Joint Mode Selection, Link Allocation and Power Control in Underlaying D2D Communication

Wei Zhang[1], **Wanbing He**[1], **Dan Wu**[1], and **Yueming Cai**[1]
[1]Institute of Communications Engineering, PLA UST
Nanjing 210007, Jiangsu – P. R. China
[e-mail: greenery_jul@sina.com]
*Corresponding author: Wei Zhang

Received June 20, 2016; revised September 19, 2016; accepted October 4, 2016;
published November 30, 2016

Abstract

Device-to-device (D2D) communication underlaying cellular networks can bring significate benefits for improving the performance of mobile services. However, it hinges on elaborate resource sharing scheme to coordinate interference between cellular users and D2D pairs. We formulate a joint mode selection, link allocation and power control optimization problem for D2D communication sharing uplink resources in a multi-user cellular network and consider the efficiency and the fairness simultaneously. Due to the non-convex difficulty, we propose a three-step scheme: firstly, we conduct mode selection for D2D pairs based on a minimum distance metric after an admission control and obtain some cellular candidates for them. And then, a cellular candidate will be paired to each D2D pair based on fairness. Finally, we use Lagrangian Algorithm to formulate a joint power control strategy for D2D pairs and their reused cellular users and a closed-form of solution is derived. Simulation results demonstrate that our proposed algorithms converge in a short time. Moreover, both the sum rate of D2D pairs and the energy efficiency of cellular users are improved.

Keywords: D2D communication, joint optimization, mode selection, link allocation, power control

---

This research was supported in part by the National Natural Science Foundation of China under Grant 61671474 and Grant 61301163.

http://dx.doi.org/10.3837/tiis.2016.11.001 **ISSN : 1976-7277**

I. Introduction

Recently, device-to-device (D2D) communication has gained more and more attention [1], [2]. It is regarded as an effective method to relieve the traffic load of cellular networks, improve resource efficiency and the local service flexibility [3]. By enabling two users in proximity to communicate directly without being relayed by a base station (BS), D2D communication guarantees four types of benefits, that is, proximity gain, reuse gain, hop gain and paring gain [4]. As a result, such a technology has been incorporated into the existing and future cellular systems to improve the performance of mobile services [5], [6]. However, introducing D2D pairs into cellular systems will cause mutual interference between D2D pairs and cellular users due to the sharing of links [7]. And furthermore, it may decrease the Quality of Service (QoS) of both users. Thus, as one of the effective methods to coordinate with the interference, an elaborate resource sharing which includes mode selection, link allocation and power control, is of paramount importance [8].

Specifically, D2D communication either chooses cellular mode or reuse mode: *i)* cellular mode: the transmission of D2D pairs is relayed to the BS when the D2D direct link is no longer feasible or beneficial, *ii)* reuse mode: D2D pairs share links with cellular users which can further improve the spectrum efficiency [9]. Once a D2D pair chooses reuse mode, link allocation should be considered, that is, which cellular user's links will be shared. Power control includes the power control strategy of D2D pairs, as well as the reused cellular users, so as to achieve optimal network performance with high power efficiency [10].

A lot of resource sharing strategies have been proposed in literature for D2D communication underlaying cellular networks. In particular, an interference limited area (ILA) is proposed in [11], where the method does not allow the coexistence (i.e., use of the same links) of cellular users and D2D pairs to avoid strong mutual interference. [12] provides a globally optimal resource sharing strategy based on the convex optimization, to maximize the rate of one D2D pair utilizing the links of all possible cellular users in the cell, and meanwhile, the quality of the cellular communication is guaranteed. However, there is only one D2D pair considered in [11], [12]. [13] considers the scenario that D2D pairs can share the links with multiple cellular users and proposes a joint reused partner selection and power allocation strategy by utilizing the Lagrange relaxation method, while only the transmit power of D2D pairs is optimized. In [14], a heuristic algorithm is proposed to match D2D pairs and cellular users, which chooses the cellular users with higher channel gain to share with D2D pairs with lower interference gain. However, in that case, coordination between both of the users which can further improve the performance of the network is not considered. [15] proposes an energy-efficient uplink resource sharing scheme based on coalition formation game for D2D communication, which derives the solution to the joint problem of mode selection, uplink reusing allocation and power management. [16] also jointly considers three respects of resource sharing to maximize the overall network throughput, while it doesn't take into account the total power limit.

Motivated by the above literature, we study the problem of joint mode selection, link allocation and power control for D2D communication underlaying a multi-user cellular network sharing uplink resources. Our goal is to design a resource sharing scheme to maximize the sum rate of D2D pairs while guaranteeing the QoS requirements of cellular users and D2D pairs. The main contributions are summarized as follows:

i) Different from most of the previous works, we target at multiple D2D pairs sharing uplink resources of multiple cellular users in a cell, rather than only one D2D pair. Besides, we model a joint mode selection, link allocation and power control optimization problem, where the interference suffered by both kinds of users and their QoS requirements are simultaneously considered to achieve the tradeoff between the mutual interference and the rate. Moreover, the total power budget of D2D pairs is embedded.

ii) We formulate the uplink resource sharing problem into a non-convex combinatorial optimization problem, and then, we transform it into a convex one by deducing step by step. Firstly, to improve the efficiency of mode selection, we decide an interference limited area (ILA) for each potential D2D pair and then conduct mode selection based on distance. Then, to achieve the tradeoff between the efficiency and the fairness of the network, we allocate the links of cellular users based on a principle of fairness to maximize the sum rate of D2D pairs. Moreover, since the performance of the network will be dramatically improved by the coordination of two kinds of users, we use Lagrangian Algorithm to simultaneously control the power allocation for both cellular users and D2D pairs. A closed-form of the solution is provided by the proposed algorithms. Moreover, the convergence of the proposed algorithms is discussed and better performance is proved.

The remainder of the paper is organized as follows. In section II, we describe the network model and formulate the optimization problem. The joint mode selection, link allocation and power control scheme is illustrated in Section III. Section IV presents the numerical results to demonstrate the performance of the proposed scheme. Finally, Section V concludes the paper.

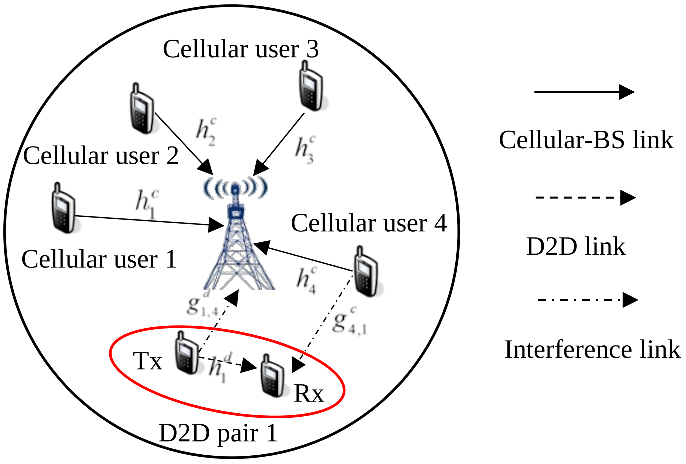


Fig. 1. System model of underlaying D2D communication.

II. Problem Formulation

As shown in **Fig. 1**, we consider a single cell with one BS, L cellular users and M potential D2D pairs, and the later form sets $\mathcal{C}=\{1,\ldots,L\}$ and $\mathcal{D}=\{1,\ldots,M\}$, respectively. We assume that there are L orthogonal frequency band and each cellular user occupies one of them to communicate with the BS. Here, we mainly consider the co-existence of cellular mode and reuse mode. For the latter, the link sharing between D2D pairs and cellular users can dramatically improve the spectrum efficiency. Moreover, the link allocation is important for making the most of the cellular links and alleviating the interference. In this way, we define the link assignment indicator as $\omega_{i,j}$. If potential D2D pair j reuses the link of cellular user i, $\omega_{i,j}=1$; Otherwise, $\omega_{i,j}=0$. In order to avoid the uncontrollable interference, we here consider that each D2D pair can only reuse the link of one cellular user and the link of each cellular user can be reused by at most one D2D pair. Accordingly, $\sum_{i=1}^{L}\omega_{i,j}\le 1,\ \forall j\in\mathcal{D}$, and $\sum_{j=1}^{M}\omega_{i,j}\le 1,\ \forall i\in\mathcal{C}$.

For the reuse mode, the Signal to Interference plus Noise Ratio (SINR) of cellular user i and D2D pair j are given respectively by

$$\gamma_i^c=\frac{p_i h_i^c}{\sigma_N^2+\sum_{j=1}^{M}\omega_{i,j}q_j g_{j,i}^d} \tag{1}$$

$$\gamma_j^d=\frac{q_j h_j^d}{\sigma_N^2+\sum_{i=1}^{L}\omega_{i,j}p_i g_{i,j}^c} \tag{2}$$

where σ_N^2 represents additive Gaussian noise variance on each frequency band of cellular users and D2D pairs. h_i^c is denoted as the channel coefficient from cellular user i to the BS. Similarly, $g_{i,j}^c$, h_j^d, and $g_{j,i}^d$ are denoted as the channel coefficients between cellular user i and the receiver of D2D pair j, between D2D pair j and between the transmitter of D2D pair j to the BS on frequency i, respectively. Denote p_i and q_j as the transmit power of cellular user i and D2D pair j, respectively.

Thus, the rate of cellular user i and D2D pair j can be expressed respectively as

$$R_i^c=log_2(1+\gamma_i^c) \tag{3}$$

$$R_j^d=log_2(1+\gamma_j^d) \tag{4}$$

In order to maximally achieve the benefits brought by D2D communications and guarantee the communication quality of the cellular users and potential D2D pairs, we propose an elaborate resource sharing scheme. That is, we maximize the sum rate of D2D pairs which reuse the links of cellular users, by means of elegant link allocation for D2D pairs and reasonable power control strategy for both potential D2D pairs and cellular users. And at the same time, we satisfy the QoS requirements of cellular users and potential D2D pairs. We formulate the joint mode selection, link allocation and power control optimization problem in D2D communication underlaying cellular networks as

$$\max_{\mathbf{p},\mathbf{q},\boldsymbol{\omega}}\ \sum_{i=1}^{L}\sum_{j=1}^{M}\omega_{i,j}R_j^d(\mathbf{p},\mathbf{q},\boldsymbol{\omega}) \tag{5}$$

$$\text{s.t}\quad R_i^c\ge R_{i,\min}^c,\ \forall i\in\mathcal{C} \tag{5a}$$

$$R_j^d\ge R_{j,\min}^d,\ \sum\nolimits_{i=1}^{L}\omega_{i,j}\ne 0,\forall j\in\mathcal{D} \tag{5b}$$

$$0\le p_i\le P_i,\ \forall i\in\mathcal{C} \tag{5c}$$

$$0\le q_j\le Q_j,\ \forall j\in\mathcal{D} \tag{5d}$$

$$\sum_{i=1}^{L}\sum_{j=1}^{M}\omega_{i,j}q_j\le Q \tag{5e}$$

$$\omega_{i,j}\in\{0,1\},\ \forall i\in\mathcal{C},\ j\in\mathcal{D} \tag{5f}$$

$$\sum_{i=1}^{L} \omega_{i,j} \le 1, \quad \forall j \in \mathcal{D} \tag{5g}$$

$$\sum_{j=1}^{M} \omega_{i,j} \le 1, \quad \forall i \in \mathcal{C} \tag{5h}$$

where we denote the power vectors of the cellular users in $\mathcal{C}$ and the potential D2D pairs in $\mathcal{D}$ as $\mathbf{p} = \left\{ p_i \middle| \ i \in \mathcal{C} \right\}$ and $\mathbf{q} = \left\{ q_j \middle| \ j \in \mathcal{D} \right\}$, and $\boldsymbol{\omega} = \left\{ \omega_{i,j} \middle| \ i \in \mathcal{C},\ j \in \mathcal{D} \right\}$ as the matrix which contains the link assignment indicator $\omega_{i,j}$. $R^c_{i,\min}$ and $R^d_{\min}$ denote the rate threshold of cellular user i and D2D pair j, respectively. The objective function in (5) characterizes the gains for forming the D2D communications, in terms of the sum rate of D2D pairs. (5a) and (5b) guarantee that cellular users and D2D pairs satisfy their minimum QoS requirements in terms of rate. (5c) and (5d) ensure that the transmit powers of cellular users and D2D pairs cannot exceed the maximum limit P_i and Q_j, respectively. (5e) indicates that Q is the total power budget of the D2D pairs sharing all links. (5g) and (5h) guarantee that each D2D pair can only reuse the link of one cellular user and each cellular user can only be reused by at most one D2D pair, respectively.

According to the problem in (5), we can obtain $\boldsymbol{\omega}$ which indicates the solution to the joint problem of mode selection and link allocation, that is, potential D2D pair j will reuse the link of cellular user i with $\omega_{i,j} = 1$. Not only that, we can determine the mode selection of potential D2D pair j based on $\boldsymbol{\omega}$. That is, if $\omega_{i,j} = 0$, the potential D2D pair chooses the cellular mode; Otherwise, it selects the reuse mode. Also, the transmit power of cellular users and potential D2D pairs is derived according to the values of $\mathbf{p}$, $\mathbf{q}$.

III. Joint Mode Selection, Link Allocation and Power Control Scheme

We formulate the joint mode selection, link allocation and power control problem into a multi-objective non-convex combinatorial one, which turns out to be strongly NP-hard. Obviously, it is very hard to obtain the optimal solution to (5) [17]. Besides, we consider the tradeoff between the efficiency and the fairness for underlaying D2D communication. Most of the existing literature decompose the optimization problem into a several sub-problems. However, they ignore fairness among D2D pairs. We will divide the optimization problem into a sequence of sub-problems involving mode selection, link allocation and power control respectively. And this way, we can transform the problem in (5) into a convex one. Then, we propose a resource sharing scheme which can guarantee that the D2D pairs even in bad channel condition can also find a cellular user to reuse the link.

Firstly, we determine whether a potential D2D pair can be admitted to reuse the links of cellular users or not and obtain a set of cellular candidates according to a minimum distance metric. Based on this, each potential D2D pair conducts mode selection. The second one is link allocation, where we find the proper reused cellular partner from the cellular candidates for each D2D pair while guaranteeing fairness. The last one is the joint power control for D2D pairs and their reused cellular users based on Lagrangian Algorithm to maximize the sum rate of D2D pairs.

A. Distance-Oriented Mode Selection for D2D Pairs

When considering which mode to choose for potential D2D pairs, we first determine whether a potential D2D pair can be admitted to reuse the uplink resources of cellular users. If a D2D pair is admitted, it selects reuse mode, and furthermore, we need to find a suitable reused cellular partner to offer its link for it. If not, it has to choose cellular mode, instead.

If a potential D2D pair is admitted to reuse a cellular user's link, $\omega_{i,j} = 1$, and the constraints in (5a), (5b), (5c), (5d) must be satisfied, which indicate that D2D pair j can share the link with cellular user i only when their QoS requirements are satisfied and their transmit power are within the maximum requirements. Consequently, we obtain a set of reused cellular candidates for D2D pair j and denote it as $\mathcal{I}_j$. It is obvious that D2D pair j can choose the reuse mode if and only if $\mathcal{I}_j \neq \varnothing$. Since the channel condition and the mutual interference mainly hinge on distance, we will determine an interference limited area (ILA) for each potential D2D pair based on the distance between cellular users and the potential D2D pair, outside of where the cellular users in $\mathcal{I}_j$ are located.

The constraints in (5a), (5b), (5c), (5d) can be easily transformed into a linear programming problem, as indicated in **Fig. 2**. The line l_c and l_d represent constraints (5a) and (5b) with equality, respectively. And $p_{i,\min}$, $q_{j,\min}$ are denoted as the minimum power of cellular user i and potential D2D pair j with no link sharing between them, respectively. As a result, there will be a feasible area, as the shadow part shown in **Fig. 2**, where both cellular user i and D2D pair j can manage an appropriate transmit power to coordinate the mutual interference, and thus, cellular user i can be a reused candidate for D2D pair j. Then, all of the cellular users satisfying ILA principle can be included in $\mathcal{I}_j$.

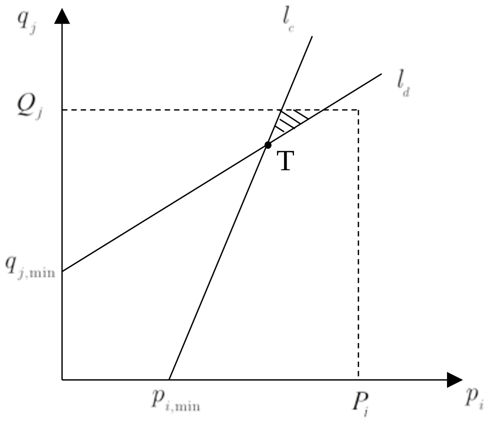


Fig. 2. Admission control of D2D pair j.

To guarantee the existence of a feasible solution of the linear programming problem, point T, the intersection of line l_c and l_d, as shown in **Fig. 2**, must be within the square area. That is, the coordinate of T, $(p_{i,T}, q_{j,T})$, which represent the transmit power of cellular user i and D2D pair

j, respectively, must meet constraints (5c) and (5d). Then, there will be a shadow area as shown in **Fig. 2**. By computing the intersection point of l_c and l_d, $p_{i,T}$ and $q_{j,T}$ can be obtained, which represent the minimum transmit power of cellular user i and D2D pair j. respectively. Thus, the admission condition is,

$$\begin{cases} 0 < p_{i,T} \le P_i \\ 0 < q_{j,T} \le Q_j \end{cases} \tag{6}$$

Combined with the distance based path loss model, $h_{i,j} = Kd_{i,j}^{-\alpha}$, we can impose a distance control between cellular users and potential D2D pairs to decide $\mathcal{I}_j$. Denote $L_{i,j}$ as the distance between cellular user i and the receiver of potential D2D pair j. And then the following proposition characterizes the admission control of potential D2D pair j.

Proposition 1. *Potential D2D pair j can be admitted to reuse the uplink resource of cellular user i, if and only if $L_{i,j} \ge L_{i,j}^{\min}$, where $L_{i,j}^{\min}$ is the minimum distance limit between cellular user i and the receiver of potential D2D pair j, and it is given by:*

$$i)\left\{\frac{KP_i(2^{R_{i,\min}^c}-1)(2^{R_{j,\min}^d}-1)g_{j,i}^d}{P_ih_i^ch_j^d-(2^{R_{i,\min}^c}-1)\sigma_N^2[h_j^d+(2^{R_{j,\min}^d}-1)g_{j,i}^d]}\right\}^{\frac{1}{\alpha}}, \text{ if } \log_2(1+\frac{P_ih_i^c}{Q_jg_{j,i}^d+\sigma_N^2}) \le R_{i,\min}^c,$$

$$ii)\ [\frac{K(2^{R_{i,\min}^c}-1)(2^{R_{j,\min}^d}-1)(Q_jg_{j,i}^d+\sigma_N^2)}{Q_jh_i^ch_j^d-(2^{R_{j,\min}^d}-1)\sigma_N^2h_i^c}]^{\frac{1}{\alpha}}, \text{ if } \log_2(1+\frac{P_ih_i^c}{Q_jg_{j,i}^d+\sigma_N^2}) \le R_{i,\min}^c.$$

Proof. The proof is given in Appendix A.

Algorithm 1: Mode Selection Algorithm (MSA)

```
for all $j \in \mathcal{D}$ do
  for all $i \in \mathcal{C}$ do
    calculate $L_{i,j}$ and $L_{i,j}^{\min}$
    if $L_{i,j} \ge L_{i,j}^{\min}$ then
      potential D2D pair $j$ is admitted to reuse the link of cellular user $i$ and
      chooses reuse mode and $i \in \mathcal{I}_j$
    else
      potential D2D pair $j$ chooses cellular mode
    end if
  end for
end for
```

The mode selection algorithm is given in Algorithm 1 as MSA. For further explanation, a D2D pair can only share links with the cellular users distributed outside of its ILA, since the D2D pair must be far away enough from the potential cellular partner, and hence, the mutual interference between them will not be too serious to decrease their QoS, as shown in line 4 to 6. However, if all of the cellular users in the network are located within a potential D2D pair's ILA, as shown in line 7 to 8, the potential D2D pair has to select cellular mode, instead.

B. Link Allocation for D2D Pairs

We have divided an ILA and obtained a set of cellular candidates for each potential D2D pair in the previous part. Here, this subsection focuses on how to choose an appropriate cellular candidate in $\mathcal{I}_j$ for each D2D pair with reuse mode to maximize the sum rate of the D2D pairs while making sure that each of them can share a cellular user's link.

Without loss of generality, we assume that each cellular user and potential D2D pair transmit with the same power respectively, which guarantees that each potential D2D pair has the same possibility to reuse the links of cellular users and as well as the same possibility for cellular users to be reused in terms of transmit power.

Algorithm 2: Link Allocation Algorithm (LAA)

```
1: $\mathcal{I}_j$ : Set of reused cellular candidates of D2D pair $j$
2: $\mathcal{M}$ : Set of D2D pairs with $\left|\mathcal{I}_j\right|$ in increasing order
3: for all $j \in \mathcal{D}$ do
4:     find $\left|\mathcal{I}_j\right| = 1$
5: end for
6: if there is more than one D2D pair that satisfies $\left|\mathcal{I}_j\right| = 1$ and the $i \in \mathcal{I}_j$ is the
7:     same then
8:     find $j^* = \arg \max_{i \in \mathcal{I}_{j^*}} R^d_{j^*}$ and D2D pair $j^*$ reuses the link of cellular
9:     user $i$, $\omega_{i,j^*} = 1$
10:    delete $i$ from other D2D pairs' $\mathcal{I}_j$ and upgrade $\left|\mathcal{I}_j\right|$ and $\mathcal{M}$
11: end if
12: for all $j \in \mathcal{M} \setminus \left\{j^*\right\}$
13:    choose cellular users in $\mathcal{I}_j$ for D2D pair $j$ in the order of $\mathcal{M}$ and determine $\boldsymbol{\omega}$
14: end for
15: find $\max \sum_{j=1}^{M} \sum_{i=1}^{L} \omega_{i,j} R^d_j(\mathbf{p}, \mathbf{q}, \boldsymbol{\omega})$
```

The link allocation algorithm is summarized in Algorithm 2. Three steps are developed in the algorithm. Firstly, we calculate $\left|\mathcal{I}_j\right|$, the number of cellular users in $\mathcal{I}_j$, and then obtain

$\mathcal{M}$, which contains the D2D pairs with $|\mathcal{I}_j|$ in increasing order. Based on this, we allocate cellular users preferentially to those with less cellular candidates to reuse and make sure each D2D pair will find a partner. However, different D2D pairs may have only one cellular candidate which may be the same. Thus, if there is more than one D2D pair that can only reuse the link of cellular candidate i, we allocate cellular user i and its link to D2D pair j^* which denotes the D2D pair achieving the maximum rate compared to others, as shown in line 4 to 11. Finally, we pair the rest of D2D pairs with their cellular candidates in the order of $\mathcal{M}$, as line 12 to 15 indicate. Then, we will find a union of D2D pairs with their reused cellular users which can maximize our optimization object $\sum_{i=1}^{L}\sum_{j=1}^{M}\omega_{i,j}R_j^d(\mathbf{p},\mathbf{q},\boldsymbol{\omega})$. Therefore, we guarantee the fairness for D2D pairs, and also achieve the maximum benefit in terms of rate in that case.

C. Optimal Power Control for D2D Pairs and Reused Cellular Users

In previous subsections, we have paired D2D pairs with proper cellular users and obtain $\boldsymbol{\omega}$. Then, for the paired cellular user i and D2D pair j, $\omega_{i,j} = 1$, and we denote $\alpha_{i,j} = h_j^c / \sigma_N^2$, $\beta_{i,j} = h_j^d / \sigma_N^2$, $\mu_{i,j} = g_{j,i}^d / \sigma_N^2$, and $\theta_{i,j} = g_{i,j}^c / \sigma_N^2$ as the normalized channel coefficients. And the rate of cellular user i and D2D pair j are respectively expressed as,

$$R_i^c = log_2(1 + \frac{p_i\alpha_{i,j}}{1 + q_j\mu_{i,j}}) \tag{9}$$

$$R_j^d = log_2(1 + \frac{q_j\beta_{i,j}}{1 + p_i\theta_{i,j}}) \tag{10}$$

This subsection will illustrate an optimal power control strategy for all these paired users to solve the problem in (5).

Proposition 2. *Denote* (p_i^*, q_j^*) *as the optimal power control strategy for problem (5) ,and* $k_j = 2^{R_{i,\min}^c} - 1$ *.Then, for all* $j = 1, \ldots, M$ *, we define* $A_{i,j} = k_j\mu_{i,j}\theta_{i,j}(\alpha_{i,j}\beta_{i,j} + k_j\mu_{i,j}\theta_{i,j})$ *,* $B_{i,j} = (\alpha_{i,j} + k_j\theta_{i,j})(2k_j\mu_{i,j}\theta_{i,j} + \alpha_{i,j}\beta_{i,j})$ *,* $C_{i,j} = (\alpha_{i,j} + k_j\theta_{i,j})(\frac{\alpha_{i,j} + k_j\theta_{i,j} - \alpha_{i,j}\beta_{i,j}}{\lambda})$ *and renew* $Q_j = \min\left\{Q_j, \frac{\alpha_{i,j}P_i - k_j}{k_j\mu_{i,j}}\right\}$. *We can get the optimal solution as, i) if* $\sum_{i=1}^{L}\sum_{j=1}^{M}\omega_{i,j}Q_j \le Q$*, then,* $q_j^* = Q_j$ *,* $p_i^* = \frac{k_j(1 + \mu_{i,j}Q_j)}{\alpha_{i,j}}$ *. ii) if* $\sum_{i=1}^{L}\sum_{j=1}^{M}\omega_{i,j}Q_j > Q$ *, then,* $q_j^* = \left[\frac{\sqrt{B_{i,j}^2 - 4A_{i,j}C_{i,j}(\lambda)} - B_{i,j}}{2A_{i,j}}\right]_0^{Q_j}$ *,* $p_i^* = \frac{k_j(1 + \mu_{i,j}q_j^*)}{\alpha_{i,j}}$*, where* $\left[\cdot\right]_0^{Q_j}$ *signifies the projection onto the interval* $\left[0, \mathrm{Q}_j\right]$.

Proof. The proof is given in Appendix B.

Algorithm 3: Power Control Algorithm (PCA)

```
for all cellular user $i \in \mathcal{C}$ and D2D pair $j \in \mathcal{D}$ do
    calculate $Q_j = \min\left\{Q_j, \frac{\alpha_{i,j}P_i - k_j}{k_j\mu_{i,j}}\right\}$
end for
if $\sum_{i=1}^{L}\sum_{j=1}^{M}\omega_{i,j}Q_j \le Q$ then
    $q_j^* = Q_j$ and $p_i^* = \frac{k_j(1+\mu_{i,j}Q_j)}{\alpha_{i,j}}$
else
    use Lagrangian Algorithm and establish the following
    $l(\mathrm{q},\lambda) = \ln 2\sum_{j=1}^{N}\log_2(1+\frac{\alpha_{i,j}\beta_{i,j}q_j}{\alpha_{i,j}+k_j\theta_{i,j}+k_j\mu_{i,j}\theta_{i,j}q_j}) + \lambda(Q-\sum_{j=1}^{N}q_j)$ to calculate $q_j^*$ and $p_i^*$
end if
```

The power control algorithm is illustrated in Algorithm 3 as PCA. For those paired D2D pairs and cellular users, we control the transmit power of them simultaneously to achieve our ultimate optimization object towards the benefits brought by the entire D2D pairs.

We can see that the optimal solution illustrated in Proposition 2 is highly bound up with the Lagrangian multiplier λ, which is extremely difficult to obtain with direct computing. We can primarily determine an interval $\left[0, \lambda_{\max}\right]$, where $\lambda_{\max} = \max_{j\in\mathcal{S}}\left\{\frac{\alpha_{i,j}\beta_{i,j}}{\alpha_{i,j}+k_j\theta_{i,j}}\right\}$, according to $C_{i,j}(\lambda) \le 0$, which guarantees $B_{i,j}^2 - 4A_{i,j}C_{i,j}(\lambda)$ in Proposition 2 positive, indicating there is a feasible solution. Besides, considering that q_j^* is monotonically decreasing with the increasing of λ, and that we can certainly find a positive solution in the feasible region, we exploit bisection method to obtain the optimal λ^*. Hence, we can finally decide an optimal λ^* to get the optimal joint power control strategy as (p_i^*, q_j^*).

IV. Numerical Results

In this section, we carry out several numerical simulation and presents some results to evaluate the performance of our proposed algorithms. The simulation setup is as follows: consider a hexagonal cell with a radius of 250m, where the BS is centered, and multiple cellular users and potential D2D pairs are randomly distributed. Each potential D2D pair is uniformly distributed at a distance of 50m. The channel power gain is modeled as $h_{i,j} = Kd_{i,j}^{-\alpha}$, where K, α are set to 10 and 4, respectively. The power spectral density of additive white Gaussian noise is 10^{-8} W/Hz, and the QoS threshold of cellular users and D2D pairs in terms of rate is $R_{i,\min}^{c} = R_{j,\min}^{d} = 1$ bit/s. It is verified in [12] that the maximum SNR of cellular user i is $\alpha_i P_i$, and without loss of generality, in order to reach the same maximum SNR, we set corresponding P_i for each cellular user according to their channel condition. Moreover, we denote the total D2D SNR as Q / σ_N^2 defined in [12].

A. Convergence

We give one realization of the single cell system with 16 cellular users and 4 potential D2D pairs, where all of the 4 potential D2D pairs are finally admitted to reuse the links of their cellular partners, according to the simulation result. We plot their transmit power with iteration times in **Fig. 3**. The transmit power is adjusted by our proposed scheme illustrated in Section III and the stopping parameter is set to be 10^{-5}. One can see that the transmit power of two kinds of users converges to equilibrium in approximately 6 iterations and the tendency of them to converge is roughly the same. From **Fig. 3** and **Fig. 4**, we can see that the transmit power, as well as the rate of D2D pairs is close to each other in our proposed algorithms. Since each D2D pair chooses an appropriate cellular partner, the mutual interference can be coordinated and a kind of balance in terms of the channel condition is achieved. Thus, limited to a total power budget, the whole process of resource sharing for those D2D pair guarantees fairness between them, which contributes to an average performance for them.

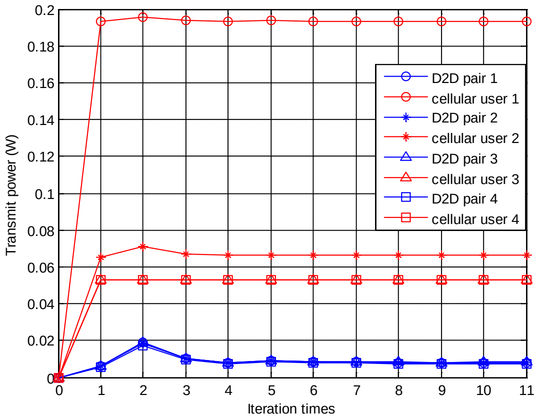


Fig. 3. Transmit power of two kinds of users with iteration times.

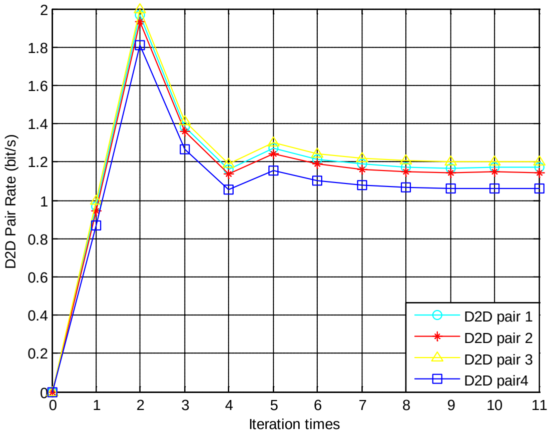


Fig. 4. D2D pair rate with iteration times.

To characterize the fairness of our proposed algorithms in a quantitative way, we formulate Jain's fairness index as $\mathcal{J}(R_1^d, R_2^d, \ldots, R_j^d, \ldots, R_N^d) = \frac{(\sum_{j=1}^{N} R_j^d)^2}{N \cdot \sum_{j=1}^{N} R_j^d}$ [18], where R_j^d represents the rate of D2D pair j, and N is the total number of D2D pairs. The result ranges from $\frac{1}{n}$ (worst case) to 1 (best case). **Table. 1** illustrates the fairness performance towards the rate of D2D pairs and we obtain Jain's fairness index with iteration times. From **Table. 1**, we can see that Jain's fairness index in our proposed algorithm is very close to 1, which corresponds to the best case. It indicates that with the proposed link allocation algorithm, together with our appropriate mode selection and power control algorithms, the performance of each D2D pair is balanced.

Table 1. Jain's fairness index with iteration times

Iteration times	Jain's fairness index	Iteration times	Jain's fairness index
1	0.9974	6	0.9980
2	0.9986	7	0.9980
3	0.9983	8	0.9980
4	0.9979	9	0.9980
5	0.9981	10	0.9980

B. Performance with Different Numbers of Cellular Users

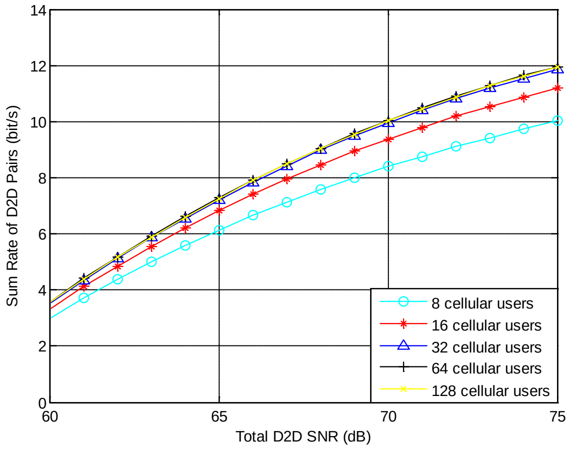


Fig. 5. The sum rate of D2D pairs with total D2D SNR for different number of cellular users.

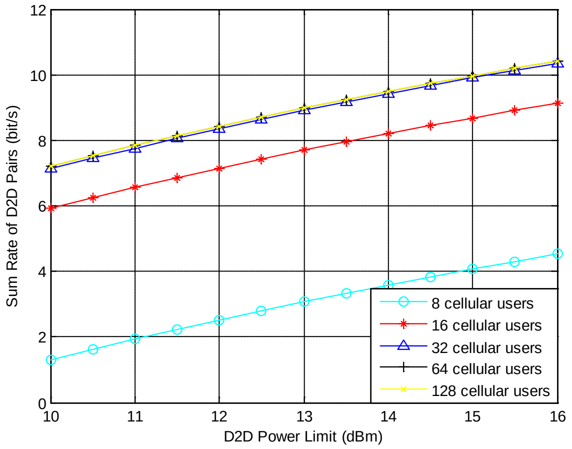


Fig. 6. The sum rate of D2D pairs with D2D power limit for different number of cellular users.

To study the effect of total power budget, we plot **Fig. 5** to display the sum rate of D2D pairs with different total D2D SNR, which represents the total power budget normalized with white Gaussian noise. Besides, we compare the performance for different number of cellular users (8,16,32,64 and 128) in a single cell, where 4 D2D pairs are potential to be introduced. We can see that all curves go up with higher total D2D SNR, since higher total D2D SNR means higher power allocated to all D2D pairs. Thus, it can lead to higher rate. With the increasing of the number of cellular users, the sum rate of D2D pairs also increases, which indicates that the number of cellular users has positive impacts on the performance of D2D pairs. However, the speed of the increasing becomes fairly slower gradually. When there are only 8 or 16 cellular users for the 4 potential D2D pairs we set in the cell, the sum rate of these D2D pairs increases obviously. However, when the number of cellular users is up to 32 or above, the performance towards the sum rate is inclined to approach. Note that, for a certain number of potential D2D pairs, if there are only a few cellular users available, there is a larger space for them to improve their performance, and their sum rate will increase remarkably as the number of cellular users increases. While if they already have well enough cellular users to reuse, they can achieve a relatively high rate on the whole. Thus, the influence of increasing the number of cellular users will be less obvious.

As **Fig. 6** shows, we take D2D power limit for the research subject, which is also a crucial parameter in D2D communication. One can see that the sum rate of D2D pairs is almost linearly increased by increasing the power limit of each D2D pair. And at a fixed power limit, the D2D pairs with more cellular users to reuse perform better. Actually, with the increasing of D2D power limit, D2D pairs will choose different cellular users to reuse and they are more likely to obtain higher rate. As such, if there exist enough of cellular users in a cellular network to reuse, we can certainly achieve the reuse gain of the D2D pairs.

C. Performance of Different Algorithms

As a comparison, we compare our proposed algorithms with two other algorithms,

i) Global search algorithm + PCA: It does not determine an ILA for each D2D pair and consider fairness in the process of matching D2D pairs and corresponding cellular users. It conducts global search when choosing a cellular user for a D2D pair. For comparison, we incorporate our mode power control algorithm into it.

ii) MSA + LAA + Suboptimal power control algorithm [12]: Based on waterfilling algorithm, it simply exploits the maximum power for the reused cellular users. And then, suitable power is allocated to the paired D2D pairs according to the channel state and the constraints in (5). Our proposed mode selection algorithm and link allocation algorithm are also incorporated to compare the proposed power control algorithm with the suboptimal power control algorithm.

We consider a cell, where 16 cellular users and 4 potential D2D pairs are distributed. **Fig. 7** illustrates that our proposed algorithm can achieve higher D2D rate than suboptimal power control strategy, while lower than global search algorithm. After determining an ILA for each D2D pair in the first step, our proposed algorithm behaves better than global search in the process of pairing, with regards to computational complexity. Moreover, the suboptimal power control only optimizes the power of D2D pairs with a fixed cellular users' power, where the coordination of these two kinds of users is not considered. As indicated in **Fig. 8**, our algorithm can also bring benefits for the cellular users. The average energy efficiency of cellular users decreases as the total D2D SNR increases, since D2D pairs transmit with a higher power which causes more serious interference to their paired cellular users. As a result, the cellular users have to increase their transmit power to reach the QoS threshold. The simulation result displays that, our proposed algorithm can perform better than two other algorithms. In global search, D2D pairs are inclined to choose cellular partners which are far away from them and also the BS, and it may generate less interference. Thus, cellular users have to use a higher transmit power to satisfy their QoS requirement. Moreover, in the suboptimal power control strategy, cellular users simply exploit their maximum power, contributing to relatively low energy efficiency. Therefore, by jointly optimizing the power of D2D pairs and cellular users while satisfying the total SNR constraint, the proposed algorithms achieve better performance.

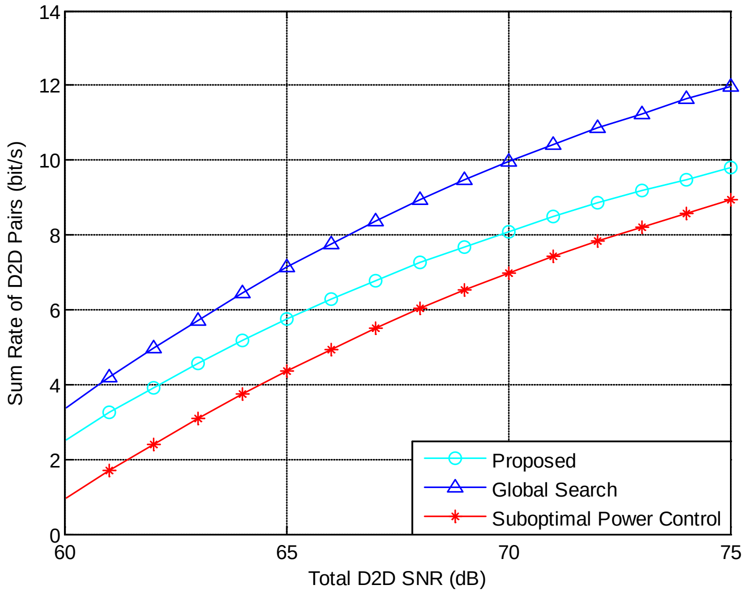


Fig. 7. The sum rate of D2D pairs with total D2D SNR.

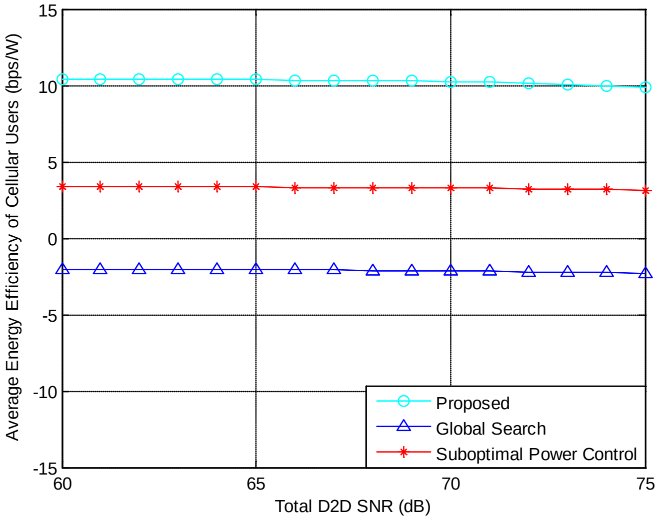


Fig. 8. Average energy efficiency of cellular users with total D2D SNR.

From **Fig. 9**, one can see that we obtain a higher sum rate of D2D pairs than suboptimal power control when D2D power limit increases, while a slightly lower one than global search. In **Fig. 10**, the performance of suboptimal power control strategy towards average energy efficiency of cellular users is closer to the proposed algorithm. However, the performance of global search remains the worst and still maintains at a low level, and the performance gap remains with larger D2D power limit. Note that, when D2D power limit is small, which also means low transmit power for D2D pairs, the transmit power of cellular users cannot be too high to affect D2D pairs in the proposed scheme. And then cellular users will achieve a low rate. In the suboptimal power control strategy, cellular users use a fixed power and their rate keeps at a fairly high level. Compromising with the effect of suboptimal power control, our proposed algorithm achieves a close performance with suboptimal power control strategy.

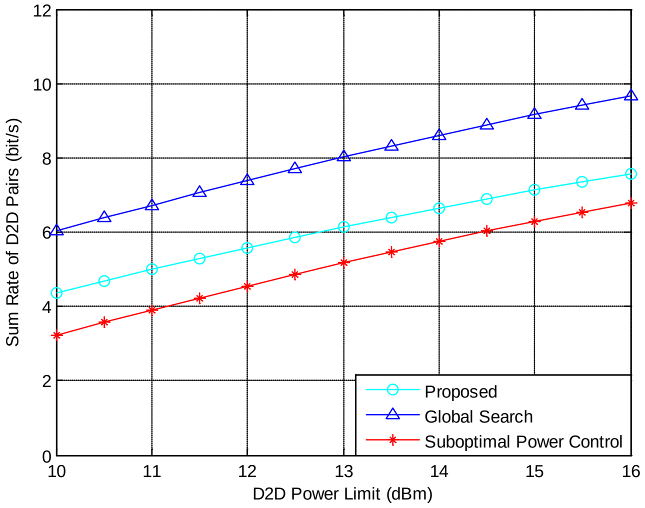


Fig. 9. The sum rate of D2D pairs with D2D power limit.

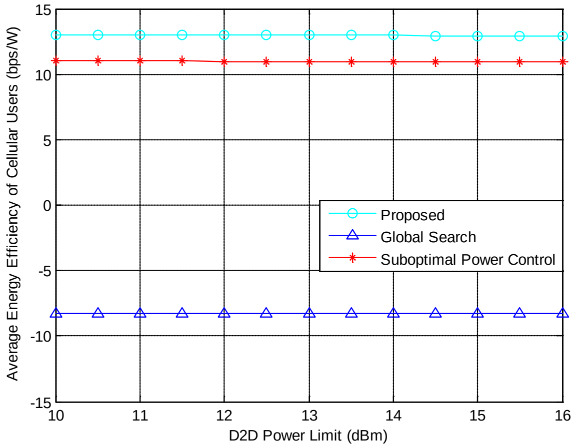


Fig. 10. Average energy efficiency of cellular users with D2D power limit.

V. Conclusion

In this paper, we studied D2D communication underlaying cellular networks in a multi-user cell by jointly considering mode selection, link allocation and power control. We proposed a joint mode selection, link allocation and power control optimization scheme to solve the resulting non-convex problem and consider the tradeoff between the efficiency and the fairness of the underlaying D2D communication. Finally, we provided a closed-form solution. In particular, we maximized the sum rate of D2D pairs with guaranteed QoS requirements of cellular users and limited power resources. Moreover, to improve efficiency in the process of mode selection, we firstly conducted admission control based on a minimum distance metric. And we set a fairness principle to ensure that each D2D pair can share the link with a cellular partner. Joint power control strategy was also proposed to coordinate between cellular users and D2D pairs. Simulation results demonstrated that the proposed scheme brings substantial performance improvements towards the sum rate of D2D pairs and also the energy efficiency of cellular users with limited resources. Besides, our algorithms can converge to a stable equilibrium within a fairly short time.

APPENDIX A

Proof of Proposition 1

According to the constraint (6), we can simplify it into the following

$$g^c_{i,j} \le \begin{cases} \dfrac{P_i h^c_i h^d_j - (2^{R^c_{i,\min}} - 1)\sigma_N^2 [h^d_j + (2^{R^d_{j,\min}} - 1) g^d_{j,i}]}{P_i (2^{R^c_{i,\min}} - 1)(2^{R^d_{j,\min}} - 1) g^d_{j,i}} = g_c \\ \dfrac{Q_j h^c_i h^d_j - (2^{\gamma_j} - 1)\sigma_N^2 h^c_i}{(2^{R^c_{i,\min}} - 1)(2^{R^d_{j,\min}} - 1)(Q_j g^d_{j,i} + \sigma_N^2)} = g_d \end{cases} \tag{7}$$

Finally, we can obtain the distance limit as

$$L_{i,\mathrm{j}}^{\min} = \begin{cases} \left\{ \dfrac{KP_i(2^{R_{i,\min}^c}-1)(2^{R_{j,\min}^d}-1)g_{j,i}^d}{P_i h_i^c h_j^d - (2^{R_{i,\min}^c}-1)\sigma_N^2[h_j^d + (2^{R_{j,\min}^d}-1)g_{j,i}^d]} \right\}^{\frac{1}{\alpha}}, \\ \text{if } \log_2(1+\dfrac{P_i h_i^c}{Q_j g_{j,i}^d + \sigma_N^2}) \le R_{i,\min}^c \\ [\dfrac{K(2^{R_{i,\min}^c}-1)(2^{R_{j,\min}^d}-1)(Q_j g_{j,i}^d + \sigma_N^2)}{Q_j h_i^c h_j^d - (2^{R_{j,\min}^d}-1)\sigma_N^2 h_i^c}]^{\frac{1}{\alpha}}, \\ \text{if } \log_2(1+\dfrac{P_i h_i^c}{Q_j g_{j,i}^d + \sigma_N^2}) \le R_{i,\min}^c \end{cases} \tag{8}$$

Then, we get Proposition 1.

APPENDIX B
Proof of Proposition 2

As defined in (10), the rate of D2D pair j is monotonically decreasing as p_i increases for fixed q_j. In the first place, we have to determine the reasonable range of p_i. To satisfy the constraint in (5a), we can get $p_i \ge \dfrac{k_j(1+\mu_{i,j}q_j)}{\alpha_{i,j}}$, and it is obvious that the optimal p_i^* is obtained when $p_i^* = p_{i,\min} = \dfrac{k_j(1+\mu_{i,j}q_j)}{\alpha_{i,j}}$. Then, we substitute it into (10) and it can be further expressed as

$$R_j^d(p_i^*, q_j) = \log_2(1+\frac{\alpha_{i,j}\beta_{i,j}q_j}{\alpha_{i,j} + k_j\theta_{i,j} + k_j\mu_{i,j}\theta_{i,j}q_j}) \tag{11}$$

Now, the center of our investigation is how to allocate q_j in (11) to finally solve the problem in (5).

Denote $h(q_j) = \dfrac{\alpha_{i,j}\beta_{i,j}q_j}{\alpha_{i,j} + k_j\theta_{i,j} + k_j\mu_{i,j}\theta_{i,j}q_j}$, and it can be easily confirmed that $h^{'}(q_j) \ge 0$ and $h^{''}(q_j) \le 0$. We can obviously see that $h(q_j)$ is a concave function and increases with the increasing of q_j, which leads to that $R_j^d(p_i^*, q_j)$ is also concave and increases by increasing q_j. Until now, we have transformed the non-convex problem in (5) into a convex one by deducing step by step.

To maximize $R_j^d(p_i^*, q_j)$ in (11), it is essential to determine the feasible region of q_j, since $R_j^d(p_i^*, q_j)$ expressed in (11) is monotonically increasing in q_j. The power constraint in (5c) is equal to $q_j \le \dfrac{\alpha_{i,j}P_i - k_j}{k_j\mu_{i,j}}$, therefore, we can renew the power limit of q_j as

$Q_j = \min\left\{Q_j, \frac{\alpha_{i,j}P_i - k_j}{k_j\mu_{i,j}}\right\}$. Given the total power budget, we should consider two different situations, *i)* if $\sum_{i=1}^{L}\sum_{j=1}^{M}\omega_{i,j}Q_j \le Q$, which means the total maximum power of the D2D pairs are within the power budget , then, we can take $q_j^* = Q_j$ for each D2D pair to maximize $R_j^d(p_i^*, q_j^*)$. And accordingly, $p_i^* = \frac{k_j(1+\mu_{i,j}Q_j)}{\alpha_{i,j}}$. *ii)* if $\sum_{i=1}^{L}\sum_{j=1}^{M}\omega_{i,j}Q_j > Q$, we use Lagrangian Algorithm to solve the optimization problem, instead. And we can write the problem as $l(\mathrm{q},\lambda) = \ln 2\sum_{j=1}^{N}\log_2(1+\frac{\alpha_{i,j}\beta_{i,j}q_j}{\alpha_{i,j}+k_j\theta_{i,j}+k_j\mu_{i,j}\theta_{i,j}q_j}) + \lambda(Q-\sum_{j=1}^{N}q_j)$ with $\lambda \ge 0$. Then, we will investigate to find the optimal power allocation strategy with the constraint $\sum_{j=1}^{N}q_j^* = Q$.

The first partial derivative of $l(\mathrm{q},\lambda)$ with respect to λ is given as,

$$\frac{\partial l(\mathrm{q},\lambda)}{\partial q_j} = \frac{\alpha_{i,j}\beta_{i,j}(\alpha_{i,j}+k_j\theta_{i,j})}{(\alpha_{i,j}+k_j\theta_{i,j}+k_j\mu_{i,j}\theta_{i,j}q_j)^2 + \alpha_{i,j}\beta_{i,j}q_j(\alpha_{i,j}+k_j\theta_{i,j}+k_j\mu_{i,j}\theta_{i,j}q_j)} - \lambda \tag{12}$$

Then, the solution of problem (5) is obtained when $\frac{\partial l(\mathrm{q},\lambda)}{\partial q_j} = 0$ and it is apparent that there will be a feasible root only when $\lambda > 0$, since the first half part on the right-hand side in (12) is always positive. Thus, we can get a quadratic equation about q_j^* expressed as $A_{i,j}q_j^{*2} + B_{i,j}q_j^* + C_{i,j}(\lambda) = 0$, with $A_{i,j} = k_j\mu_{i,j}\theta_{i,j}(\alpha_{i,j}\beta_{i,j} + k_j\mu_{i,j}\theta_{i,j})$, $B_{i,j} = (\alpha_{i,j}+k_j\theta_{i,j})(2k_j\mu_{i,j}\theta_{i,j}+\alpha_{i,j}\beta_{i,j})$, and $C_{i,j} = (\alpha_{i,j}+k_j\theta_{i,j})(\frac{\alpha_{i,j}+k_j\theta_{i,j}-\alpha_{i,j}\beta_{i,j}}{\lambda})$, and the projection of the positive solution is onto $\left[0, \mathrm{Q}_j\right]$ for q_j^*.

References

[1] S. Xiang, T. Peng, Z. Liu, W. Wang, “A distance-dependent mode selection algorithm in heterogeneous D2D and IMT-Advanced network,” in *Proc. of 2012 IEEE Globecom Workshops*, pp. 416-420, Anaheim, Dec, 2012. Article (CrossRef Link)

[2] M. Jo, T. Maksymyuk, R. l. Batista, T. F. Maciel, A. L. F. de Almeida, and M. Klymash, “A survey of converging solutions for heterogeneous mobile networks,” *IEEE Wireless Commun.*, vol. 21, no. 6, pp.54-62, Dec 2014. Article (CrossRef Link)

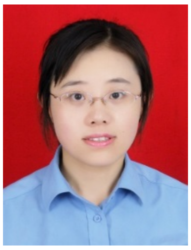
[3] M. Jo, T. Maksymyuk, B. Strykhalyuk, and C.-H. Cho, “Device-to-device based heterogeneous radio access network architecture for mobile cloud computing,” *IEEE Wireless Commun.*, vol.22, no.3, pp. 50-58, June 2015. Article (CrossRef Link)

[4] L. Wei, R. Q. Hu, Y. Qian, and G. Wu, “Enable device-to-device communications underlaying cellular networks: challenges and research aspects,” *IEEE Commun. Mag.*, vol. 52, no. 6, pp. 90-96, Jun. 2014. Article (CrossRef Link)

[5] H. Tang, Z. Ding, “Resource allocation in mixed mode device-to-device communications,” in *Proc. of IEEE International Conference on Communications (ICC)*, pp. 2554-2559, London, Jun. 2015. Article (CrossRef Link)
[6] D. Wu, L. Zhou, Y. Cai, and R. Q. Hu, “The role of mobility for D2D communications in LTE-advanced networks: energy vs. bandwidth efficiency,” *IEEE Wireless Commun. Mag.*, vol. 21, no. 2, pp. 66-71, Apr. 2014. Article (CrossRef Link)
[7] F. Wang, C. Xu, L. Song, and Z. Han, “Energy-efficient resource allocation for device-to-device underlay communication,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 2082– 2092, Apr. 2015. Article (CrossRef Link)
[8] D. Feng, L. Lu, Y. Y. Wu, G. Y. Li, S. Li, and G. Feng, “Device-to-device communications in cellular networks,” *IEEE Commun. Mag.*, vol. 52, no. 4, pp. 49-55, Apr. 2014. Article (CrossRef Link)
[9] A. Asadi, Q. Wang, and V. Mancuso, “A Survey on device-to-device communication in cellular networks,” *IEEE Commun. Surveys Tuts.*, vol. 16, no. 4, pp. 1801-1818, 2014. Article (CrossRef Link)
[10] D. Wu, Y. Cai, R. Q. Hu, and Y. Qian, “Dynamic distributed resource sharing for mobile D2D communications,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 10, pp. 5417-5429, Oct. 2015. Article (CrossRef Link)
[11] H. Min, J. Lee, S. Park, and D. Hong, “Capacity enhancement using an interference limited area for device-to-device uplink underlaying cellular networks,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 3995– 4000, Dec. 2011. Article (CrossRef Link)
[12] J. Wang, D. Zhu, C. Zhao, G. Y. Li, and M. Lei, “Resource sharing of underlaying device-to device and uplink cellular communications,” *IEEE Commun. Lett.*, vol. 17, no. 6, pp. 1148-1151, Jun. 2013. Article (CrossRef Link)
[13] L. Xu, G. Yu, and R. Yin, “Joint power allocation and reuse partner selection for device-to-device communications,” in *Proc. of IEEE 81st Vehicular Technology Conference (VTC Spring)*, pp. 1-5, Glasgow, May. 2015. Article (CrossRef Link)
[14] H. Meshgi, D. Zhao, and R. Zheng, “Joint channel and power allocation in underlay multicast device-to-device communications,” in *Proc. of IEEE International Conference on Communications (ICC)*, pp. 2937-2942, London, Jun. 2015. Article (CrossRef Link)
[15] D. Wu, J. Wang, R. Q. Hu, Y. Cai, and L. Zhou, “Energy-efficient resource sharing for mobile device-to-device multimedia communications,” *IEEE Trans. Veh. Technol.*, vol.63, no.5, pp. 2093-2103, Apr. 2014. Article (CrossRef Link)
[16] D. Feng, L. Lu, Y. Y. Wu, G. Y. Li, G. Feng, and S. Li, “Device-to-device communications underlaying cellular networks,” *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 3541-3551, Aug. 2013. Article (CrossRef Link)
[17] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge University Press, 2004.
[18] R. Jain, D. Chiu, and W. Hawe, “A quantitative measure of fairness and discrimination for resource allocation in shared systems,” Digital Equipment Corporation, DEC-TR-301, Tech. Rep., 1984. [Online]. Available: http://www1.cse.wustl.edu/～jain/papers/ftp/fairness.pdf.

Wei Zhang received her B.S. degree from the Institute of Communications Engineering, PLA University of Science and Technology, Nanjing, P. R. China. She is currently pursuing a master's degree at the same university. Her research interests are mainly in D2D communication, mobile communication, resource allocation and management.

Wanbing He received her B.S. degree from the Institute of Communications Engineering, PLA University of Science and Technology, Nanjing, P. R. China. She is currently pursuing a master's degree at the same university. Her research interests are mainly in D2D communication, communication security and game theory.



Dan Wu is an associate professor at the PLA University of Science and Technology, Nanjing, P. R. China. She received her B.S., M.S., and Ph.D. degrees from the Institute of Communications Engineering, PLA University of Science and Technology, Nanjing, P. R. China in 2006, 2009, and 2012, respectively. Her research interests are mainly in resource allocation and management, game theory, cooperative communications, and wireless sensor networks

Yueming Cai is a professor at the PLA University of Science and Technology, Nanjing, P. R. China. He received his B.S. degree in physics from Xiamen University, P. R. China, in 1982, and his M.S. degree in micro-electronics engineering and Ph.D. degree in communications and information systems from Southeast University, Nanjing, P. R. China, in 1988 and 1996, respectively. His current research interests include MIMO systems, signal processing in communications, cooperative communications, and wireless sensor networks.