

Decentralized Moving Average Filtering with Uncertainties

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Abstract

A filtering algorithm based on the decentralized moving average Kalman filter with uncertainties is proposed in this paper. The proposed filtering algorithm presented combines the Kalman filter with the moving average strategy. A decentralized fusion algorithm with the weighted sum structure is applied to the local moving average Kalman filters (LMAKFs) of different window lengths. The proposed algorithm has a parallel structure and allows parallel processing of observations. Hence, it is more reliable than the centralized algorithm when some sensors become faulty. Moreover, the choice of the moving average strategy makes the proposed algorithm robust against linear discrete-time dynamic model uncertainties. The derivation of the error cross-covariances between the LMAKFs is the key idea of studied. The application of the proposed decentralized fusion filter to dynamic systems within a multisensor environment demonstrates its high accuracy and computational efficiency.

Keywords: Decentralized fusion, fusion formula, Kalman filter, multisensor, moving average

1. INTRODUCTION

There is a growing concern about the potential application of multisensor data fusion in diverse areas such as guidance, robotics, aerospace, target tracking, signal processing, and control [1,2]. Swift progress in communications, low-power computing, and sensing hardware have resulted in an abundance of commercially available sensor nodes. The main challenge now is to develop efficient methods for the automatic fusion and interpretation of the information generated by multisensor data fusion.

Multisensor data fusion is typically carried out for 1) reducing the overall redundant information obtained from different sensors, 2) increasing information gain by using multiple sensors, and 3) increasing the accuracy and decreasing the uncertainty of the system. Further, multisensor data fusion can provide benefits such as extended temporal and spatial coverage, reduced ambiguity, enhanced spatial resolution, and increased dimensionality of the measurement space. In general, two basic fusion methods are

commonly used to process measured sensor data; these methods are discussed below.

The first approach is called centralized fusion estimation. The fusion center directly receives all measurement data from all local sensors and processes them in real time. One advantage of the centralized estimation is that it causes minimal information loss. However, the centralized estimation approach has several serious drawbacks, including poor survivability and reliability, as well as heavy communication and computational burden. Further the multisensory environment must be considered ideal, as mentioned above.

The second approach is called decentralized fusion estimation, in which every local sensor is attached to a local processor. In this method, the processor optimally estimates an object based on its own local measurements and then transmits its local estimate to the fusion center. Finally, the fusion center optimally estimates the object by using all received local estimates. Recently, to overcome the disadvantages of the centralized estimation, various decentralized and parallel versions of the standard Kalman filters have been proposed for linear dynamic systems with a multisensor environment [2-8]. The advantage of this decentralized fusion of filters is that these parallel structures can lead to an increase in input data rates and make fault detection and isolation easier. However, the accuracy of the decentralized filter is generally lower than that of the centralized filter.

In estimation problems attempting to achieve robustness

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against temporary uncertainty, numerous strategies have been suggested, rigorously investigated, and have been implemented over the past few decades. Among them, the moving average technique has been popular and successful. It is robust against temporal uncertainty, and thus it has been rigorously investigated. It has been a general rule that local moving average Kalman filters (LMAKFs) are typically more robust against dynamic model uncertainties and numerical errors than standard local Kalman filters, which utilize all measurements [9-12].

In addition, decentralized moving average fusion filtering for multiple sensors with equal moving average time intervals (moving average sizes) has been proposed in [13]. In this case, all fused LMAKFs with the same moving average time interval utilize finite measurements over the most recent time interval [9-12].

In this study, we investigate a generalization of [13] for arbitrary non-equal moving average sizes. This design of decentralized filters for sensor measurements with non-equal moving average sizes is more complicated than that for sensor measurements with equal sizes owing to a lack of common time intervals that contain all the sensor data; in this case, it is impossible to design a centralized filtering algorithm. We propose using a decentralized moving average filter for a set of local sensors with non-equal moving average sizes. Then, we derive the key differential equations for error cross-covariances between LMAKFs using the different moving average sizes.

The remainder of this paper is organized as follows. The problem setting is described in Section 2. In Section 3, we present the main results pertaining to decentralized moving average filtering for a multisensor environment. Here, the key equations for determining the cross-covariances between the local moving average filtering errors are derived. In Section 4, an example for discrete-time dynamic systems within a multisensor environment is provided to illustrate the main results, and the concluding remarks are given in Section 5

2. PROBLEM SETTING

Consider the linear discrete-time dynamic system with N sensors

$$x_{k+1} = F_k x_k + G_k v_k, \quad k = 0, 1, 2, \dots, \tag{1}$$

$$y_k^{(i)} = H_k^{(i)} x_k + w_k^{(i)}, \quad i = 1, \dots, N, \tag{2}$$

where $x_k \in \mathfrak{R}^n$ is the state, and $y_k^{(i)} \in \mathfrak{R}^{m_i}$ is the measurement. The

system noise $v_k \in \mathfrak{R}^r$ and the measurement noises $w_k^{(i)} \in \mathfrak{R}^{m_i}$, $i = 1, \dots, N$ are uncorrelated white Gaussian noises with zero mean and covariances Q_k and $R_k^{(i)}$, respectively, and $F_k, G_k, H_k^{(i)}$ are matrices with compatible dimensions. Superscript (i) denotes the i th sensor, and N is the total number of sensors.

The initial state $x_0 \sim \mathbf{N}(m_0; P_0)$, $m_0 = E(x_0)$, $P_0 = \text{cov}\{x_0, x_0\}$ is assumed to be Gaussian and uncorrelated with v_k and $w_k^{(i)}$, $i = 1, \dots, N$.

Our purpose is to find the decentralized fusion estimate of the state x_k based on the overall moving average sensor measurements Y_k with different moving average time intervals $\Delta_i, i = 1, \dots, N$, i.e.,

$$Y_k = \{y_k^{(1)}, \dots, y_k^{(N)}\}, \tag{3}$$

$$y_k^{(i)} = \{y_s^{(i)} : s = k - \Delta_i, k - \Delta_i + 1, \dots, k\}, \quad i = 1, \dots, N.$$

3. DECENTRALIZED MOVING AVERAGE FILTER

Now we show that the fusion formula (FF) [8,14] can serve as the basis for designing a decentralized fusion filter. A *new decentralized fusion moving average filter with non-equal moving average sizes (NE-filter)* includes two stages: LMAKFs (estimates) $\hat{x}_{k|k}^{(1)}, \dots, \hat{x}_{k|k}^{(N)}$ that are computed at the first stage are linearly fused at the second stage based on the FF.

First step ("Calculation of LMAKFs").

$$\begin{cases} x_{k+1} = F_k x_k + G_k v_k, \quad x_0 \sim N(m_0; P_0), \\ y_k^{(i)} = H_k^{(i)} x_k + w_k^{(i)}, \end{cases} \tag{4}$$

where the number i of a local subsystem is fixed.

Let $\hat{x}_{k|k}^{(i)}$ denote local moving average estimate of the state x_k based on the individual sensor measurements $Y_k^{(i)} = \{y_s^{(i)} : s \in [k - \Delta_i; k]\}$. To find $\hat{x}_{k|k}^{(i)}$, we can apply the optimal MAKF to the subsystem (4) [9-12]. We obtain the following differential equations:

$$\begin{aligned} \hat{x}_{s+1|s}^{(i)} &= F_s \hat{x}_{s|s}^{(i)}, \\ \hat{x}_{s+1|s+1}^{(i)} &= \hat{x}_{s+1|s}^{(i)} + K_{s+1}^{(i)} [y_{s+1}^{(i)} - H_{s+1}^{(i)} \hat{x}_{s+1|s}^{(i)}], \\ P_{s+1|s}^{(ii)} &= F_s P_{s|s}^{(ii)} F_s^T + \tilde{Q}_s, \quad \tilde{Q}_s = G_s Q_s G_s^T, \\ K_{s+1}^{(i)} &= P_{s+1|s}^{(ii)} H_{s+1}^{(i)T} [H_{s+1}^{(i)} P_{s+1|s}^{(ii)} H_{s+1}^{(i)T} + R_{s+1}^{(i)}]^{-1}, \\ P_{s+1|s+1}^{(ii)} &= [I_n - K_{s+1}^{(i)} H_{s+1}^{(i)}] P_{s+1|s}^{(ii)}, \\ P_{s|s}^{(ii)} &= \text{cov}\{e_s^{(i)}, e_s^{(i)}\}, \quad e_s^{(i)} = x_s - \hat{x}_{s|s}^{(i)}, \\ & \quad s = k - \Delta_i, k - \Delta_i + 1, \dots, k, \quad i = 1, \dots, N \end{aligned} \tag{5}$$

with the following moving average initial conditions:

$$\begin{aligned} \hat{x}_{k-\Delta_i}^{(i)} &= m_{k-\Delta_i} = E(x_{k-\Delta_i}), \\ P_{k-\Delta_i|k-\Delta_i}^{(i)} &= P_{k-\Delta_i} = \text{cov}\{x_{k-\Delta_i}, x_{k-\Delta_i}\} \end{aligned} \quad (6)$$

determined by the Lyapunov equations [12] on the interval $\tau \in [0; k-\Delta_i]$:

$$\begin{aligned} m_{\tau+1} &= F_\tau m_\tau, \quad m_0 = E(x_0), \\ P_{\tau+1} &= F_\tau P_\tau F_\tau^T + \tilde{Q}_\tau, \quad P = \text{cov}\{x_0; x_0\}, \\ \tau &= 0, 1, \dots, k - \Delta_i. \end{aligned} \quad (7)$$

Thus, we get N LMAKFs $\hat{x}_{k|k}^{(1)}, \dots, \hat{x}_{k|k}^{(N)}$ with the corresponding local error covariances $P_{k|k}^{(11)}, \dots, P_{k|k}^{(NN)}$.

Second step (“Fusion of LMAKFs”).

To express the final NE-filter (estimate) \hat{x}_k^{fus} for the state x_k in terms of the LMAKFs $\hat{x}_{k|k}^{(1)}, \dots, \hat{x}_{k|k}^{(N)}$, we use the FF. We have

$$\hat{x}_k^{fus} = \sum_{i=1}^N c_k^{(i)} \hat{x}_{k|k}^{(i)}, \quad \sum_{i=1}^N c_k^{(i)} = I_n, \quad (8)$$

where I_n is the identity matrix, and $c_k^{(1)}, \dots, c_k^{(N)}$ are the time-varying weighted matrices determined by the mean square criterion.

Theorem 1 [8, 14]. (a) The optimal weights $c_k^{(1)}, \dots, c_k^{(N)}$ satisfy the linear algebraic equations

$$\sum_{i=1}^N c_k^{(i)} [P_{k|k}^{(ij)} - P_{k|k}^{(iN)}] = 0, \quad \sum_{i=1}^N c_k^{(i)} = I_n, \quad (9)$$

and they can be explicitly written in the following form

$$c_k^{(i)} = \sum_{j=1}^N W_k^{(ij)} \left(\sum_{l,n=1}^N W_k^{(ln)} \right)^{-1}, \quad i = 1, \dots, N, \quad (10)$$

where $W_k^{(ij)}$ is the (ij) th $(n \times n)$ submatrix of the $(nN \times nN)$ block matrix P_k^{-1} , $P_k = [P_{k|k}^{(ij)}]_{i,j=1}^N$

(b) The fusion error covariance $P_k^{fus} = \text{cov}\{e_k^{fus}, e_k^{fus}\}$, $e_k^{fus} = x_k - \hat{x}_k^{fus}$ is given by

$$P_k^{fus} = \sum_{i,j=1}^N c_k^{(i)} P_{k|k}^{(ij)} c_k^{(j)T}. \quad (11)$$

Equations (9)–(11) defining the unknown weights $c_k^{(i)}$ and fusion error covariance P_k^{fus} depend on the local covariances $P_{k|k}^{(ij)}$, which have been determined by (5), and the local cross-covariances given by

$$P_{k|k}^{(ij)} = \text{cov}\{e_k^{(i)}, e_k^{(j)}\}, \quad i, j = 1, \dots, N, \quad i \neq j, \quad (12)$$

as given in Theorem 2.

Theorem 2. (a) The local cross-covariances (12) satisfy the following recursive equations:

$$\begin{aligned} P_{s+1|s+1}^{(ij)} &= \tilde{F}_s^{(i)} P_{s|s}^{(ij)} \tilde{F}_s^{(j)T} + \tilde{G}_s^{(i)} Q_s \tilde{G}_s^{(j)T}, \quad s \in [k-\Delta_i; k], \\ \tilde{F}_s^{(i)} &= (I_n - K_{s+1}^{(i)} H_{s+1}^{(i)}) F_s, \quad \tilde{G}_s^{(i)} = (I_n - K_{s+1}^{(i)} H_{s+1}^{(i)}) G_s, \\ l &= i, j; \quad i, j = 1, \dots, N; \quad i \neq j \end{aligned} \quad (13)$$

with the following moving average initial conditions:

$$P_{k-\Delta_i|k-\Delta_i}^{(ij)} = P_{k-\Delta_i} - \text{cov}\{x_{k-\Delta_i}, \hat{x}_{k-\Delta_i}^{(j)}\}. \quad (14)$$

(b) The covariance $\text{cov}\{x_{k-\Delta_i}, \hat{x}_{k-\Delta_i}^{(j)}\}$ in (14) represents the non-diagonal element of the block covariance matrix $D_\tau^{(j)}$, which is expressed as follows:

$$D_\tau^{(j)} = \begin{bmatrix} \text{cov}\{x_\tau, x_\tau\} & \text{cov}\{x_\tau, \hat{x}_\tau^{(j)}\} \\ \text{cov}\{\hat{x}_\tau^{(j)}, x_\tau\} & \text{cov}\{\hat{x}_\tau^{(j)}, \hat{x}_\tau^{(j)}\} \end{bmatrix} \quad (15)$$

at $\tau = t - \Delta_i$, which is described by the Lyapunov recursive equation.

$$\begin{aligned} D_{\tau+1}^{(j)} &= A_\tau^{(j)} D_\tau^{(j)} A_\tau^{(j)T} + B_\tau^{(j)} Q_\tau^{(j)} B_\tau^{(j)T}, \\ A_\tau^{(j)} &= \begin{bmatrix} F_\tau & 0 \\ K_{\tau+1}^{(j)} H_{\tau+1}^{(j)} F_\tau & \tilde{F}_{\tau+1}^{(j)} \end{bmatrix}, \quad B_\tau^{(j)} = \begin{bmatrix} G_\tau & 0 \\ K_{\tau+1}^{(j)} H_{\tau+1}^{(j)} G_\tau & K_{\tau+1}^{(j)} \end{bmatrix}, \\ Q_\tau^{(j)} &= \begin{bmatrix} Q_\tau & 0 \\ 0 & R_{\tau+1}^{(j)} \end{bmatrix}, \quad \tau \in [t - \Delta_i; k - \Delta_i] \end{aligned} \quad (16)$$

with the initial condition

$$D_{k-\Delta_i}^{(j)} = \begin{bmatrix} \text{cov}(x_{k-\Delta_i}, x_{k-\Delta_i}) & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} P_{k-\Delta_i} & 0 \\ 0 & 0 \end{bmatrix}, \quad (17)$$

determined by (7).

Thus, equations (5)–(17) completely define the NE-filter.

Remark 1. The LMAKFs $\hat{x}_k^{(i)}$, $i = 1, \dots, N$ are separated for different types of sensors, i.e., each local estimate $\hat{x}_k^{(i)}$ is calculated independently of other estimates. Therefore, the LMAKFs can be implemented in parallel for different sensors (2).

Remark 2. We may note that the local error covariances $P_k^{(ij)}$, $i, j = 1, \dots, N, i \neq j$, and the weights $c_k^{(i)}$ may be pre-computed, since they do not depend on the sensor measurements (3), but only on the noise statistics Q_k and $R_k^{(i)}$ and the system matrices $F_k, G_k, H_k^{(i)}$, which are part of the system model (1), (2). Thus, once the measurement schedule has been settled, the real-time implementation of the NE-filter requires only the computation of the LMAKFs $\hat{x}_k^{(i)}$, $i = 1, \dots, N$ and the final suboptimal fusion estimate \hat{x}_k^{fus} .

4. NUMERICAL EXAMPLE

Here we verify the NE-filter using a linear model of the motion of the Ground Moving Target Indicator (GMTI), maintaining straight and fixed level moving at a constant velocity with

measurements taken at sampling interval Δt [15]. The state vector $X \in \mathfrak{R}^4$ consists of position and velocity in each of the two dimensions; that is, $X = [x \ v_x \ y \ v_y]^T$. The system noise $\xi_k \in \mathfrak{R}^2$ is zero-mean white Gaussian noise with covariance $Q_k = \text{diag}[q_1 q_2]$.

Then the discretized system model is given by

$$X_{k+1} = \begin{bmatrix} 1 & \Delta t + 0.5\delta_k & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} X_k + \begin{bmatrix} \Delta t^2/2 & 0 \\ \Delta t & 0 \\ 0 & \Delta t^2/2 \\ 0 & \Delta t \end{bmatrix} \xi_k, \quad (18)$$

where δ_k is an uncertain model parameter, a GMTI with the initial position $(x, y) = (20, 10)$ [m] and velocity $(v_x, v_y) = (0.5, -0.08)$ [m/s] yielding $\bar{X}_0 = [20 \ 0.5 \ 10 \ -0.08]^T$, and the initial covariance of the GMTI is $P_0 = \text{diag}[2 \ 1 \ 2 \ 1]^T$. The covariances are subjected to $Q_k = \text{diag}[0.1, 0.1]$, and sampling interval $\Delta t = 0.1$. For simplicity, we assume that the uncertain model parameter δ_k takes the form

$$\delta_k = \begin{cases} \sin(k), & k \in T_{UI} = [1; 3], \\ 0, & \text{otherwise}, \end{cases} \quad (19)$$

where T_{UI} is the *uncertainty interval*.

The measurement system includes three sensors $Y_k^{(1)}, Y_k^{(2)}$, and $Y_k^{(3)}$, all of them measure position along x- and y-axes. The first coordinate $x_{1,k}$, related to the x-position, is observable through a measurement model having three identical local sensors, one of which is the main sensor, and the others are reserve sensors. We have

$$Y_k^{(i)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} X_k + w_k^{(i)}, \quad i=1,2,3, \quad (20)$$

where the measurement noises $w_k^{(i)} \in \mathfrak{R}^2$ are also zero-mean white Gaussian noises with intensities $R_k^{(1)} = \text{diag}[0.5 \ 0.5]$, $R_k^{(2)} = \text{diag}[1 \ 1]$, and $R_k^{(3)} = \text{diag}[2.5 \ 2.5]$.

The LMAKFs for each sensor $y_k^{(i)}, i=1,2,3$, are designed for moving average sizes of $\Delta_1 = 0.4, \Delta_2 = 0.5$, and $\Delta_3 = 0.6$, respectively. The simulation results of two decentralized fusion moving average filters with *non-equal* (NE-filter) and *equal* (EQ-filter) moving average sizes and three LMAKFs are shown in Figs. 1 and 2. All simulations are then evaluated in terms of the mean square errors (MSEs) of 1000 Monte Carlo runs. In particular, we focus on the MSEs of the first coordinate $x_{1,k}$, the x-position, because the time uncertainty δ_k from (18) only appears at this coordinate. Here,

$$P_{11,k} = E(x_{1,k} - \hat{x}_{1,k})^2, \quad (21)$$

where $\hat{x}_{1,k} = \hat{x}_{1,k}^{NE-filter}, \hat{x}_{1,k}^{EQ-filter}$ or $\hat{x}_{1,k}^{LRHKF}$.

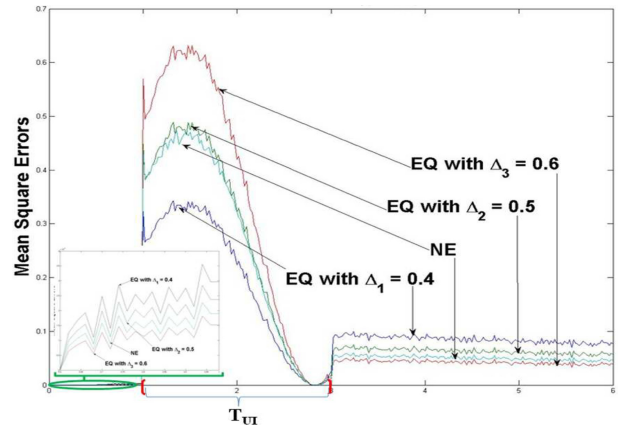


Fig. 1. MSEs comparison between NE-filter and three EQ-filters.

Fig. 1 compares the MSEs of NE-filter (“NE”) with three EQ-filters (“EQ”) having common moving average sizes $\Delta = \Delta_i, i=1,2,3$.

Our point of interest is the behavior of the aforementioned filters, both inside and outside the time interval $T_{UI} = [1;3]$.

From Fig. 1, we can observe that inside the time interval T_{UI} , the NE-filter is more accurate than the two EQ-filters with moving average sizes $\Delta_2 = 0.5$ and $\Delta_3 = 0.6$. However, the NE-filter performs slightly worse than the EQ-filter with a moving average size of $\Delta_1 = 0.4$, such that

$$P_{11,k}^{EQ}(\Delta_1) < P_{11,k}^{NE} < P_{11,k}^{EQ}(\Delta_2) < P_{11,k}^{EQ}(\Delta_3), \quad k \in T_{UI}. \quad (22)$$

The reason for the presence of such a robust property (22) is to compensate for the given uncertainty δ_k , as the common moving average size Δ_{com} for all local sensors (common memory of LMAKFs) should be minimal. In this case they are equal, i.e., $\Delta_{com} = \Delta_1 = 0.4$.

On the other hand, outside the T_{UI} , the differences between the EQ-filters and NE-filters are negligible. In this case, the EQ-filter with the maximum common moving average size $\Delta_{com} = \Delta_3 = 0.6$ is less accurate than the NE-filter, i.e.,

$$P_{11,k}^{EQ}(\Delta_3) < P_{11,k}^{NE} < P_{11,k}^{EQ}(\Delta_2) < P_{11,k}^{EQ}(\Delta_1), \quad k \notin T_{UI}. \quad (23)$$

Fig. 2 illustrates the time histories of the MSEs for the NE-filter and the three LMAKFs (“LKF”). The figure shows that inside the T_{UI} , the MSE of the NE-filter is better than that of the LMAKFs having moving average sizes $\Delta_2 = 0.5$ and $\Delta_3 = 0.6$, and that it is worse than the LMAKF having $\Delta_1 = 0.4$, i.e.,

$$P_{11,k}^{LKF}(\Delta_1) < P_{11,k}^{NE} < P_{11,k}^{LKF}(\Delta_2) < P_{11,k}^{LKF}(\Delta_3), \quad k \in T_{UI}. \quad (24)$$

However, outside the T_{UI} , the NE-filter is better than all LMAKFs, as shown in Fig. 2, i.e.,

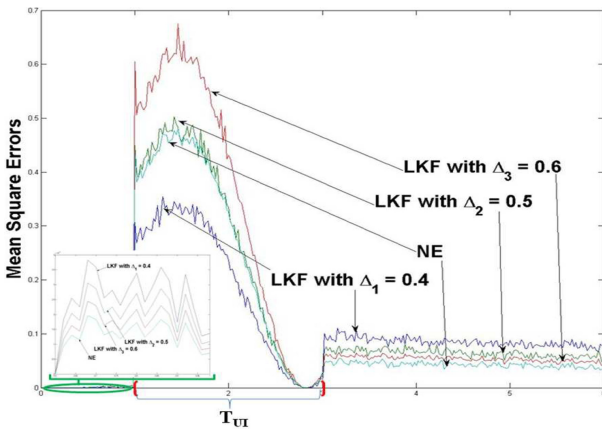


Fig. 2. MSEs comparison between NE-filter and three LMAKFs.

$$P_{11,k}^{NE} < P_{11,k}^{LKF}(\Delta_3) < P_{11,k}^{LKF}(\Delta_2) < P_{11,k}^{LKF}(\Delta_1), \quad k \notin T_{UI}. \quad (25)$$

Note that the reduction in the moving average size to zero ($\Delta_i \rightarrow 0$) inside the uncertainty interval is impossible owing to the loss of sensor measurements (20). Thus, the problem in finding the optimal moving average size Δ_i for each individual LMAKFs is quite complex.

Summarizing the simulation results in Figs. 1 and 2, and using (22)–(25), we can infer the following relationships between MSEs inside/outside of the T_{UI} :

$$P_{11,k}^{EQ}(\Delta_1) < P_{11,k}^{LKF}(\Delta_1) < P_{11,k}^{NE} < P_{11,k}^{EQ}(\Delta_2) < P_{11,k}^{LKF}(\Delta_2), \quad k \in T_{UI},$$

and

$$P_{11,k}^{EQ}(\Delta_3) < P_{11,k}^{NE} < P_{11,k}^{EQ}(\Delta_2) < P_{11,k}^{LKF}(\Delta_3) < P_{11,k}^{LKF}(\Delta_1), \quad k \notin T_{UI}. \quad (26)$$

Since in actual situations, T_{UI} is not known in advance, Eq. (26) shows that the NE-filter is the best choice among all other filters.

5. CONCLUSIONS

In this paper, we have proposed a new decentralized moving average filter for a set of local sensors with non-equal moving average sizes. Further, we have derived the key differential equations for local cross-covariances between the LMAKFs with different moving average sizes.

Simulation results and comparison the NE-filter with the EQ-filters and LMAKFs verify the good estimation accuracy and robustness of the proposed filter.

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