\mathrm{div}(|\nabla U|^{\gamma-2}\nabla U) \ge 0, \tag{2.1}$$

in our arguments is contained in the following lemma.

Lemma 2.1. *Let* $U \ge 0$ *belong to* $C^2(\mathbb{R}^n)$. *If* $U(x) = u(r)$ *is a radial solution of* (2.1), *then*

(i) $u'(r) \le 0$ *for* $r > 0$;

(ii) $u(r) \geq l := \lim_{R\to\infty} u(R)$ *for* $r \geq 0$.

Proof. Clearly, if U solves (2.1), then u is a radial solution of

$$-(r^{n-1}|u'|^{\gamma-2}u')' \geq 0, \quad r \geq 0. \tag{2.2}$$

By integrating on both sides of (2.2) from 0 to R with $R > 0$, we obtain

$$|u'(R)|^{\gamma-2}u'(R) \leq 0,$$

and hence (i) is verified.

In addition, u is nonincreasing and nonnegative. Thus, l is well-defined and (ii) is proved. □

The following lemma plays the key role in this paper. The idea of the proof comes from [12] which appears in Souplet's earlier paper (cf. Lemma 2.7 of [14]).

Lemma 2.2. *Assume that* f, g *satisfy*

$$f(X,Y) \geq g(X,Y), \quad 0 \leq X \leq Y. \tag{2.3}$$

If (u, v) *is a nonnegative radial solution of* (1.5) *such that*

$$\lim_{R\to\infty} \inf v(R) = 0, \tag{2.4}$$

then $v \leq u$ *in* $\mathbb{R}^n$.

Proof. Let $w = v - u$. By (2.3), we have

$$\Delta_\gamma v - \Delta_\gamma u = f - g \geq 0 \quad \text{in} \ \ \{w \geq 0\}. \tag{2.5}$$

We prepare a standard smooth replacement of the positive part function. Let $H \in C^2(\mathbb{R})$ be a function with the following properties

$$0 \leq H(t) \leq t_+ = \max(t, 0) \text{ for } t \in \mathbb{R}, \quad H'(t), H''(t) > 0 \text{ for } t > 0. \tag{2.6}$$

We then set

$$h(R) := H(w)(R) \quad \text{for} \ \ R > 0.$$

Using (2.6), we have

$$0 \leq h(R) \leq w_+(R) \leq v(R), \quad R > 0 \tag{2.7}$$

Consequently, in view of (2.4), we have

$$\lim_{r\to\infty} \inf h(r) = 0.$$

It follows that there exists a sequence $R_i \to \infty$ such that $h'(R_i) < 0$.

According to Lemma 2.1(i) the integral mean value theorem, we get

$$|\partial_r u|^{\gamma-1} - |\partial_r v|^{\gamma-1} = (\gamma - 1)\int_0^1 [t|\partial_r u| + (1-t)|\partial_r v|]^{\gamma-2} dt \partial_r w.$$

In view of $h'(R_i) < 0$, there holds

$$\begin{aligned}
&\int_{\partial B_{R_i}} (|\partial_r u|^{\gamma-1} - |\partial_r v|^{\gamma-1}) H'(w) d\theta \\
&= (\gamma - 1)|S^{n-1}| h'(R_i) R_i^{n-1} \int_0^1 [t|\partial_r u| + (1-t)|\partial_r v|]^{\gamma-2} dt \leq 0.
\end{aligned} \tag{2.8}$$

On the other hand, when $\gamma > 2$, we have

$$|\nabla w|^\gamma \leq c(|\nabla v|^{\gamma-2}\nabla v - |\nabla u|^{\gamma-2}\nabla u)\nabla(v-u); \tag{2.9}$$

and when $1 < \gamma \leq 2$, we have

$$(|\nabla v| + |\nabla u|)^{\gamma-2}|\nabla w|^2 \leq c(|\nabla v|^{\gamma-2}\nabla v - |\nabla u|^{\gamma-2}\nabla u)\nabla(v-u). \tag{2.10}$$

Therefore,
(i) when $\gamma > 2$, by (2.8) and (2.9), we have

$$\begin{aligned}
0 &\leq \int_{B_{R_i}} H''(w)|\nabla w|^\gamma dx \\
&\leq \int_{B_{R_i}} H''(w)(|\nabla v|^{\gamma-2}\nabla v - |\nabla u|^{\gamma-2}\nabla u)\nabla(v-u) dx \\
&= \int_{B_{R_i}} (|\nabla v|^{\gamma-2}\nabla v - |\nabla u|^{\gamma-2}\nabla u)\nabla H'(w) dx \\
&= \int_{\partial B_{R_i}} (|\partial_r u|^{\gamma-1} - |\partial_r v|^{\gamma-1}) H'(w) ds - \int_{B_{R_i}} (\Delta_\gamma v - \Delta_\gamma u) H'(w) dx \\
&\leq -\int_{B_{R_i}} (f-g) H'(w) dx \leq 0.
\end{aligned}$$

(ii) When $1 < \gamma \leq 2$, by the same argument of (i), from (2.10) we also deduce that

$$0 \leq \int_{B_{R_i}} (|\nabla v| + |\nabla u|)^{\gamma-2} H''(w)|\nabla w|^2 dx \leq 0.$$

Therefore, for $\gamma > 1$, we always have $\nabla w = 0$ on $\mathbb{R}^n$, which implies that w is a constant. Going back to (2.7) and (2.4), we conclude that $w_+ = 0$, and hence $v \leq u$. □

Proof of Theorem 1.2. In view of Lemma 2.2, it suffices to show that either (u, v) is semitrivial or

$$\lim_{R\to\infty} \inf u(R) = \lim_{R\to\infty} \inf v(R) = 0. \tag{2.11}$$

Assume, for instance, that the first limit does not hold. Then there exists $C > 0$ such that $u \geq C$ in $\mathbb{R}^n$ by (ii) of Lemma 2.1. Thus, $-\Delta_\gamma v \geq \tilde{c} v^r$ in $\mathbb{R}^n$ by assumption (1.7). According to the Liouville type results in [13], it is known that $v \equiv 0$ by virtue of $r \leq \frac{(n-1)\gamma}{n-\gamma} - 1$. The proof is complete. □

Remark 2.1. A couple $(u, 0)$ is a semitrivial solution of (1.8) if and only if $r > 0$, and either $q > 0$ and u is a p-harmonic function (i.e., it solves $-\mathrm{div}(|\nabla u|^{\gamma-2}\nabla u) = 0$), or $q = 0$, $n > \gamma$, $p \geq \frac{n\gamma}{n-\gamma} - 1$, and u solves

$$-\mathrm{div}(|\nabla u|^{\gamma-2}\nabla u) = u^p \quad \text{in} \quad \mathbb{R}^n$$

(the existence is showed in [13]). A symmetric statement of course still holds for semitrivial solutions of the form $(0, v)$.

3. Proof of Theorem 1.5

First we state a Pohozaev type result.

Lemma 3.1. *If the boundary value problem*

$$\begin{cases} -\Delta_\gamma v = f(v) = (1+v)^p v^q, & x \in B_R, \\ v = 0, \ x \in \partial B_R, \end{cases} \tag{3.1}$$

has positive radial solutions, then

$$\int_{B_R} v f(v) dx < \frac{n\gamma}{n-\gamma} \int_{B_R} F(v) dx.$$

Here $B_R = B_R(0)$ *and* $F(v) = \int_0^v f(t)dt$.

Proof. Let v be the positive solution of the boundary value problem (3.1).

We multiply the equation in (3.3) by $(x \cdot \nabla v)$ and integrate over B_R. Using integration by parts, we obtain

$$\begin{aligned} & -\int_{B_R} \Delta_\gamma v(x \cdot \nabla v) dx \\ = & -\int_{\partial B_R} (|\nabla v|^{\gamma-2}\nabla v \cdot \nu)(\nabla v \cdot x) ds + \int_{B_R} (|\nabla v|^{\gamma-2}\nabla v)\nabla(\nabla v \cdot x) dx \\ = & -R\int_{\partial B_R} (|\nabla v|^{\gamma-2}|\frac{\partial v}{\partial \nu}|^2) ds + \int_{B_R} |\nabla v|^\gamma dx + \frac{1}{\gamma}\int_{B_R} x \cdot \nabla(|\nabla v|^\gamma) dx \\ = & -\frac{n-\gamma}{\gamma}\int_{B_R} |\nabla v|^\gamma dx + \frac{1-\gamma}{\gamma} R \int_{\partial B_R} |\nabla v|^\gamma ds. \end{aligned}$$

The last equality is deduced by the radial symmetry of v. In addition, we get

$$\begin{aligned} \int_{B_R} f(v)(x \cdot \nabla v) dx &= \int_{B_R} x \cdot \nabla F(v) dx \\ &= R\int_{\partial B_R} F(v) ds - n\int_{B_R} F(v) dx \\ &= -n\int_{B_R} F(v) dx \end{aligned}$$

by using $F(v) = 0$ on ∂B_R. Thus,

$$-\frac{n-\gamma}{\gamma}\int_{B_R} |\nabla v|^\gamma dx + \frac{1-\gamma}{\gamma} R \int_{\partial B_R} |\nabla v|^\gamma ds = -n\int_{B_R} F(v) dx.$$

Noting $\gamma > 1$, we obtain

$$\frac{1-\gamma}{\gamma} R \int_{\partial B_R} |\nabla v|^\gamma ds < 0,$$

and hence

$$\frac{n-\gamma}{\gamma} \int_{B_R} |\nabla v|^\gamma dx < n \int_{B_R} F(v) dx.$$

On the other hand, multiply the equation in (3.3) by v. Integrating by parts, we obtain

$$\int_{B_R} v f(v) dx = -\int_{B_R} (\Delta_\gamma v) v dx = \int_{B_R} |\nabla v|^\gamma dx.$$

Combining with the result above, we complete the proof easily. □

Proof of Theorem 1.5. (i) In view of Lemma 2.2, it is sufficient to check that either $\lim_{R\to\infty} \inf u(R) = 0$ or $\lim_{R\to\infty} \inf v(R) = 0$, which implies $u \le v$ or $v \le u$ by Lemma 2.2. If these were not the case, then $u, v \ge C > 0$ in $\mathbb{R}^n$ by (ii) of Lemma 2.1. Thus, $-\Delta_\gamma u \ge c > 0$ in $\mathbb{R}^n$. Namely,

$$-R^{1-n}(R^{n-1}|u'|^{\gamma-2}u')' \ge c.$$

Multiplying by R^{n-1} and integrating from 0 to R, we see that

$$|u'(R)|^{\gamma-2}u'(R) \le -cR \quad \text{for} \quad R > 0. \tag{3.2}$$

Noting (i) of Lemma 2.1, we get $-u' > 0$ which implies

$$|u'(R)|^{\gamma-2}u'(R) = -(-u'(R))^{\gamma-1}.$$

Combining with (3.2) yields

$$u'(R) \le -(cR)^{\frac{1}{\gamma-1}} \quad \text{for} \quad R > 0.$$

Integrating from r_0 to r, we get

$$u(r) \le u(r_0) - \frac{\gamma-1}{\gamma} c^{\frac{1}{\gamma-1}} r^{\frac{\gamma}{\gamma-1}}.$$

When r is sufficiently large, u is negative. It is a contradiction.

Moreover, without loss of generality, we assume $u \le v$. Then from (1.8) it follows that

$$-\operatorname{div}(|\nabla u|^{\gamma-2}\nabla u) \ge u^{p+q} \quad \text{in} \quad \mathbb{R}^n.$$

According to the Liouville type results in [13], we get $u \equiv 0$ in view of $p + q \le \frac{(n-1)\gamma}{n-\gamma} - 1$.

(ii) We look for a solution such that $u = v + 1$. Then system (1.8) becomes equivalent to the single equation

$$-\Delta_\gamma v = f(v) = (1+v)^p v^q, \quad x \in \mathbb{R}^n. \tag{3.3}$$

Let $F(t) = \int_0^t f(\tau)d\tau$. We claim that

$$tf(t) - \frac{n\gamma}{n-\gamma} F(t) \ge 0, \quad t \ge 0. \tag{3.4}$$

In fact, in view of (3.3) and integrating by parts, we have

$$\begin{aligned} F(t) &= \int_0^t (1+\tau)^p \tau^q d\tau \\ &= \frac{t^{q+1}(1+t)^p}{q+1} - \int_0^t \frac{\tau^{q+1}}{q+1} d((1+\tau)^p) \\ &= \frac{1}{q+1}[(1+t)^p t^{q+1} - \int_0^t p(1+\tau)^{p-1}\tau^{q+1} d\tau] \\ &\le \frac{1}{q+1} t f(t). \end{aligned}$$

Since $q \ge \frac{n\gamma}{n-\gamma} - 1$, we obtain

$$tf(t) \ge (q+1)F(t) \ge \frac{n\gamma}{n-\gamma}F(t).$$

Therefore, (3.4) is verified.

According to Lemma 3.1, we know the boundary value problem (3.1) does not admit any positive solution by noting (3.4).

Next, consider the following initial value problem

$$\begin{cases} -t^{1-n}(t^{n-1}|v'|^{\gamma-2}v')' = f(v), \quad t > 0, \\ v(0) = 1, v'(0) = 0. \end{cases} \tag{3.5}$$

Clearly, one of the following two cases holds

Case 1: $v > 0, v' \le 0$ for all $t > 0$;

Case 2: v has the first zero R_*.

We claim that Case 2 does not happen, since this would contradict the above nonexistence statement on the ball B_{R_*}. We conclude that problem (3.5) and hence (3.3), admits a positive entire solution v which is decaying to zero. More precisely, according to the results in [6] and [7], v decays fast with the rate $\frac{n-\gamma}{\gamma-1}$ when $q = \frac{n\gamma}{n-\gamma} - 1$ and slowly with rate $\frac{\gamma}{q-(\gamma-1)}$ when $q > \frac{n\gamma}{n-\gamma} - 1$.

(iii) Let $(u, v) = (cw, c^{-1}w)$ with c a positive constant. Then system (1.8) becomes equivalent to

$$-\Delta_\gamma w = w^{2p-\gamma+1}.$$

According to the existence results in [13], we see that this equation admits positive solutions by virtue of $2p - \gamma + 1 \ge \frac{n\gamma}{n-\gamma} - 1$. □

References

[1] J. Bourgain, *Global solutions of Nonlinear Schrödinger Equations*, Amer. Math. Soc. Colloq. Publ. **46** AMS, Providence, RI, 1999.

[2] J. Byeon, L. Jeanjean, and M. Maris, *Symmetry and monotonicity of least energy solutions*, Calc. Var. PDEs **36** (2009), no. 4, 481–492.

[3] T. Cazenave, *Semilinear Schrödinger equations*, Courant Lecture Notes in Mathematics, 10. New York University, Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 2003.

[4] W. Chen and C. Li, *Classification of solutions of some nonlinear elliptic equations*, Duke Math. J. **63** (1991), no. 3, 615–622.
[5] ______, *Classification of positive solutions for nonlinear differential and integral systems with critical exponents*, Acta Math. Sci. Ser. B Engl. Ed. **29** (2009), no. 4, 949–960.
[6] M. Franca, *Classification of positive solutions of p-Laplace equation with a growth term*, Arch. Math. (BRNO), **40** (2004), no. 4, 415–434.
[7] N. Kawano, E. Yanagida, and S. Yotsutani, *Structure theorems for positive radial solutions to* $\mathrm{div}(|Du|^{m-2}Du) + K(|x|)u^q = 0$ *in* $\mathbb{R}^n$, J. Math. Soc. Japan **45** (1993), no. 4, 719–742.
[8] Y. Lei and C. Li, *Integrability and asymptotics of positive solutions of a* γ*-Laplacian system*, J. Differential Equations **252** (2012), no. 3, 2739–2758.
[9] Y. Lei, C. Li, and C. Ma, *Decay estimation for positive solutions of a* γ*-Laplace equation*, Discrete Contin. Dyn. Syst. **30** (2011), no. 2, 547–558.
[10] C. Li and L. Ma, *Uniqueness of positive bound states to Schrödinger systems with critical exponents*, SIAM J. Math. Anal. **40** (2008), no. 3, 1049–1057.
[11] T. Lin and J. Wei, *Ground state of* N *coupled nonlinear Schrödinger equations in* $\mathbb{R}^n$, $n \leq 3$, Commun. Math. Phys. **255** (2005), no. 3, 629–653.
[12] P. Quittner and Ph. Souplet, *Symmetry of components for semilinear elliptic systems*, SIAM J. Math. Anal. **44** (2012), no. 4, 2545–2559.
[13] J. Serrin and H. Zou, *Cauchy-Liouville and universal boundedness theorems for quasilinear elliptic equations and inequalities*, Acta Math. **189** (2002), no. 1, 79–142.
[14] Ph. Souplet, *The proof of the Lane-Emden conjecture in four space dimensions*, Adv. Math. **221** (2009), no. 5, 1409–1427.

Institute of Mathematics
School of Mathematical Sciences
Nanjing Normal University
Nanjing, 210023, P. R. China
E-mail address: 407650186@qq.com