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ON THE TRIANGULAR EQUILIBRIUM POINTS IN THE ELLIPTIC RESTRICTED THREE-BODY PROBLEM UNDER RADIATION AND OBLATENESS EFFECTS

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ABSTRACT

This work considers the elliptic restricted three-body problem under effects of radiation of the bigger primary, and an oblate spheroid for the smaller primary to mimic an exoplanetary system with a gas giant planet. Under the influences of both effects we look for the existence of the triangular equilibrium points and the influences of the radiation and oblateness on the locations and motion of the points. We set the system in a normalized rotating coordinate system and derive equations of motion for the third infinitesimal object. Our study shows that the effects modify the equilateral/isosceles triangle shape with respect to the primaries. The triangular points also have non-planar motion with period depending on the value of the planet oblateness.

Key words: elliptic restricted three-body problem: triangular points: radiation: oblateness

1. INTRODUCTION

As of mid 2014 more than a thousand exoplanetary systems have been discovered. Many of them consist of gas giant planets with elliptical orbits. To mimic an exoplanetary system that has a single gas giant planet, it is adequate to set the system in the Restricted Three-Body Problem (R3BP). The problem consists of three bodies, i.e. two primary bodies with finite masses and a third one that is a negligible-mass (infinitesimal) body whose motion is influenced by the primaries. The problem can be made more realistic by adding the effects of radiation pressure of the star and the oblateness of the gas giant planet. In addition, a more general case of the problem is the Elliptic Restricted Three-Body Problem (ER3BP) because it describes the dynamical system under the realistic assumption that the primaries move elliptical orbits (Kumar & Narayan, 2012).

The combined effects of radiation and oblateness or triaxial-shape have also been investigated recently, e.g. Singh & Umar (2012) and Singh (2013). In this work we set a problem belonging to ER3BP with the bigger primary having radiative effects and the smaller primary having an oblate spheroid shape. We look for the existence of the triangular points in this system and study the influences of both effects on the locations and motion of the points especially in a non-planar fashion.

We derive the equations of motion in the Section 2. After looking for the existence of the triangular points in the planar ER3BP in Section 3, we then describe and discuss the possible non-planar motion of the points in Section 4, and finally give conclusions in Section 5.

2. EQUATIONS OF MOTION

Let m_1 and m_2 be the masses of the primaries. The usual practice is to choose a system of units so that the gravitational constant and the sum of mass of the primaries is unity. This yields

$$m_1 = 1-\mu, \quad m_2 = \mu, \quad \mu = \frac{m_2}{(m_1+m_2)} \leq \frac{1}{2}.$$

In the rotational frame, the coordinates of the primaries are (ξ_1, η_1, ζ_1), and (ξ_2, η_2, ζ_2), and of the object is (ξ, η, ζ). In the dimensionless rotational coordinate system, we normalize the instantaneous distance r between the two primaries, such that

$$\bar{\xi} = \frac{\xi}{r}, \ \bar{\eta} = \frac{\eta}{r}, \ \bar{\zeta} = \frac{\zeta}{r}.$$

The equations of motion for the potential function V in general are

$$\begin{aligned} \frac{d^2\bar{\xi}}{df^2} - 2\frac{d\bar{\eta}}{df} &= \frac{dV}{d\bar{\xi}}, \\ \frac{d^2\bar{\eta}}{df^2} + 2\frac{d\bar{\xi}}{df} &= \frac{dV}{d\bar{\eta}}, \end{aligned} \tag{1}$$

$$\frac{d^2\bar{\zeta}}{df^2} + \bar{\zeta} = \frac{dV}{d\bar{\zeta}},$$

where f is the true anomaly of the system. In this work, the potential function is developed by taking into account the effects of radiation of the star and oblateness of the planet.

The radiation effect acting on the infinitesimal object can be described as follows. The radiation pressure force F_p changes with distance by the same law as the gravitational attraction force of the star (F_{grav1}) and acts opposite to it. The radiation exerts a small acceleration on the third body in the opposite direction to the gravitational acceleration of the star. We can then introduce the radiation factor (q_1) for the star

$$F_{grav1} - F_p = q_1 F_{grav1}, \quad q_1 = 1 - \frac{F_p}{F_{grav1}} \leq 1.$$

On the other hand, the gravitational attraction force of the planet (F_{grav2}) includes the oblateness coefficient (A_2), such that $0 \leq A_2 \ll 1$.

$$F_{grav2} = \frac{m_2}{{r_2}^2} + \frac{3m_2}{2{r_2}^4} A_2 \left(1 - \frac{5\bar{\zeta}^2}{{r_2}^2}\right),$$
$$A_2 = \frac{AE^2 - AP^2}{5R^2},$$

where AE, AP, and R are, respectively, the dimensional equatorial and polar radii, and the effective radius of the smaller primary.

Both effects enhance the potential experienced by the infinitesimal object. The potential function becomes

$$\begin{aligned} V &= (1 + e\,\cos f)^{-1} \left[\frac{1}{2}\left(\bar{\xi}^2 + \bar{\eta}^2 + \bar{\zeta}^2\right)\right. \\ &\left. + \frac{1}{n^2}\left[\frac{q_1(1-\mu)}{r_1} + \frac{\mu}{r_2}\left[1 + \frac{A_2}{2{r_2}^2}\left(1 - \frac{3\bar{\zeta}^2}{{r_2}^2}\right)\right]\right]\right], \end{aligned} \tag{2}$$

where e is the eccentricity of the primaries, n is the mean motion of the elliptical orbit with oblateness

$$n^2 = 1 + \frac{3}{2}A_2, \tag{3}$$

and

$$\begin{aligned} r_1 &= \sqrt{\left(\bar{\xi} - \mu\right)^2 + \bar{\eta}^2 + \bar{\zeta}^2}, \\ r_2 &= \sqrt{\left(\bar{\xi} - \mu + 1\right)^2 + \bar{\eta}^2 + \bar{\zeta}^2}. \end{aligned}$$

3. Location of Triangular Equilibrium Points

The equilibrium points are solutions of the set of equations

$$V_{\bar{\xi}} = 0, \quad V_{\bar{\eta}} = 0, \quad V_{\bar{\zeta}} = 0,$$

where subscripts ξ, η, ζ denote the partial derivatives with respect to components ξ, η, ζ, respectively. This implies

$$\begin{aligned} \bar{\xi} - \tfrac{1}{n^2}\left[\tfrac{q_1(1-\mu)(\mu-\bar{\xi})}{r_1^3} - \tfrac{15\mu A_2\bar{\zeta}^2(\mu-1-\bar{\xi})}{2{r_2}^7}\right. \\ \left. + \tfrac{3\mu A_2(\mu-1-\bar{\xi})}{2{r_2}^5} + \tfrac{\mu(\mu-1-\bar{\xi})}{{r_2}^3}\right] &= 0, \\ \bar{\eta}\left[1 - \tfrac{1}{n^2}\left[\tfrac{q_1(1-\mu)}{{r_1}^3} + \tfrac{\mu}{{r_2}^3} + \tfrac{3\mu A_2}{2{r_2}^5} - \tfrac{15\mu A_2\bar{\zeta}^2}{2{r_2}^7}\right]\right] &= 0, \\ \bar{\zeta}\left[1 - \tfrac{1}{n^2}\left[\tfrac{q_1(1-\mu)}{{r_1}^3} + \tfrac{\mu}{{r_2}^3} + \tfrac{9\mu A_2}{2{r_2}^5} - \tfrac{15\mu A_2\bar{\zeta}^2}{2{r_2}^7}\right]\right] &= 0. \end{aligned} \tag{4}$$

The ER3BP with radiation and oblateness effects has been studied to illustrate the motion of an infinitesimal object in an exoplanetary system that has a gas giant planet. In this work we find that the triangular points exist, and the locations of the points are shifted closer to the star with decreasing values of the radiation factor (more energetic star) and the increasing value of oblateness (more oblate shape of the planet). We also show that there is a non-planar harmonic motion around the points whose period depends on the value of oblateness coefficient. The radiation effect has no contribution to this motion.

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