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THE UNIQUENESS OF MEROMORPHIC FUNCTIONS WHOSE DIFFERENTIAL POLYNOMIALS SHARE SOME VALUES†

CHAO MENG* AND XU LI

Abstract. In this article, we deal with the uniqueness problems of meromorphic functions concerning differential polynomials and prove the following theorem. Let f and g be two nonconstant meromorphic functions, $n \geq 12$ a positive integer. If $f^n(f^3-1)f'$ and $g^n(g^3-1)g'$ share $(1,2)$, f and g share ∞ IM, then $f \equiv g$. The results in this paper improve and generalize the results given by Meng (C. Meng, Uniqueness theorems for differential polynomials concerning fixed-point, Kyungpook Math. J. 48(2008), 25-35), I. Lahiri and R. Pal (I. Lahiri and R. Pal, Nonlinear differential polynomials sharing 1-points, Bull. Korean Math. Soc. 43(2006), 161-168), Meng (C. Meng, On unicity of meromorphic functions when two differential polynomials share one value, Hiroshima Math.J. 39(2009), 163-179).

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1. Introduction, definitions and results

Let f be a nonconstant meromorphic function defined in the open complex plane C. Set $E(a,f) = \{z : f(z) - a = 0\}$, where a zero point with multiplicity m is counted m times in the set. If these zeros points are only counted once, then we denote the set by $\overline{E}(a,f)$. Let f and g be two nonconstant meromorphic functions. If $E(a,f) = E(a,g)$, then we say that f and g share the value a CM; if $\overline{E}(a,f) = \overline{E}(a,g)$, then we say that f and g share the value a IM. We assume that the reader is familiar with the notations of Nevanlinna theory that can be found, for instance, in [4] or [14].

Let m be a positive integer or infinity and $a \in C \cup \{\infty\}$. We denote by $E_{m)}(a,f)$ the set of all a-points of f with multiplicities not exceeding m, where

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*Corresponding author.

an a-point is counted according to its multiplicity. Also we denote by $\overline{E}_{m)}(a,f)$ the set of distinct a-points of f with multiplicities not greater than m. We denote by $N_{k)}(r,1/(f-a))$ the counting function for zeros of $f-a$ with multiplicity $\leq k$, and by $\overline{N}_{k)}(r,1/(f-a))$ the corresponding one for which multiplicity is not counted. Let $N_{(k}(r,1/(f-a))$ be the counting function for zeros of $f-a$ with multiplicity at least k and $\overline{N}_{(k}(r,1/(f-a))$ the corresponding one for which multiplicity is not counted. Set

$$N_k(r,\frac{1}{f-a})=\overline{N}(r,\frac{1}{f-a})+\overline{N}_{(2}(r,\frac{1}{f-a})+...+\overline{N}_{(k}(r,\frac{1}{f-a})\,.$$

By the above definition, we have

$$\overline{N}\left(r,\frac{1}{h}\right)+\overline{N}_{(2}\left(r,\frac{1}{h}\right)=N_2\left(r,\frac{1}{h}\right)\leq N\left(r,\frac{1}{h}\right)\,.$$

Definition 1.1 ([17])**.** Let F and G be two nonconstant meromorphic functions such that F and G share the value 1 IM. Let z_0 be a 1-point of F with multiplicity p, a 1-point of G with multiplicity q. We denote by $N_L(r,\frac{1}{F-1})$ the counting function of those 1-points of F and G where $p>q$, by $N_E^{1)}(r,\frac{1}{F-1})$ the counting function of those 1-points of F and G where $p=q=1$ and by $N_E^{(2}(r,\frac{1}{F-1})$ the counting function of those 1-points of F and G where $p=q\geq 2$, each point in these counting function being counted only once.

We also require the following notion of weighted sharing which was introduced by I. Lahiri.

Definition 1.2 ([5, 6])**.** For a complex number $a\in C\cup\{\infty\}$, we denote by $E_k(a,f)$ the set of all a-points of f where an a-point with mutiplicity m is counted m times if $m\leq k$ and $k+1$ times if $m>k$. For a complex number $a\in C\cup\{\infty\}$, such that $E_k(a,f)=E_k(a,g)$, then we say that f and g share the value a with weight k.

The definition implies that if f, g share a value a with weight k then z_0 is a zero of $f-a$ with multiplicity $m(\leq k)$ if and only if it is a zero of $g-a$ with multiplicity $m(\leq k)$ and z_0 is a zero of $f-a$ with multiplicity $m(>k)$ if and only if it is a zero of $g-a$ with multiplicity $n(>k)$, where m is not necessarily equal to n. We write f, g share (a,k) to mean that f, g share the value a with weight k. Clearly if f, g share (a,k) then f, g share (a,p) for all integer p, $0\leq p<k$. Also we note that f, g share a value a IM or CM if and only if f, g share $(a,0)$ or (a,∞) respectively. We call f and g share (z,k) if $f-z$ and $g-z$ share $(0,k)$.

It is well known that if f and g share four distinct values CM, then f is a fractional transformation of g. In 1997, corresponding to one famous question of Hayman, C.C. Yang and X.H. Hua showed the similar conclusions hold for certain types of differential polynomials when they share only one value. They proved the following result.

Theorem 1.3 ([13]). *Let f and g be two nonconstant meromorphic functions, $n \geq 11$ an integer and $a \in C - \{0\}$. If $f^n f'$ and $g^n g'$ share the value a CM, then either $f = dg$ for some $(n+1)$th root of unity d or $g(z) = c_1 e^{cz}$ and $f(z) = c_2 e^{-cz}$, where c, c_1 and c_2 are constants and satisfy $(c_1 c_2)^{n+1} c^2 = -a^2$.*

In 2001, M.L. Fang and W. Hong obtained the following result.

Theorem 1.4 ([3]). *Let f and g be two transcendental entire functions, $n \geq 11$ an integer. If $f^n(f-1)f'$ and $g^n(g-1)g'$ share the value 1 CM, then $f \equiv g$.*

In 2004, W.C. Lin and H.X. Yi extended the above theorem in view of the fixed-point. They proved the following result.

Theorem 1.5 ([8]). *Let f and g be two transcendental meromorphic functions, $n \geq 13$ an integer. If $f^n(f-1)^2 f'$ and $g^n(g-1)^2 g'$ share z CM, then $f \equiv g$.*

In 2008, the first author relaxed the nature of fixed-point to IM and proved

Theorem 1.6 ([10]). *Let f and g be two transcendental meromorphic functions, $n \geq 28$ an integer. If $f^n(f-1)^2 f'$ and $g^n(g-1)^2 g'$ share z IM, then $f \equiv g$.*

Some works have already been done in this direction [**?**],[9]. In 2006, I. Lahiri and R. Pal proved the following result.

Theorem 1.7 ([7]). *Let f and g be two nonconstant meromorphic functions and let $n(\geq 14)$ be an integer. If $E_{3)}(1, f^n(f^3-1)f') = E_{3)}(1, g^n(g^3-1)g')$, then $f \equiv g$.*

Naturally, we consider the following question: Can the nature of the sharing value be relaxed in the above theorem?

In 2009, the first author gave a positive answer to the above Question and proved

Theorem 1.8 ([11]). *Let f and g be two nonconstant meromorphic functions such that $f^n(f^3-1)f'$ and $g^n(g^3-1)g'$ share $(1, l)$, where n be a positive integer such that $n+1$ is not divisible by 3. If (1) $l = 2$ and $n \geq 14$; (2) $l = 1$ and $n \geq 17$; (3) $l = 0$ and $n \geq 35$, then $f \equiv g$.*

In this paper, we study the uniqueness problems of meromorphic functions concerning differential polynomials and prove the following results

Theorem 1.9. *Let f and g be two nonconstant meromorphic functions, $n \geq 12$ a positive integer. If $f^n(f^3-1)f'$ and $g^n(g^3-1)g'$ share $(1, 2)$, f and g share ∞ IM, then $f \equiv g$.*

Theorem 1.10. *Let f and g be two nonconstant meromorphic functions, $n \geq 19$ a positive integer. If $f^n(f^3-1)f'$ and $g^n(g^3-1)g'$ share 1 IM, f and g share ∞ IM, then $f \equiv g$.*

2. Some Lemmas

In this section, we present some lemmas which will be needed in the sequel. We will denote by H the following function:

$$H=\left(\frac{F''}{F'}-\frac{2F'}{F-1}\right)-\left(\frac{G''}{G'}-\frac{2G'}{G-1}\right),$$

where F and G are two meromorphic functions.

Lemma 2.1 ([12]). *Let f be a nonconstant meromorphic function, and let $a_1, a_2, ..., a_n$ be finite complex numbers, $a_n \neq 0$. Then*

$$T(r, a_n f^n+\cdots+a_2 f^2+a_1 f+a_0)=nT(r,f)+S(r,f).$$

Lemma 2.2 ([1]). *If F and G share $(1,2)$ and (∞,k), where $0\le k\le\infty$, then one of the following cases holds.*

$$(1) T(r,F)\le N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)+\overline{N}(r,F)+\overline{N}(r,G)$$
$$+\overline{N}_*(r,\infty;F,G)+S(r,F)+S(r,G),$$

the same inequality holds for $T(r,G)$; (2) $F\equiv G$; (3) $FG\equiv 1$. Here $\overline{N}_*(r,\infty;F,G)$ is the reduced counting function of those a-points of F whose multiplicities differ from the multiplicities of the corresponding a-points of G.

Lemma 2.3 ([16]). *Let f be a nonconstant meromorphic function. Then*

$$N\left(r,\frac{1}{f^{(k)}}\right)\le N\left(r,\frac{1}{f}\right)+k\overline{N}(r,f)+S(r,f).$$

Lemma 2.4 ([7]). *Let f and g be two nonconstant meromorphic functions. Then $f^n(f^3-1)f'g^n(g^3-1)g'\not\equiv 1$, where n is a positive integer.*

Lemma 2.5 ([7]). *Let $F^*=f^{n+1}\left(\frac{f^3}{n+4}-\frac{1}{n+1}\right)$, $G^*=g^{n+1}\left(\frac{g^3}{n+4}-\frac{1}{n+1}\right)$, where $n(\ge 2)$ is an integer. If $F^*\equiv G^*$, then $f\equiv g$.*

Lemma 2.6 ([18]). *Suppose that two nonconstant meromorphic function F and G share 1 and ∞ IM. Let H be given as above. If $H\not\equiv 0$, then*

$$T(r,F)+T(r,G)\le 3\overline{N}(r,F)+N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)$$
$$+N_E^{1)}\left(r,\frac{1}{F-1}\right)+2N_E^{(2}\left(r,\frac{1}{F-1}\right)+3N_L\left(r,\frac{1}{F-1}\right)$$
$$+3N_L\left(r,\frac{1}{G-1}\right)+S(r,F)+S(r,G).$$

Lemma 2.7 ([15]). *Let H be defined as above. If $H\equiv 0$ and*

$$\limsup_{r\to\infty}\frac{\overline{N}(r,\frac{1}{F})+\overline{N}(r,\frac{1}{G})+\overline{N}(r,F)+\overline{N}(r,G)}{T(r)}<1, r\in I$$

where I is a set with infinite linear measure and $T(r)=\max\{T(r,F),T(r,G)\}$, then $FG\equiv 1$ or $F\equiv G$.

3. Proof of Theorem 1.9

Let

$$F=f^n(f^3-1)f' \ ,G=g^n(g^3-1)g' \,, \tag{1}$$

and

$$F^*=f^{n+1}\left(\frac{f^3}{n+4}-\frac{1}{n+1}\right) \ ,G^*=g^{n+1}\left(\frac{g^3}{n+4}-\frac{1}{n+1}\right) .$$

Thus we obtain that F and G share $(1,2)$. If the case (1) in Lemma 2.2 occur, that is

$$\begin{aligned} T(r,F)\le N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)+\overline{N}(r,F)+\overline{N}(r,G)\\ +\overline{N}_*(r,\infty;F,G)+S(r,F)+S(r,G)\,. \end{aligned} \tag{2}$$

Moreover, by Lemma 2.1, we have

$$T(r,F^*)=(n+4)T(r,f)+S(r,f) \ , \tag{3}$$

$$T(r,G^*)=(n+4)T(r,g)+S(r,g)\,. \tag{4}$$

Since $(F^*)'=F$, we deduce

$$m\left(r,\frac{1}{F^*}\right)\le m\left(r,\frac{1}{F}\right)+S(r,f)\,, \tag{5}$$

and by the first fundamental theorem

$$T(r,F^*)\le T(r,F)+N\left(r,\frac{1}{F^*}\right)-N\left(r,\frac{1}{F}\right)+S(r,f)\,. \tag{6}$$

Note that

$$N\left(r,\frac{1}{F^*}\right)=(n+1)N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{f^3-\frac{n+4}{n+1}}\right) , \tag{7}$$

$$N\left(r,\frac{1}{F}\right)=nN\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{f'}\right)+N\left(r,\frac{1}{f^3-1}\right) . \tag{8}$$

It follows from $(6)-(8)$ that

$$\begin{aligned} T(r,F^*)\le T(r,F)+N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{f^3-\frac{n+4}{n+1}}\right)\\ -N\left(r,\frac{1}{f'}\right)-N\left(r,\frac{1}{f^3-1}\right)+S(r,f)\,. \end{aligned} \tag{9}$$

It follows from (1) that

$$N_2\left(r,\frac{1}{F}\right)+\overline{N}(r,F)\le 2\overline{N}\left(r,\frac{1}{f}\right)+N_2\left(r,\frac{1}{f'}\right) \tag{10}$$

$$+N_2\left(r,\frac{1}{f^3-1}\right)+\overline{N}(r,f)\,,$$

$$N_2\left(r,\frac{1}{G}\right)+\overline{N}(r,G)\leq 2\overline{N}\left(r,\frac{1}{g}\right)+N_2\left(r,\frac{1}{g'}\right) \tag{11}$$

$$+N_2\left(r,\frac{1}{g^3-1}\right)+\overline{N}(r,g)\,.$$

From (2), (9), (10) and (11) we obtain

$$T(r,F^*)\leq 3N\left(r,\frac{1}{f}\right)+3\overline{N}(r,f)+N\left(r,\frac{1}{f^3-\frac{n+4}{n+1}}\right)+2N\left(r,\frac{1}{g}\right)$$

$$+N\left(r,\frac{1}{g'}\right)+N\left(r,\frac{1}{g^3-1}\right)+S(r,f)\,. \tag{12}$$

By Lemma 2.3 we have

$$N\left(r,\frac{1}{g'}\right)\leq N(r,g)+N\left(r,\frac{1}{g}\right)\leq 2T(r,g)+S(r,g)\,. \tag{13}$$

We have from (12) and (13) that

$$(n-3)T(r,f)\leq 8T(r,g)+S(r,g)\,. \tag{14}$$

In the same manner as above, we have

$$(n-3)T(r,g)\leq 8T(r,f)+S(r,g)\,. \tag{15}$$

Therefore by (14) and (15), we obtain that $n\leq 11$, which contradicts $n\geq 12$. Thus by Lemma 2.2, we get $F\equiv G$ or $FG\equiv 1$. If $FG\equiv 1$, that is

$$f^n(f^3-1)f'g^n(g^3-1)g'\equiv 1\,.$$

By Lemma 2.4, we get a contradiction. If $F\equiv G$, that is

$$F^*=G^*+c\,, \tag{16}$$

where c is a constant. It follows that $T(r,f)=T(r,g)+S(r,f)$. Suppose that $c\neq 0$, by the second fundamental theorem, we have

$$(n+4)T(r,g)=T(r,G^*)<\overline{N}\left(r,\frac{1}{G^*}\right) \tag{17}$$

$$+\overline{N}\left(r,\frac{1}{G^*+c}\right)+\overline{N}(r,G^*)+S(r,g)$$

$$\leq\overline{N}\left(r,\frac{1}{g}\right)+\overline{N}\left(r,\frac{1}{g^3-\frac{n+4}{n+1}}\right)+\overline{N}(r,g)+\overline{N}(r,\frac{1}{f})$$

$$+\overline{N}\left(r,\frac{1}{f^3-\frac{n+4}{n+1}}\right)+S(r,f)\leq 9T(r,f)+S(r,f)\,,$$

which contradicts the assumption. Therefore $F^*\equiv G^*$. Thus by Lemma 2.5, we have $f\equiv g$. This completes the proof of Theorem 1.9.

4. Proof of Theorem 1.10

Let

$$F=f^n(f^3-1)f'\ ,G=g^n(g^3-1)g'\,, \tag{18}$$

and

$$F^*=f^{n+1}\left(\frac{f^3}{n+4}-\frac{1}{n+1}\right)\ ,G^*=g^{n+1}\left(\frac{g^3}{n+4}-\frac{1}{n+1}\right)\,.$$

Thus we obtain that F and G share 1 IM. If possible, we suppose that $H\not\equiv 0$. Thus, by Lemma 2.6, we have

$$\begin{aligned}T(r,F)+T(r,G)\le 3\overline{N}(r,F)+N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)\\ +N_E^{1)}\left(r,\frac{1}{F-1}\right)+2N_E^{(2}\left(r,\frac{1}{F-1}\right)+3N_L\left(r,\frac{1}{F-1}\right)\\ +3N_L\left(r,\frac{1}{G-1}\right)+S(r,F)+S(r,G)\,.\end{aligned} \tag{19}$$

Also we have

$$\begin{aligned}N_E^{1)}\left(r,\frac{1}{F-1}\right)+2N_E^{(2}\left(r,\frac{1}{F-1}\right)+N_L\left(r,\frac{1}{F-1}\right)\\ +2N_L\left(r,\frac{1}{G-1}\right)\le N\left(r,\frac{1}{G-1}\right)\le T(r,G)+O(1)\,.\end{aligned} \tag{20}$$

We get from (19) and (20) that

$$\begin{aligned}T(r,F)\le 3\overline{N}(r,F)+N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)\\ +2N_L\left(r,\frac{1}{F-1}\right)+N_L\left(r,\frac{1}{G-1}\right)+S(r,F)+S(r,G)\,.\end{aligned} \tag{21}$$

It's obvious that

$$\begin{aligned}2N_L\left(r,\frac{1}{F-1}\right)\le 2N\left(r,\frac{F}{F'}\right)\le 2N\left(r,\frac{F'}{F}\right)+S(r,f)\\ \le 2\overline{N}(r,F)+2\overline{N}\left(r,\frac{1}{F}\right)+S(r,f)\,,\end{aligned} \tag{22}$$

$$\begin{aligned}N_L\left(r,\frac{1}{G-1}\right)\le N\left(r,\frac{G}{G'}\right)\le N\left(r,\frac{G'}{G}\right)+S(r,g)\\ \le \overline{N}(r,G)+\overline{N}\left(r,\frac{1}{G}\right)+S(r,g)\,.\end{aligned} \tag{23}$$

Combining (21), (22) and (23), we deduce

$$T(r,F)\le 6\overline{N}(r,F)+N_4\left(r,\frac{1}{F}\right)+N_3\left(r,\frac{1}{G}\right)+S(r,f)+S(r,g)\,. \tag{24}$$

Moreover, by Lemma 2.1, we have

$$T(r, F^*) = (n+4)T(r, f) + S(r, f) \ , \tag{25}$$

$$T(r, G^*) = (n+4)T(r, g) + S(r, g) \, . \tag{26}$$

Since $(F^*)' = F$, we deduce

$$m\left(r, \frac{1}{F^*}\right) \leq m\left(r, \frac{1}{F}\right) + S(r, f) \, , \tag{27}$$

and by the first fundamental theorem

$$T(r, F^*) \leq T(r, F) + N\left(r, \frac{1}{F^*}\right) - N\left(r, \frac{1}{F}\right) + S(r, f) \, . \tag{28}$$

Note that

$$N\left(r, \frac{1}{F^*}\right) = (n+1)N\left(r, \frac{1}{f}\right) + N\left(r, \frac{1}{f^3 - \frac{n+4}{n+1}}\right) \ , \tag{29}$$

$$N\left(r, \frac{1}{F}\right) = nN\left(r, \frac{1}{f}\right) + N\left(r, \frac{1}{f'}\right) + N\left(r, \frac{1}{f^3 - 1}\right) \ . \tag{30}$$

It follows from (24), (28), (29) and (30) that

$$\begin{aligned} T(r, F^*) \leq T(r, F) + N\left(r, \frac{1}{f}\right) + N\left(r, \frac{1}{f^3 - \frac{n+4}{n+1}}\right) \\ -N\left(r, \frac{1}{f'}\right) - N\left(r, \frac{1}{f^3 - 1}\right) + S(r, f) + S(r, g) \, . \end{aligned} \tag{31}$$

We have from (25) and (31) that

$$(n-10)T(r, f) \leq 8T(r, g) + S(r, g) \, . \tag{32}$$

In the same manner as above, we have

$$(n-10)T(r, g) \leq 8T(r, f) + S(r, g) \, . \tag{33}$$

Therefore by (32) and (33), we obtain that $n \leq 18$, which contradicts $n \geq 19$. Therefore $H \equiv 0$. That is

$$\frac{F''}{F'} - 2\frac{F'}{F-1} \equiv \frac{G''}{G'} - 2\frac{G'}{G-1} \, . \tag{34}$$

By integration, we have from (34)

$$\frac{1}{G-1} = \frac{A}{F-1} + B \, , \tag{35}$$

where $A(\neq 0)$ and B are constants. Thus

$$T(r, F) = T(r, G) + S(r, f) \, . \tag{36}$$

From (18), we have

$$\overline{N}\left(r, \frac{1}{F}\right) + \overline{N}\left(r, \frac{1}{G}\right) + \overline{N}(r, F) + \overline{N}(r, G) \tag{37}$$

$$\leq \overline{N}\left(r,\frac{1}{f}\right)+\overline{N}\left(r,\frac{1}{f^3-1}\right)+\overline{N}\left(r,\frac{1}{f'}\right)+\overline{N}(r,f)$$
$$+\overline{N}\left(r,\frac{1}{g}\right)+\overline{N}\left(r,\frac{1}{g^3-1}\right)+\overline{N}\left(r,\frac{1}{g'}\right)+\overline{N}(r,g).$$

Note that

$$\overline{N}\left(r,\frac{1}{f'}\right)\leq T(r,f')-m\left(r,\frac{1}{f'}\right) \tag{38}$$
$$\leq 2T(r,f)-m\left(r,\frac{1}{f'}\right)+S(r,f)\,,$$

and

$$T(r,F)+m\left(r,\frac{1}{f'}\right)=T\left(r,f^n(f^3-1)f'\right)+m\left(r,\frac{1}{f'}\right) \tag{39}$$
$$\geq T(r,f^n(f^3-1))\,.$$

From (37) − (39), we apply Lemma 2.7 and get $F\equiv G$ or $FG\equiv 1$. Proceeding as in the proof of Theorem 1.9, we get the conclusion. This completes the proof of Theorem 1.10.

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Chao Meng received Ph.D at Shandong University. Since 2009 he has been at Shenyang Aerospace University. His research interests include meromorphic function theory and fixed point theory.

School of Science, Shenyang Aerospace University, Shenyang 110136, China.
e-mail: mengchaosau@163.com

Xu Li received M.Sc. from Liaoning University. She is currently an engineer at AVIC SAC Commercial Aircraft Company Limited since 2009. Her research interests are robust contrl theory and engeerning mechanics.

Department of Research and Development Center, AVIC SAC Commercial Aircraft Company Limited, Shenyang 110003, China.
e-mail: li.xu@sacc.com.cn