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# IMPACT-TIME-CONTROL GUIDANCE LAWS FOR COOPERATIVE ATTACK OF MULTIPLE MISSILES

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ABSTRACT. Two major simultaneous attack strategies have been introduced, as one of cooperative attack of multiple missiles. One strategy is an undesignated time attack, in which the missiles communicate among themselves to synchronize the arrival times by reducing the mutual differences of times-to-go of multiple missiles during the homing. The other is a designated time attack, in which a common impact time is commanded to all members in advance, and thereafter each missile tries to home on the target on time independently. For this individual homing, Impact-Time-Control Guidance (ITCG) law is required. After introducing cooperative proportional navigation (CPN) for the first strategy, this article presents a new closed-form ITCG guidance solution for the second strategy. It is based on the linear formulation, employing base trajectories driven by PNG with various navigation constants. Nonlinear simulation of several engagement situations demonstrates the performance and feasibility of the proposed ITCG law.

## 1. INTRODUCTION

A group of well-organized low-cost multiple vehicles might be far superior to a single high-technology and high-cost UAV in effectiveness. Tactical missile systems as well as UAVs provide more capabilities when they are organized as a coordinated group than when they are operated independently. Modern anti-ship missiles need to be able to penetrate the formidable defensive systems of battleships such as anti-air defense missile systems and close-in weapon system (CIWS). CIWS is a naval shipboard weapon system for detecting and destroying incoming anti-ship missiles and enemy aircraft at short range. These defensive weapons with powerful fire capability and various strategies seriously intimidate the survivability of the conventional anti-ship missiles. Hence, anti-ship missile developers have made great efforts to develop a high-performance missile system with ultimate sea-skimming flight and terminal evasive maneuvering capabilities despite a huge cost. On the other hand, cooperative attack strategies have been studied to enhance survivability of the conventional ones. Here, a cooperative attack means that multiple missiles attack a single target or multiple targets cooperatively or, in a specific case, simultaneously [1, 2]. Clearly, it is difficult to defend a group of attackers bursting into sight at the same time, even though each member is the conventional one in

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performance. So the simultaneous attack of multiple missiles is a cost-effective and efficient cooperative attack strategy.

A simultaneous attack of a group of missiles against a single common target can be achieved by two ways. The first approach is an undesignated time attack, in which the missiles try to synchronize the arrival times communicating among themselves. In this case it does not matter what the final flight time is. In other words, the missiles with larger times-to-go try to take shortcuts, whereas others with shorter times-to-go take detours to delay the arrival times. The second approach is a designated time attack, in which a common impact time is commanded to all members in advance, and thereafter each missile tries to home on the target on time independently. The first needs online data links throughout the engagement but the second concept requires determination of a suitable common impact time before homing.

Despite a number of studies on guidance problems related to time-to-go [3-7], studies on guidance laws to control impact time for a simultaneous attack are still rare. Proportional navigation (PN) is a well-known homing guidance method in which the rate of turn of the interceptor is made proportional with a navigation ratio N to the rate of turn of the line of sight (LOS) between the interceptor and the target. The navigation constant N is a unit-less gain chosen in the range from 3 to 5 [8]. Although PN with $N = 3$ is known to be energy-optimal, an arbitrary $N > 3$ is also optimal if a time-varying weighting function is included into the cost function of the linear quadratic energy-optimal problem [9, 10]. In general, the navigation ratio is held fixed. In some cases, however, it can be considered as a control parameter to achieve a desired terminal heading angle [11]. Although PN results in successful intercepts under a wide range of engagement conditions, its control-efficiency is not optimal, in general, especially for the case of maneuvering targets [12]. Augmented proportional navigation, a variant of PN, is useful in cases in which target maneuvers are significant [8]. Biased proportional navigation is also commonly used to compensate for target accelerations and sensor noises or to achieve a desired attitude angle at impact [13]. Even if PN and its variants are already well known and widely used, they are not directly applicable to many-to-one engagements.

For the undesignated time attack strategy, the homing guidance law named cooperative proportional navigation (CPN) was introduced [14]. CPN implements the concept in which the missiles synchronize the arrival times by reducing the mutual differences of times-to-go of multiple missiles during the homing. It has the same structure as conventional PN except that it has a time-varying navigation gain that is adjusted based on the onboard time-to-go and the times-to-go of the other missiles. CPN uses the time-varying navigation gain as a control parameter for reducing the variance of times-on-target of multiple missiles.

For the designated time attack strategy, a completely different type of guidance law is required from the designated time attack strategy. For this, an impact-time-control guidance law (ITCG) for anti-ship missiles was developed in [1] and, as an extension of this study, a guidance law to control both impact time and angle (ITACG) was presented in [15, 16]. In addition, the other form of impact time guidance laws was introduced in [17], in which the law has a form of a logical summation of the conventional PN law and an additional term of an impact-time error multiplied by some gain.

In this article, we introduce two representative guidance laws for a simultaneous attack of multiple missiles. For comprehensive explanation, CPN is first introduced as a preliminary study for the undesignated time attack, based on [14]. Then we propose a more general ITCG law than that of the previous study in [1], which is more general in that this study employs base trajectories driven by PNG with various navigation constants while the previous study did with only one navigation constant.

## 2. Background on cooperative attack strategies

2.1. **Cooperative Attack Strategies.** A cooperative attack in naval warfare means that multiple missiles attack their common target cooperatively to achieve their common goals. Figs. 1 and 2 illustrate the simultaneous attack for two missiles. Note that the terminal impact times, $T_{1f}$ and $T_{2f}$ , should be the same. Mutual on-line communication of maneuvering information is of course required for accomplishing their common goal, simultaneous approach, in a desirable manner. One can consider the case in which the desired or aiming impact-time for multiple missiles is commonly specified beforehand. This paper names this specific situation as a designated time attack. For example, Fig.1 represents the designated time attack for two missiles. Note that current maneuvering of a missile does not have an effect on the maneuver of the other missile at all and unique information shared with each other is a designated impact-time, $T_d$ . Each missile tries to home on the given target in an individual manner.

Next let us consider the situation in which the desired or aiming impact-time for multiple missiles is not specified in advance. Since the common impact-time is not specified, every missile may try to reduce the difference of the expected impact-time from the others. The undesignated time attack for two missiles is illustrated in Fig.2. Here the current maneuvering of a missile does have an effect on the maneuvering of the other missile and thus information is supposed to be exchanged with each other continuously during the homing.

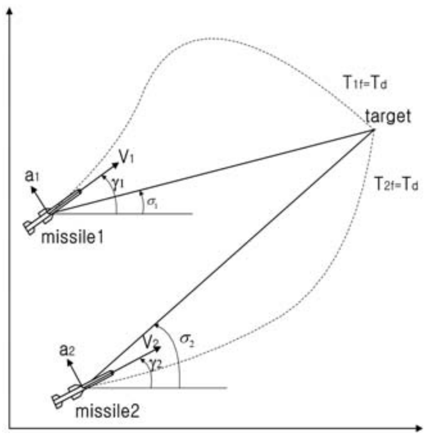


FIGURE 1.
Designated time attack.

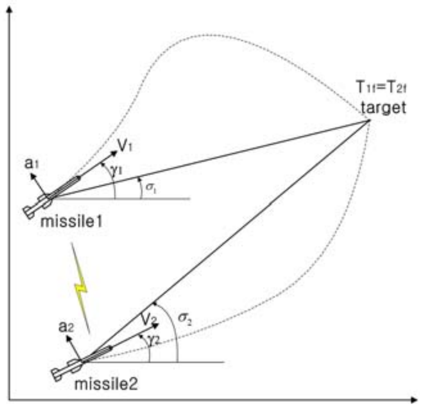


FIGURE 2.
Undesignated time attack.

2.2. **Time-To-Go Approximation.** To find the time-to-go expression associated with an initial range-to-go and a heading angle to LOS, we consider the inertial coordinate system shown in Fig.3. The origin of the coordinate system lies on the initial position of missile. Here, the x axis is directed to the target position and the y axis is normal to x axis. The governing equations are given by

$$\dot{x} = V_M \cos\gamma_M, \quad \dot{y} = V_M \sin\gamma_M, \quad \dot{\gamma} = a_M/V_M \tag{2.1}$$

where the dot denotes the differentiation with respect to time. Under the assumptions of small $\gamma_M$ angle, constant speed $V_M$, and acceleration command $a_M$ normal to velocity, substituting $x/V_M$ with time $t$ leads to

$$y' = \gamma_M, \quad \gamma'_M = a_M/V_M^2 \tag{2.2}$$

where the prime represents the derivative with respect to the downrange $x$. Since the line-of-sight angle $\sigma$ can be approximated as $-y/(x_f - x)$, the PN command $a_M$ is

$$a_M(x) = NV_M\sigma' = -\frac{NV_M^2}{(x_f - x)^2}y - \frac{NV_M^2}{(x_f - x)}y' \tag{2.3}$$

Substitute (2.3) into (2.2) to get

$$y'' + \frac{N}{(x_f - x)}y' + \frac{N}{(x_f - x)^2}y = 0 \tag{2.4}$$

where the initial conditions are given by $y(0) = 0$ and $y'(0) = 0$ . The solution of this Cauchy equation can be easily obtained as

$$y(x) = \frac{\gamma_{M_0}}{N-1}(x_f - x)\left(1 - \left(1 - \frac{x}{x_f}\right)^{N-1}\right) \tag{2.5}$$

Also differentiating (2.5) with respect to $x$ yields

$$\gamma_M(x) = -\frac{\gamma_{M_0}}{N-1}\left(1 - N\left(1 - \frac{x}{x_f}\right)^{N-1}\right) \tag{2.6}$$

The length of the trajectory of (2.5), $s$, is given by

$$s = V_M t_f = \int_0^{x_f} \sqrt{1 + y'^2}dx \tag{2.7}$$

If $y'(= \gamma_M)$ is assumed to be small, (2.7) can be approximated as

$$V_M t_f \approx \int_0^{x_f} (1 + \frac{1}{2}y'^2)dx = x_f\left(1 + \frac{\gamma_{M_0}^2}{2(2N-1)}\right) \tag{2.8}$$

Thus,

$$t_f \approx \frac{x_f}{V_M}\left(1 + \frac{\gamma_{M_0}^2}{2(2N-1)}\right) \tag{2.9}$$

where $t_f$ can be regarded as a time-to-go estimation at initial time.

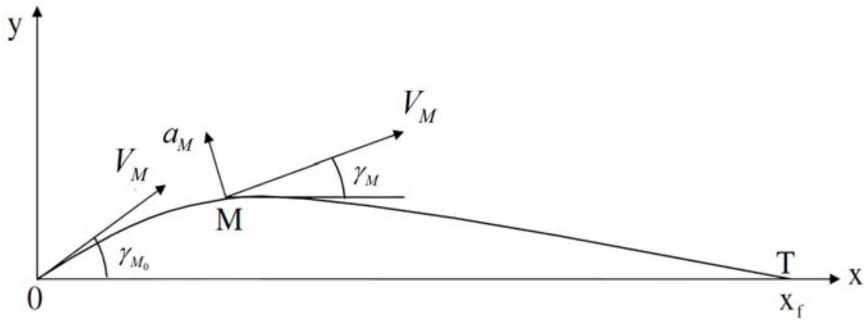


FIGURE 3. Guidance geometry for time-to-go calculation

## 2.3. **Guidance law for Undesignated Time Attack.**

2.3.1. *Cooperative Proportional Navigation.* Consider the homing guidance geometry of a missile attacking a stationary target as shown in Fig.4. The target is modeled stationary. The missile speed $V_M$ is assumed to be constant during the engagement. Let us suppose that the missile uses well-known PN for homing as

$$a_M = N V_M \dot{\sigma} \tag{2.10}$$

where $N$ denotes the navigation constant and $\dot{\sigma}$ the rate of line-of-sight angle, respectively.

Then a flight path angle is calculated from

$$\dot{\gamma_M} = a_M / V_M = N\dot{\sigma} \tag{2.11}$$

The rate of the line-of-sight can be obtained as

$$\dot{\sigma} = -\frac{V_M \sin\rho}{r} \tag{2.12}$$

where $r$ is a range-to-go and $\rho = \gamma_M - \sigma$ . Thus the governing equations in this homing problem can be expressed in terms of two states of $r$ and $\rho$ as

$$\dot{r} = -V_M \cos\rho \tag{2.13}$$

$$\dot{\rho} = -\frac{(N-1)V_M \sin\rho}{r} \tag{2.14}$$

Assume that $\rho$ is small and define a time-to-go for straight flight along LOS as $\hat{t}_{goL} = r/V_M$ , then the time-to-go of PN with navigation constant $N$ can be approximated from (2.9) as

$$\hat{t}_{go} = \hat{t}_{goL}(1+\delta) \tag{2.15}$$

where $\delta = \rho^2/(2(2N-1))$ represents an increment of the time-to-go by the initial heading angle $\rho$ . If $N$ remains constant for an entire homing, the time-to-go associated with normally used $N = 3 \sim 5$ is not much widely distributed. Note that the sensitivity of time-to-go to

N decreases as N increases. Further, $\hat{t}_{go} \to \hat{t}_{goL}$ as $N \to \infty$, so cannot be reduced less than $\gamma/V_M$ even though $N \to \infty$. Thus a time-invariant N may not be an effective design parameter for timing control. For this reason, a time-varying gain beyond the normally used navigation gain, especially even a negative gain, should be allowed.

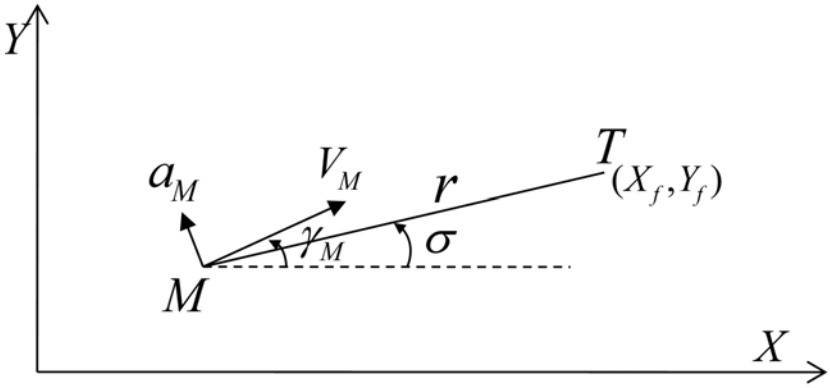


FIGURE 4. Homing guidance geometry of i-th missile.

Now suppose that $m$ missiles participate in a cooperative attack. Figure 5 shows the guidance geometry on many-to-one engagement scenario. Here the speed $V_M$ of one missile is also assumed to be constant during the engagement, but not necessarily to be the same as the speeds of the other missiles. Although each missile has different initial conditions, their common aim is to reach the target at the same instance. To achieve their common goal, conceptually, a missile far away from the target should try to home on the target as faster as possible and another missile relatively near the target should delay the impact-time by making a detour intentionally. Therefore, each missile is supposed to share its acceleration not only for a homing on the target but also for timing control with the others by this cooperative strategy.

The problem being posed here is to find a time-varying navigation gain $\overline{N}_i$ of PN as

$$a_i = \overline{N}_i V_{M_i} \dot{\sigma}_i \tag{2.16}$$

in order that a group of missiles participating in a cooperative attack can achieve a simultaneous attack in a cooperative manner, where the subscript i represents the $i$-th missile. First, consider a time-varying navigation gain $\overline{N}$ with a form of

$$\overline{N}_i(t) = N(1 - \Omega_i(t)), \quad i = 1, \ldots, m \tag{2.17}$$

where $N$ is navigation constant and $\Omega_i(t)$ is a time-varying gain ratio. Then the problem of finding a time-varying navigation gain $\overline{N}_i$ is transformed to that of finding a time-varying gain ratio $\Omega_i(t)$. Next, let us define a relative time-to-go error as

$$\hat{\varepsilon}_i(t) = \left( \frac{1}{m-1} \sum_{j=1, j \neq i}^{m} \hat{t}_{goj}(t) \right) - \hat{t}_{goi}(t) \tag{2.18}$$

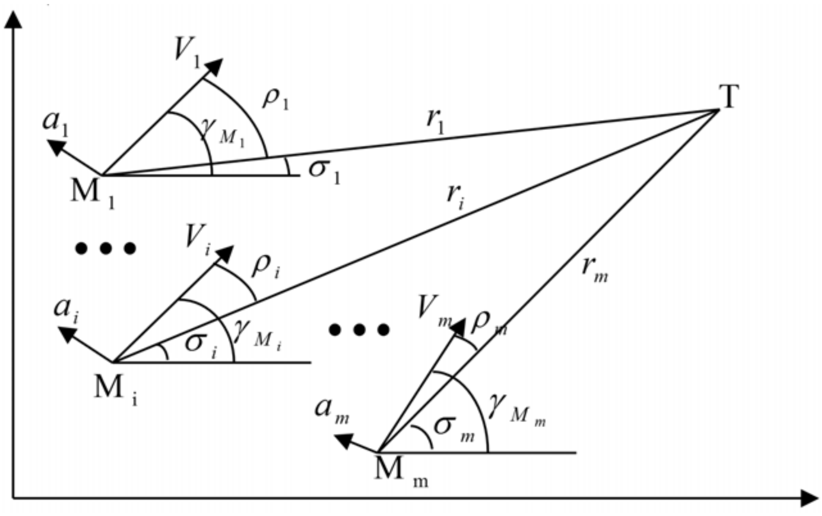


FIGURE 5. Guidance geometry on a many-to-one engagement scenario.

where, from (2.15),

$$\hat{t}_{goi}(t) \approx \frac{r_i(t)}{V_{M_i}} \left(1 + \frac{\rho_i^2(t)}{2(2N-1)}\right) \tag{2.19}$$

The relative time-to-go error is the difference between a time-to-go of one missile and a mean of times-to-go of the others. It can also be written as

$$\hat{\varepsilon}_i(t) = \frac{m}{m-1}(\bar{t}_{go}(t) - \hat{t}_{goi}(t)) \tag{2.20}$$

where

$$\bar{t}_{go}(t) = \frac{1}{m}\sum_{j=1}^{m} \hat{t}_{goj}(t) \tag{2.21}$$

which is the mean of times-to-go of all missiles. Then a variance, $\sum^2(t)$, of times-to-go of multiple missiles is expressed as

$$\sum{}^2(t) = \frac{1}{m}\sum_{j=1}^{m}(\bar{t}_{go}(t) - \hat{t}_{goj}(t))^2 \tag{2.22}$$

Note that all missiles have the same times-to-go, i.e., they might home on the target simultaneously,if $\sum^2(t)$ at any time during the homing. So the variance can be regarded as a performance indicator for a cooperative attack of multiple missiles employing PN with navigation constant of $N$. For convenience this paper call it the -variance. It is a central idea of this paper that a simultaneous attack can be achieved by means of decreasing the -variance by the time-varying navigation gain $\overline{N}_i(t)$ of (2.17) and further making $\overline{N}_i(t)$ itself become $N$ as

$r_i \to 0$ during the homing. If the time-varying navigation gain $\overline{N}_i(t)$ of (2.19) is applied, the governing equations can be written as, from (2.13) and (2.14),

$$\dot{r}_i(t) = -V_{M_i}\cos\rho_i(t) \tag{2.23}$$

$$\dot{\rho}_i(t) = -\frac{(N-1)V_{M_i}\sin\rho_i(t)}{r_i(t)} + \frac{NV_{M_i}\sin\rho_i(t)}{r_i(t)}\Omega_i(t) \tag{2.24}$$

where the terminal conditions are given by $r_i(t_{ft}) = 0$.

Now, we consider a homing guidance law, called CPN, in a very concise form as

$$\Omega_i(t) = Kr_i(t)\hat{\varepsilon}_i(t) \tag{2.25}$$

Note that $\overline{N}_i \to N$ as $r_i(t) \to 0$ , since $\Omega_i(t) \to 0$ as $r_i(t) \to 0$ . For a positive gain $K$, the proposed law of (2.25) subject to (2.23) and (2.24) makes the $N$-variance $\sum^2(t)$ of (2.22) decrease during the homing [14].

The CPN law makes a compromise with two independent purposes: the homing of one missile via well-known conventional navigation constant, and the reduction of the $N$-variance via trajectory reshaping by the gain ratio. Two purposes can be compromised from the fact that the trajectory reshaping is effective only when the range-to-go is long enough, i.e., at initial homing stage, but the homing takes precedence of trajectory reshaping near the target since CPN becomes conventional PN as the range-to-go reduces from (2.25). Note that CPN can be implemented without predetermination of the desired impact-time, and the law has a flexibility to allow a missile group with different speeds respectively to carry out a cooperative attack. And also CPN requires no additional information to normal PN for implementation except time-to-go calculations of missiles and the range-to-go of own missile.

2.3.2. *Numerical Simulation.* Suppose that four $(m = 4)$ missiles attack a single target equipped with CIWS. The initial conditions of each missile are given in Table 1. Initial ranges-to-go are 13 km, 13 km, 12 km, and 12 km, respectively. Missile has a speed from 270 to 300 m/s, respectively, as shown in Table 1.

TABLE 1. Engagement scenario for four missiles

| Parameters | Missile 1 | Missile 2 | Missile 3 | Missile 4 |
|---|---|---|---|---|
| Position (km) | $(1.74, 6.50)$ | $(0.20, 2.26)$ | $(1.18, -2.08)$ | $(2.16, -6.00)$ |
| Heading (deg) | 0 | 15 | 30 | 45 |
| Velocity (m/s) | 270 | 280 | 290 | 300 |
| Target Position (km) | $(13, 0)$ | $(13, 0)$ | $(13, 0)$ | $(13, 0)$ |

To apply the proposed law to this scenario, the gain K is properly chosen as $K = 40/(\overline{r}_0\overline{t}_{go0})$ where the subscript 0 denotes an initial time and the superscript bar represents the mean value. Initial times-to-go calculated by (3.16) are distributed from 40.3 to 49.5 sec as can be seen in the second column of Table 2.

TABLE 2. Simulation results (unit: sec)

| | $\hat{t}_{go0}$ | $t_f(PN)$ | $t_f(CPN)$ |
|---|---|---|---|
| Missile 1 | 49.5 | 49.5 | 48.3 |
| Missile 2 | 47.3 | 47.3 | 46.7 |
| Missile 3 | 41.9 | 41.9 | 47.5 |
| Missile 4 | 40.3 | 40.3 | 47.5 |
| $\bar{t}_{go}$(mean) | 44.7 | 44.7 | 47.5 |
| $\sum$ (std.) | 3.8 | 3.8 | 0.6 |

The mean value of times-to-go is 44.7 sec and the standard deviation is 3.8 sec. These values are quite consistent with the simulation results of the third column of Table 2. Figure 6 shows the flight trajectories of four missiles compared with PN-homing cases. CPN can guide four missiles to home on the target nearly at the same time where the mean value of impact-times is 47.5 sec and the standard deviation is only 0.6 sec as shown in the last column of Table 2. Figure 7 depicts the navigation gains. Each missile tries to reduce cooperatively the difference of times-to-go at early stage and then homes on the target with normal navigation constant of $N = 3$.

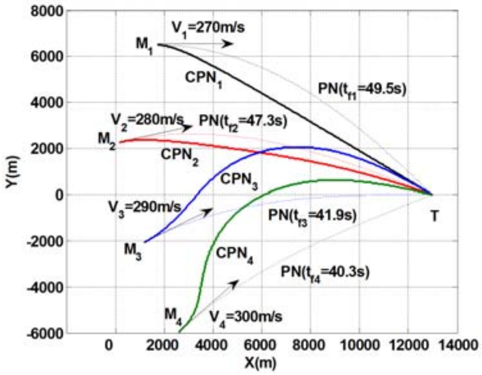


FIGURE 6.
Cooperative attack by CPN

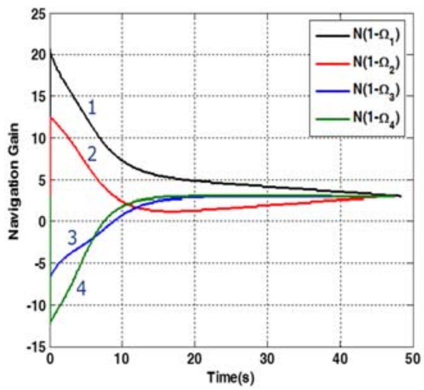


FIGURE 7. Navigation gains.

Figure 8 shows the time history of the standard deviation of the difference of times-to-go. One can see that the compromise between timing control and terminal homing occurs around 15 sec in this scenario. Figure 9 shows the time histories of acceleration commands of four missiles

## 3. GUIDANCE LAW FOR DESIGNATED TIME ATTACK

3.1. **Problem Statement.** Let us consider a typical cooperative attack scenario. Although each missile has a different missile-to-target range and an initial heading angle, their common

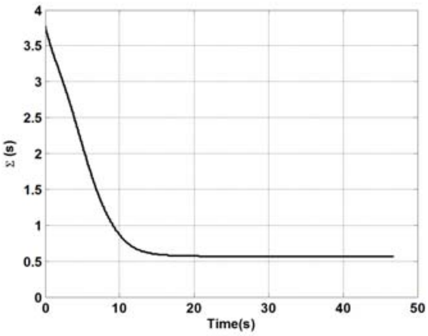


FIGURE 8.
Standard deviation of the difference of times-to-go.

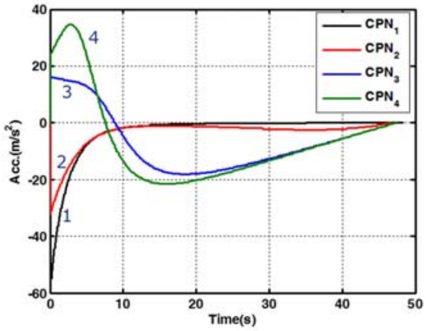


FIGURE 9.
Acceleration commands.

aim is to reach the target simultaneously. Two assumptions are mentioned here for realistic cooperative attack situations: The first one is that each of the missiles has a relatively similar range-to-target with each other. The second is that an initial velocity vector of each missile points towards not far away from the target. Under these assumptions, the discrepancies of the impact times by PNG are not to be so conspicuous. Thus in this work it is assumed that the designated impact time $(T_d)$ for cooperative attack is determined as

$$T_d \geq \max_i\{\hat{T}_i\} \text{ for any } i \in \{1, 2, \ldots, n\} \tag{3.1}$$

where $\hat{T}_i$ is the estimated impact time of the $i$-th missile guided by PNG. Here the trajectory generated by PNG is regarded as a base trajectory. It is reasonable to choose $T_d$ to be equal or larger than the maximum of $\{\hat{T}_i\text{'s}\}$ since a part of missiles may not satisfy the impact time requirement otherwise. It can be easily understood that while one can always increases the flight time by make a detour from the nominal path in principle, it is restricted to decrease the path length too much for reducing the flight time.

Let us consider the homing guidance geometry of one of the missiles against the single target as shown in Fig.4. The target is modelled stationary. And it is assumed that the missile speed is constant through the engagement and the autopilot lag is negligible. In Fig.4, $X$, $Y$, and $\gamma_M$ denote the missile positions and flight path angle in the inertial reference frame, respectively. The subscripts 0 and $f$ represent the initial and final times, respectively. Note that the terminal time is designated as $T_d$ in (3.1), then the governing equations in this homing problem are given by

$$\begin{aligned} \dot{X} &= V_M \cos\gamma_M, \quad X(0) = X_0, \quad X(T_f) = X_f \\ \dot{Y} &= V_M \sin\gamma_M, \quad Y(0) = Y_0, \quad Y(T_f) = Y_f \\ \dot{\gamma}_M &= a_M/V_M = (a_B + a_F)/V_M, \quad \gamma_M(0) = \gamma_{M0} \end{aligned} \tag{3.2}$$

where the terminal miss distance constraints are as $X(T_f) = X_f$ and $T(T_f) = Y_f$. The missile is controlled by the acceleration command, $a_M$, which is perpendicular to missile velocity. Here the acceleration command is composed of two different commands: the first one is a feedback command $a_B$ for reducing the miss-distance, and the second one is an additional command $a_F$ for adjusting the impact time.

First, we will solve an optimal control problem in which the control effort

$$J = \frac{1}{2}\int_0^{T_f} \frac{a_B^2}{(T_f - t)^m} dt \tag{3.3}$$

is minimized subject to (3.2) under the assumption that the feed-forward command $a_F$ is constant. And then, we will determine the additional command,$a_F$, which makes the length of homing route equal to the required range-to-go, i.e., the multiple of the missile speed and the designated time-to-go.

3.2. **Solution of Optimal Problem for Feedback Command.** The linearized state equations can be written as

$$\begin{bmatrix} dy/dx \\ d\gamma_M/dx \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \gamma_M \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_B + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_F \tag{3.4}$$

where the downrange $x$ is the independent parameter, and $\gamma_M$ is assumed to be small. Each variable is non-dimensionalized as

$$t = T/T_f, \quad x = X/(V_M T_f), \quad y = Y/(V_M T_f),$$
$$u_B = a_B/(V_M/T_f), \quad u_F = a_F/(V_M/T_f)$$

As a cost corresponding to (3.3), we consider the following cost as

$$J' = \frac{1}{2}\int_{x_0}^{x_f} \frac{u_B^2}{(x_f - x)^m} dx \tag{3.5}$$

Boundary conditions of each state variables are also non-dimensionalized as

$$x(0) = X_0/(V_M T_f) = x_0, \quad x(1) = X_f/(V_M T_f) = x_f,$$
$$y(x_0) = Y_0/(V_M T_f) = y_0, \quad y(x_f) = Y_f/(V_M T_f) = y_f,$$

Since $V_M$ is constant, $V_M T_f$ means the length of the flight trajectory at the final time, so

$$\begin{aligned} V_M T_f &= \int_0^{S_f} dS = \int_0^{S_f} \sqrt{dX^2 + dY^2} \\ &= \int_{X_0}^{X_f} \sqrt{1 + (dY/dX)^2} dX = \int_{X_0}^{X_f} \sqrt{1 + \tan^2 \gamma_M} dX \end{aligned} \tag{3.6}$$

Dividing the both sides of (3.6) by $V_M T_f$ and non-dimensionalizing $X$ yields

$$1 = \int_{x_0}^{x_f} \sqrt{1 + \tan^2 \gamma_M} dx \tag{3.7}$$

Therefore, the constraint on impact time under the assumption of the small $\gamma_M$ angle can be expressed with respect to downrange as

$$1 = \int_{x_0}^{x_f} \sqrt{1+\gamma_M^2(x)}dx \tag{3.8}$$

$\gamma_M^2$ in LHS is an approximation of $\tan^2\gamma_M$ representing the curvature of trajectory. This relation is of importance since the given impact-time constraint is changed into the length constraint of trajectory so that we can treat the intractable independent variable more easily in an indirect manner.

This linear optimization problem can be solved analytically by applying the minimum principle [10]. Let us write the Hamiltonian $H$ for the problem as

$$H = \frac{1}{2}\frac{u_B^2}{(x_f-x)^m} + \lambda_y\gamma_M + \lambda_\gamma(u_B+u_F) \tag{3.9}$$

where $\lambda_y$ and $\lambda_\gamma$ are the co-states. The optimality condition $\partial H/\partial u_B = 0$ gives the optimal acceleration command

$$u_B = -(x_f-x)^m\lambda_\gamma = -\nu_y(x_f-x)^{m+1} \tag{3.10}$$

where $\nu_y$ is a constant to be determined to satisfy the final conditions. Integration of (3.4) leads to

$$\nu_y = -\frac{(m+3)\left(y_f - \gamma_{M_0}x_f - u_F\frac{x_f^2}{2}\right)}{x_f^{m+3}} \tag{3.11}$$

Hence, the feedback guidance command at the initial time can be expressed as a function of $u_F$ :

$$u_B = (m+3)\frac{(y_f-\gamma_{M_0}x_f)}{x_f^2} - \frac{m+3}{2}u_F \tag{3.12}$$

The continuous-time feedback law is then obtained as

$$u_B = u_P - \frac{m+3}{2}u_F \tag{3.13}$$

where $u_P \equiv (m+3)(y_{go}-\gamma_M x_{go})/x_{go}^2$, $x_{go} \equiv x_f - x$ and $y_{go} \equiv y_f - y$. Note that $u_P$ of (3.13) is the linear approximation of PNG with the navigation constant of $N = m+3$.

3.3. **Determination of the Additional Command.** We start from the condition of (3.8), which says that the range-to-go has to be equal to the multiple of the constant missile speed and the time-to-go specified by $T_d$ , which is the difference between the designated impact time and the current time. The heading angle, $\gamma_M$ can be expressed as a polynomial of $x$ from (3.4) and (3.13)

$$\gamma_M(x) = \frac{\nu_y(x_f-x)^{m+2}}{m+2} + u_F x + \gamma_{M_0} - \frac{\nu_y x_f^{m+2}}{m+2} \tag{3.14}$$

Taylor series expansion of (3.8) over $\gamma_M$ with the higher order terms neglected yields

$$\bar{t}_{go} = \int_{x_0}^{x_f} \sqrt{1+\gamma_M^2(\eta)}d\eta \approx \int_{x_0}^{x_f} 1 + \frac{1}{2}\gamma_M^2(\eta)d\eta \tag{3.15}$$

where $\bar{t}_{go}$ is the designated time-to-go. Substituting (3.14) into (3.15) and integrating (3.15) yields

$$u_F^2 + \frac{6u_P u_F}{(m+3)(m+1)} + \frac{12(m+4)(2m+5)}{(m+1)^2 x_f^3}\left(\left(1+\frac{1}{2}\gamma_{M_0}^2\right)x_f + \frac{x_f^2\gamma_{M_0}u_P}{m+3} - \frac{x_f^3 u_P^2}{(2m+5)(m+3)}\right) - \frac{12(m+4)(2m+5)}{(m+1)^2}\frac{\bar{t}_{go}}{x_f^3} = 0 \tag{3.16}$$

Letting $u_F$ be zero in (3.16) results in the useful approximation of the time-to-go of PNG, $\hat{t}_{go}$, as follows:

$$\hat{t}_{go} = \left(1+\frac{1}{2}\gamma_{M_0}^2\right)x_f + \frac{1}{m+3}x_f^2\gamma_{M_0}u_P - \frac{1}{(2m+5)(m+3)}x_f^3 u_P^2 \tag{3.17}$$

Then (3.16) is written as

$$u_F^2 + \left(\frac{6u_P}{(m+3)(m+1)}\right)u_F + \frac{12(m+4)(2m+5)}{(m+1)^2}\frac{(\hat{t}_{go}-\bar{t}_{go})}{x_f^3} = 0 \tag{3.18}$$

The solutions of (3.18) with respect to $u_F$ are given by

$$u_F = -\frac{3u_P}{(m+3)(m+1)} \pm \sqrt{\left(\frac{3u_P}{(m+3)(m+1)}\right)^2 + \frac{12(m+4)(2m+5)}{(m+1)^2}\frac{\varepsilon_T}{x_{go}^3}} \tag{3.19}$$

where $\varepsilon_T$ is an impact time error, i.e., $\varepsilon_T = \bar{t}_{go} - \hat{t}_{go}$. It is natural to choose the sign in expression of (3.19) as to nullify when $\varepsilon_T = 0$, which means an achievement of the goal of impact time control guidance. Recall that a zero $u_F$ implies PNG with navigation constant $(m+3)$ from (3.13). Thus we obtain the additional command as

$$u_F = -\frac{3u_P}{(m+3)(m+1)} + s(u_P)\sqrt{\left(\frac{3u_P}{(m+3)(m+1)}\right)^2 + \frac{12(m+4)(2m+5)}{(m+1)^2}\frac{\varepsilon_T}{x_{go}^3}} \tag{3.20}$$

where $s(\cdot)$ is kind of a sign function defined by

$$s(x) = \begin{cases} +1 & x>0 \\ \pm 1 & x=0 \\ -1 & x<0 \end{cases} \tag{3.21}$$

Combining (3.13) with (3.20) yields the acceleration command, which is the proposed ITCG law based on the linear formulation, as follows:

$$u_B + u_F = \frac{2m+9}{2m+6}u_P - s(u_P)\frac{1}{2}\sqrt{\left(\frac{3u_P}{(m+3)}\right)^2 + \frac{12(m+4)(2m+5)}{x_{go}^3}\varepsilon_T} \tag{3.22}$$

As the missile approaches the target, the control command may blow up as (3.22) implies that the sensitivity of $u_F$ to the time-to-go error is inversely proportional to $x_{go}$ . To circumvent this undesirable property of ITCG, ITCG is switched to PNG once the calculated impact time error is reduced below a certain threshold. A small impact time error means that the range-to-go by PNG is approximately equal to that of ITCG. So conversion to PNG yields smooth homing with acceptable impact time errors.

The expression on ITCG with physical units corresponding to (3.22) for nonlinear simulation in the time domain is as follows:

$$a_M = a_B + a_F = \frac{2m+9}{2}V_M\dot{\sigma} - \frac{1}{2}s(\dot{\sigma})\sqrt{(3V_M\dot{\sigma})^2 + \frac{12(m+4)(2m+5)V_M^5}{R_{go}^3}\varepsilon_T} \tag{3.23}$$

where $\varepsilon_T = \overline{T}_{go} - \hat{T}_{go}$ and $R_{go} = \sqrt{X_{go}^2 + Y_{go}^2}$. Employing (2.9), we can get a different simple form of $\hat{t}_{go}$ from (3.17) by setting the axis of Fig.3 to be consistent with the current LOS in Fig.5. So from (2.9), the time-to-go estimate is given by

$$\hat{t}_{go} \approx \left(1 + \frac{(\gamma_M - \sigma)^2}{2(2N+1)}\right) r_{go} \tag{3.24}$$

where $N = m + 3$ , $r_{go} = \sqrt{x_{go}^2 + y_{go}^2}$, and $\sigma$ is a LOS angle. So the time-to-go estimation with physical units can be expressed as

$$\hat{T}_{go} = \frac{R_{go}}{V_M}\left(1 + \frac{(\gamma_M - \sigma)^2}{2(2N+1)}\right) \tag{3.25}$$

Since this approximation becomes accurate when the angle between the missile velocity and LOS is reduced, the accuracy of estimation increases as a missile approaches to a target. If we set $N = 3$, i.e. $m = 0$ in (3.23), then (3.23) is equal to the result given in [1] as

$$a_M = \frac{3}{2}(3V_M\dot{\sigma}) - \frac{1}{2}s(\dot{\sigma})\sqrt{(3V_M\dot{\sigma})^2 + \frac{240V_M^5}{R_{go}^3}\varepsilon_T} \tag{3.26}$$

In conclusion, we suggest (3.23) as a general ITCG law based on base trajectories driven by PNG with navigation constant of $N = m + 3$.

3.4. **Numerical Examples.** In order to investigate the characteristics of the proposed law, nonlinear simulations are performed. The target is fixed by 10km from the missile. Initial heading angle is 30deg and the speed of missile is 300m/e. In the case of PN with $N = 3$, the impact time is observed in simulation to be about $34.28$ sec as shown in Fig.10, which is almost the same as the approximated impact time (34.25 sec) of (3.25). Note that this is quite different from the designated impact time of 40 sec.

In the case of ITCG, however, the impact time is observed to be about 40.0 sec, quite equal to the desired impact time. The solid-line in Fig.5 depicts the trajectory by ITCG, producing much longer route than PNG. Figure 10 also compares the heading angle profiles of PNG and ITCG. The heading angle increases at the beginning and then decreases for ITCG, while it decreases monotonically for PNG.

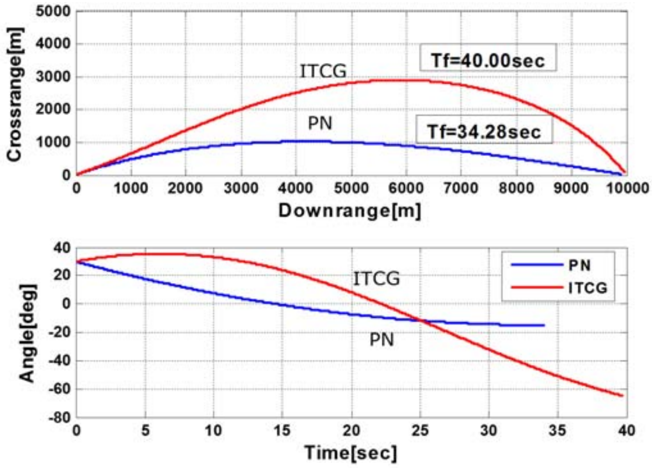


FIGURE 10. Trajectories by ITCG.

Figure 11 shows clearly that the impact-time error decreases from the early stage with ITCG while it does not with PNG. As a result, it is observed in Fig.12 that ITCG employs more control energy than PNG.

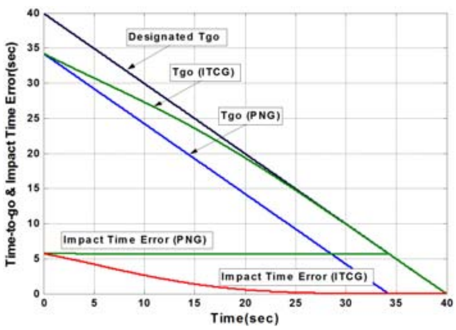


FIGURE 11.
Time-to-go and impact time errors.

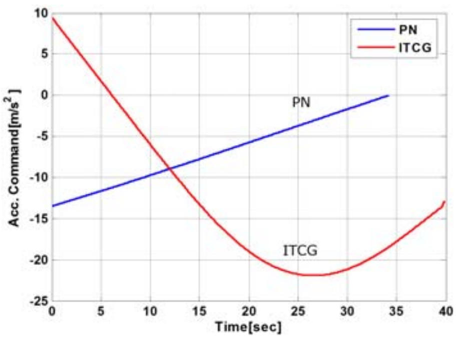


FIGURE 12.
Acceleration Commands.

Figure 13-15 illustrate the case of ITCG by (3.23) and (3.25) based on PNG with N=2, 3, 4, 5, respectively. They all satisfy the impact time requirement. As N becomes larger, ITCG employs more control energy at initial stage as expected from the PNG trajectories.

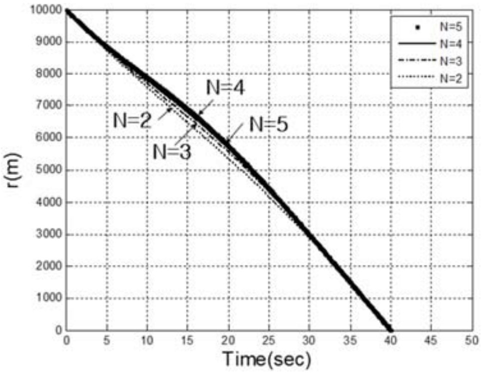


FIGURE 13. Range-to-go by ITCG (N=2 5).

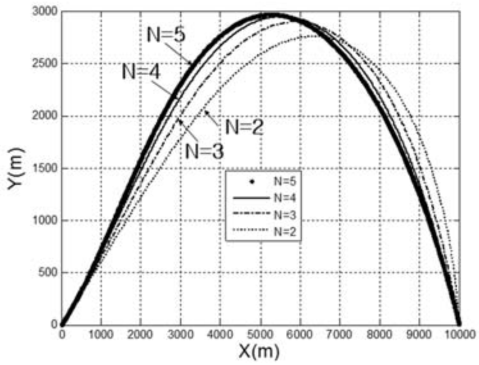


FIGURE 14. Trajectories by ITCG (N=2 5).

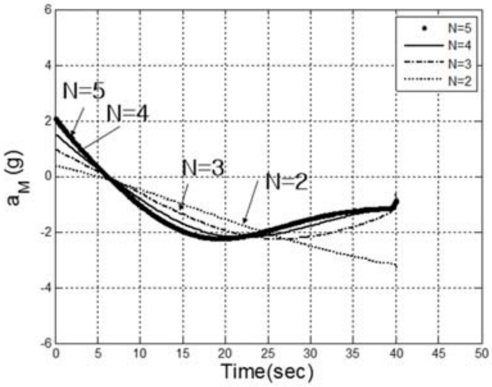


FIGURE 15. Acceleration commands by ITCG (N=2 5).

Suppose that four missiles attack a single target with different initial conditions, respectively, as shown in Table 1 (in Section 2). Each missile has the different speed, from 270 up to 300 m/s, respectively, and the designated impact time is determined as 50 sec, which is selected to be slightly larger than the maximum of estimated impact times by PNG $(N = 3)$. Figure 16 illustrates the cooperative attack trajectories of four missiles by PNG and shows large discrepancies in the impact time of each missile. In this scenario, it is observed that the dispersion of impact time is about 9.2 seconds. On the other hand, ITCG explicitly demonstrates that it can drive the four missiles to hit the target nearly simultaneously at the desired impact time.

In this example, ITCG reduces the impact time dispersion within about 0.1 seconds around the designated value

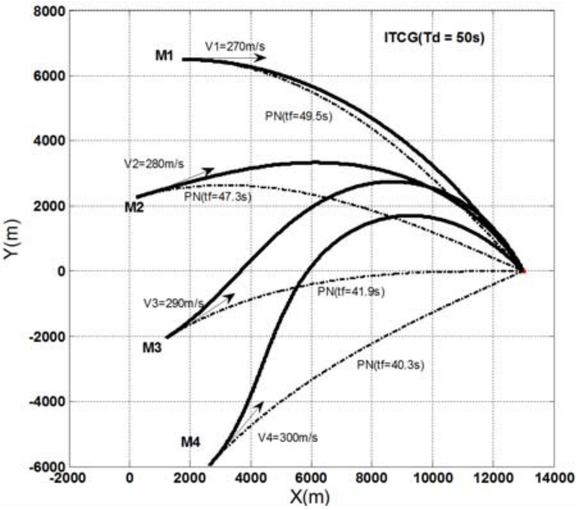


FIGURE 16. Cooperative attack by ITCG.

## 4. CONCLUSION

This paper introduces two important simultaneous attack strategies of multiple missiles and corresponding homing guidance laws. For comprehensive explanation, the CPN (cooperative proportional navigation) guidance law is first introduced as an undesignated impact time strategy, which can achieve a simultaneous attack by decreasing the t-go variance cooperatively till the intercept. As main result in this article, the new ITCG (Impact-time-control guidance) law is presented for the designated time attack strategy. This law guides a missile to the stationary target at the predetermined designated-impact-time. As multiple missiles share their common designated-impact-time at homing starting point at once, they can achieve cooperative simultaneous attack independently by the proposed law. The proposed law is regarded more general form of ITCG than that of the previous study, in that it employs base trajectories driven by PNG with various navigation constants while the previous study did with only one navigation constant. The performance and the feasibility of the laws are demonstrated through nonlinear simulations of various engagements.

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