CORROSION SCIENCE AND TECHNOLOGY, Vol.14, No.4(2015), pp.161~165
pISSN: 1598-6462 / eISSN: 2288-6524
DOI: http://dx.doi.org/10.14773/cst.2015.14.4.161

Stress Analysis in Polymeric Coating Layer Deposited on Rigid Substrate

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(Received February 03, 2015; Revised August 06, 2015; Accepted August 24, 2015)

This paper presents an analysis of thermal stress induced along the interface between a polymeric coating layer and a steel substrate as a result of uniform temperature change. The epoxy layer is assumed to be a linear viscoelastic material and to be theromorheologically simple. The viscoelastic boundary element method is employed to investigate the behavior of interface stresses. The numerical results exhibit relaxation of interface stresses and large stress gradients, which are observed in the vicinity of the free surface. Since the exceedingly large stresses cannot be borne by the polymeric coating layer, local cracking or delamination can occur at the interface corner.

Keywords : *thermal stress, coating layer, polymeric layer, interface stress*

1. Introduction

Polymeric materials such as an epoxy are widely used as coating layers.[1,3] Thermal stresses in coating layer/substrate structures are an unavoidable result of the difference in coefficients of thermal expansion between coating layer and substrate. Such stresses result from the polymeric coating layer having a higher thermal coefficient of expansion than the substrate. The deformation of the coating layer is constrained by the substrate, and hence thermal stresses are built up in the coating layer. These thermal stresses can play a very important role during subsequent loading of the coating layer/substrate system. Such thermal stresses may cause premature failure upon external loading.

When the temperature change occurs, adhesion of epoxy coating layer to substrates such as steel tends to fall off as shown Fig. 1. It is well known that the interface

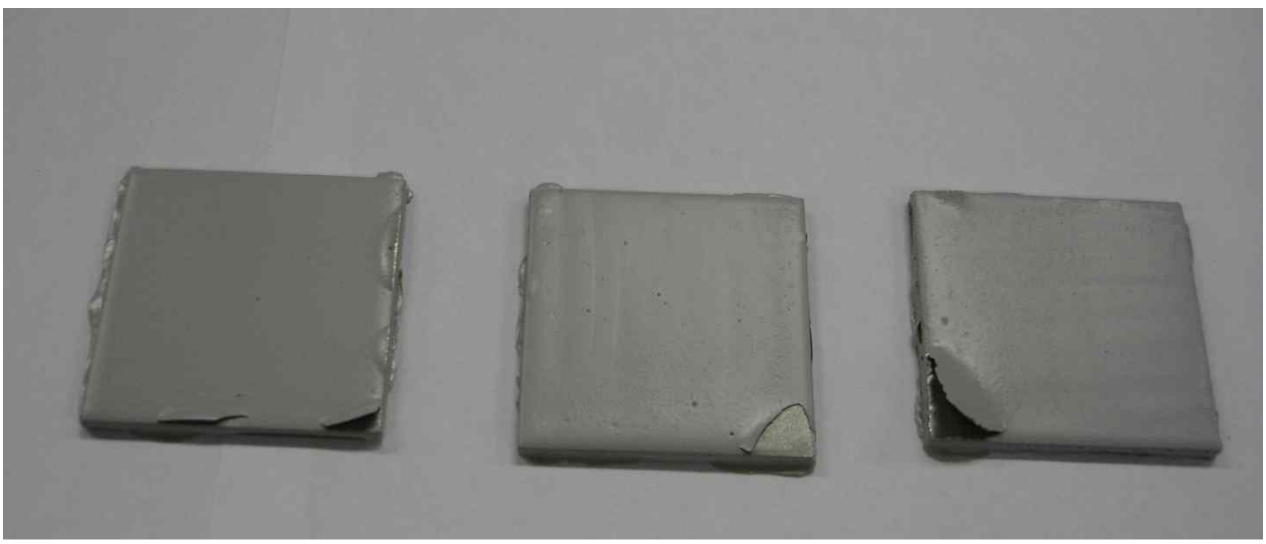

Fig. 1. Epoxy layers separated from steel substrates.

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of bonded quarter planes suffers from a stress system in the vicinity of the free surface under the external loading.[4,5] In such a region two interacting free surface effects occur, and singular interface stresses can be produced.

Polymeric layers in general respond in a viscoelastic manner under loads and their time-dependent behavior is affected by temperature. Therefore, a time-dependent stress analysis of the bi-material system is essential in understanding and predicting failure in such systems. The interface stresses in viscoelastic layer have been studied by several investigators. Weitsman[6] analyzed the mechanical behavior of an epoxy adhesive layer as the adhesive absorbs moisture from the ambient environment. Delale and Erdogan[7] presented the viscoelastic behavior of an adhesively bonded lap joint. Lee[8] performed the boundary element analysis of the stress singularity for the viscoelastic adhesive layer under transverse tensile strain. Because the order of the singularity is material-dependent, it tends to change with time in viscoelastic materials. Lee[9] also analyze the osmotic blistering behavior of polymeric coating film which is contact with an aqueous environment.

In this study, the thermal stress singularity at the interface corner between an elastic steel substrate and a viscoelastic epoxy coating layer subjected to a uniform temperature change is investigated. The substrate is assumed to be rigid as it is much stiffer than the viscoelastic layer. The boundary element method (BEM) is employed to investigate the interface stresses in the viscoelastic coating layer.

2. The Stress Singularity

The order of the stress singularity at the interface corner between the substrate and the viscoelastic layer can be

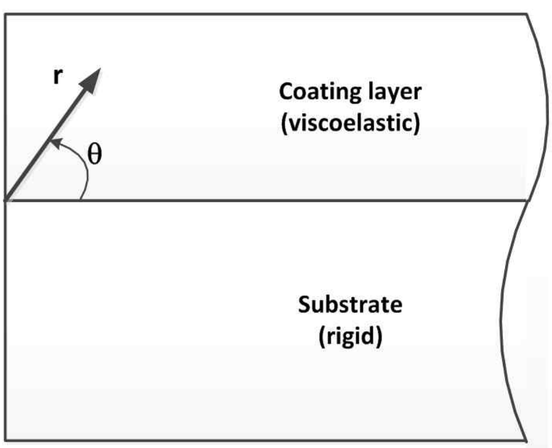

Fig. 2. Region near interface corner between the coating layer and the substrate.

determined using a method similar to that described in Ref.[8,10] Fig. 2 shows the region near the interface corner between perfectly bonded viscoelastic and rigid quarter planes. The free surfaces are assumed to be traction-free and the uniform temperature change provides the only loading.

Epoxy coating layers in general are thin. For sufficiently thin layer it is possible to neglect the temperature gradient through the thickness and consider the transient case of uniform temperature $T(\mathbf{x},t) = T(t)$. It is further assumed that the coating layer is thermorheologically simple.

In the following, a condition of plane strain is considered. A solution of

$$\nabla^4 \phi(r,\theta;t) = 0 \tag{1}$$

or, equivalently,

$$\nabla^4 \phi(r,\theta;\xi) = 0 \tag{2}$$

is to be found such that the normal stress, $\sigma_{\theta\theta}$, and shear stress, $\tau_{r\theta}$,vanish along $\theta = \pi/2$, further that the displacements are zero across the common interface line $\theta = 0$. In Eq. 2, ξ is the *reduced time* defined as follows:

$$\xi = \xi(t) = \int_0^t A_T(T(\rho))d\rho \tag{3}$$

where A_T is the shift function, a function of temperature history.

In the present study, a constant temperature change $\Delta TH(t)$ is considered. Under the constant temperature change, the reduced time ξ of Eq. 3 becomes

$$\xi = A_T t \tag{4}$$

The solution of this problem can be facilitated by the Laplace transform. With temperature change $\Delta TH(t)$ in layer, it is convenient to use the Laplace transform with respect to reduced time ξ, instead of *real time* t. Then, Eq. 2 can be rewritten as follows:

$$\nabla^4 \phi^*(r,\theta;s) = 0 \tag{5}$$

where ϕ^* denotes the Laplace transform of ϕ with respect to ξ and s is the transform parameter.

Using a method similar to that described by Williams [10], the Laplace transformed characteristic equation is obtained as follows:

$$\frac{2\lambda^2}{s} - 8s\left[\nu^*(s)\right]^2 + 12\nu^*(s) - \frac{5}{s} - \left[\frac{3}{s} - 4\nu^*(s)\right]\cos(\lambda\pi) = 0 \tag{6}$$

where λ is the stress singularity parameter and s is the transform parameter. $\nu^*(s)$ is Laplace transform of the viscoelastic Poisson's ratio $\nu(\xi)$.

The time-dependent behavior of the problem is recovered by inverting Eq. 6 into the time space. The coating layer considered here is characterized by a standard solid shear relaxation modulus and an elastic bulk modulus as follows:

$$K(\xi) = k_o, \quad \mu(\xi) = \mu_0 + \mu_1 \exp\left(-\frac{\xi}{t^*}\right) \tag{7}$$

where $\mu(\xi)$ is a shear relaxation modulus, $K(\xi)$ is a bulk modulus, μ_0, μ_1 and k_o are positive constants, and t^* is the relaxation time. Introducing Eq. 7 into Eq. 6 and inverting the resulting equation, we have

$$2\lambda^2 - 8M_1(\xi) + 12M_2(\xi) - 5 - \left[3 - 4M_2(\xi)\right]\cos(\lambda\pi) = 0 \tag{8}$$

where

$$M_1(\xi) = \frac{1}{4}\left[\frac{3k_0 - 2\mu(0)}{3k_0 + \mu(0)}\right]^2 \left[N_1^2 + \left(1 - N_1^2 + N_2\frac{\xi}{t^*}\right)\exp\left(-N_3\frac{\xi}{t^*}\right)\right]$$
$$M_2(\xi) = \frac{1}{2}\left[\frac{3k_0 - 2\mu(0)}{3k_0 + \mu(0)}\right]\left[N_1 + (1 - N_1)\exp\left(-N_3\frac{\xi}{t^*}\right)\right] \tag{9a}$$

and

$$N_1 = \frac{\left[3k_0 + \mu(0)\right]\left[3k_0 - 2\mu_0\right]}{\left[3k_0 - 2\mu(0)\right]\left[3k_0 + \mu_0\right]}$$
$$N_2 = 2\frac{3k_0 - 2\mu_0}{3k_0 - 2\mu(0)} - \frac{3k_0 + \mu_0}{3k_0 + \mu(0)} - \frac{3k_0 + \mu(0)}{3k_0 + \mu_0}\left[\frac{3k_0 - 2\mu_0}{3k_0 - 2\mu(0)}\right]^2$$
$$N_3 = \frac{3k_0 + \mu_0}{3k_0 + \mu(0)} \tag{9b}$$

The singularity at the interface corner has a form of $r^{1-\lambda}$. Roots of Eq. 8 with $0 < \mathrm{Re}(\lambda) < 1$ are of main interest. The calculation of the zeros of Eq. 8 can be carried out numerically for given values of material properties. For $0 < \nu(\xi) < 0.5$, there is at most one root λ with $0 < \mathrm{Re}(\lambda) < 1$, and that root is real .

The numerical values used in this example are as follows:

$$\mu(0) = 1.0\times10^3 \quad MPa; \quad \mu(\infty) = 0.5\times10^3 \quad MPa$$
$$K_0 = 2.0\times10^3 \quad MPa; \quad t^* = 10^2 \quad \text{min.}$$
$$\Delta T = 57^o C; \quad \alpha = 6\times10^{-5} \quad /^o C; \quad A_T = 100 \tag{10}$$

Fig. 3 shows the variation of the order of the singularity with the real time for the material properties given by Eq. 10.

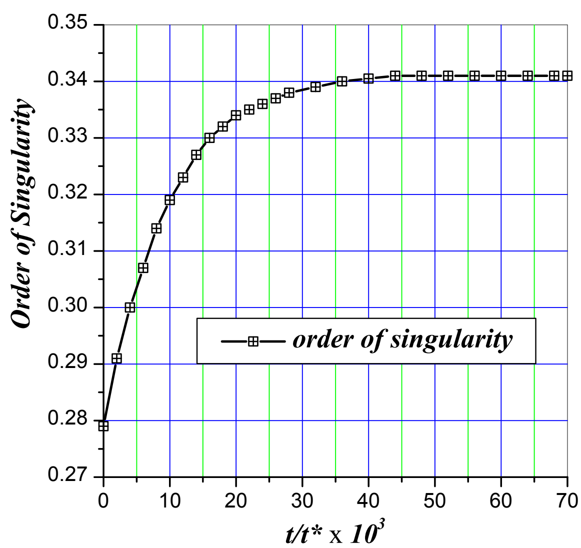

Fig. 3. Variation of the order of the singularity.

3. Boundary Element Analysis

A viscoelastic layer bonded to a thick rigid substrate is shown in Fig. 4(a). The coating layer has thickness h and length $2L$. Due to symmetry, only one half of the layer needs to be modeled. Fig. 4(b) represents the two-dimensional plane strain model for analysis of the interface stresses between the layer and the substrate. Calculations are performed for $L/h = 20$.

A uniform temperature change $\Delta TH(t)$ in the film is equivalent to increasing the tractions by $\gamma(t)n_j$ [11] where

$$\gamma(t) = 3K\alpha\Delta TH(t) \tag{11}$$

Here, K is the bulk modulus; n_j are the components of the unit outward normal to the boundary surface; and α is the coefficient of thermal expansion of the viscoelastic layer.

With a uniform thermal change in the layer, it is convenient to write the boundary integral equations with respect to *reduced time* ξ, instead of *real time* t. Then, the boundary integral equations without any other body forces are written as follows[11]:

$$\begin{aligned}
&c_{ij}(\mathbf{y})u_j(\mathbf{y},\xi) \\
&+\int_S\left[u_j(\mathbf{y}',\xi)T_{ij}(\mathbf{y},\mathbf{y}';0+)+\int_{0+}^{\xi}u_j(\mathbf{y}',\xi-\xi')\frac{\partial T_{ij}(\mathbf{y},\mathbf{y}';\xi')}{\partial\xi'}d\xi'\right]dS\,(\mathbf{y}') \\
&=\int_S\left[t_j(\mathbf{y}',\xi)U_{ij}(\mathbf{y},\mathbf{y}';0+)+\int_{0+}^{\xi}t_j(\mathbf{y}',\xi-\xi')\frac{\partial U_{ij}(\mathbf{y},\mathbf{y}';\xi')}{\partial\xi'}d\xi'\right]dS\,(\mathbf{y}') \\
&+\int_S\left[\gamma(\xi)n_jU_{ij}(\mathbf{y},\mathbf{y}';0+)+\int_{0+}^{\xi}\gamma(\xi-\xi')n_j\frac{\partial U_{ij}(\mathbf{y},\mathbf{y}';\xi')}{\partial\xi'}d\xi'\right]dS\,(\mathbf{y}')
\end{aligned} \tag{12}$$

where u_j and t_j represent displacement and traction, and S is the boundary of the given domain. $c_{ij}(\mathbf{y})$ is dependent only upon the local geometry of the boundary. For $\mathbf{y}$ on a smooth surface, the free term $c_{ij}(\mathbf{y})$ is simply a diagonal matrix $0.5\delta_{ij}$. The viscoelastic fundamental solutions, $U_{ij}(\mathbf{y},\mathbf{y}';\xi)$ and $T_{ij}(\mathbf{y},\mathbf{y}';\xi)$, can be obtained by applying the elastic-viscoelastic correspondence principle to Kelvin's fundamental solutions of linear elasticity.

Eq. 12 can solved in a step by step fashion in time by using the modified Simpson's rule for the time integrals and employing the standard BEM for the surface integrals. Eq. 12 can be rewritten in matrix form to give the global system of equations:

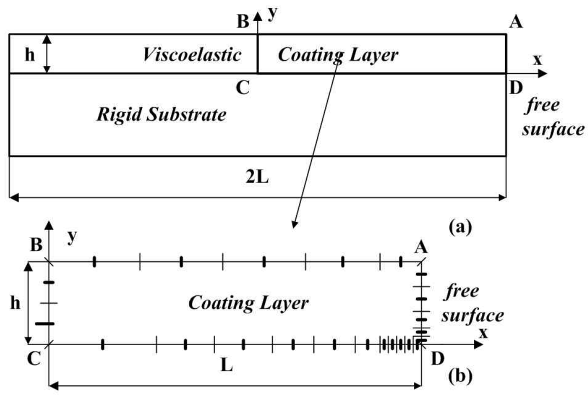

Fig. 4. Boundary element analysis model.

$$[\mathbf{H}]\{\mathbf{u}\} = [\mathbf{G}]\{\mathbf{t}\} + \{\mathbf{B}_T + \mathbf{R}\} \tag{13}$$

where $\mathbf{H}$ and $\mathbf{G}$ are the influence matrices; $\mathbf{B}_T$ is the known input due to temperature change; $\mathbf{R}$ is the hereditary effect due to the viscoelastic history. Solving Eq. 13 under boundary conditions leads to determination of all boundary displacements and tractions.

In order to examine the viscoelastic behavior along the interface line of the film, the numerical values given by Eq. 10 were used. The boundary element discretization consisting of 46 line elements was employed. The refined mesh was used near the interface corner. Quadratic shape functions were used to describe both the geometry and functional variations. The solutions were obtained in terms of reduced time. The final solution was then obtained by converting to real time. Viscoelastic stress profiles were plotted along interface to investigate the nature of stresses. Fig. 5 shows the distribution of normal stress σ_{yy} and shear stress τ_{xy} on the interface at nondimensional times $t/t^* = 0$ and 0.07. The numerical results exhibit the relaxation of interface stresses and large gradients are observed in the vicinity of the free surface. Singular stresses may cause interface debonding in the absence of applied external stresses or local cracking. To characterize the singularity levels near the free-edge, the stress singularity factor needs to be determined.

The stress singularity factor is normalized by the quantity h^{s+2}, giving it stress unit, as follows:

$$K_{ij} = \lim_{r\to 0}\left(\frac{r}{h}\right)^{s+2}\sigma_{ij}(r,\theta;t)\Big|_{\theta=0} \tag{14}$$

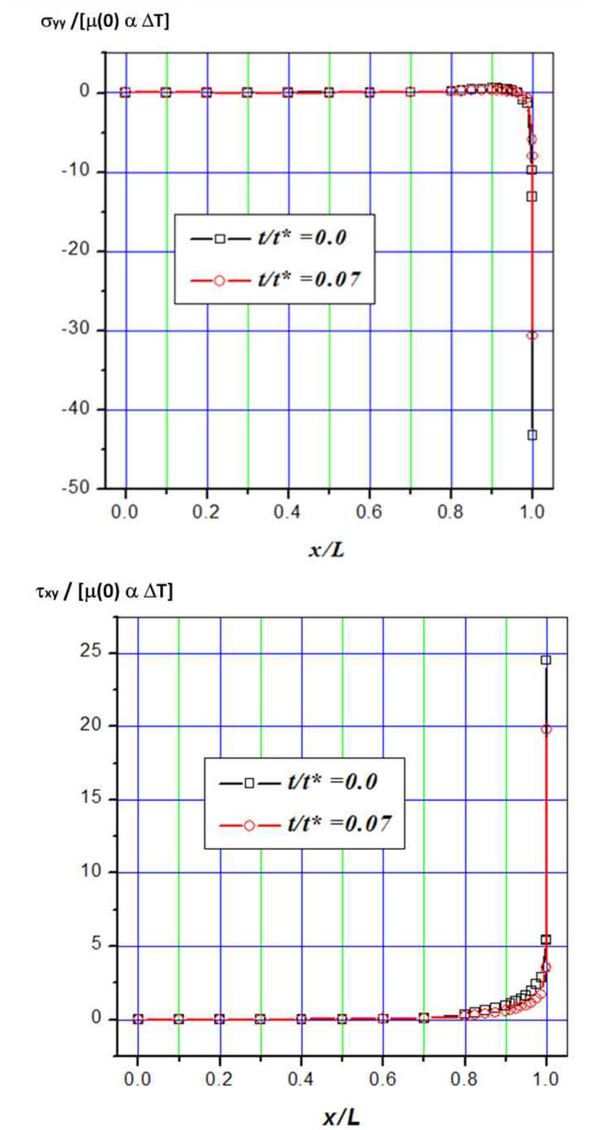

Fig. 5. Distribution of interface normal stresses and shear stresses at times $t/t^* = 0$ and 0.07.

Fig. 6 shows the variation of the stress singularity factor. It is shown that the stress singularity factor is relaxed with time while the order of the singularity increases first and then remains constant with time as shown in Fig. 3.

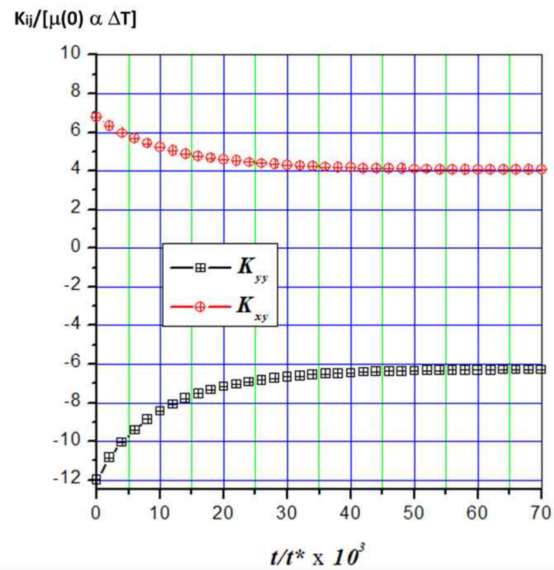

Fig. 6. Variation of the free-edge stress intensity factors.

4. Conclusions

The stress singularity at the interface corner between the viscoelastic film and the elastic substrate has been investigated. The interface stresses have been calculated using BEM. Localized but large gradients have been observed in the vicinity of the free surface. The order of the singularity has been obtained numerically for a given viscoelastic model. Since the exceedingly large stresses cannot be borne by a viscoelastic coating layer, local cracking or the delamination can occur at the interface corner as shown Fig. 1.

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