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THE FRACTAL DIMENSION OF THE ρ OPHIUCUS MOLECULAR CLOUD COMPLEX

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Abstract: We estimate the fractal dimension of the ρ Ophiuchus Molecular Cloud Complex, associated with star forming regions. We selected a cube (v, l, b) database, obtained with $J = 1-0$ transition lines of ^{12}CO and ^{13}CO at a resolution of $22''$ using a multibeam receiver system on the 14-m telescope of the Five College Radio Astronomy Observatory. Using a code developed within IRAF, we identified slice-clouds with two threshold temperatures to estimate the fractal dimension. With threshold temperatures of 2.25 K (3σ) and 3.75 K (5σ), the fractal dimension of the target cloud is estimated to be D = 1.52–1.54, where $P \propto A^{D/2}$, which is larger than previous results. We suggest that the sampling rate (spatial resolution) of observed data must be an important parameter when estimating the fractal dimension, and that narrower or wider dispersion around an arbitrary fit line and the intercepts at NP = 100 should be checked whether they relate to rms noise level or characteristic structure of the target cloud. This issue could be investigated by analysing several high resolution databases with different quality (low or moderate sensitivity).

Key words: ISM: Ophiuchus Molecular Cloud – ISM: structure – ISM: fractal dimension – ISM: turbulence

1. INTRODUCTION

Molecular gas and dust have been found to be organized into irregular, but self-similar hierarchical structures over a wide range of scales through their integrated intensity maps or infrared images (Falgarone et al. 1991). This self-similarity has been interpreted as the signature of an underlying fractal geometry, relating to the physical processes supporting the generation of structures in the interstellar medium (Elmegreen & Scalo 2004). Wiliams et al. (2000) also reported that the invariance of the fractal dimension of these clouds may imply self- similar structure of the molecular clouds, and turbulent motion within them; A self-similar object is exactly or approximately similar to a part of itself. Embedded physical processes are supposed to be a consequence of turbulence, although self-gravity could also play an important role (de Vega et al. 1996). Any similarities or differences could be valuable clues to the structure of the molecular clouds and the surrounding turbulent environment. Most observed molecular clouds have highly supersonic linewidths, implying turbulent motion in them, for which one would naturally expect a fractal structure (Mandelbrot 1983). In fact, fractal structure of the interstellar medium is consistent with many observed features, including cloud size and mass distribution functions, its properties, and even the stellar mass function (Sánchez et al. 2005). One way of characterizing the structure of the molecular clouds, the birth place of all stars, is based on their fractal dimensions. Fractal dimension D of a molecular cloud can be determined from the perimeter-area relation of $P \propto A^{D/2}$. Studies of the molecular interstellar medium found a dimension of $D \sim 1.4$ (Falgarone et al. 1991; Wiliams et al. 2000). Falgarone et al. (1991) observed clouds at two different distances at high angular resolution using several transitions of CO molecule. Cloud edge regions were selected for the study to avoid the spatial crowding of emitting components which obscure the structure of cloud cores, and it was found that spatial structure existed on all scales down to linear resolution of 0.02 pc. Their final estimate of the fractal dimension was $D = 1.36$. Recently its best value was determined by $D = 1.4$ (Wiliams et al. 2000). However, these are all for the small clouds located in the Inner Galaxy. No study has been reported on local clouds with high resolution in large scale.

In this paper, we will study the fractal structure of one of the closest local dark cloud complexes, ρ Ophiuchus Molecular Cloud Complex, based on ^{12}CO as well as ^{13}CO molecular emission cube data obtained at an unprecedented high resolution of 22” to identify differences of fractal dimension depending upon the observed resolution. In Section 2, we describe the database, and we define the fractal dimension in Section 3. In Section 4, we present our results and discussion. We summarize our study in the final section.

2. DATABASE

The ρ Ophiuchus Molecular Cloud Complex is a dark nebula complex of gas and dust, and one of the closest star forming regions to the sun. Estimated distance to this cloud is about 135 pc (Mamajek 2008). This

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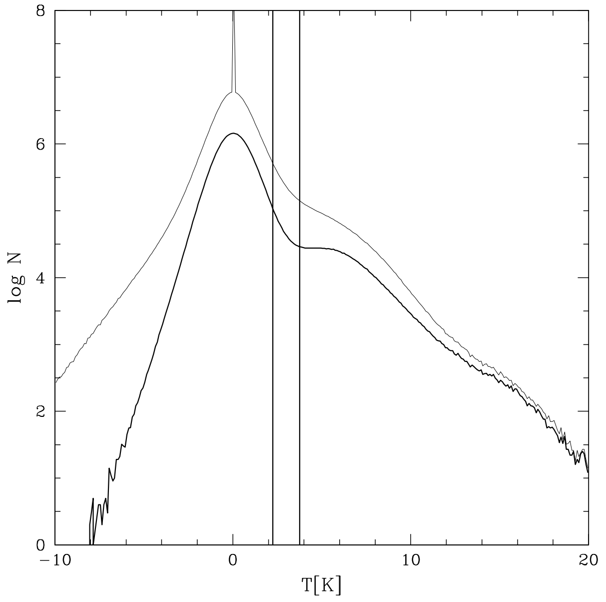

Figure 1. Distribution of ^{12}CO (1-0) brightness temperatures in all channels of the spectra (voxels) is shown on a logarithmic scale. Thin solid line represents the whole region of Rho Ophiuchus Cloud Complex including unmapped region, and the thick solid line is for the rectangular box marked on Figures 2 and 3. The shape of the negative brightness temperature of the rectangular region shows a complete Gaussian distribution, estimated rms noise of which is $\sigma = 0.75$ K. Two straight lines are the 3σ and 5σ, which are used as threshold temperatures for estimating fractal dimensions. The spike feature in the thin solid curve is from the unmapped region.

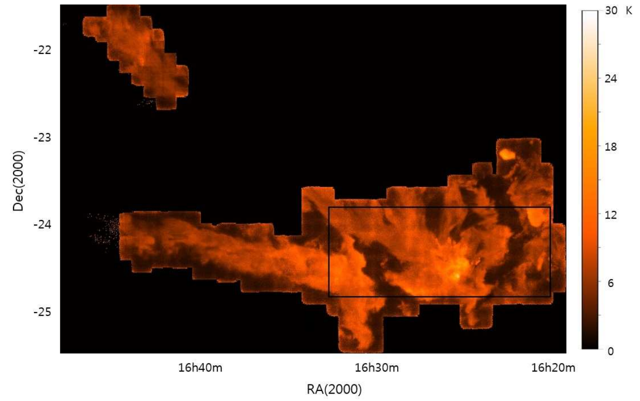

Figure 2. The ^{12}CO (1-0) peak temperature map of Rho Ophiuchus Molecular Cloud Complex. Temperature is color-coded from black (0 K) and white yellow (30 K). Unmapped region can be recognized as black color (0 K). A solid line box inside the main image is marked for estimating the rms noise level excluding the boundary noise values, which is caused by OTF observation mode.

cloud complex covers an angular area of 6.5×4.5 degrees, located far above the Galactic Plane toward the Galactic Center region. Its proximity and displacement from the Galactic Plane ($b \sim +17°$) afford higher spatial resolution views of a star-forming region with little confusion from the background stars and gas. It consists of two major regions of dense gas and dust. The first contains a star-forming cloud (L1688) and two filaments (L1709 and L1755), while the second has a star-forming region (L1689) and a filament (L1712-L1729). These filaments extend up to $10-17.5$ parsecs in length and can be as narrow as 0.24 parsecs in width. Some of the structures within the complex appear to be the result of a shock front passing through the clouds from the direction of the neighboring Sco OB2 association (Loren 1989). Temperatures of the clouds range from 10 to 25 K, and their total mass is about 3,000 times in solar mass. Over half of the mass of the complex is concentrated around the L1688 cloud, and this is the most active star-forming region (Bontemps et al. 2001).

Li & Goldsmith (2016) observed this cloud complex in $J = 1 - 0$ transition lines of ^{12}CO and ^{13}CO at an unprecedented high resolution of $22''$ using a multi-beam receiver system on 14 m telescope of Five College Radio Astronomy Observatory. With the deployment of heterodyne focal plane arrays at millimeter and sub-millimeter wavelengths mounted on the 14-m radio telescope, it is possible to construct high spatial dynamic range spectroscopic images of molecular line emission from interstellar clouds. Velocity resolution per channel is 0.0635 $\mathrm{km\ s^{-1}}$, and the full velocity range is about 35 $\mathrm{km\ s^{-1}}$. The fine resolution database are a result of fully sampled mapping of the molecular clouds with the state-of-art receiver system equipped with 32-beam and an efficient observation mode of On-The-Fly (OTF). OTF mapping generates a set of data that is densely but irregularly sampled on the sky. To construct regularly sampled spectroscopic data cubes and to co-add spatially redundant measurements, the data were convolved into an output grid using a kernel that accounts for the edge taper of the 14 m antenna to minimize noise aliasing, and retains the full angular resolution of the telescope. One pixel ($22''$) corresponds to 0.015 pc in for the target cloud, which is one of the finest resolutions achieved for molecular cloud complexes.

A histogram of the number of (l, b, v) data points (voxels) versus brightness temperature of the ^{12}CO data for the whole voxels of the target region is shown in Figure 1 in logarithmic scale. The thin solid line represents the whole region of the Ophiuchus molecular cloud complex (Figures 2, 3; see below), and the thick solid line is for the region marked with rectangular box inside the mapped region. When using OTF map mode, the boundaries of the mapped region or OTF blocks have much higher signal to noise ratio. Thus, when estimating the meaningful rms of the data, one should use the data, not affected by the boundary values. The negative part of Figure 1 shows a well-defined Gaussian distribution, which shows the consistency of noise level of the region selected. A Gaussian curve can be fit for the negative voxels as the positive voxels contain the real emission of the cloud; small number of voxels at the skirt of the curve are just spurious noise which can be

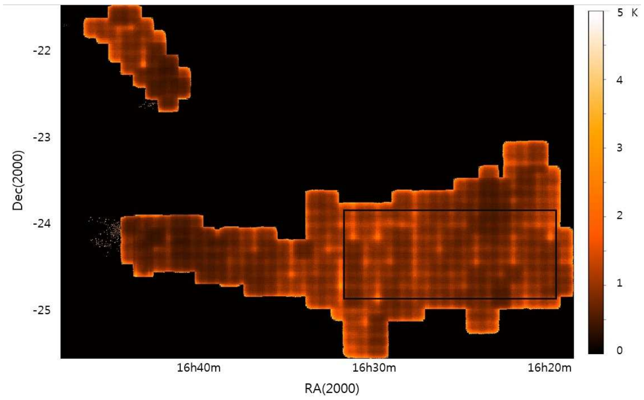

Figure 3. The ^{12}CO (1-0) rms map of Rho Ophiuchus Molecular Cloud Complex. Temperature is color-coded from black (0 K) and white yellow (5 K). A solid line box inside the main image is the same as that of Figure 2.

ignored. We do not show the ^{13}CO distribution since it was almost the same as the ^{12}CO distribution, and the fractal dimension results using ^{13}CO was also similar to that of ^{12}CO . Figure 2 is the peak temperature map of ^{12}CO represented in heat color. The bright color is for higher temperature, and dark color is low temperature; white yellow color is 30 K and black color is 0 K. Thus, the mapped area is distinguished from the unmapped one (black). Again, the boundary of the mapped area has more noise features, thus when estimating the rms noise level, one should use the inner part of the mapped area substantially away from the boundary. In, addition, we also present the rms map of the whole region in Figure 3. It shows relatively high rms noise along the boundary of the OTF blocks, and especially the boundary of the mapped region. The constructed database seems to be moderate in quality as shown by the estimated rms noise level of 0.75 K per channel (^{12}CO), and 0.33 K (^{13}CO), even though they were obtained on an extremely fine grid of $22''$.

3. Estimate of Fractal Dimension

The fractal dimension of the molecular clouds or atomic clouds is one way to characterize their fractal structure. Estimate of the fractal dimension of an object would be possible when we have a three dimensional database. For the interstellar medium, however, it is impossible to get spatial three dimensional data. For example, in (l, b, z)-space, the direction of line of sight (z) cannot be observed directly. Instead, we can construct arbitrary three dimensional database with spectroscopic observation, in the form of (l, b, v) or (v, l, b) when observing in Galactic coordinates. A new way of estimating fractal dimension had been reported by Lee et al. (2008) using slice-clouds instead of projected clouds. In the following, we will describe their method.

Integrated intensity maps or channel maps of a cloud are projected on two-dimensional space. Several factors may affect estimates of fractal dimensions when projected integrated intensity maps are used. Spatial resolution and data sensitivity (rms noise level) would be the most influential (Lee 2004). Moreover, the number of pixels becomes an issue at threshold levels when using integrated intensity map. At higher threshold values, the number of pixels would be much less than at lower threshold values, which may affect the slope of the perimeter-area relationship. Thus, instead of using integrated intensity maps, we generated slice-clouds, and estimated the area and perimeter of each slice-cloud.

Lee (2004) developed a fractal code, which works as a user task within IRAF (Image Reduction and Analysis Facility), to effectively identify clouds at each velocity slice (or each channel), and to calculate their perimeters and areas in pixel units from the 3-dimensional cube database; From a cube database we define a slice-cloud to be an object composed of all pixels in longitude, and latitude that are simply connected and that lie above some threshold temperature. Above the arbitrary threshold temperature, only those clouds with 4 or more pixels (l, b) space were retained. Ideally, one would like to define clouds with a 0 K threshold temperature. However, 0 K threshold temperature is impractical in view of the noise level in the spectra. On the other hand, with too high a threshold temperature, regions are severely truncated, and it is impossible to obtain a reliable estimate of the size. To define clouds we should choose a reasonable threshold temperature. Lee (2004) suggested that the proper threshold would be 3σ of rms.

4. Results and Discussion

We used two threshold temperatures of 3σ (2.25 K) and 5σ (3.75 K), and identified 900 and 948 slice-clouds for Rho Ophiuchus Molecular Cloud Complex. The reason for two thresholds is to see whether there is any variation of the fractal dimension. Two threshold temperatures (solid lines) are shown in Figure 1. As mentioned in Section 2, there is a kink in the distribution around the 5σ line, distinguished from a Gaussian curve. This could imply that the cloud boundary starts at a 5σ of the rms noise level. The emission sum above 3σ of the rms noise level is found to be 75.5% of the total emission of the selected region, and 60.1% above 5σ. The relatively low percentage of emission sum above threshold temperatures may be due to spurious noise near the boundary of the mapped area. For comparison, Lee et al. (2008) reported that the emission sum of Sh 156 above 3σ of the rms noise level was 82.0% of the total emission of the selected region, and 65.1% above 5σ.

We present relationships between the perimeters and the areas of the slice-clouds in logarithmic scale using two threshold temperatures (Figure 4 and Figure 5). The perimeters (P) and areas (NP) are in units of pixels. Their fractal dimension is D = 2 for rods (straight part), and D = 1 for circles. The number of pixels of the largest slice-clouds are 52 820 for 3σ, and 39 723 for 5σ. These numbers are the largest among similar studies of molecular clouds (Falgarone et al. 1991; Lee et al. 2008; Walch et al. 2011). Figures 4 and 5 show good correlation between two parameters for both cases, though there is some dispersion from an arbitrary fit line. A bi-

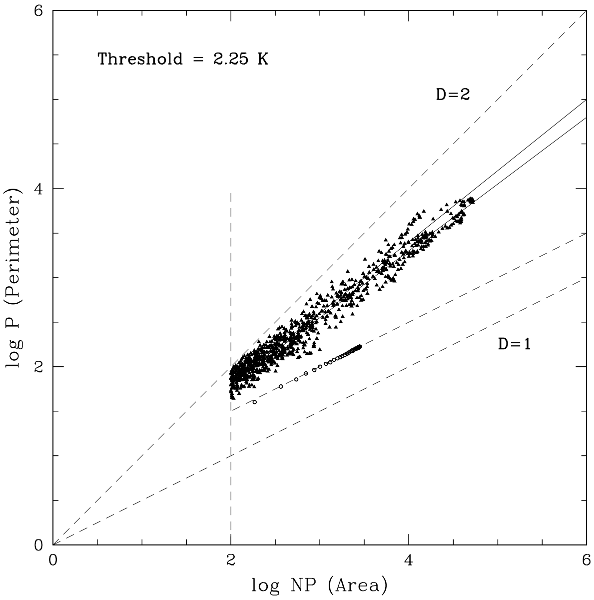

Figure 4. This is a log-log plot of measured perimeter versus area in units of pixels in (l, b) space for the slice-clouds using a threshold temperature of 2.25 K (3σ), represented with filled triangles. For comparison, we show a dashed line with circles on it indicating the arbitrary circular cloud, fractal dimension of which is $D = 1$. The two dashed lines have slopes $D = 1$ and 2, while the two solid lines have slopes $D = 1.5$ and 1.6. Dashed vertical line remarks the pixel number at NP = 100.

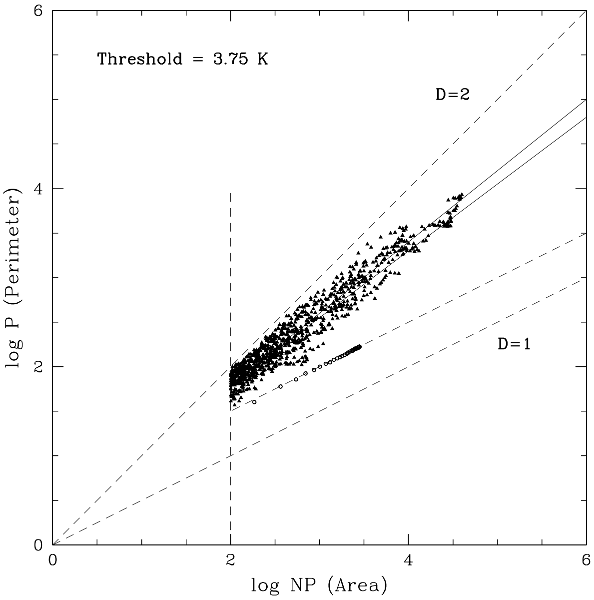

Figure 5. The same plot as Figure 4 except for threshold temperature of 3.75 K (5σ).

sector fit (Isobe et al. 1990) applied to each case yields a best fit.

$$\log(P) = 1.52(\pm 0.06)/2 \log(NP) + 0.25(\pm 0.02) \quad (1)$$

$$\log(P) = 1.54(\pm 0.06)/2 \log(NP) + 0.24(\pm 0.02) \quad (2)$$

We estimate fractal dimension of clouds having more than 100 pixels since clouds with small number do not give reliable estimates. In fact, Lee et al. (2008) used smaller number of pixels (NP = 40) as the limit. The largest number of pixels of their slice-cloud was less than 2000, while that of our target slice-cloud in this study is more than 39 000. Thus, the threshold number of pixels (NP = 100) appears reasonable. The slopes (equal to $D/2$ by definition; see below) for two threshold temperatures are found to be almost the same; $D = 1.52 \sim 1.54$. In the meanwhile, we also estimated fractal dimension using ^{12}CO (1-0) integrated intensity map, and obtained a similar result of $D = 1.5$, as mentioned in our previous studies (Lee 2004; Lee et al. 2008). Results using ^{13}CO (1-0) are not shown since they show almost the same trend. Width of dispersion from an arbitrary fit line should be examined whether they have correlations with rms noise level of databases. This issue could be examined by analyzing several databases with different quality (low or moderate rms noise levels) and obtained in high resolution. The CO database of Rho Ophiuchus Molecular Cloud Complex in this study would be one example illustrating this issue, as its rms noise level is moderate of 0.75 K per channel. We suggest that the nature of dispersion may be clarified by using a database with much lower rms noise, such as 0.3 K or lower. The intercepts at NP = 100 in Figures 4 and 5 could be related issue, as they are obviously related to structural characteristic of clouds as well as their rms levels and resolution. In a forthcoming study, we will attack these issues when we construct high quality and fine resolution database from nearby clouds.

Several studies of fractal dimensions had been reported; Orion A ($D = 1.3$; Sánchez et al. 2007), Taurus complex ($D = 1.4$; Scalo 1990), Chamaeleon complex ($D = 1.3$ 1.5; Hetem & Lepine 1993). Falgarone et al. (1991) reported $D = 1.36$ for a wide range of cloud sizes. Our estimate ($D = 1.53$) is somewhat higher than these studies, and similar to that for a molecular cloud associated with H II region Sh156 (Lee et al. 2008). As this is just one case of higher dimension, more study is required for more clouds with a well-sampled database and high sensitivity. In addition, relationship between fractal dimensions and the physical processes determining the interstellar medium structure is still an open issue, as well as the dispersion and the intercept at NP = 100.

5. SUMMARY

We have estimated the fractal dimension of ρ Ophiuchus Molecular Cloud Complex, associated with star forming regions. We selected a cube (v, l, b) database, obtained with $J = 1 - 0$ transition lines of ^{12}CO at a resolution of $22''$ using a multibeam receiver system on the 14-m telescope of the Five College Radio Astronomy Observatory. Using a code developed within IRAF, we identified slice-clouds with two threshold temperatures to estimate the fractal dimension. With the threshold temperatures of 2.25 K (3σ), and 3.75 K (5σ), we identified 900 slice- clouds and 948 slice-clouds, respectively. There seems to be a turn-over location in fractional dimension slope around NP (area) = 100. The number of pixels (areas) of the largest slice clouds are 52 820 for $T_{th} = 2.25$ K, and 39 723 for $T_{th} = 3.75$ K. The fractal dimension was estimated to be $D = 1.52$–1.54, which is slightly larger than other results, or in the upper limits of the range of fractal dimension of other studies. The spatial resolution of observed data must be an important parameter when estimating fractal dimension. Size of dispersion from the fit line should be examined whether they have correlations with sensitivity of databases. In addition, the intercepts at NP=100 should be studied whether it is related to sensitivity or characteristic structure of the target cloud. These issues can be addressed adequately by analysing several database with different qualities (low or moderate sensitivity) in high resolution.

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