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ON SOME PROPERTIES OF SOFT α-IDEALS

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Abstract. The notion of soft α-ideals and α-idealistic soft BCI-algebras is introduced and their basic properties are discussed. Relations between soft ideals and soft α-ideals of soft BCI-algebras are provided. Also idealistic soft BCI-algebras and α-idealistic soft BCI-algebras are being related. The restricted intersection, union, restricted union, restricted difference and "AND" operation of soft α-ideals and α-idealistic soft BCI-algebras are established. The characterizations of (fuzzy) α-ideals in BCI-algebras are given by using the concept of soft sets. Relations between fuzzy α-ideals and α-idealistic soft BCI-algebras are discussed.

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1. Introduction

classical methods, out of which the inability of the parametrization tool for these methods is one of he main reasons, Molodtsov [16] gave the concept of soft set theory as a new tool for dealing with such type of problems. Now a days, soft set theory is considered to be the one of the most reliable method for dealing with uncertainties. Here, first of all we discuss some developments in different fields of life which have been done by using the soft set theory.

De Morgans laws have been verified in soft set theory. Sezgin and Atagun [17] proved that certain De Morgans law holds in soft set theory with respect to different operations on soft sets. Herawan et al. [8] defined an attribute reduction based on the notion of multi-soft sets and "AND" operation. Gong [7] et al. introduced the concept of bijective soft set and also defined restricted "AND", relaxed "AND" operations on bijective soft set. This work revealed an application of bijective soft set in decision making problems. Babitha and Sunil [3] introduced soft set relations as a sub-soft set of cartesian product of the soft sets. They also discussed related concepts like equivalent soft set relation,

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partition, composition and function. Kharal and Ahmad [12] discussed mappings on soft classes and also images and inverse images of soft sets which are used for medical diagnosis in medical expert systems. Çağman et al. [4] defined soft matrices and their operations and constructed a max-min decision making method. Chen et al. [5] proposed parametrization reduction of soft sets and compared it with the concept of attributes reduction in rough sets theory. Aktaş and Çağman [2] discussed the basics of soft set theory and its differences with fuzzy and rough set theories. They also derived the notion of soft groups. Feng et al. [6] established a connection between rough sets and soft sets. Yang et al. [18] defined the operation on fuzzy soft sets , which are based on three fuzzy logic operations: negation, triangular norm and triangular co-norm.

Jun [10] was the first person who discussed the applications of soft sets in BCK/BCI-algebras. He introduced the notion of soft BCK/BCI-algebras and soft subalgebras. Ali et al. [1] disproved certain definitions and results discussed by Maji et al. in [15] and defined some new operations. In this paper, we introduce the notion of soft α-ideals and α-idealistic soft BCI-algebras and discuss various operations introduced in [1] on these concepts. Using soft sets, we give characterizations of (fuzzy) α-ideals in BCI-algebras. We provide relations between fuzzy α-ideals and α-idealistic soft BCI-algebras.

2. BCI-algebras

Y. Imai and K. Iséki [9], introduced the idea of BCK-algebras in 1966, after exploring the properties of set difference. Y. Imai generalized the concept of BCK-algebras and gave the idea of BCI-algebras.

We define BCI-algebra as an algebra $(X, *, 0)$ of type $(2, 0)$, in which the following axioms hold:

(I) $(x * y) * (x * z) \leq (z * y)$
(II) $x * (x * y) \leq y$
(III) $x \leq x$
(IV) $x \leq y$ and $y \leq x$ imply $x = y$

for all $x, y, z \in X$. Here a partial ordering " $\leq$ " is defined by putting, $x \leq y$ if and only if $x * y = 0$.

If a BCI-algebra X satisfies the identity:
(V) $0 \leq x = 0$,
for all $x \in X$, then X is called a BCK-algebra.

In any BCI-algebra the following hold:

(VI) $(x * y) * z = (x * z) * y$
(VII) $x * 0 = x$

(VIII) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$
(IX) $0 * (x * y) = (0 * x) * (0 * y)$
for all $x, y, z \in X$.

A non-empty subset S of a BCI-algebras X is called a subalgebra of X if $x * y \in S$ for all $x, y \in S$. A non-empty subset I of a BCI-algebra X is called an ideal of X if for any $u \in X$

(I1) $0 \in I$
(I2) $u * v \in I$ and $v \in I$ implies $u \in I$

A non-empty subset I of a BCI-algebra X is called an α-ideal of X if it satisfies (I1) and

$$\text{(I3)}\ (u * w) * (0 * v) \in I \text{ and } w \in I \Rightarrow v * u \in I \text{ for all } u, v \in X.$$

It can be observed that every α-ideal of a BCI-algebra X is an ideal of X.

3. Preliminaries

Molodtsov in [16] defined the soft sets as under: "Let U be an universal set and P be a set of parameters. Let $\mathfrak{P}(U)$ denotes the power set of U and $A \subset P$".

Definition 3.1 (Molodtsov [16]). A pair $(\mathcal{F},\ A)$ is called a soft set over U, where $\mathcal{F}$ is a mapping given by

$$\mathcal{F} : A \to \mathfrak{P}(U)$$

In other words, a soft set over U is a family of parameters of subsets of universal set U.

Definition 3.2 (Maji et al. [15]). For two soft sets $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ over a common universe U, the union of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$, denoted by $(\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B)$, is defined to be the soft set $(\mathcal{H},\ C)$ satisfying the following conditions:

(i) $C = A \cup B$
(ii) for all $c \in C$,

$$\mathcal{H}(c) = \begin{cases} \mathcal{F}(c) & if\ c \in A - B \\ \mathcal{G}(c) & if\ c \in B - A \\ \mathcal{F}(c) \cup \mathcal{G}(x) & if\ c \in A \cap B \end{cases}$$

Definition 3.3 (Ali et al. [1]). For two soft sets $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ over a common universe U, the extended intersection of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$, denoted by $(\mathcal{F},\ A) \cap_{\xi}\ (\mathcal{G},\ B)$, is defined to be the soft set $(\mathcal{H},\ C)$ satisfying the following conditions:

(i) $C = A \cup B$
(ii) for all $c \in C$,

$$\mathcal{H}(c) = \begin{cases} \mathcal{F}(c) & if\ c \in A - B \\ \mathcal{G}(c) & if\ c \in B - A \\ \mathcal{F}(c) \cap \mathcal{G}(c) & if\ c \in A \cap B \end{cases}$$

Definition 3.4 (Ali et al. [1])**.** For two soft sets $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ over a common universe U such that $A \cap B \neq \emptyset$, the restricted intersection of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$, denoted by $(\mathcal{F},\ A) \cap_{\Re} (\mathcal{G},\ B)$, is defined to be the soft set $(\mathcal{H},\ C)$ satisfying the following conditions:

(i) $C = A \cap B$
(ii) $\mathcal{H}(c) = \mathcal{F}(c) \cap \mathcal{G}(c)$ for all $c \in C$.

Definition 3.5 (Ali et al. [1])**.** For two soft sets $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ over a common universe U such that $A \cap B \neq \emptyset$, the restricted union of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$, denoted by $(\mathcal{F},\ A) \cup_{\Re} (\mathcal{G},\ B)$, is defined to be the soft set $(\mathcal{H},\ C)$ satisfying the following conditions:

(i) $C = A \cap B$
(ii) $\mathcal{H}(c) = \mathcal{F}(c) \cup \mathcal{G}(c)$ for all $c \in C$.

Definition 3.6 (Ali et al. [1])**.** For two soft sets $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ over a common universe U such that $A \cap B \neq \emptyset$, the restricted difference of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$, denoted by $(\mathcal{F},\ A) \smile_{\Re} (\mathcal{G},\ B)$, is defined to be the soft set $(\mathcal{H},\ C)$ satisfying the following conditions:

(i) $C = A \cap B$
(ii) $\mathcal{H}(c) = \mathcal{F}(c) - \mathcal{G}(c)$ for all $c \in C$ (set difference of $F(c)$ and $G(c)$).

Definition 3.7 (Maji et al. [15])**.** Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two soft sets over a common universe U. Then "$(\mathcal{F},\ A)\ AND\ (\mathcal{G},\ B)$" denoted by $(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B)$ is defined as $(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B) = (\mathcal{H},\ A \times B)$, where $\mathcal{H}(x, y) = \mathcal{F}(x)\ \cap\ \mathcal{G}(y)$ for all $(x, y) \in A \times B$.

Definition 3.8 (Maji et al. [15])**.** Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two soft sets over a common universe U. Then "$(\mathcal{F},\ A)\ OR\ (\mathcal{G},\ B)$" denoted by $(\mathcal{F},\ A)\ \tilde{\vee}\ (\mathcal{G},\ B)$ is defined as $(\mathcal{F},\ A)\ \tilde{\vee}\ (\mathcal{G},\ B) = (\mathcal{H},\ A \times B)$, where $\mathcal{H}(x, y) = \mathcal{F}(x)\ \cup\ \mathcal{G}(y)$ for all $(x, y) \in A \times B$.

4. Soft α-ideals

Let U and M be a BCI-algebra and a nonempty set, respectively and R will refer to an arbitrary binary relation between an element of M and an element of U, that is, R is a subset of $M \times U$ without otherwise specified. "A set valued function $\mathcal{F} : M \to \mathfrak{P}(U)$ can be defined as $\mathcal{F}(x) = \{y \in U \mid xRy\}$ for all $x \in M$. The pair $(\mathcal{F},\ M)$ is then a soft set over U".

Definition 4.1 (Jun and Park [11])**.** Let V be a subalgebra of U. A subset I of U is called an ideal of U related to V (briefly, V-ideal of U), denoted by $I \lhd V$, if it satisfies:

(i) $0 \in I$
(ii) $x * y \in I$ and $y \in I \Rightarrow x \in I$ for all $x \in V$.

Definition 4.2. Let V be a subalgebra of U. A subset I of U is called an α-ideal of U related to V (briefly, V-α-ideal of U), denoted by $I \lhd_\alpha V$, if it satisfies:

(i) $0 \in I$
(ii) $(x * z) * (0 * y)) \in I$ and $z \in I \Rightarrow y * x \in I$ for all $x, y \in V$.

Example 4.3. Consider a BCI-algebra $X = \{0, a, b, c\}$ defined by the following Cayley table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then $Q = \{0, a\}$ is a subalgebra of X and $I = \{0, a, b\}$ is an Q-α-ideal of X.

Note that every S-α-ideal of X is an S-ideal of X.

Definition 4.4 (Jun [10])**.** Let $(\mathcal{F},\ V)$ be a soft set over U. Then $(\mathcal{F},\ V)$ is called a soft BCI-algebra over U if $\mathcal{F}(x)$ is a subalgebra of U for all $x \in V$.

Definition 4.5 (Jun and Park [11])**.** Let $(\mathcal{F},\ V)$ be a soft BCI-algebra over U. A soft set $(\mathcal{G},\ I)$ over U is called a soft ideal of $(\mathcal{F},\ V)$, denoted $(\mathcal{G},\ I)\ \tilde{\lhd}\ (\mathcal{F},\ V)$, if it satisfies:

(i) $I \subset V$
(ii) $\mathcal{G}(x) \lhd \mathcal{F}(x)$ for all $x \in I$.

Definition 4.6. Let $(\mathcal{F},\ V)$ be a soft BCI-algebra over U. A soft set $(\mathcal{G},\ I)$ over U is called a soft α-ideal of $(\mathcal{F},\ V)$, denoted $(\mathcal{G},\ I)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ V)$, if it satisfies:

(i) $I \subset V$
(ii) $\mathcal{G}(x) \lhd_\alpha \mathcal{F}(x)$ for all $x \in I$.

The following example will be helpful to understand the above example.

Example 4.7. Consider a BCI-algebra $X = \{0, a, b, c\}$ which is given in Example 4.3. Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \{0\} \cup \{y \in X \mid y * (y * x) \in \{0, a\}\}$$

for all $x \in A$. Then $\mathcal{F}(0) = \mathcal{F}(a) = X,\ \mathcal{F}(b) = \mathcal{F}(c) = \{0\}$, which are subalgebras of X. Hence $(\mathcal{F},\ A)$ is a soft BCI-algebra over X. Let $I = \{0, a, b\} \subset A$ and $\mathcal{G} : I \to \mathfrak{P}(X)$ be a set-valued function defined by:

$$\mathcal{G}(x) = \begin{cases} Z(\{0, a\}) & if\ x = b \\ \{0\} & if\ x \in \{0, a\} \end{cases}$$

where $Z(\{0, a\}) = \{x \in X \mid 0 * (0 * x) \in \{0, a\}\}$. Then $\mathcal{G}(0) = \{0\} \lhd_\alpha X = \mathcal{F}(0),\ \mathcal{G}(a) = \{0\} \lhd_\alpha X = \mathcal{F}(a),\ \mathcal{G}(b) = \{0, a\} \lhd_\alpha \{0\} = \mathcal{F}(b)$. Hence $(\mathcal{G},\ I)$ is a soft α-ideal of $(\mathcal{F},\ A)$.

Note that every soft α-ideal is a soft ideal but the converse is not true as seen in the following example.

Example 4.8. Consider a BCI-algebra $X = \{0, a, b, c, d\}$ defined by the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	b	0
c	c	c	c	0	0
d	d	d	c	b	0

Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = X$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(u) = \{y \in X \mid y * (y * u) \in \{0, b\}\}$$

for all $u \in A$. Then $\mathcal{F}(0) = X,\ \mathcal{F}(a) = \mathcal{F}(b) = \{0, b, c, d\},\ \mathcal{F}(c) = \mathcal{F}(d) = \{0, b\}$, which are subalgebras of X. Hence $(\mathcal{F},\ A)$ is a soft BCI-algebra over X.
Let $(\mathcal{G},\ I)$ be a soft set over X, where $I = \{b, c, d\} \subset A$ and $\mathcal{G} : I \to \mathfrak{P}(X)$ be a set-valued function defined by:

$$\mathcal{G}(u) = \{y \in X \mid y * u = 0\}$$

for all $u \in I$. Then $\mathcal{G}(b) = \{0, a, b\} \lhd \{0, b, c, d\} = \mathcal{F}(b),\ \mathcal{G}(c) = \{0, a, c\} \lhd \{0, b\} = \mathcal{F}(c),\ \mathcal{G}(d) = X \lhd \{0, b\} = \mathcal{F}(d)$. Hence $(\mathcal{G},\ I)$ is a soft ideal of $(\mathcal{F},\ A)$ but it is not a soft α-ideal of $(\mathcal{F},\ A)$ because $\mathcal{G}(b)$ is not an $\mathcal{F}(b)$-α-ideal of X since $(b * b) * (0 * d) = 0 \in \mathcal{G}(b)$ and $b \in \mathcal{G}(b)$ but $d * b = c \notin \mathcal{G}(b)$.

Theorem 4.9. *Let $(\mathcal{F},\ A)$ be a soft BCI-algebra over X and $(\mathcal{G},\ I)$ and $(\mathcal{H},\ J)$ are two soft sets over X such that $I \cap J \neq \emptyset$. If $(\mathcal{G},\ I) \tilde{\lhd}_\alpha (\mathcal{F},\ A)$, $(\mathcal{H},\ J) \tilde{\lhd}_\alpha (\mathcal{F},\ A)$, then $((\mathcal{G},\ I) \cap_\Re (\mathcal{H},\ J)) \tilde{\lhd}_\alpha (\mathcal{F},\ A)$.*

Proof. Using Definition 3.4, we can write

$$(\mathcal{G},\ I) \cap_{\Re}\ (\mathcal{H},\ J) = (\mathcal{R},\ U)$$

where $U = I \cap J$ and $\mathcal{R}(e) = \mathcal{G}(e) \cap \mathcal{H}(e)$ for all $e \in U$. Obviously, $U \subset A$ and $\mathcal{R} : U \to \mathfrak{P}(X)$ is a mapping. Hence $(\mathcal{R},\ U)$ is a soft set over X. Since $(\mathcal{G},\ I)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$ and $(\mathcal{H},\ J)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$, it follows that $\mathcal{G}(e) \lhd_\alpha \mathcal{F}(e)$ for all $e \in I$ and $\mathcal{H}(e) \lhd_\alpha \mathcal{F}(e)$ for all $e \in J$. Therefore $\mathcal{R}(e) = \mathcal{G}(e) \cap \mathcal{H}(e) \lhd_\alpha \mathcal{F}(e)$ for all $e \in U = I \cap J$. Hence

$$(\mathcal{G},\ I) \cap_{\Re}\ (\mathcal{H},\ J) = (\mathcal{R},\ U)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A) \qquad .$$

This completes the proof. □

Corollary 4.10. *Let $(\mathcal{F},\ A)$ be a soft BCI-algebra over X and $(\mathcal{G},\ I)$ and $(\mathcal{H},\ I)$ are two soft sets over X. If $(\mathcal{G},\ I)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$, $(\mathcal{H},\ I)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$, then $(\mathcal{G},\ I) \cap_{\Re}\ (\mathcal{H},\ I)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$.*

Proof. Straightforward. □

Theorem 4.11. *Let $(\mathcal{F},\ A)$ be a soft BCI-algebra over X and $(\mathcal{G},\ I)$ and $(\mathcal{H},\ J)$ are two soft sets over X. If $(\mathcal{G},\ I)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$, $(\mathcal{H},\ J)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$, then $((\mathcal{G},\ I) \cap_{\xi}\ (\mathcal{H},\ J))\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$.*

Proof. By means of Definition 3.3, we can write $(\mathcal{G},\ I) \cap_{\xi}\ (\mathcal{H},\ J) = (\mathcal{R},\ U)$, where $U = I \cup J \subset A$ and for every $e \in U$,

$$\mathcal{R}(x) = \begin{cases} \mathcal{G}(e) & if\ e \in I - J \\ \mathcal{H}(e) & if\ e \in J - I \\ \mathcal{G}(e) \cap \mathcal{H}(e) & if\ e \in I \cap J \end{cases}$$

For every $e \in U$ such that $e \in I \backslash J$, $\mathcal{R}(e) = \mathcal{G}(e) \lhd_\alpha \mathcal{F}(e)$ since $(\mathcal{G},\ I)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$. Similarly for any $e \in U$ such that $e \in J \setminus I$, $\mathcal{R}(e) = \mathcal{H}(e) \lhd_\alpha \mathcal{F}(e)$ since $(\mathcal{H},\ J)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$. Moreover for some $e \in U$ such that $e \in I \cap J$, $\mathcal{R}(e) = \mathcal{G}(e) \cap \mathcal{H}(e)$. Since $\mathcal{G}(e)$ is an α-ideal of X related to $\mathcal{F}(e)$ for all $e \in I$ and $\mathcal{H}(e)$ is an α-ideal of X related to $\mathcal{F}(e)$ for all $e \in J$, it follows that $\mathcal{G}(e) \cap \mathcal{H}(e) = \mathcal{R}(e)$ is an α-ideal of X related to $\mathcal{F}(e)$ for all $e \in I \cap J = U$. Thus $\mathcal{R}(e) \lhd_\alpha \mathcal{F}(e)$ for all $e \in U$. Hence $(\mathcal{G},\ I) \cap_{\xi}\ (\mathcal{H},\ J) = (\mathcal{R},\ U)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$. □

Theorem 4.12. *Let $(\mathcal{F},\ A)$ be a soft BCI-algebra over X. For any soft sets $(\mathcal{G},\ I)$ and $(\mathcal{H},\ J)$ over X in which I and J are disjoint, we have*
$(\mathcal{G},\ I)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$, $(\mathcal{H},\ J)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A) \Rightarrow (\mathcal{G},\ I)\ \tilde{\cup}\ (\mathcal{H},\ J)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$

Proof. Assume that $(\mathcal{G},\ I)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$ and $(\mathcal{H},\ J)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$. By means of Definition 3.2, we can write $(\mathcal{G},\ I)\ \tilde{\cup}\ (\mathcal{H},\ J) = (\mathcal{R},\ U)$, where $U = I \cup J$ and for every $e \in U$,

$$\mathcal{R}(x) = \begin{cases} \mathcal{G}(e) & if\ e \in I - J \\ \mathcal{H}(e) & if\ e \in J - I \\ \mathcal{G}(e) \cup \mathcal{H}(e) & if\ e \in I \cap J \end{cases}$$

Since $I \cap J = \emptyset$, either $e \in I \setminus J$ or $e \in J \setminus I$ for all $e \in U$. If $e \in I \setminus J$, then $\mathcal{R}(e) = \mathcal{G}(e) \lhd_\alpha \mathcal{F}(e)$ since $(\mathcal{G},\ I)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$. If $e \in J \setminus I$, then $\mathcal{R}(e) = \mathcal{H}(e) \lhd_\alpha \mathcal{F}(e)$ since $(\mathcal{H},\ J)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$. Thus $\mathcal{R}(e) \lhd_\alpha \mathcal{F}(e)$ for all $e \in U$ and so

$$(\mathcal{G},\ I)\ \tilde{\cup}\ (\mathcal{H},\ J) = (\mathcal{R},\ U)\ \tilde{\lhd}_\alpha\ (\mathcal{F},\ A)$$

□

It I and J are not disjoint in Theorem 4.12, then Theorem 4.12 is not true in general as seen in the following example.

Example 4.13. Consider a BCI-algebra $X = \{0, 1, a, b, c\}$ defined by the following Cayley table:

$*$	0	1	a	b	c
0	0	0	a	b	c
1	1	0	a	b	c
a	a	a	0	c	b
b	b	b	c	0	a
c	c	c	b	a	0

Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = \{0, 1\}$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(u) = \{y \in X \mid y * u = y\}$$

for all $u \in A$. Then $\mathcal{F}(0) = X$ and $\mathcal{F}(1) = \{0, a, b, c\}$, which are subalgebras of X. Hence $(\mathcal{F},\ A)$ is a soft BCI-algebra over X.
If we take $I = A$ and define a set valued function $\mathcal{G} : I \to \mathfrak{P}(X)$ by:

$$\mathcal{G}(u) = \{y \in X \mid u * (u * y) \in \{0, b\}\}$$

for all $u \in I$. Then $\mathcal{G}(0) = \{0, 1, b\} \lhd_\alpha X = \mathcal{F}(0)$ and $\mathcal{G}(1) = \{0, b\} \lhd_\alpha \{0, a, b, c\} = \mathcal{F}(1)$. Hence $(\mathcal{G},\ I)$ is a soft α-ideal of $(\mathcal{F},\ A)$.
Now consider $J = \{0\}$ which is not disjoint with I and let $\mathcal{H} : J \to \mathfrak{P}(X)$ be a set valued function by:

$$\mathcal{H}(u) = \{y \in X \mid u * (u * y) \in \{0, c\}\}$$

for all $u \in J$. Then $\mathcal{H}(0) = \{0, 1, c\} \lhd_\alpha X = \mathcal{F}(0)$. Hence $(\mathcal{H},\ J)$ is a soft α-ideal of $(\mathcal{F},\ A)$. But if $(\mathcal{R},\ U) = (\mathcal{G},\ I)\ \tilde{\cup}\ (\mathcal{H},\ J)$, then $\mathcal{R}(0) =$

$\mathcal{G}(0) \cup \mathcal{H}(0) = \{0, 1, b, c\}$, which is not an α-ideal of X related to $\mathcal{F}(0)$, since $(b * b) * (0 * c) = c \in \mathcal{R}(0)$ and $b \in \mathcal{R}(0)$ but $c * b = a \notin \mathcal{R}(0)$. Hence $(\mathcal{R}, U) = (\mathcal{G}, I) \tilde{\cup} (\mathcal{H}, J)$ is not a soft α-ideal of $(\mathcal{F}, A)$.

Remark. (i) It should be noted that the restricted difference of two soft α-ideals is not a soft α-ideal in general as is the case in the above example, i.e,
We define $(\mathcal{G}, I) \smile_{\Re} (\mathcal{H}, J) = \mathcal{R}, U)$, where $U = I \cap J$ and $\mathcal{R}(u) = \mathcal{G}(u) - \mathcal{H}(u)$ for all $u \in U$. Then $\mathcal{R}(0) = \mathcal{G}(0) - \mathcal{H}(0) = \{b\}$, which is not an α-ideal of X related to $\mathcal{F}(0)$. Hence $(\mathcal{G}, I) \smile_{\Re} (\mathcal{H}, J) = (\mathcal{R}, U)$ is not a soft α-ideal of $(\mathcal{F}, A)$.

(ii) From the Example 4.13, it is also clear that the restricted union of any two soft α-ideals is not a soft α-ideal in general.
Since by Definition 3.5, $(\mathcal{G}, I) \cup_{\Re} (\mathcal{H}, J) = \mathcal{R}, U)$, where $U = I \cap J$ and $\mathcal{R}(u) = \mathcal{G}(u) \cup \mathcal{H}(u)$ for all $u \in U$. Then $\mathcal{R}(0) = \mathcal{G}(0) \cup \mathcal{H}(0) = \{0, 1, b, c\}$, which is not an α-ideal of X related to $\mathcal{F}(0)$ since $(b * b) * (0 * c) = c \in \mathcal{R}(0)$ and $b \in \mathcal{R}(0)$ but $c * b = a \notin \mathcal{R}(0)$. So $(\mathcal{G}, I) \cup_{\Re} (\mathcal{H}, J) = \mathcal{R}, U)$ is not a soft α-ideal of $(\mathcal{F}, A)$.

5. α-idealistic soft BCI-algebras

Definition 5.1 (Jun and Park [11])**.** Let $(\mathcal{F}, V)$ be soft set over U. Then $(\mathcal{F}, V)$ is called an idealistic soft BCI-algebra over U if $\mathcal{F}(x)$ is an ideal of U for all $x \in V$.

Definition 5.2. A soft set $(\mathcal{F}, V)$ over U is called an α-idealistic soft BCI-algebra over U if $\mathcal{F}(v)$ is an α-ideal of U for all $v \in V$.

Example 5.3. Let $X = \{0, a, b, c\}$ be the BCI-algebra which is defined by the Cayley table given in Example 4.3. Let $(\mathcal{F}, A)$ be a soft set over X, where $A = X$ and define a set valued function, $\mathcal{F} : A \to \mathfrak{P}(X)$ as:

$$\mathcal{F}(x) = \begin{cases} Z(\{0, a\}) & if\ x \in \{b, c\} \\ X & if\ x \in \{0, a\} \end{cases}$$

where $Z(\{0, a\}) = \{u \in X \mid 0 * (0 * u) \in \{0, a\}\}$. Then $(\mathcal{F}, A)$ will be an α-idealistic soft BCI-algebra over X.

It is easy to see that α-idealistic soft BCI-algebra over X is an idealistic soft BCI-algebra over X. But in general, the converse is not true and it can be observed by the following example.

Example 5.4. Le X $= \{0, a, b, c, d\}$ be the BCI-algebra defined by the Cayley table given in Example 4.8. Let $(\mathcal{F}, B)$ be a soft set over X, where $B = \{b, c, d\}$ and the set valued functon $\mathcal{F} : B \to \mathfrak{P}(X)$ is defined as:

$$\mathcal{F}(u) = \{y \in X \mid y * u = 0\}$$

for all $u \in B$. Then

$\mathcal{F}(b) = \{0, a, b\}$, $\mathcal{F}(c) = \{0, a, c\}$, $\mathcal{F}(d) = X$, which are ideals of X. Hence $(\mathcal{F},\ B)$ is an idealistic soft BCI-algebra over X but it is not an α-idealistic soft BCI-algebra over X because $\mathcal{F}(b)$ is not an α-ideal of X since, $(b * b) * (0 * d) = 0 \in \mathcal{F}(b)$ and $b \in \mathcal{F}(b)$ but $d * b = c \notin \mathcal{F}(b)$.

Theorem 5.5. *For two α-idealistic soft BCI-algebras $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ over X such that $A \cap B \neq \emptyset$, the restricted intersection $(\mathcal{F},\ A) \cap_{\Re} (\mathcal{G},\ B)$ is also an α-idealistic soft BCI-algebra over X.*

Proof. Using Definition 3.4, we can write

$$(\mathcal{F},\ A) \cap_{\Re} (\mathcal{G},\ B) = (\mathcal{H},\ C)$$

where $C = A \cap B$ and $\mathcal{H}(e) = \mathcal{F}(e) \cap \mathcal{G}(e)$ for all $e \in C$. Note that $\mathcal{H} : C \to \mathfrak{P}(X)$ is a mapping, therefore $(\mathcal{H},\ C)$ is a soft set over X. Since $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ are α-idealistic soft BCI-algebras over X, it follows that $\mathcal{F}(e)$ is an α-ideal of X for all $e \in A$ and $\mathcal{G}(e)$ is an α-ideal of X for all $e \in b$. Then $\mathcal{F}(e) \cap \mathcal{G}(e) = \mathcal{H}(e)$ is also an α-ideal of X for all $e \in A \cap B = C$. Hence $(\mathcal{H},\ C) = (\mathcal{F},\ A) \cap_{\Re} (\mathcal{G},\ B)$ is an α-idealistic soft BCI-algebra over X. □

Corollary 5.6. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ A)$ be two α-idealistic soft BCI-algebras over X. Then their restricted intersection $(\mathcal{F},\ A) \cap_{\Re} (\mathcal{G},\ A)$ is also an α-idealistic soft BCI-algebra over X.*

Proof. Straightforward. □

Theorem 5.7. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two α-idealistic soft BCI-algebras over X. Then the extended intersection $(\mathcal{F},\ A) \cap_{\xi} (\mathcal{G},\ B)$ is also an α-idealistic soft BCI-algebra over X.*

Proof. By means of Definition 3.3, we can write $(\mathcal{F},\ A) \cap_{\xi} (\mathcal{G},\ B) = (\mathcal{H},\ C)$, where $C = A \cup B$ and for every $e \in C$,

$$\mathcal{H}(x) = \begin{cases} \mathcal{F}(e) & if\ e \in A - B \\ \mathcal{G}(e) & if\ e \in B - A \\ \mathcal{F}(e) \cap \mathcal{G}(e) & if\ e \in A \cap B \end{cases}$$

For every $e \in C$ such that $e \in A \setminus B$, we get $\mathcal{H}(e) = \mathcal{F}(e)$ which is an α-ideal of X as $(\mathcal{F},\ A)$ is an α-idealistic soft BCI-algebra over X. Similarly for any $e \in C$ such that $e \in B \setminus A$, $\mathcal{H}(e) = \mathcal{G}(e)$, which is an α-ideal of X since $(\mathcal{G},\ B)$ is an α-idealistic soft BCI-algebra over X. Moreover for some $e \in C$ such that $e \in A \cap B$, $\mathcal{H}(e) = \mathcal{F}(e) \cap \mathcal{G}(e)$. Since $\mathcal{F}(e)$ is an α-ideal of X for all $e \in A$ and $\mathcal{H}(e)$ is an α-ideal of X for all $e \in B$, it follows that $\mathcal{F}(e) \cap \mathcal{G}(e) = \mathcal{H}(e)$ is an α-ideal of X for all $e \in A \cap B$. Thus $\mathcal{H}(e)$ is an α-ideal of X for all $e \in C$. Hence $(\mathcal{F},\ A) \cap_{\xi} (\mathcal{G},\ B) = (\mathcal{H},\ C)$ is an α-idealistic soft BCI-algebra over X. □

Theorem 5.8. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two α-idealistic soft BCI-algebras over X. If A and B are disjoint, then the union $(\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B)$ is also an α-idealistic soft BCI-algebra over X.*

Proof. By means of Definition 3.2, we can write $(\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B) = (\mathcal{H},\ C)$, where $C = A \cup B$ and for every $e \in C$,

$$\mathcal{H}(x) = \begin{cases} \mathcal{F}(e) & if\ e \in A - B \\ \mathcal{G}(e) & if\ e \in A - B \\ \mathcal{F}(e) \cup \mathcal{G}(e) & if\ e \in A \cap B \end{cases}$$

Since $A \cap B = \emptyset$, either $e \in A \setminus B$ or $e \in B \setminus A$ for all $e \in C$. If $e \in A \setminus B$, then $\mathcal{H}(e) = \mathcal{F}(e)$ is an α-ideal of X since $(\mathcal{F},\ A)$ is an α-idealistic soft BCI-algebra over X. If $e \in B \setminus A$, then $\mathcal{H}(e) = \mathcal{G}(e)$ is an α-ideal of X since $(\mathcal{G},\ B)$ is an α-idealistic soft BCI-algebra over X. Hence $(\mathcal{H},\ C) = (\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B)$ is an α-idealistic soft BCI-algebra over X. □

Example 5.9. We consider the case when we have a non-empty intersection of the set of parameters A and B. Let $X = \{0, a, b, c\}$ be the BCI-algebra defined by the Cayley table given in Example 4.3. Let $(\mathcal{F},\ A)$ be a soft set over X, where $A = \{0, a, b\}$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by:

$$\mathcal{F}(x) = \begin{cases} Z(\{0, a\}) & if\ x = b \\ \{0\} & if\ x \in \{0, a\} \end{cases}$$

where $Z(\{0, a\}) = \{x \in X \mid 0 * (0 * x) \in \{0, a\}\}$. Then $\mathcal{F}(0) = \{0\},\ \mathcal{F}(a) = \{0\}$ and $\mathcal{F}(b) = \{0, a\}$, which are α-ideals of X. Thus $(\mathcal{F},\ A)$ is an α-idealistic soft BCI-algebra over X.
Now take $(\mathcal{G},\ B)$ as another soft set over X, where $B = \{b\}$ and $\mathcal{G} : B \to \mathfrak{P}(X)$ is a set-valued function defined by:
$\mathcal{G}(u) = \{0\} \cup \{y \in X \mid x \leq y\}$ for all $u \in B$. Then $\mathcal{G}(b) = \{0, b\}$, which is an α-ideal of X. Hence $(\mathcal{G},\ B)$ is also an α-idealistic soft BCI-algebra over X.
By Definition 3.2, for the union, $(\mathcal{F},\ A)\ \tilde{\cup}\ (\mathcal{G},\ B) = (\mathcal{H},\ C)$, we get $\mathcal{H}(b) = \mathcal{F}(b) \cup \mathcal{G}(b) = \{0, a, b\}$, which is not an α-ideal of X since $(a*b)*(0*b) = a \in \mathcal{H}(b)$ and $b \in \mathcal{H}(b)$ but $b * a = c \notin \mathcal{H}(b)$.
Thus the union two α-idealistic soft BCI-algebras will be an α-idealistic soft BCI-algebra provided that the set of parameters of these soft sets are disjoint.

Remark. (i) It should be noted from the above example that the restricted difference of two α-idealistic soft BCI-algebras is not an α-idealistic soft BCI-algebra in general, i.e,
$(\mathcal{F},\ A) \smile_{\Re} (\mathcal{G},\ B) = (\mathcal{H},\ C)$, where $C = A \cap B \neq \Phi$ and $\mathcal{H}(c) = \mathcal{F}(c) - \mathcal{G}(c)$ for all $c \in C$. Then $\mathcal{H}(b) = \mathcal{F}(b) - \mathcal{G}(b) = \{a\}$, which is not an α-ideal of X. Therefore $(\mathcal{F},\ A) \smile_{\Re} (\mathcal{G},\ B) = (\mathcal{H},\ C)$ is not an α-idealistic soft BCI-algebra over X.

(ii) It can also be observed that, in general, the restricted union of two α-idealistic soft BCI-algebras is not an α-idealistic soft BCI-algebra, i.e,
$(\mathcal{F},\ A) \cup_{\Re} (\mathcal{G},\ B) = (\mathcal{H},\ C)$, where $C = A \cap B \neq \Phi$ and $\mathcal{H}(c) = \mathcal{F}(c) \cup \mathcal{G}(c)$

for all $c \in C$. Then $\mathcal{H}(b) = \mathcal{F}(b) \cup \mathcal{G}(b) = \{0, a, b\}$, which is not an α-ideal of X since $(a * b) * (0 * b) = a \in \mathcal{H}(b)$ and $b \in \mathcal{H}(b)$ but $b * a = c \notin \mathcal{H}(b)$. Hence $(\mathcal{F},\ A) \cup_{\Re} (\mathcal{G},\ B) = (\mathcal{H},\ C)$ is not an α-idealistic soft BCI-algebra over X.

Theorem 5.10. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be two α-idealistic soft BCI-algebras over X, then $(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B)$ is an α-idealistic soft BCI-algebra over X.*

Proof. Since by Definition 3.4, we know that

$$(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B) = (\mathcal{H},\ A \times B),$$

where $H(x, y) = \mathcal{F}(x)\ \cap\ \mathcal{G}(y)$ for all $(x, y) \in A \times B$. Since $\mathcal{F}(x)$ and $\mathcal{G}(y)$ are α-ideals of X, the intersection $\mathcal{F}(x)\ \cap\ \mathcal{G}(y)$ is also an α-ideal of X. Hence $H(x, y)$ is an α-ideal of X for all $(x, y) \in A \times B$.
Hence $(\mathcal{F},\ A)\ \tilde{\wedge}\ (\mathcal{G},\ B) = (\mathcal{H},\ A \times B)$ is an α-idealistic soft BCI-algebra over X. □

Definition 5.11 (Liu and Zhang [14])**.** A fuzzy set μ in X is called a fuzzy α-ideal of X, if for all $x, y, z \in X$,

(i) $\mu(0) \geq \mu(x)$
(ii) $\mu(y * x) \geq min\{\mu((x * z) * (0 * y)),\ \mu(z)\}$

The transfer principle for fuzzy sets described in [13] suggest's the following theorem.

Lemma 5.12 (Liu and Zhang [14])**.** *A fuzzy set μ in X is a fuzzy α-ideal of X if and only if for any $t \in [0, 1]$, the level subset $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is either empty or an α-ideal of X.*

Theorem 5.13. *For every fuzzy α-ideal μ of X, there exists an α-idealistic soft BCI-algebra $(\mathcal{F},\ A)$ over X.*

Proof. Let μ be a fuzzy α-ideal of X. Then $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is an α-ideal of X for all $t \in Im(\mu)$. If we take $A = Im(\mu)$ and consider a set valued function $\mathcal{F} : A \to \mathfrak{P}(X)$ given by $\mathcal{F}(t) = U(\mu; t)$ for all $t \in A$, then $(\mathcal{F},\ A)$ is an α-idealistic soft BCI-algebra over X. □

Conversely, it can be easily observed that the following theorem holds.

Theorem 5.14. *For any fuzzy set μ in X, if an α-idealistic soft BCI-algebra $(\mathcal{F},\ A)$ over X is given by $A = Im(\mu)$ and $\mathcal{F}(t) = U(\mu; t)$ for all $t \in A$, then μ is a fuzzy α-ideal of X.*

Let μ be a fuzzy set in X and let $(\mathcal{F},\ A)$ be a soft set over X in which $A = Im(\mu)$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is a set-valued function defined by

$$\mathcal{F}(t) = \{x \in X \mid \mu(x) + t > 1\} \ (5.2)$$

for all $t \in A$. Then there exists $t \in A$ such that $\mathcal{F}(t)$ is not an α-ideal of X as seen in the following example.

Example 5.15. For any BCI-algebra X, define a fuzzy set μ in X by $\mu(0) = t_\circ < 0.5$ and $\mu(x) = 1 - t_\circ$ for all $x \neq 0$. Let $A = Im(\mu)$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ be a set-valued function defined by (5.2). Then $\mathcal{F}(1 - t_\circ) = X \setminus \{0\}$, which is not an α-ideal of X.

Theorem 5.16. *Let μ be a fuzzy set in X and let $(\mathcal{F},\ A)$ be a soft set over X in which $A = [0, 1]$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is given by (5.2). Then the following assertions are equivalent:*

(1) μ is a fuzzy α-ideal of X.

(2) for every $t \in A$ with $\mathcal{F}(t) \neq \emptyset$, $\mathcal{F}(t)$ is an α-ideal of X.

Proof. Assume that μ is a fuzzy α-ideal of X. Let $t \in A$ be such that $\mathcal{F}(t) \neq \emptyset$. Then for any $x \in \mathcal{F}(t)$, we have $\mu(0) + t \geq \mu(x) + t > 1$, that is, $0 \in \mathcal{F}(t)$. Let $(x * z) * (0 * y) \in \mathcal{F}(t)$ and $z \in \mathcal{F}(t)$ for any $t \in A$ and $x, y, z \in X$. Then $\mu((x * z) * (0 * y)) + t > 1$ and $\mu(z) + t > 1$. Since μ is a fuzzy α-ideal of X, it follows that

$$\begin{aligned}\mu(y * x) + t &\geq min\{\mu((x * z) * (0 * y)),\ \mu(z)\} + t \\ &= min\{\mu((x * z) * (0 * y)) + t,\ \mu(z) + t\} > 1\end{aligned}$$

so that $y * x \in \mathcal{F}(t)$. Hence $\mathcal{F}(t)$ is an α-ideal of X for all $t \in A$ such that $\mathcal{F}(t) \neq \emptyset$.

Conversely, suppose that (2) is valid. If there exists $x_\circ \in X$ such that $\mu(0) < \mu(x_\circ)$, then there exists $t_\circ \in A$ such that $\mu(0) + t_\circ \leq 1 < \mu(x_\circ) + t_\circ$. It follows that $x_\circ \in \mathcal{F}(t_\circ)$ and $0 \notin \mathcal{F}(t_\circ)$, which is a contradiction. Hence $\mu(0) \geq \mu(x)$ for all $x \in X$. Now assume that

$$\mu(y_\circ * x_\circ) < min\{\mu((x_\circ * z_\circ) * (0 * y_\circ)),\ \mu(z_\circ)\}$$

for some $x_\circ, y_\circ, z_\circ \in X$. Then there exists some $s_\circ \in A$ such that

$$\mu(y_\circ * x_\circ) + s_\circ \leq 1 < min\{\mu((x_\circ * z_\circ) * (0 * y_\circ)),\ \mu(z_\circ)\} + s_\circ$$

$$\Rightarrow \mu(y_\circ * x_\circ) + s_\circ \leq 1 < min\{\mu((x_\circ * z_\circ) * (0 * y_\circ)) + s_\circ,\ \mu(z_\circ) + s_\circ\}$$

which implies that $(x_\circ * z_\circ) * (0 * y_\circ) \in \mathcal{F}(s_\circ)$ and $z_\circ \in \mathcal{F}(s_\circ)$ but $y_\circ * x_\circ \notin \mathcal{F}(s_\circ)$. This is a contradiction. Therefore

$$\mu(y * x) \geq min\{\mu((x * z) * (0 * y)),\ \mu(z)\}$$

for all $x, y, z \in X$ and thus μ is fuzzy α-ideal of X. □

Corollary 5.17. *Let μ be a fuzzy set in X such that $\mu(x) > 0.5$ for all $x \in X$ and let $(\mathcal{F},\ A)$ be a soft set over X in which*

$$A := \{t \in Im(\mu) \mid t > 0.5\}$$

and $\mathcal{F} : A \to \mathfrak{P}(X)$ is given by (5.2). If μ is a fuzzy α-ideal of X, then $(\mathcal{F},\ A)$ is an α-idealistic soft BCI-algebra over X.

Proof. Straightforward. □

Theorem 5.18. *Let μ be a fuzzy set in X and let $(\mathcal{F},\ A)$ be a soft set over X in which $A = (0.5, 1]$ and $\mathcal{F} : A \to \mathfrak{P}(X)$ is defined by*

$$\mathcal{F}(t) = U(\mu; t) \text{ for all } t \in A$$

Then $\mathcal{F}(t)$ is an α-ideal of X for all $t \in A$ with $\mathcal{F}(t) \neq \emptyset$ if and only if the following assertions are valid:

(1) $max\{\mu(0),\ 0.5\} \geq \mu(x)$ for all $x \in X$.

(2) $max\{\mu(y * x),\ 0.5\} \geq min\{\mu((x * z) * (0 * y)),\ \mu(z)\}$ for all $x, y, z \in X$.

Proof. Assume that $\mathcal{F}(t)$ is an α-ideal of X for all $t \in A$ with $\mathcal{F}(t) \neq \emptyset$. If there exists $x_\circ \in X$ such that $max\{\mu(0),\ 0.5\} < \mu(x_\circ)$, then there exists $t_\circ \in A$ such that $max\{\mu(0),\ 0.5\} < t_\circ \leq \mu(x_\circ)$. It follows that $\mu(0) < t_\circ$, so that $x_\circ \in \mathcal{F}(t_\circ)$ and $0 \notin \mathcal{F}(t_\circ)$. This is a contradiction. Therefore (1) is valid. Suppose that there exist $a, b, c \in X$ such that

$$max\{\mu(b * a),\ 0.5\} < min\{\mu((a * c) * (0 * b)),\ \mu(c)\}$$

Then there exists $s_\circ \in A$ such that

$$max\{\mu(b * a),\ 0.5\} < s_\circ \leq min\{\mu((a * c) * (0 * b)),\ \mu(c)\}$$

which implies $(a * c) * (0 * b) \in \mathcal{F}(s_\circ)$ and $c \in \mathcal{F}(s_\circ)$, but $b * a \notin \mathcal{F}(s_\circ)$. This is a contradiction. Hence (2) is valid.
Conversely, suppose that (1) and (2) are valid. Let $t \in A$ with $\mathcal{F}(t) \neq \emptyset$. Then for any $x \in \mathcal{F}(t)$, we have

$$max\{\mu(0),\ 0.5\} \geq \mu(x) \geq t > 0.5$$

which implies $\mu(0) \geq t$ and thus $0 \in \mathcal{F}(t)$. Let $(x * z) * (0 * y)) \in \mathcal{F}(t)$ and $z \in \mathcal{F}(t)$, for any $x, y, z \in X$. Then $\mu((x * z) * (0 * y)) \geq t$ and $\mu(z) \geq t$. It follows from the second condition that

$$max\{\mu(y*x),\ 0.5\} \geq min\{\mu((x*z)*(0*y)),\ \mu(z)\} \geq t > 0.5$$

So that $\mu(y*x) \geq t$, i.e., $y*x \in \mathcal{F}(t)$. Therefore $\mathcal{F}(t)$ is an α-ideal of X for all $t \in A$ with $\mathcal{F}(t) \neq \emptyset$. □

6. CONCLUSION

- The union of two soft α-ideals $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ is a soft α-ideal if $A \cap B = \phi$. Similarly the union of two α-idealistic soft BCI-algebras $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ is an α-idealistic soft BCI-algebra provided that $A \cap B = \phi$
- The restricted intersection and restricted difference of any two soft α-ideals are not soft α-ideals in general. The same is the case for the restricted intersection and restricted difference of any two α-idealistic soft BCI-algebras.
- Fuzzy α-ideals can be characterized using the concept of soft sets.
- For every fuzzy α-ideal, there exists an α-idealistic soft BCI-algebra.
- For a soft set $(\mathcal{F},\ A)$ over X, a fuzzy set μ in X is a fuzzy α-ideal of X if and only if for every $t \in A$ with $\mathcal{F}(t) = \{x \in X \mid \mu(x) + t > 1\} \neq \emptyset$, $\mathcal{F}(t)$ is an α-ideal of X.

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