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EXISTENCE OF RADIAL POSITIVE SOLUTIONS FOR A QUSILINEAR NON-POSITONE PROBLEM IN A BALL†

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Abstract. In this paper, we prove existence of radial positive solutions for the following boundary value problem

$$\begin{cases} -\triangle_p u = \lambda f(u(x)), & x \in \Omega; \\ u(x) = 0, & x \in \partial\Omega. \end{cases}$$

where $\lambda > 0$, Ω denotes a ball in $\mathbf{R}^N$; f has more than one zero and $f(0) < 0$(the nonpositone case).

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1. Introduction

Let us consider the the existence of radial positive solutions of the problem

$$\begin{cases} -\triangle_p u = \lambda f(u(x)), & x \in \Omega, \\ u(x) = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\lambda > 0$, Ω denotes a ball in $\mathbf{R}^N$; $f(0) < 0$, f has more than one zero and is not strictly increasing entirely on $[0,\infty)$. $\triangle_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ $(1 < p \le N)$ is the p-Laplacian operator of u.

The problem (1.1) arises in the theory of quasiregular and quasiconformal mappings or in the study of non-Newtonian fluids. In the latter case, the quantity p is a characteristic of the medium.Media with $p > 2$ are called dilatant fluids and these with $p < 2$ are called pseudoplastics(see[18,19]). If $p = 2$, they are Newtonian fluid. When $p \ne 2$, the problem becomes more complicated certain nice properties inherent to the case $p = 2$ seem to be lost or at least difficult to verify. The main differences between $p = 2$ and $p \ne 2$ can be founded in [9,11].

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In recent years, the asymptotic behavior, existence and uniqueness of the positive solutions for the quasilinear eigenvalue problems:

$$\begin{cases} -\mathrm{div}(|\nabla u|^{p-2}\nabla u) = \lambda f(u), & x \in \Omega, \\ u(x) = 0, & x \in \partial\Omega, \end{cases} \tag{1.2}$$

with $\lambda > 0, p > 1, \Omega \in \mathbf{R}^N, N \geq 2$ have been considered by a number of authors, see [5-15, 20-24, 26-28] and the references therein. In [11], Guo and Webb proved existence and uniqueness results of (1.2) for λ large when $f \geq 0$, $(f(x)/x^{p-1})' < 0$ for $x > 0$ and f satisfies some p-sublinearity conditions at 0 and ∞, generalizing a result in [11] where Ω is a ball. When $p = 2$, uniqueness results for semilinear equations were obtained in [29, 30] where the assumption $(f(x)/x)' < 0$ is required only for large x. Similar results for systems were discussed in [31]. Related results for the superlinear case when $f \geq 0$ can be found in [26, 32]. When $p = 2$, $f(0) < 0$, $f(s)$ has only one zero and Ω being a unit ball or an annulus in $\mathbf{R}^N$ the related results have been obtained by Castro and Shivaji [2], Arcoya and Zertiti[1]. The case when $f(0) < 0$ and $p = 2$ was treated in [33], in which uniqueness of positive solution to single equation of (1.1) for λ large was established for sublinear f. See also [34] where this result was extended to the case when Ω is any bounded domain with convex outer boundary.

In this paper, we study this problem for $p \neq 2$, $f(0) < 0$ and Ω being a unit ball in $\mathbf{R}^N$. It extends and complements previous results in the literature [1].

The paper is organized as follows. In section 2, we recall some facts that will be needed in the paper and give the main results. In section 3, we give the proofs of the main results in this paper.

2. Main results

We consider radial solution of (1.1), then, the existence of radial positive solutions of (1.1) is equivalent to the existence of positive solutions of the problem

$$\begin{cases} -(r^{N-1}|u'(r)|^{p-2}u'(r))' = \lambda r^{N-1} f(u(r)), & r \in (0,1), \\ u'(0) = 0, u(1) = 0 & . \end{cases} \tag{2.1}$$

where Ω is the unit ball of $\mathbf{R}^N$ and $\lambda > 0$. Here $f : [0, +\infty) \longrightarrow R$ satisfies the following assumptions:

(H1) $f \in C^1([0, +\infty), R)$ such that $f' \geq 0$ on $[\beta, +\infty)$, where β is the greatest zero of f;

(H2) $f(0) < 0$;

(H3) $\lim_{u\to+\infty} \frac{f(u)}{u^q} = +\infty$, where $p-1 < q < p^*-1$, $p^* = \frac{Np}{N-p}$ if $1 < p < N$; $p^* = \infty$ if $p \geq N$;

(H4) For some $k \in (0,1)$, $\lim_{d\to+\infty}(\frac{d^{p-1}}{f(d)})^{\frac{N}{p}}(F(kd) - \frac{N-p}{Np} df(d)) = +\infty$, where $F(x) = \int_0^x f(s)ds$.

Remark 2.1. We note that in hypothesis (H1), there is no restriction on the function $f(u)$ for $0 < u < \beta$.

Remark 2.2. If f satisfies (H1), any nonnegative solution u of (2.1) is positive in Ω, radial symmetric and radially decreasing, that is

$$\begin{cases} u > 0, & \text{in } \Omega; \\ u = u(r), & r = \|x\|; \\ \frac{\partial u}{\partial r} < 0, & \text{in } \Omega. \end{cases} \tag{2.2}$$

By a modification of the method given in [1], we obtain the following results.

Theorem 2.1. *Let assumptions (H1)-(H4) be satisfied. Then there exists a positive real number λ_0 such that if $\lambda \in [0, \lambda_0]$, problem (1.1) has at least one radial positive solution which is decreasing on [0,1].*

The proof of the theorem is based on the following preliminaries and four Lemmas.

Lemma 2.2. *Let $u(r)$ be a solution of (2.1) in $(r_1,r_2) \subset (0,\infty)$ and let a be an arbitrary constant, then for each $r \in (r_1, r_2)$ we have*

$$\frac{d}{dr}[r^N\{(1-\frac{1}{p})|u^{'}|^p + F(u) + \frac{a}{r}uu^{'}|u^{'}|^{p-2}\}] = r^{N-1}[NF(u) - auf(u) + (a+1-\frac{N}{p})|u^{'}|^p].$$

Remark 2.3. The identity of Pohozaev type was obtained by Ni and Serrin [6].

By a modification of the method given in [1], we first introduce the notations and the following preliminaries. Let F be defined as $F(x) = \int_0^x f(s)ds$, and θ denotes the greatest zero of F.

From (H4), we have $\gamma \geq \{\frac{\beta}{k}, \frac{\theta}{k}\}$ such that

$$pNF(kd) - (N-p)df(d) \geq 1, \text{ for } \forall d \geq \gamma. \tag{2.3}$$

Given $d \in R$, $\lambda \in R$, we define

$$\begin{cases} E(r,d,\lambda) = \frac{p-1}{p}|u^{'}(r,d,\lambda)|^p + \lambda F(u(r,d,\lambda)) & ; \\ H(r,d,\lambda) = rE(r) + \frac{N-p}{p}uu^{'}|u^{'}|^{p-2}. \end{cases}$$

By Lemma 2.2, we show the following Pohozaev identity on (r_0, r_1)

$$\begin{aligned} & r_1^{N-1}H(r_1,d,\lambda) - r_0^{N-1}H(r_0,d,\lambda) \\ = & \int_{r_0}^{r_1} \lambda r^{N-1}\{NF(u(r,d,\lambda)) - \frac{N-p}{p}f(u(r,d,\lambda))u(r,d,\lambda)\}dr. \end{aligned} \tag{2.4}$$

Moreover, for $d \geq \gamma$, there exists t_0 such that

$$u(t_0,d,\lambda) = kd, 0 < k < 1;$$

$$kd \leq u(r,d,\lambda) \leq d, \forall r \in [0,t_0]. \tag{2.5}$$

Next from (H1), we obtain that f is nondecreasing on $[kd, d] \subset (\beta, +\infty)$, and from (2.1) we have $u^{'}(r,d,\lambda) = -(\lambda r^{1-N}\int_0^r s^{N-1}f(u(s))ds)^{\frac{1}{p-1}}$, then we obtain

$$(\frac{\lambda rf(kd)}{N})^{\frac{1}{p-1}} \leq -u^{'} \leq (\frac{\lambda rf(d)}{N})^{\frac{1}{p-1}}$$

Integrating on $[0, t_0]$, which implies

$$C_1(\frac{d^{p-1}}{\lambda f(d)})^{\frac{1}{p}} \leq t_0 \leq C_1(\frac{d^{p-1}}{\lambda f(kd)})^{\frac{1}{p}}, \tag{2.6}$$

where $C_1 = [(\frac{p}{p-1})^{p-1}(1-k)^{p-1}N]^{\frac{1}{p}} > 0$.

Hence, taking $r_0 = 0$,$r_1 = t_0$ in (2.4), and using (2.5)-(2.6), we find

$$\begin{aligned} t_0^{N-1}H(t_0, d, \lambda) &= \int_0^{t_0} \lambda r^{N-1}(NF(u) - \frac{N-p}{p}f(u)u)dr \\ &\geq \lambda[NF(kd) - \frac{N-p}{p}df(d)]\frac{t_0^N}{N} \\ &\geq \lambda C_1^N[NF(kd) - \frac{N-p}{p}df(d)](\frac{d^{p-1}}{\lambda f(d)})^{\frac{N}{p}} \\ &\geq C_2\lambda^{1-\frac{N}{p}}[NF(kd) - \frac{N-p}{p}df(d)](\frac{d^{p-1}}{f(d)})^{\frac{N}{p}} \end{aligned} \tag{2.7}$$

where $C_2 = C_1^N$

Lemma 2.3. *There exists $\lambda_1 > 0$ such that if $\lambda \in (0, \lambda_1)$, then $u(r, \gamma, \lambda) \geq \beta$, for $\forall r \in [0, 1]$.*

Proof. Let $r^* = \sup\{0 \leq r \leq 1 \ : u(r, \gamma, \lambda) \geq \beta\}$. For u is decreasing on $[0, r^*]$, then $\beta \leq u(r, \gamma, \lambda) \leq u(0, \gamma, \lambda) = \gamma$, $\forall r \in [0, r^*]$

Moreover, since $f \geq 0$ on $[\beta, +\infty)$ and $u^{'}(r, \gamma, \lambda) = -(\lambda r^{1-N}\int_0^r s^{N-1}f(u(s))ds)^{\frac{1}{p-1}}$, we obtain

$$\begin{aligned} |u^{'}(r, \gamma, \lambda)| &= |\lambda r^{1-N}\int_0^r s^{N-1}f(u(s))ds|^{\frac{1}{p-1}} \\ &\leq |\lambda r^{1-N}\int_0^r s^{N-1}f(\gamma)ds|^{\frac{1}{p-1}} \leq |\frac{\lambda f(\gamma)}{N}|^{\frac{1}{p-1}}. \end{aligned}$$

Then for $\lambda < \lambda_1 = \frac{N(\gamma-\beta)^{p-1}}{f(\gamma)}$, we have

$$|u^{'}(r, \gamma, \lambda)| \leq \gamma - \beta. \tag{2.8}$$

Next, by using the mean value theorem and (2.8), there exists $\tilde{r} \in (0, r^*)$ such that

$$u(r^*, \gamma, \lambda) - u(0, \gamma, \lambda) = u^{'}(\tilde{r}, \gamma, \lambda)(r^* - 0) \geq -(\gamma - \beta)r^*$$

Assume that $r^* < 1$, we have

$$u(r^*, \gamma, \lambda) > -(\gamma - \beta)r^* + \gamma = \beta,$$

which contradicts the definition of r^*. Then, the lemma is proved for $r^* = 1$. $\square$

Lemma 2.4. *There exists $\lambda_2 > 0$ such that for $\lambda \in (0, \lambda_2)$*

$$u(r, d, \lambda)^2 + u^{'}(r, d, \lambda)^2 > 0, \forall r \in [0, 1], \forall d \geq \gamma$$

Proof. From Lemma 2.2, we have the following Pohozaev identity on (r, t_0)

$$r^{N-1}H(r) = t_0^{N-1}H(t_0) + \lambda \int_{t_0}^{r} s^{N-1}[NF(u) - \frac{N-p}{p} f(u)u]ds. \tag{2.9}$$

Extending f by $f(x) = f(0) < 0$, for $x \in (-\infty, 0]$, then there exists $B < 0$ such that

$$NF(s) - \frac{N-p}{p} f(s)s \geq B, \forall s \in R.$$

For sufficiently large γ, from (H4), we deduce

$$(F(kd) - \frac{N-p}{Np} df(d))\{\frac{d^{p-1}}{f(d)}\}^{\frac{N}{p}} \geq 1, \forall d \geq \gamma$$

By (2.7) and (2.9), we get

$$\begin{aligned} r^{N-1}H(r) &= t_0^{N-1}H(t_0) + \lambda \int_{t_0}^{r} s^{N-1}[NF(u) - \frac{N-p}{p} f(u)u]ds \\ &\geq C_2\lambda^{1-\frac{N}{p}}\{F(kd) - \frac{N-p}{Np} f(d)d\}\{\frac{d^{p-1}}{f(d)}\}^{\frac{N}{p}} + \lambda B\frac{r^N - r_0^N}{N} \end{aligned} \tag{2.10}$$

Then, there exists λ_2 such that

$$r^{N-1}H(r) \geq C_2\lambda^{1-\frac{N}{p}} + \frac{\lambda B}{N} = \lambda(C_2\lambda^{-\frac{N}{p}} + \frac{B}{N}) > 0, \forall r \in [t_0, 1].$$

Hence, for all $\lambda \in (0, \lambda_2)$ and $r \in [0, 1]$,$H(t) > 0, \forall d \geq \gamma$. This also implies that $u(r, d, \lambda)^2 + u^{'}(r, d, \lambda)^2 > 0$, for all $t \in [0, 1]$ and all $d \geq \gamma$. $\square$

Lemma 2.5. *For $r \in [0, 1]$, there exists $d \geq \gamma$ such that $u(r, d, \lambda) < 0$.*

Proof. By contradiction, let $d \geq \gamma$, we assume that $u(r, d, \lambda) \geq 0$ for $\forall r \in [0, 1]$.

Let $\bar{r} = \sup\{r \in (0, 1) : u(\cdot, d, \lambda)$ is decreasing on $(0, r)\}$. Define ω be the solution of the following equation:

$$\begin{cases} (|\omega^{'}|^{p-2}\omega^{'})^{'} + \frac{N-1}{r}(|\omega^{'}|^{p-2}\omega^{'}) + \delta(|\omega|^{p-2}\omega) = 0, \quad r \in (0, 1); \\ \omega(0) = 1, \omega^{'}(0) = 0, \end{cases} \tag{2.11}$$

where δ is chosen such that the first zero of ω is $\frac{\bar{r}}{4}$ and ω satisfies $\frac{1}{|\omega^{'}|}\omega < \frac{1}{|u^{'}|}u$, $r \in (0, 1)$.

From (H3), there exists $d_0 \geq \gamma$ such that

$$\frac{f(s)}{s^q} \geq \frac{\delta}{\lambda}, \forall s \geq d_0. \tag{2.12}$$

Since

$$\begin{cases} (|(d\omega)^{'}|^{p-2}(d\omega)^{'})^{'} + \frac{N-1}{r}(|(d\omega)^{'}|^{p-2}(d\omega)^{'}) + \delta(|d\omega|^{p-2}(d\omega)) = 0, \\ (|u^{'}|^{p-2}u^{'})^{'} + \frac{N-1}{r}(|u^{'}|^{p-2}u^{'}) + \lambda f(u) = 0. \end{cases}$$

Let $v = d\omega$,

$$\begin{cases} -(r^{N-1}|v^{'}|^{p-2}v^{'})^{'} = r^{N-1}\delta v^{p-1}, \\ -(r^{N-1}|u^{'}|^{p-2}u^{'})^{'} = r^{N-1}\lambda f(u). \end{cases}$$

Then, we obtain

$$r^{N-1}(u^{p-1}|v|'^{p-2}v' - v^{p-1}|u|'^{p-2}u') + \int_0^r (p-1)s^{N-1}(|u'|^{p-2}v^{p-2} - |v'|^{p-2}u^{p-2})u'v'ds$$
$$= \int_0^r s^{N-1}(\frac{\lambda f(u)}{u^{p-1}} - \delta)u^{p-1}v^{p-1}ds. \quad (2.13)$$

Suppose $u(r,d,\lambda) \geq d_0$ for all $r \in [0, \frac{\bar{r}}{4}]$, from (2.12),

$$\int_0^r s^{N-1}[\frac{\lambda f(u)}{u^{p-1}} - \delta]u^{p-1}v^{p-1}ds > 0. \quad (2.14)$$

On the other hand, from the quality of ω,i.e.$\frac{1}{|\omega'|}\omega < \frac{1}{|u'|}u$,$r \in (0,1)$, we know that $|u'|^{p-2}v^{p-2} - |v'|^{p-2}u^{p-2} < 0$, then

$$\int_0^r (p-1)s^{N-1}(|u'|^{p-2}v^{p-2} - |v'|^{p-2}u^{p-2})u'v'ds < 0. \quad (2.15)$$

From (2.13)-(2.15), we obtain

$$u^{p-1}v'^{p-1} - v^{p-1}u'^{p-1} > 0, \forall r \in [0, \frac{\bar{r}}{4}]. \quad (2.16)$$

On the other hand, since $v(\frac{\bar{r}}{4}) = 0, v'(\frac{\bar{r}}{4}) < 0$,

$$u^{p-1}(\frac{\bar{r}}{4})v'^{p-1}(\frac{\bar{r}}{4}) - v^{p-1}(\frac{\bar{r}}{4})u'^{p-1}(\frac{\bar{r}}{4}) < 0,$$

which is contradiction with (2.16).
Hence, there exists $\hat{r}$ in $(0, \frac{\bar{r}}{4})$ such that $u(\hat{r},d,\lambda) = d_0$.
And since $d_0 \geq \gamma > \beta$, there exists $r_1 \in (\hat{r},\bar{r})$ such that

$$\beta \leq u(r,d,\lambda) \leq d_0, \forall r \in (\hat{r}, r_1). \quad (2.17)$$

Now, we consider t_0 defined in (2.5), also $t_0 < \bar{r}$.
On $[0,t_0]$, from (H1), F is nondecreasing on $[\beta,+\infty)$ and $u(r,d,\lambda) \geq kd \geq \beta, \forall r \in (0,t_0]$. We have

$$E(r,d,\lambda) = \frac{p-1}{p}|u'(r,d,\lambda)|^p + \lambda F(u(r,d,\lambda)) \geq \lambda F(kd), \forall r \in [0,t_0]. \quad (2.18)$$

On the other hand, since $u(r,d,\lambda)u'(r,d,\lambda) \leq 0, \forall r \in (t_0,\bar{r}]$ then

$$r^N E(r,d,\lambda) = r^{N-1}H(r,d,\lambda) - \frac{N-p}{p}r^{N-1}uu'|u'|^{p-2} \geq r^{N-1}H(r,d,\lambda)$$

Hence, by (2.10) we get

$$r^N E(r,d,\lambda) \geq C_2\lambda^{1-\frac{N}{p}}\{F(kd) - \frac{N-p}{Np}f(d)d\}\{\frac{d^{p-1}}{f(d)}\}^{\frac{N}{p}} + \lambda B\frac{r^N - r_0^N}{N}. \quad (2.19)$$

From (H4), (2.18), (2.19)

$$\lim_{d \longrightarrow +\infty} E(r,d,\lambda) = +\infty, \forall r \in [0,\bar{r}]$$

Therefore, there exists $d_1 \geq d_0$ such that for $d \geq d_1$, we get

$$E(r, d, \lambda) \geq \lambda F(d_0) + \frac{p-1}{p} \frac{d_0^p}{(r_1 - \hat{r})^p}$$

By (2.17), (2.18)

$$\begin{aligned} \frac{p-1}{p} |u^{'}(r, d, \lambda)|^p &= E(r, d, \lambda) - \lambda F(u(r, d, \lambda)) \\ &\geq \lambda F(d_0) - \lambda F(u(r, d, \lambda)) + \frac{p-1}{p} \frac{d_0^p}{(r_1 - \hat{r})^p}, \forall r \in (\hat{r}, r_1) \end{aligned}$$

Which implies

$$u^{'}(r, d, \lambda) \leq -\frac{d_0}{r_1 - \hat{r}}, \forall r \in (\hat{r}, r_1)$$

The mean value theorem gives us $C \in (\hat{r}, \frac{\hat{r}+r_1}{2})$ such that

$$u(\frac{\hat{r}+r_1}{2}) - u(\hat{r}) = u^{'}(C)\frac{r_1 - \hat{r}}{2} \leq -\frac{d_0}{r_1 - \hat{r}} \cdot \frac{r_1 - \hat{r}}{2} = -\frac{d_0}{2}$$

hence $u(\frac{\hat{r}+r_1}{2}) \leq 0$ and since $u^{'}(\frac{\hat{r}+r_1}{2}) \leq -\frac{d_0}{r_1-\hat{r}} < 0$, there exists $T \in (0,1)$ such that $u(T, d, \lambda) < 0$, which contradicts with the assuming, the lemma is proved. □

3. Proof of the Main Results

The proof of Theorem 2.1. Let $\lambda_0 = \min\{\lambda_1, \lambda_2\}$, for $\forall \lambda \in (0, \lambda_0)$. Define $\hat{d} = \sup\{d \geq \gamma : u(r, d, \lambda) \geq 0, \forall r \in (0,1]\}$. From Lemma 2.3, we obtain that the set $\{d \geq \gamma : u(r, d, \lambda) \geq 0, \forall r \in (0,1]\}$ is nonempty. From Lemma 2.5 implies that $\hat{d} < +\infty$.

Then we claim that $u(r, \hat{d}, \lambda)$ is the solution of problem (1.1). Moreover, the solution $u(r, \hat{d}, \lambda)$ satisfies the following properties:

(i) $u(r, \hat{d}, \lambda) > 0$, for all $r \in [0, 1)$;

(ii) $u(1, \hat{d}, \lambda) = 0$;

(iii) $u^{'}(1, \hat{d}, \lambda) < 0$;

(iv) u is decreasing in [0,1].

For (i). By contradiction, if there exists $0 \leq R_1 < 1$ such that $u(R_1, \hat{d}, \lambda) = 0$, from Lemma 2.4, $u^{'}(R_1, \hat{d}, \lambda) \neq 0$, then we can suppose $u^{'}(R_1, \hat{d}, \lambda) < 0$.

Hence from $u(R_1, \hat{d}, \lambda) = 0$, $u(1, \hat{d}, \lambda) = 0$ and $u^{'}(R_1, \hat{d}, \lambda) < 0$, we find there exists $R_2 \in (R_1, 1)$ such that $u(R_2, \hat{d}, \lambda) < 0$ which contradicts with the definition of $\hat{d}$,

So $u(r, \hat{d}, \lambda) > 0$, for all $r \in [0, 1)$.

For (ii). By contradiction,we assume $u(1, \hat{d}, \lambda) > 0$, then from (i) there exists η such that $u(r, \hat{d}, \lambda) > \eta$, for $\forall r \in (0, 1)$, moreover, there exists $\delta > 0$ such that

$u(t,\hat{d}+\delta,\lambda) \geq \frac{\eta}{2}$ for $\forall t \in (0,1]$, which is a contradiction with the definition of $\hat{d}$, (ii) is proved.

For (iii). From (2.1),$u^{'}(r,d,\lambda) = -(\lambda r^{1-N}\int_0^r s^{N-1}f(u(s))ds)^{\frac{1}{p-1}}$. Taking into account Lemma 2.3 and (H1), we have for $\forall\lambda \in (0,\lambda_0), u(r,\gamma,\lambda) \geq \beta$ and $f(s) > 0$, for $\forall s \in (\beta,+\infty)$, which implies for $\forall\lambda \in (0,\lambda_1), \hat{d} \geq d \geq \gamma$,

$$u^{'}(r,d,\lambda) = -(\lambda r^{1-N}\int_0^r s^{N-1}f(u(s))ds)^{\frac{1}{p-1}} < 0, \forall r \in (0,1].$$

So $u^{'}(1,\hat{d},\lambda) < 0$, (iv) is also proved.

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