

## GUIDANCE LAW FOR IMPACT TIME AND ANGLE CONTROL WITH CONTROL COMMAND RESHAPING

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**ABSTRACT.** In this article, a more generalized form of the impact time and angle control guidance law is proposed based on the linear quadratic optimal control methodology. For the purpose on controlling an additional constraint such as the impact time, we introduce an additional state variable that is defined to be the jerk (acceleration rate). Additionally, in order to provide an additional degree of freedom in choosing the guidance gains, the performance index that minimizes the control energy weighted by an arbitrary order of time-to-go is considered in this work. First, the generalized form of the impact angle control guidance law with an additional term which is used for the impact time control is derived. And then, we also determine the additional term in order to achieve the desired impact time. Through numbers of numerical simulations, we investigate the superiority of the proposed guidance law compared to previous guidance laws. In addition, a salvo attack scenario with multiple missile systems is also demonstrated.

### 1. INTRODUCTION

The guided flight vehicle systems such as the missile systems and the unmanned aerial vehicle(UAV) systems have been designed to intercept a target or to pass a numbers of waypoints in order to spy on the movement of the enemy. Also, it is well-understood that the primary goal of the guided weapon systems is to effectively intercept a designated target. To this end, a guidance law to carry the missile systems to a target is needed, and proportional navigation guidance(PNG) or optimal guidance law(OGL) have been received a great attention for the terminal homing guidance laws. Not only there have been extensive studies on these guidance laws but also these guidance laws have been successfully applied to various missile systems during the course of several years [1, 2]. Also, the guidance laws providing the terminal constraints on the flight path angle as well as the interception have been also studied by using the concept of the optimal control and the biased PNG in [3, 4, 5]. These days, those guidance laws are accepted for the effective terminal guidance laws of the anti-tank or -ship systems because of their essential properties.

Meanwhile, in general, the air density is varied according to change of the operating altitude. Accordingly, the drag force and the efficient of the control energy can be different in that

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case. Additionally, in the end of the homing phase, the employing guidance laws should appropriately produce its guidance command in order to achieve the main goal of the guidance law even though they sacrifice some control energies. Most established guidance laws have been devised from the control energy minimization standpoint, while a guidance law that allows us to distribute the control effort evenly during the entire flight has also been studied in [6] from this perspective.

In addition, the survivability enhancement has become an important requirement of the missile systems because the defense systems have been grown recently. To minimize the reflection of the radar signal or the heat energy such as the stealth technique is one of effective ways to enhance the survivability of the missile against the defense systems because such technique can significantly reduce the chances of detection from the tracking radar.

From the flight control standpoint, in order to enhance the survivability against the defense systems, a single vehicle capable of evasive maneuver has been also studied. However, this approach generally spends a lot of the control energies as well as makes the achievement of the intercept so difficult in the vicinity of a target because the evasive maneuver may try to increase the miss distance in that time [7]. In another approach, using a group of missile systems is also considered. This strategy can increase the survivability by means of saturating the resource of the defense systems. In this approach, although a multiple missile systems simultaneously attack a single target, each missile system is implicitly working to achieve the goal. Additionally, this approach cannot guarantee the improving of sufficient survivability, when the limited numbers of the missile systems are engaged.

In general, a high value target such as the battle ship has a strong defense system called the close in weapon system(CIWS) that can fire a several thousands of bullets within just one second. However, due to the limitation of install space, CIWS cannot defense all direction in general. Therefore, even though the battle ship is protected by a strong defense system, it can be neutralized by means of simultaneous attack with different approaching angles using a group of missile systems.

Based on this idea, in most recent, a new guidance law neutralizing the resource of the defense system has been suggested. In [8], Jeon et. al. have devised the guidance law to control the terminal impact time in the operations of the multiple missile systems, which is called the impact time control guidance(ITCG). ITCG is consists of two command terms: the first term is well-known PNG command to minimize the miss distance and the second term is proposed to control the flight time of the missile. In addition, Lee et. al. have proposed the guidance law to control the impact time and the impact angle simultaneously, called impact time and angle control guidance(ITACG, [9]). In this paper, the authors have introduced an additional state variable such as jerk to provide additional degree of freedom. In reference [10], a homing guidance law to control the impact time for multiple missile systems was newly proposed. Unlike the previous works related this issue, the concept of an explicit cooperation has been taken in this paper to achieve the main goal.

In the previous version of ITACG, it is not allowed to adjust its guidance gain properly. As mentioned before, this is not a desirable structure for shaping the guidance command in order to take into account the efficiency of the control energy. Accordingly, in this paper, we propose an

extended version of ITACG that can provide the desired impact time and angle in conjunction with capable of shaping its guidance command appropriately. To this end, in the derivation process of ITACG, we use a different cost function that can provide the additional degree of freedom in shaping the guidance command. Namely, the optimal impact angle control is first derived using the concept of jerk and the performance index weighted by the time-to-go. And then, based on this result, a more general form of ITACG is suggested in this paper.

This paper is structured in five sections as follows. In Section 2, we first review of IACG. Then, in Section 3, the proposed guidance law is discussed. The performance of the proposed guidance law is shown in Section 4. Finally, we conclude our study.

## 2. CONVENTIONAL IMPACT ANGLE CONTROL LAW

In this section, we review on the conventional impact angle control guidance law for the readers who are not familiar with that concept of the guidance law. Let us consider a planar homing engagement scenario as shown in Fig. 1. We assume that the missile flies with a constant speed  $V_M$  and the target is almost stationary. In this figure, the position of the missile is expressed by the notation as  $(X, Y)$  in the inertial frame and the variable  $\gamma_M$  denotes the flight path angle. The X-axis is defined in a way that leads to a small value of the terminal flight path angle. And,  $a_M$  represents the missile acceleration command to change the velocity vector, which is normal to  $V_M$  as shown in Fig. 1. Other variables in this engagement geometry are self-explanatory.

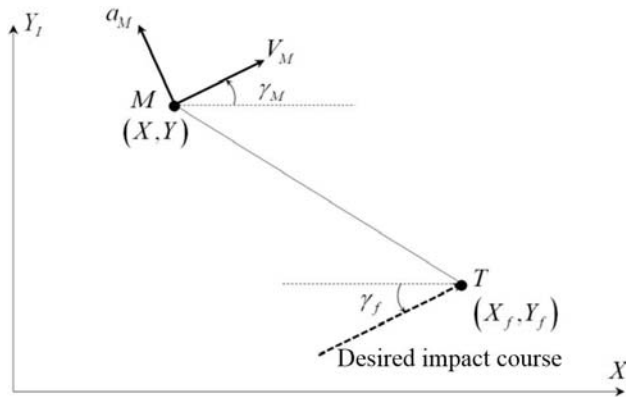


FIGURE 1. Guidance geometry

In this homing geometry, the equations of motion for the homing problem can be written by

$$dY/dt = V_M \sin \gamma_M \quad (2.1)$$

$$dX/dt = V_M \cos \gamma_M \quad (2.2)$$

$$d\gamma_M/dt = a_M/V_M \quad (2.3)$$

where  $t$  is the flight time. The boundary conditions for the impact angle control are given as follows:

$$\begin{aligned} X(t_0) &= X_0, Y(t_0) = Y_0, \gamma_M(t_0) = \gamma_{M,0} \\ X(t_f) &= X_f, Y(t_f) = Y_f, \gamma_M(t_f) = \gamma_{M,f} \end{aligned}$$

In these descriptions, the subscript 0 and  $f$  denote the initial time and the terminal time, respectively. Note that the desired value of the flight path angle at the terminal time is prescribed before the engagement in this problem. As stated in the introduction, in this paper, we derive more general form of ITACG by extending the work on [9]. Here, the generalization means that the guidance law can provide more options of choosing guidance gains in order to achieve the specified guidance objective. In this section, we consider a generalized impact angle first. And then, by introducing the jerk term, a new impact angle control guidance law capable of shaping its guidance gains as well as providing additional degree of freedom in controlling the impact time. Based on this result, we finally derive the generalized guidance law to control both impact angle and time. Compared to [9], the proposed guidance law in here can also provide addition options for choosing its guidance gain to the designer.

In order to derive the generalized impact angle control, we suppose the optimal control problem with the following cost function [6]:

Find time-to-go weighted  $a_M(t)$  which minimizes

$$J = \frac{1}{2} \int_{t_0}^{t_f} \frac{1}{t_{go}^N} a_M(s)^2 ds \quad (2.4)$$

subject to Eqs. (2.1) to (2.3). The reason why we use the performance index as shown in Eq. (2.4) is to reduce the demanded guidance command as the missile approaches the target, which is desirable from the standpoint of saving some operational margin to cope with unexpected situation near the target. It is noted that this cost function is often used to design the guidance law because it can provide an effective guidance law in the vicinity of the target.

Hereafter, we assume a small value of  $\gamma_M$  which is allowed by means of choosing a proper reference frame. Then, we can linearize Eqs. (2.1) to (2.3) as

$$\begin{bmatrix} y' \\ \gamma_M' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \gamma_M \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a \quad (2.5)$$

where  $y$  denotes the lateral separation in the linearized kinematics. The prime  $'$  represents the derivatives of variables with respect to  $x$  which denotes the non-dimensionalized relative range between the missile and the target in the linearized formulation. By introducing this transformation, in this paper, each variable is non-dimensionalized to relieve the complexity in the derivation of the proposed guidance law as

$$\begin{aligned} x &= X/L_f, y = Y/L_f \\ a &= a_M/(V_M/t_f), \tau = t/t_f \end{aligned}$$

where  $L_f(\equiv V_M t_f)$  denotes the total length of flight.  $a$  is a non-dimensionalization of  $a_M$ . Also, the boundary conditions can be now expressed as

$$\begin{aligned} y(x_0) &= y_0, \quad \gamma_M(x_0) = \gamma_{M,0} \\ y(x_f) &= y_f, \quad \gamma_M(x_f) = \gamma_f \end{aligned}$$

In a similar way, the cost function corresponding to Eq. (2.4) is rewritten as

$$\bar{J}_1 = \frac{1}{2} \int_{x_0}^{x_f} \frac{1}{(x_f - s)^N} a(s)^2 ds. \quad (2.6)$$

The final solution of this optimal guidance problem, known as the time-to-go weighted optimal guidance law in the non-dimensionalized form can be determined as follows:

$$a = \frac{(N+2)(N+3)y_{go}}{x_{go}^2} - \frac{2(N+2)\gamma_M}{x_{go}} - \frac{(N+1)(N+2)\gamma_f}{x_{go}} \quad (2.7)$$

where  $y_{go} = y_f - y$ ,  $x_{go} = x_f - x$ . Note that this guidance law cannot satisfy the desired impact time explicitly. This guidance law has an interesting feature the command as shown in Eq. (2.7) is identical to the conventional impact angle control guidance law with the gains of  $[6 \quad -4 \quad -2]$  in the case of  $N = 0$ .

### 3. GENERALIZED IACG AND ITACG LAWS

Hereafter, we discuss the generalized versions of IACG and ITACG laws. To provide an additional degree of freedom for the impact time control, we introduce an additional state variable which is the rate of the acceleration command (i.e., jerk) and is defined to be the control input.

$$da_M/dt = G(t) \quad (3.1)$$

where  $G(t) = g(t) + g_0$ . And, the parameter  $g_0$  is defined to be an arbitrary constant while  $g(t)$  is given by a function of time. In that case, additional initial condition is needed as

$$a_M(t_0) = a_{M,0} \quad (3.2)$$

where  $a_{M,0}$  represents the initial acceleration command. Then, let us look at the following optimal guidance problem in consideration on the impact angle constraint: find  $g(t)$  which minimizes the following performance index.

$$J_2 = \frac{1}{2} \int_{t_0}^{t_f} \frac{1}{(t_f - s)^N} g(s)^2 ds. \quad (3.3)$$

subject to Eqs. (2.1) through (2.3) and Eq. (3.1).

Note that the impact time control guidance law generally takes a detour at the initial time to enlarge the flight time as much as the desired value. The reason why we introduce this performance index is to take a detour as fast as possible in the beginning of the homing phase. Then, the obtained guidance law can produce a small acceleration command in the terminal phase. We can predict that the obtained guidance law may generate a huge acceleration command as the parameter  $N$  increases. Additionally, note that the performance index as shown in Eq. (3.3)

is associated with  $g(t)$  only. Hereafter, we first derive the optimal impact angle control law that minimizes the quadratic performance index with a free design parameter  $g_0$ . After that, we determine  $g_0$  to satisfy the impact time constraint also, i.e.,  $t_f = t_d$  where  $t_d$  represents a prescribed impact time. The issue on how to satisfy the impact time will be discussed in the end of this section. Let the control command with the non-dimensionalized input variable be defined as [9]

$$\Xi = \eta + \eta_0 \quad (3.4)$$

where the parameters  $\eta$  and  $\eta_0$  are new non-dimensionalized variables, which are defined as  $\Xi = G/(V_M/t_f^2)$ ,  $\eta = g/(V_M/t_f^2)$  and  $\eta_0 = g_0/(V_M/t_f^2)$ , respectively. In addition, the initial condition of the acceleration is given by  $a(x_0) = a_0$  in the non-dimensional form. In a similar way as shown in Eq. (2.5), under the assumption of small  $\gamma_M$ , linearizing Eqs. (2.1) to (2.3) with Eq. (3.1) yields the augmented governing equation as follows:

$$\xi' = A\xi + B\Xi \quad (3.5)$$

where  $\xi = [y \quad \gamma_M \quad a]^T$ . The matrix  $A$  and  $B$  are given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (3.6)$$

**3.1. Derivation of generalized IACG and ITACG.** To obtain the control input, as shown in Eq. (3.4), that minimizes the considering performance index as given in Eq. (3.3) subject to Eq. (3.5), we define the Hamiltonian  $H$  as follows:

$$H = \frac{1}{2} \frac{1}{(x_f - x)^N} \eta^2 + \lambda_y \gamma + \lambda_{\gamma_M} a + \gamma_a (\eta + \eta_0) \quad (3.7)$$

The first-order necessary conditions to accomplish the optimality involve the following adjoint equations are determined by

$$\lambda'_y = 0, \quad \lambda'_{\gamma_M} = -\lambda_y, \quad \lambda'_a = -\lambda_{\gamma_M} \quad (3.8)$$

The boundary conditions are given by

$$\lambda_{y_f} = \nu_y, \quad \lambda_{\gamma_f} = \nu_{\gamma_M}, \quad \lambda_{a_f} = 0 \quad (3.9)$$

Integrating Eq. (3.8) backward from the terminal range  $x_f$  gives

$$\begin{aligned} \lambda_y &= \nu_y, \quad \lambda_{\gamma_M} = \nu_y(x_f - x) + \nu_{\gamma_M}, \\ \lambda_a &= \frac{1}{2} \nu_y (x_f - x)^2 + \nu_{\gamma_M} (x_f - x) \end{aligned} \quad (3.10)$$

From the well-known optimal control theory, the optimal control is determined by imposing the optimal condition as  $\partial H/\partial \eta = 0$ . From this condition, we have

$$\eta = -\frac{1}{2}\nu_y(x_f - x)^{N+2} - \nu_{\gamma_M}(x_f - x)^{N+1} \quad (3.11)$$

Then, substituting  $\eta$  and  $\eta_0$  into Eq. (3.4) and integrating Eq. (3.4) from  $x$  to  $s$  for  $s \in [x, x_f]$ , we can get explicit expressions of  $a(s)$ ,  $\gamma_M(s)$ , and  $y(s)$  as follows:

$$\begin{aligned} a'(s) &= \eta + \eta_0 = \eta_0 - \frac{1}{2}\nu_y(x_f - s)^{N+2} - \nu_{\gamma_M}(x_f - s)^{N+1} \\ a(s) &= a + \eta_0(s - x) - \frac{1}{(N+2)}\nu_{\gamma_M}(x_f - x)^{N+2} - \frac{1}{2(N+3)}\nu_y(x_f - x)^{N+3} \end{aligned} \quad (3.12)$$

$$\begin{aligned} &+ \frac{1}{(N+2)}\nu_{\gamma_M}(x_f - s)^{N+2} + \frac{1}{2(N+3)}\nu_y(x_f - s)^{N+3} \\ \gamma_M(s) &= \gamma_f - \left[ a + \eta_0(x_f - x) - \frac{1}{(N+2)}\nu_{\gamma_M}(x_f - x)^{N+2} - \frac{1}{2(N+3)}\nu_y(x_f - x)^{N+3} \right] \times \\ &(x_f - s) + \frac{1}{2}\eta_0(x_f - s)^2 - \frac{1}{(N+2)(N+3)}\nu_{\gamma_M}(x_f - s)^{N+3} \\ &- \frac{1}{2(N+3)(N+4)}\nu_y(x_f - s)^{N+4} \end{aligned} \quad (3.13)$$

and

$$\begin{aligned} y(s) &= y_f - \gamma_f(x_f - s) \\ &+ \frac{1}{2} \left[ a + \eta_0(x_f - x) - \frac{1}{(N+2)}\nu_{\gamma_M}(x_f - x)^{N+2} - \frac{1}{2(N+3)}\nu_y(x_f - x)^{N+3} \right] \times \\ &(x_f - s)^2 - \frac{1}{6}\eta_0(x_f - s)^3 + \frac{1}{(N+2)(N+3)(N+4)}\nu_{\gamma_M}(x_f - s)^{N+4} \\ &+ \frac{1}{2(N+3)(N+4)(N+5)}\nu_y(x_f - s)^{N+5} \end{aligned} \quad (3.14)$$

where  $a$  denotes  $a(x)$ . By evaluating these expressions at  $x = x_0$ , we can determine the terminal values  $\nu_{\gamma_M}$  and  $\nu_y$  as

$$\begin{bmatrix} \nu_y \\ \nu_{\gamma_M} \end{bmatrix} = W_\xi \xi + W_f \xi_f + W_\eta \eta_0 \quad (3.15)$$

where  $\xi_f = [y_f \ \gamma_f]^T$  and

$$\begin{aligned}
 W_\xi &= \begin{bmatrix} \frac{4(N+4)^2(N+5)}{x_{go}^{N+5}} & \frac{2(N+4)(N+5)^2}{x_{go}^{N+4}} & \frac{2(N+4)(N+5)}{x_{go}^{N+3}} \\ -\frac{2(N+3)(N+4)(N+5)}{x_{go}^{N+4}} & -\frac{(N+3)(N+4)(N+6)}{x_{go}^{N+3}} & -\frac{(N+3)(N+4)}{x_{go}^{N+2}} \end{bmatrix} \\
 W_f &= \begin{bmatrix} -\frac{4(N+4)^2(N+5)}{x_{go}^{N+5}} & \frac{2(N+3)(N+4)(N+5)}{x_{go}^{N+4}} \\ \frac{2(N+3)(N+4)(N+5)}{x_{go}^{N+4}} & -\frac{(N+3)(N+4)^2}{x_{go}^{N+3}} \end{bmatrix} \\
 W_\eta &= \begin{bmatrix} -\frac{(N+1)(N+4)(N+5)}{3x_{go}^{N+2}} \\ \frac{(N+2)(N+3)(N+4)}{6x_{go}^{N+1}} \end{bmatrix}
 \end{aligned} \tag{3.16}$$

Substituting Eq. (3.15) into Eq. (3.11) leads to the optimal control as

$$\begin{aligned}
 \eta &= \begin{bmatrix} -\frac{2(N+4)(N+5)}{x_{go}^3} & -\frac{(N+4)(N+7)}{x_{go}^2} & -\frac{2(N+4)}{x_{go}} \end{bmatrix} \xi \\
 &+ \begin{bmatrix} \frac{2(N+4)(N+5)}{x_{go}^3} & -\frac{(N+3)(N+4)}{x_{go}^2} \end{bmatrix} \xi_f + \frac{(N-1)(N+4)}{6} \eta_0
 \end{aligned} \tag{3.17}$$

Hence, from Eqs. (3.4) and (3.17), we can obtain ITACG which is given by the state feedback form as

$$\Xi_{ITACG} = K\xi + K_f\xi_f + K_\eta\eta_0 \tag{3.18}$$

where

$$\begin{aligned}
 K &= \begin{bmatrix} -\frac{2(N+4)(N+5)}{x_{go}^3} & -\frac{(N+4)(N+7)}{x_{go}^2} & -\frac{2(N+4)}{x_{go}} \end{bmatrix} \\
 K_f &= \begin{bmatrix} \frac{2(N+4)(N+5)}{x_{go}^3} & -\frac{(N+3)(N+4)}{x_{go}^2} \end{bmatrix} \\
 K_\eta &= \frac{(N+1)(N+2)}{6}
 \end{aligned} \tag{3.19}$$

In this equation, the parameters  $K$ ,  $K_f$ , and  $K_\eta$  are defined to be the guidance gains which are also given by the function of the design parameter  $N$ . From here, by setting  $\eta_0$  in Eq. (3.18), we finally obtain the generalized form of IACG for the augmented system as shown in Eq. (3.6).

$$\Xi_{IACG} = K\xi + K_f\xi_f \tag{3.20}$$



**3.2. Derivation of time-to-go ( $\hat{\tau}_{go,IACG}$ ) of generalized IACG.** The primary goal of this paper is to determine the optimal solution of IACG in conjunction with the specific additional term satisfying the desired impact time. It means that the length of estimated trajectory divided by the missile speed should be equal to the desired time-to-go. Then the impact angle and the impact time requirements are met simultaneously.

The impact time constraint in the non-dimensional form can be written by the term of the curvature of the estimated trajectory [11, 12]:

$$\bar{\tau}_{go} = \int_x^{x_f} \sqrt{1 + \gamma_M^2(s, \eta_0)} ds \quad (3.21)$$

where  $\bar{\tau}_{go}$  denotes the normalized desired time-to-go, i.e., the difference between the desired impact time and the current flight time. In our formulation, if  $\eta_0$  is properly chosen to satisfy the condition as shown in Eq. (3.21), the impact angle and impact time requirements will be satisfied.

From Eq. (3.21) the estimate in the normalized time-to-go for generalized IACG can be expressed by simply setting  $\eta_0 = 0$  as follows:

$$\hat{\tau}_{go,IACG} = \int_x^{x_f} \sqrt{1 + \gamma_M^2(s; \eta_0 = 0)} ds \quad (3.22)$$

Letting  $\zeta = x_f - s$  for convenience and taking the first two terms of Taylor series expansion of the integrand with respect to the heading angle provides

$$\begin{aligned} \hat{\tau}_{go,IACG} &= \int_x^{x_f} \sqrt{1 + \gamma_M^2} ds \approx \int_x^{x_f} \left( 1 + \frac{\gamma_M^2}{2} \right) ds \\ &= \int_0^{x_f-x} \left( 1 + \frac{1}{2} \bar{\gamma}_M(\zeta)^2 \right) d\zeta \end{aligned} \quad (3.23)$$

in which from Eq. (3.13)

$$\begin{aligned} \bar{\gamma}_M(\zeta) &= \gamma_f - \left[ a - \frac{1}{(N+2)} \nu_\gamma (x_f - x)^{N+2} - \frac{1}{2(N+3)} \nu_y (x_f - x)^{N+3} \right] \zeta \\ &\quad - \frac{1}{(N+2)(N+3)} \nu_{\gamma_M} \zeta^{N+3} - \frac{1}{2(N+3)(N+4)} \nu_y \zeta^{N+4} + \frac{1}{2} \eta_0 \zeta^2 \end{aligned} \quad (3.24)$$

From Eqs. (3.15) and (3.23), we obtain the estimate in the normalized time-to-go for IACG by letting  $\eta_0 = 0$  as follows:

$$\hat{\tau}_{go,IACG} = C + \Delta^{-1} (\xi^T P \xi + \xi^T Q \xi_f + \xi_f^T R \xi) \quad (3.25)$$

where

$$\Delta = 1080(N+5)(N+6)(N+7)(2N+7)(2N+9) \quad (3.26)$$

and

$$C = x_{go} \quad (3.27)$$

$$P = 90(N + 7) \times \begin{bmatrix} 8(N + 4)(N + 5)^2(4N + 19)/x_{go} & 2(N + 5)(14N^2 + 124N + 275) \\ 2(N + 5)(14N^2 + 124N + 275) & 2x_{go}(N + 4)(15N^2 + 151N + 376) \\ 2x_{go}(N + 5)(4N + 19) & x_{go}^2(3N + 13)(4N + 19) \\ 2x_{go}(N + 5)(4N + 19) & \\ x_{go}^2(3N + 13)(4N + 19) & \\ 2x_{go}^3(3N + 13) & \end{bmatrix} \quad (3.28)$$

$$Q = 180 \begin{bmatrix} -2(N + 4)(N + 5)^2(N + 7)(4N + 19)/x_{go} \\ -2(N + 5)(N + 7)(14N^2 + 124N + 275) \\ -2(N + 5)(N + 7)(4N + 19)x_{go} \\ 2(N + 3)(N + 5)(N + 7)(4N^2 + 26N + 37) \\ -2(N + 3)(N + 7)(N^2 + 14N + 43)x_{go} \\ -(N + 3)(N + 7)(4N + 19)x_{go}^2 \end{bmatrix} \quad (3.29)$$

$$R = 180(N + 7) \begin{bmatrix} 4(N + 4)(N + 5)^2(4N + 19)/x_{go} \\ -(N + 3)(N + 5)(4N^2 + 26N + 37) \\ -(N + 3)(N + 5)(4N^2 + 26N + 37) \\ (N + 3)^2(N + 4)(4N + 19)/x_{go} \end{bmatrix} \quad (3.30)$$

**3.3. Derivation of generalized ITACG**( $\eta_0 \neq 0$ ). Similarly, the estimate in the normalized time-to-go for the generalized ITACG using an arbitrary input  $\eta_0 \neq 0$  can be determined as follows:

$$\hat{\tau}_{go,ITACG} = \int_x^{x_f} \sqrt{1 + \gamma_M(s; \eta_0)^2} ds = \int_x^{x_f-x} \left( 1 + \frac{1}{2} \bar{\gamma}_M(\zeta^2) \right) d\zeta \quad (3.31)$$

$$\begin{aligned} \bar{\gamma}_M(\zeta) = \gamma_f - & \left[ a + \eta_0(x_f - x) - \frac{1}{(N + 2)} \nu_{\gamma_M}(x_f - x)^{N+2} - \frac{1}{2(N + 3)} \nu_y(x_f - x)^{N+3} \right] \zeta \\ & - \frac{1}{(N + 2)(N + 3)} \nu_{\gamma_M} \zeta^{N+3} - \frac{1}{2(N + 3)(N + 4)} \nu_y \zeta^{N+4} + \frac{1}{2} \eta_0 \zeta^2 \end{aligned} \quad (3.32)$$

By substituting Eq. (3.32) into Eq. (3.31), we also yield the normalized time-to-go estimation for ITACG in terms of  $\hat{\tau}_{go,ITACG}$  as follows:

$$\hat{\tau}_{go,ITACG} = \alpha \eta_0^2 + \beta \eta_0 + \hat{\tau}_{go,ITACG} \quad (3.33)$$

$$\alpha = \Delta^{-1}(N + 1)^2(N + 2)^2(8N + 35)x_{go}^5 \quad (3.34)$$

$$\begin{aligned} \beta = \Delta^{-1}30(N + 1)(N + 2)x_{go}^2 \times & \\ & \left[ -(N + 5)(4N^2 + 20N + 7) \quad 2x_{go}(5N^2 + 49N + 119) \quad x_{go}^2(8N + 35) \right] \xi \\ & + \Delta^{-1}30(N + 1)(N + 2)x_{go}^2 \times \left[ (N + 5)(4N^2 + 20N + 7) \right. \\ & \left. - x_{go}(N + 3)(4N^2 + 38N + 91) \right] \xi_f \end{aligned} \quad (3.35)$$

**3.4. Time-to-go error.** Now let us define the impact time error  $\varepsilon_\tau$  as follows.

$$\varepsilon_\tau = \hat{\tau}_{go,ITACG} - \hat{\tau}_{go,IACG} \quad (3.36)$$

From Eq. (3.33), we can easily observe that  $\hat{\tau}_{go,ITACG}$  becomes to  $\hat{\tau}_{go,IACG}$  when  $\eta_0$  vanishes. By using Eq. (3.33), the impact time error can be also written as

$$\eta_0^2 + (\beta/\alpha)\eta_0 = (\varepsilon_\tau/\alpha) \quad (3.37)$$

Let us introduce new variables  $\eta_L = (\beta/\alpha)$  and  $\eta_E = (4/\alpha)\varepsilon_\tau$  for simplicity, then the additional control command denoted by  $\eta_0$  satisfying Eq. (3.22) can be determined as

$$\eta_0 = -\frac{1}{2} \left( \eta_L \pm \sqrt{\eta_L^2 + \eta_E} \right) \quad (3.38)$$

where

$$\eta_L = M^T \xi + M_f^T \xi_f \quad (3.39)$$

$$\eta_E = N_E \varepsilon_\tau \quad (3.40)$$

Note that the term of  $\eta_E$  as given in Eq. (3.40) is used to control the impact time error  $\varepsilon_\tau$  by which  $\hat{\tau}_{go,ITACG}$  is replaced with the desired time-to-go in the normalized form of  $\bar{\tau}_{go}$ . Each gains of  $\eta_L$  and  $\eta_E$  are given by

$$M = \frac{30}{(N+1)(N+2)(8N+35)} \times \left[ \begin{array}{ccc} -\frac{(N+5)(4N^2+20N+7)}{x_{go}^3} & \frac{2(5N^2+49N+119)}{x_{go}^2} & \frac{(8N+35)}{x_{go}} \end{array} \right]^T \quad (3.41)$$

$$M_f = \frac{30}{(N+1)(N+2)(8N+35)} \left[ \begin{array}{c} \frac{(N+5)(4N^2+20N+7)}{x_{go}^3} \\ -\frac{(N+3)(4N^2+38N+91)}{x_{go}^2} \end{array} \right]^T \quad (3.42)$$

$$N_E = \frac{4320(N+5)(N+6)(N+7)(2N+7)(2N+9)}{x_{go}^5(N+1)^2(N+2)^2(8N+35)} \quad (3.43)$$

From here, note that the solution of Eq. (3.38) exists in the case of  $\varepsilon_\tau > 0$ . Generally, in the salvo attack scenarios, the desired impact time, which is the common goal of multiple missiles, is chosen to be larger than the maximum value of the estimated impact times under IACG of each missile. Hence, the impact time error  $\varepsilon_t$  has always a positive value in the initial time. Then, the value of  $\varepsilon_\tau$  approaches zero by employing ITACG. For the numerical stability in the actual implementation,  $\varepsilon_\tau$  in Eq. (3.38) can be computed by

$$\varepsilon_\tau = \max(\varepsilon_\tau, 0) \quad (3.44)$$

Also note that the sign of the second term on the right-hand-side (RHS) is decided to satisfy such condition that  $\eta_0 = 0$  when  $\varepsilon_\tau$  is null. This condition results in selecting the smaller one in absolute value out of two available  $\eta_0$  s, i.e., positive sign for  $\eta_L \geq 0$ , which is suitable for

the original objective of the minimization of jerk-control energy. Also this condition matches with the fact that if the impact time error is zero, the time-to-go of IACG is the same as the desired time-to-go and thus there is no need for the additional control term to control the impact time error. The optimal solution as given in Eq. (3.4) in conjunction with the additional control command term as given in Eq. (3.38) can simultaneously satisfy the impact time and angle constraints.

**3.5. ITACG with original state variable.** In the derivation of the proposed guidance law, we use the non-dimensionalized variables for convenience. Therefore, we need to write the guidance command using the original state variables. The non-dimensionalized variables used in this work are given by

$$\begin{aligned} x &= X/L_f, \quad y = Y/L_f, \quad a = a_M/(V_M/t_f), \\ \eta &= G/(V_M/t_f^2), \quad \tau = t/t_f \end{aligned} \quad (3.45)$$

The control input of Eqs. (3.1) and (3.4) can be rewritten in terms of the original states as follows:

$$G = \tilde{K}\tilde{\xi} + \tilde{K}_f\tilde{\xi}_f + \tilde{K}_0g_0 \quad (3.46)$$

where

$$\begin{aligned} \tilde{\xi} &= [ Y \quad \gamma_M \quad a_M ]^T \\ \tilde{\xi}_f &= [ Y_f \quad \gamma_f ] \end{aligned} \quad (3.47)$$

Accordingly, the feedback gains are also given by

$$\begin{aligned} \tilde{K} &= \left[ \begin{array}{ccc} -\frac{2(N+4)(N+5)V_M^3}{X_{go}^3} & -\frac{(N+4)(N+7)V_M^3}{X_{go}^2} & -\frac{2(N+4)V_M}{X_{go}} \end{array} \right] \\ \tilde{K}_f &= \left[ \begin{array}{cc} \frac{2(N+4)(N+5)V_M^3}{X_{go}^3} & -\frac{(N+3)(N+4)V_M^3}{X_{go}^2} \end{array} \right] \\ \tilde{K}_0 &= \frac{(N+1)(N+2)}{6} \end{aligned} \quad (3.48)$$

Consequently, the normalized time-to-go for IACG as given in Eq. (3.24) is rewritten as

$$\hat{\tau}_{go,IACG} = C + \Delta^{-1}(\xi^T P \xi + \xi^T Q \xi_f + \xi_f^T R \xi_f) \quad (3.49)$$

where  $\xi$ ,  $C$ ,  $P$ ,  $Q$ , and  $R$  are defined in Eq. (3.5), and Eqs. (3.27) through (3.30), respectively. Similarly, the estimation of the actual time-to-go by IACG can be converted as

$$\hat{t}_{go,IACG} = t_f \hat{\tau}_{go,IACG} = \tilde{C} + \tilde{\Delta}^{-1}(\tilde{\xi}^T \tilde{P} \tilde{\xi} + \tilde{\xi}^T \tilde{Q} \tilde{\xi}_f + \tilde{\xi}_f^T \tilde{R} \tilde{\xi}_f) \quad (3.50)$$

where

$$\tilde{C} = X_{go}/V_M \quad (3.51)$$

$$\tilde{P} = 90(N+7) \times \begin{bmatrix} 8(N+4)(N+5)^2(4N+19)/V_M X_{go} \\ 2(N+5)(14N^2+124N+275)/V_M \\ 2(N+5)(4N+19)X_{go}/V_M^3 \\ 2(N+5)(14N^2+124N+275)/V_M & 2(N+5)(4N+19)/X_{go}V_M^3 \\ 2(N+4)(15N^2+151N+376)X_{go}/V_M & (3N+13)(4N+19)X_{go}^2/V_M^3 \\ (3N+13)(4N+19)X_{go}^2/V_M^3 & 2(3N+13)X_{go}^3/V_M^5 \end{bmatrix} \quad (3.52)$$

$$\tilde{Q} = 180 \begin{bmatrix} -2(N+4)(N+5)^2(N+7)(4N+19)/V_M X_{go} \\ -2(N+5)(N+7)(14N^2+124N+275)/V_M \\ -2(N+5)(N+7)(4N+19)/V_M^3 \\ 2(N+3)(N+5)(N+7)(4N^2+26N+37)/V_M \\ -2(N+3)(N+7)(N^2+14N+43)X_{go}/V_M \\ -(N+3)(N+7)(4N+19)x_{go}^2/V_M^3 \end{bmatrix} \quad (3.53)$$

$$\tilde{R} = 180(N+7) \begin{bmatrix} 4(N+4)(N+5)^2(4N+19)/V_M X_{go} \\ -(N+3)(N+5)(4N^2+26N+37)/V_M \\ -(N+3)(N+5)(4N^2+26N+37)/V_M \\ (N+3)^2(N+4)(4N+19)X_{go}/V_M \end{bmatrix} \quad (3.54)$$

Now we can estimate the actual impact time error  $\varepsilon_t$  as

$$\varepsilon_t = t_f \varepsilon_\tau = \hat{t}_{go,ITACG} - \hat{t}_{go,IACG} \quad (3.55)$$

The additional control command  $g_0$  can also be rewritten as

$$g_0 = -\frac{1}{2} \left( \tilde{\eta}_L \pm \sqrt{\tilde{\eta}_L^2 + \tilde{\eta}_E} \right) \quad (3.56)$$

where

$$\tilde{\eta}_L = \tilde{M}^T \tilde{\xi} + \tilde{M}_f^T \tilde{\xi}_f \quad (3.57)$$

$$\tilde{\eta}_E = \tilde{N}_E \varepsilon_t \quad (3.58)$$

Here, each gains of  $\tilde{\eta}_L$  and  $\tilde{\eta}_E$  are given by

$$\tilde{M} = \frac{30}{(N+1)(N+2)(8N+35)} \times \begin{bmatrix} -\frac{(N+5)(4N^2+20N+7)V_M^3}{X_{go}^3} & \frac{2(5N^2+49N+119)V_M^3}{X_{go}^2} & \frac{V_M(8N+35)}{X_{go}} \end{bmatrix}^T \quad (3.59)$$

$$\tilde{M}_f = \frac{30}{(N+1)(N+2)(8N+35)} \times \left[ \frac{(N+5)(4N^2+20N+7)V_M^3}{X_{go}^3} - \frac{(N+3)(4N^2+38N+91)V_M^3}{X_{go}^2} \right]^T \quad (3.60)$$

$$\tilde{N}_E = \frac{4320(N+5)(N+6)(N+7)(2N+7)(2N+9)}{(N+1)^2(N+2)^2(8N+35)} \frac{V_M^7}{X_{go}^5} \quad (3.61)$$

Finally, the acceleration command to achieve the impact time and angle constraints together is obtained by taking integration of the jerk command as given in Eq. (3.46).

#### 4. NUMERICAL SIMULATION

**4.1. Performance of the proposed generalized ITACG.** In this section, we perform a number of numerical simulations in order to reveal the performance and characteristics of the proposed guidance law. In the first simulation, the performance of the proposed guidance law according to changes of the design parameter  $N$ . Note that the proposed guidance law with  $N = 0$  is identical to the previous version of ITACG. In this simulation, we consider a homing engagement scenario in which the missile flies with a constant speed of  $V = 250 \text{ m/s}$  and the target is assumed to be a stationary battle ship. The initial missile positions are chosen to be  $(0 \text{ km}, 0 \text{ km})$  and  $(0 \text{ km}, 10 \text{ km})$ , respectively. In this engagement scenario, the initial heading angle is set to be  $20 \text{ deg}$  and the desired impact angle is set to be  $-40 \text{ deg}$ , respectively. The prescribed impact time is chosen as  $t_d = 50 \text{ sec}$ . Additionally, the initial guidance command and the initial value of the integrator are set to zero. Under this engagement case, we perform simulations with and without in order to find out that how the design parameter can affect the performance of the proposed law.

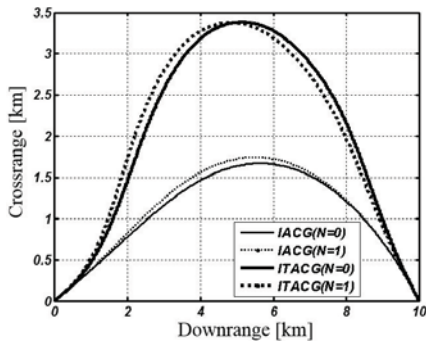


FIGURE 2. Trajectories by IACG and ITACG.

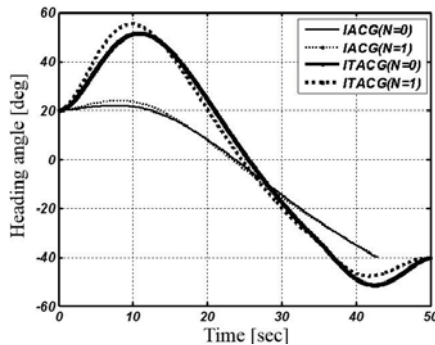


FIGURE 3. Heading angles.

Figure 2 represents the flight trajectories of these simulation cases. In the condition of IACG which is not supposed to control the impact time, the flight times are recorded as  $42.7 \text{ sec}$  for

$N = 0$  and  $42.9$  deg for  $N = 1$ , respectively. On the other hand, we can observe that ITACG can successfully satisfy the desired impact time  $t_d = 50$  sec as shown in Fig. 2. Under ITACG, it initially takes a detour in order to match its flight time. The heading angle as shown in Fig. 3 also shows the same tendency. In this figure, additionally, we can observe that ITACG can also achieve the desired impact angle value (i.e.,  $-40$  deg) as well.

Figure 4 describes the acceleration profiles. This figure shows that ITACG needs more control energy than IACG because ITACG should take a detour in the beginning of the homing phase to enlarge the flight trajectory. However, it is noted that ITACG needs less control energy near a target because ITACG generally takes a linear trajectory in the vicinity of the target to nullify the impact time error.

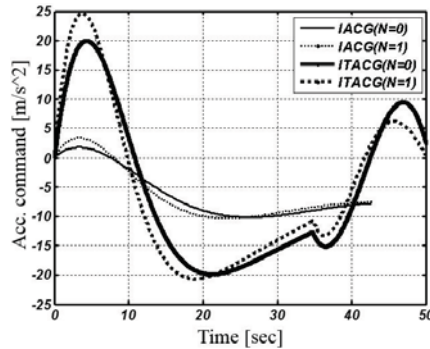


FIGURE 4. Acceleration commands

Figure 5 shows the jerk profile which is defined to be the input variable under the proposed method. In the case of ITACG, there is an abrupt command change around  $34.8$  deg due to the change of sign in Eq. (3.56). Namely, in the selection of two solutions, the proposed guidance law is designed to choose the solution that minimizes the control energy. Meanwhile, Fig. 6 represents the predicted impact time error which is computed from Eqs. (3.25) and (3.33). Compare with IACG, the predicted impact time error under ITACG goes to zero as  $t \rightarrow t_f$ .

Especially, if we use  $N = 1$  instead of  $N = 0$ , then we can achieve a small acceleration command around a target in the proposed guidance law as shown in Fig. 5 and 6. As shown in Eq. (3.3), the cost value in the case of  $N = 1$  gradually expensive as the time-to-go goes to zero. Accordingly, the proposed guidance law with  $N = 1$  tries to correct the guidance error early in the flight phase

**4.2. Application of generalized ITACG to a salvo attack scenario.** In this section, we apply the proposed guidance law to salvo attack scenario to show the validity of the proposed method. In this hear, the salvo attack means that each missile systems start from the different initial positions and after that all missile systems simultaneously attack a common target with various designated impact angles and single designated impact time. In this simulation, we assume that the target is placed on the origin as  $(0 \text{ km}, 0 \text{ km})$ . We consider the three missile systems with different initial conditions as provided in Table. 1. Note that all flight times are also set to

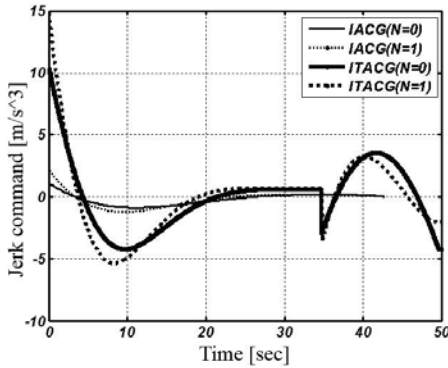


FIGURE 5. Jerk commands.

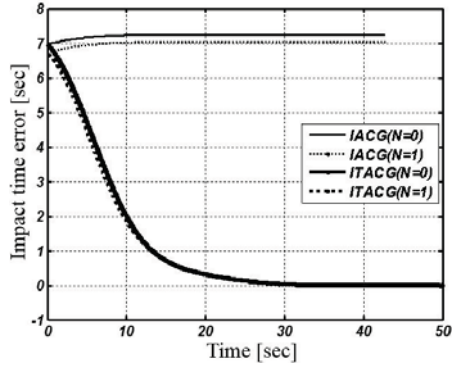


FIGURE 6. Time-to-go errors.

$t_d = 50$  sec. Additionally, the proposed guidance laws with  $N = 2$  are applied to all missile systems.

TABLE 1. Scenario for Salvo Attack

	$(X_0, Y_0), [km]$	$(\gamma_0, \gamma_f), [deg]$	$t_d, [sec]$
Target	(0, 0)	-	-
Missile #1	(-10, 1)	(30, 0)	50
Missile #2	(6, 6)	(180, -90)	50
Missile #3	(11, 3)	(180, 180)	50

Figure 7 shows the flight trajectories of multiple missile systems in the case of salvo attack scenario. As shown in this figure, under IACG, the flight times of each missile systems can be varied as 41.6 sec, 38.8 sec, and 46.1 sec. while, in the case of ITACG, all flight times are coincident with the desired value  $t_d = 50$  sec. In addition, a numbers of missile systems which are guided by the proposed guidance law can successfully achieve the desired impact angle values as well.

### 5. CONCLUSION

In this work, we propose a more generalized guidance to control the impact time and angle. The proposed guidance law can provide an additional degree of option to choose the guidance gain, then the designers can select the proper guidance gain as they want according to changes of the considering engagement cases. For example, the proposed guidance law enables the distribution of control effort near a target and the consideration of changing control efficiency according to changes of the altitude. Namely, since we suggest a new guidance law considering the control effort distribution, the proposed method can be applicable to the problem which demands enlarging of the flight envelope.



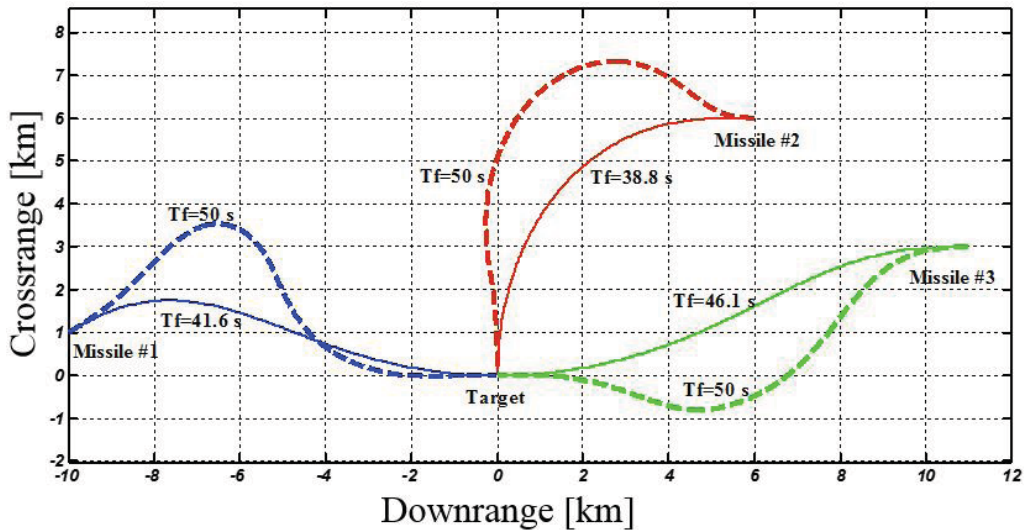


FIGURE 7. Salvo attack by IACG(solid) and ITACG(dotted)

#### REFERENCES

- [1] A. E. Bryson and Y. C. Ho, *Applied Optimal Control*. New York: Halsted Press Book, 1975.
- [2] P. Zarchan, *Tactical and Strategic Missile Guidance* (3rd ed.). Progress in Astronautics and Aeronautics, AIAA, 1997, 11–29.
- [3] M. Kim and K. V. Grider, *Terminal guidance for impact attitude angle constrained flight trajectories*, IEEE Transactions on Aerospace and Electronic Systems, **9**(6) (1973), 852–859.
- [4] B. S. Kim, J. G. Lee, and S. H. Han, *Biased PNG law for impact with angular constraint*, IEEE Transactions on Aerospace and Electronic Systems, **34**(1) (1998), 277–288.
- [5] C. K. Ryoo, H. J. Cho, and M. J. Tahk, *Optimal guidance laws with terminal impact angle constraint*, Journal of Guidance, Control, and Dynamics, **28**(4) (2005), 724–732.
- [6] C. K. Ryoo, H. H. Cho, and M. J. Tahk, *Time-to-go weighted optimal guidance with impact angle constraints*, IEEE Transactions on Control Systems Technology, **14**(2) (2006), 483–492.
- [7] Y. H. Kim, C. K. Ryoo, and M. J. Tahk, *Guidance synthesis for evasive maneuver of anti-ship missiles against close-in weapon systems*, IEEE Transactions on Aerospace and Electronic Systems, **46**(3) (2010), 1376–1388.
- [8] I. S. Jeon, J. I. Lee, and M. J. Tahk, *Impact-time-control guidance law for anti-ship missiles*, IEEE Transactions on Control Systems Technology, **14**(2) (2006), 260–266.
- [9] J. I. Lee, I. S. Jeon, and M. J. Tahk, *Guidance Law to Control Impact Time and Control*, IEEE Transactions on Aerospace and Electronic Systems, **43**(1) (2007), 301–310.
- [10] I. S. Jeon, J. I. Lee, and M. J. Tahk, *Homing guidance law for cooperative attack of multiple missiles against a single target*, Journal of Guidance, Control, and Dynamics, **33**(1) (2010), 275–280.
- [11] D. G. Hull, J. J. Radke, and R. E. Mack, *Time-to-go prediction for homing missiles based on minimum-time intercepts*, Journal of Guidance, Control, and Dynamics, **14**(5) (1991), 865–871.
- [12] M. J. Tahk, C. K. Ryoo, and H. Cho, *Recursive time-to-go estimation for homing guidance missiles*, IEEE Transactions on Aerospace and Electronic Systems, **38**(1) (2002), 13–24.