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Challenges Faced by a Mathematically Strong Student in Transferring His Success in Mathematics to Statistics: A Case Study

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Ⅰ. INTRODUCTION

Statisticians have noted that their discipline is by nature distinct from mathematics in several aspects (Garfield & Ben-Zvi, 2008; Wild & Pfannkuch, 1999). One difference lies in the goals for problem-solving in each discipline. While the main goal of doing mathematics problems is to better understand structure and content (Schoenfeld, 1992), the goal of doing statistics problems is to gain insights from data (Moore & Notz, 2005; Rossman, Chance, & Medina, 2006). This difference in goals is connected to the different types of research activities undertaken in the two disciplines. A typical mathematics research activity is to construct a conjecture and provide its proof (Lakatos, 1998). Although theoretical statisticians may provide proofs of conjectures, statisticians more often explore procedures empirically. They design and conduct experiments, analyze data and draw inferences using hypotheses about a given population. Most frequently, statistics research is grounded in the desire to make sense of data.

Another difference between statistics and mathematics lies in whether or not the problem-solving context is essential. In statistics, data are necessarily placed in context. The procedures that statisticians choose to use to understand given data are determined by the context in which their hypothesis is embedded (Cobb & Moore, 1997; Rossman et al., 2006), and in data analysis, context provides meaning. Even though context-dependent examples are occasionally used to promote student understanding in mathematics, mathematical concepts often have meaning free of context. Also, while statistical procedures can be taught in an abstract way, similar to mathematical procedures, statisticians are much more concerned with problems where the context matters. In addition, the context of a mathematics problem can obscure its underlying structure (Cobb & Moore, 1997). For example, students commonly find it challenging to figure out mathematical relationships between concepts embedded in different contexts (Wagner, 2010). In contrast, most statistics problems originate from real-world situations and are embedded in a specific context that provides meaning for the numbers used (Cobb & Moore, 1997). In statistics, data cannot be meaningfully analyzed without careful consideration of context, including how the data were collected and what they represent (Garfield & Ben-Zvi, 2008). Acknowledging the differences between mathematics and statistics, both in problem-solving goals and in how context is considered, is important because these distinctions give rise to dissimilarities in the kind of critical thinking required in each discipline, which is the main

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focus of this paper.

While there are various definitions of critical thinking among cognitive psychologists, one that is widely accepted is that of Ennis (1989, 1993): *critical thinking* is reasonable reflective thinking focused on deciding what to believe and do. Several cognitive psychological studies (for example, McPeck, 1981; Ennis 1989, 1993; Facione, Sánchez, Facione, & Gainen, 1995) have considered two aspects of critical thinking: (1) skills/ability and (2) habits/disposition. It is the latter aspect that is the main concern of this study. The habitual or dispositional aspect of critical thinking, which we refer to as *critical thinking disposition* in this paper, is defined as a collection of habits of mind, intellectual virtues and attitudes (Facione, Sánchez, Facione, & Gainen, 1995).

According to Ennis (1989), many cognitive psychologists hold that critical thinking is subject specific: the skills/abilities and habits/disposition that support the development of the kind of critical thinking needed in one discipline are different from those that support the kind of critical thinking required in another discipline. Considering that *statistics* and *mathematics* are distinct disciplines, the two may require, in particular, different types of dispositions that support critical thinking in their subjects. A disposition that supports critical thinking in mathematics would promote understanding of the structure of mathematical content while, in statistics, it would promote understanding the context of data collection and gaining insights from data (that is, interpreting the results of data analysis). Because of the potentially different dispositions that support the subject specific critical thinking required for the two different disciplines - mathematics and statistics - students who have been successful in mathematics could experience difficulty in translating that success to their first statistics course. Also, as Ennis argued, simple transfer of critical thinking dispositions and skills/abilities from one discipline to another discipline is unlikely. This means that it may take time and effort for students who have studied mathematics for twelve years to adjust to the new culture of statistics.

In this study, we ask how differences in subject-specific critical thinking dispositions between mathematics and statistics might explain the phenomenon that a mathematically strong student (Aaron) could fail to transfer his success in mathematics to statistics. More specifically, the question we address in this study is, "In the case of a student who is strong in mathematics, how do his dispositional characteristics that support the critical thinking he has needed in the study of mathematics relate to his struggles in his first elementary statistics course?"

By exploring this question in a qualitative case study, we aim to identify possible issues in the transfer process. This study will lay a foundation for a larger scale study with a similar research inquiry and for studies that attempt to argue for the need for statistics instruction that allows students to develop a critical thinking disposition specific to learning statistics.

II. RELATED LITERATURE

In his seminal work on critical thinking with regard to subject matter knowledge, Ennis (1989) noted that many epistemologists believe different thinking skills and problem-solving strategies are required to promote critical thinking in different subjects. Ennis set forth three principles regarding critical thinking dispositions and abilities that promote subject-specific critical thinking. The first principle established the relationship between subject matter knowledge and the development of subject-specific critical thinking:

> [K]nowledge about a topic is ordinarily a *necessary* condition for thinking critically in the topic. ... [W]e must beware, however, of inferring carelessly from this necessary "condition-ship" that subject matter knowledge is a sufficient condition for good thinking. (p. 6)

Although subject matter knowledge may be obtained through rote memorization, it is not necessarily accompanied by deep understanding that leads to subject-specific critical thinking. Ennis's second principle states: "Simple transfer of critical thinking dispositions and abilities from one domain to another domain is unlikely" (p. 5). Ennis further asserts that subject-specific critical thinking can only be transferred to another subject when there is sufficient practice in each domain and explicit instruction about transfer. Most cognitive psychologists agree with these two principles. According to the third principle, which is more controversial than the first two, general critical thinking instruction is not likely to be effective in developing subject-specific critical thinking. This claim is held only by strong domain specificists. While Ennis's first principle claims that subject matter knowledge influences the development of critical thinking specific to a subject, it does not explicitly include subject-specific problem-solving skills and strategies as factors in the development of a disposition that supports subject-specific critical thinking.

In the field of statistics, several studies have explored subject-specificity in critical thinking. Aizikovitsh-Udi and Amit (2010) explored the effects of incorporating critical thinking training in a probability course. In a subsequent study, Aizikovitsh-Udi (2012) asked "whether critical thinking skills were general or depended on content and the systems of concepts specific to that particular content" (p. 1034) and claimed that "the construction and teaching of critical thinking skills are determined by specific contents and tasks the teacher uses" (p. 1040). Although these studies are valuable in that they addressed critical thinking skills in the context of acquiring statistical knowledge, their focus was limited to critical thinking skills that promote the learning of probability or statistics. It remains unexplored how disposition might matter in the study of probability and statistics.

In light of the difference as disciplines between mathematics and statistics and of the difficulty in transferring subject-specific critical thinking disposition between distinct subjects, it is worth identifying what learning dispositions, if any, promote the learning of mathematics but hinder the learning of statistics, and vice versa. It is also important to test Ennis's (1989) statement that a disposition that supports subject-specific critical thinking is affected by subject matter knowledge by investigating how the different types of subject matter knowledge in mathematics and statistics require different studying strategies and critical thinking skills. Although this study does not fully address these questions, it attempts to help us understand the possible issues around these questions.

In the paragraphs that follow, we provide the definition of critical thinking disposition, drawing on Ennis's (1989) definition and Halpern's (1998) description of the characteristics of a critical thinking disposition. We then present a theoretical framework that includes our definition of a critical thinking disposition and our view of the process of how critical thinking disposition transfers between two domains, as well as a model of how subject matter knowledge and problem-solving skills and strategies affect the transfer of subject-specific critical thinking to a different subject.

Conditions for developing a critical thinking disposition

In studies of critical thinking, many cognitive psychologists have considered it in terms of two dimensions: cognitive skills/abilities and disposition (Facione, 1990). For example, McPeck (1981) considered skills and propensity; Ennis (1989, 1993), abilities and disposition; Facione, Sánchez, Facione and Gainen (1995), skills and disposition. In particular, Facione, Sánchez, Facione and Gainen described the *disposition to think critically* as "a characterological profile, a constellation of attitudes, a set of intellectual virtues, or ... a group of habits of mind" (p. 1).

Although not every cognitive psychologist agrees, many view the dispositional dimension of critical thinking as containing an affective component as well as a cognitive component (Facione, 1990). While the cognitive component directly correlates with cognitive skills if they are exercised appropriately, the affective component, which is the focus of this study, relates to habits of mind and attitudes that seem to characterize good critical thinkers. Specifically, such personal traits refer to "a critical spirit, a probing inquisitiveness, a keenness of mind, a zealous dedication to reason, and a hunger or eagerness for reliable information" (p. 11). Aligning with Facione's description, we define *critical thinking disposition* as a probing inquisitiveness, a keenness of mind, a zealous dedication to reason, and a hunger or eagerness for reliable information that enable reasonable reflective thinking that leads to "the deliberate use of skills and strategies that increase the probability of a desirable outcome" (Halpern, 1998, p. 449). Further, we define *subject-specific critical thinking disposition* by confining the *desirable outcomes* and the reflective thinking within a given subject.

Halpern (1998) asserted that critical thinkers exhibit certain attitudes in behaviors such as engaging with complex tasks, showing flexibility or open-mindedness and abandoning or modifying nonproductive strategies in attempts to self-correct, among others. In line with Halpern's view of what attitudes a critical thinker exhibits, we hold that an individual's subject-specific critical thinking disposition can be represented by three characteristics: *eagerness*, *flexibility* and *willingness*. *Eagerness* refers to one's enthusiasm to engage in and persist at a complex task in a domain. Routine problems tend to be solved with habitual solutions and without the problem solver's conscious awareness of the process, but critical thinking requires the conscious exertion of mental effort. Therefore, an eagerness to engage in and persist at a complex task in a domain would be expected from one with a probing inquisitiveness and a zealous dedication to reason in that domain. *Flexibility* refers to one's ability to apply problem-solving knowledge, skills and strategies developed within the study of a subject to problems that require the same strategies or skills in a new or different context in a different subject. Gaining such am ability requires reinterpretation of the problem-solving knowledge, skills and strategies in the new context and reflective thinking on how the knowledge, skills and strategies apply to the problems in the new subject. It is reasonable to consider that an individual with keenness of mind would be successful in such "reinterpretation" and "reflective thinking." *Willingness* refers to whether one is ready to distinguish necessary critical thinking skills from unnecessary ones. In a general context, a critical thinking disposition is more than the successful use of a particular skill in an appropriate context, encompassing an attitude to recognize when a skill is needed and the willingness to apply it. Analogously, a subject-specific critical thinking disposition is more than the successful use of a particular skill in an appropriate context of the subject. It would encompass an attitude to recognize when a skill is needed and the willingness to apply it in problem-solving within the domain.

Due to the different natures of problem-solving in mathematics and statistics, a student with an *eagerness* for mathematics problems may not have developed a similar *eagerness* for statistics problems; a student who has developed problem-solving skills and strategies for mathematics problems may not have the *flexibility* to apply those skills and strategies to statistics problems; and a student may not have the *willingness* to distinguish the problem-solving skills and strategies commonly used in mathematics that are not useful in statistics from those that are, or to seek out new strategies that might work. This means that a learner who has developed a disposition that supports critical thinking in mathematics may not be able to automatically transfer that disposition to statistics. Our goal in this study is to examine what challenges were faced by a student who had been successful in his mathematics courses and failed to transfer that success to his first statistics course, and to interpret how this student's challenges relate to his subject-specific critical thinking disposition in terms of the three characteristics of *eagerness*, *flexibility* and *willingness*.

The transfer of a disposition

Perspectives on developing subject-specific critical thinking can be divided into four categories: (1) general, (2) infusion, (3) immersion and (4) mixed (Ennis, 1989). Cognitive psychologists who work from a general perspective on critical thinking believe that critical thinking abilities and dispositions can be taught "separately from the presentation of the content of existing subject-matter offerings, with the purpose of teaching critical thinking" (p. 4). From an infusion perspective, critical thinking requires "deep, thoughtful, well understood subject matter instruction in which students are encouraged to think critically in the subject, and in which general principles of critical thinking dispositions and abilities are made explicit" (p. 5). Immersion is similar to infusion in that it is a thought-provoking kind of subject matter instruction in which students do get deeply immersed in the subject, but it differs in that general critical thinking principles are not made explicit. And the mixed perspective describes a general view paired with either the infusion or the immersion perspective.

We hold that a mixed approach that takes both the general and infusion perspectives is the most effective way to develop subject-specific critical thinking. We agree with Ennis's (1989) first two principles, which claim that subject matter knowledge is necessary to develop subject-specific critical thinking and that the "simple transfer of critical thinking dispositions and abilities from one domain to another domain is unlikely" (p. 5). We further agree that the transfer of a disposition that supports critical thinking from one domain to another is more likely when accompanied by instruction and practice that focus on transfer.

The differing natures of mathematics and statistics require students to learn different types of problem-solving skills and strategies. A mixed infusion perspective suggests that a disposition that supports subject-specific critical thinking for learning mathematics may not automatically transfer to statistics; it is necessary to design instruction and practice that focus on transfer. As Halpern (1998, p. 452) wrote, "some people may have excellent critical-thinking skills and may recognize when the skills are needed, but they also may choose not to engage in the effortful process of using them." This is a distinction between what people can do and what they actually do in real-world contexts. It is important to note that we distinguish a disposition that supports critical thinking from the ability to think critically.

Ⅲ. METHODS

This study is a case study that is largely phenomenological and descriptive in nature in that the study seeks an explanation or description of the phenomenon of how a student who successfully completed many mathematics courses could experience challenges in his first statistics course. We use a case study design because we agree with Merriam (1998) that it is ideal for *examining* and *interpreting* educational phenomena systematically.

The study emerges from another study that investigated students' understanding of statistics symbols, for which the data were collected from eight students from two introductory statistics classes at a mid-sized public university in the United States. One class was a non-calculus-based statistics course that served first-year students and met the general education requirement of the university, and the other was calculus-based and was designed to serve mathematics and statistics majors. Both courses took place during the spring semester of 2012. While the curricular organization of the courses in this study conformed to those typically found in a reform-oriented classroom, the instruction itself was essentially traditional. The instructors had almost total responsibility for daily classroom activities, and they primarily used lectures to deliver content.

To collect data for the original study we employed a survey assessment and follow-up interviews, commonly used in phenomenological studies. The survey had fourteen items. Some of these items were modified from items in *Assessment Resource Tools for Improving Statistical Thinking* (ARTIST), which was developed by Garfield, DelMas and Chance (2006) at the University of Minnesota. The rest of the items were created by the research team of this study based on their teaching experience and discussion with statistics instructors. These assessment items were initially chosen to evaluate student understanding of statistical symbols and the concepts that the symbols represent. Students were mainly assessed on their semiotic and lexical understanding of statistics terms such as *variable*, *mean*, *median*, *standard deviation* and *sampling distribution of sample means*, as well as related statistics symbols. The questions addressed interpretation of the terms, understanding of the symbols, and understanding of mathematical notions that underlie statistics concepts. However, computational proficiency was not addressed. The surveys took the participating students 15 - 25 minutes to complete.

For each student, the survey assessment was followed by an interview, which lasted 20 - 35 minutes. All interviews were audio-recorded and transcribed. We analyzed both the survey and the interviews qualitatively. To develop our conclusions, we assessed each utterance for the information it gave about a student's symbolic understanding. Based on the results of the content survey and comments made during the interviews, the eight students ranged from low to high in their understanding of statistics and mathematics concepts.

In this study, we seek to answer the question of how a mathematically strong student's dispositional characteristics, which have supported his critical thinking development in mathematics, cause him to struggle in his first elementary statistics course. The data of this study mainly come from one of the eight participants, Aaron, a mathematics major and one of the six students enrolled in the calculus-based statistics class. We chose to focus on Aaron in this study because, unlike the other seven students, he voluntarily shared the challenges he faced in learning statistics in comparison to learning mathematics, taking extra time during the interview to do so. His comments about his challenges conveyed his cognitive and met a cognitive thinking about the kinds of struggles he was engaged in during the course and the reasons he was struggling. We occasionally refer to remarks made by some of the other students, but for the most part their interviews were less illuminating, as they simply answered the interviewer's questions and did not comment on any struggles they might have been experiencing.

In order to explore the nature of Aaron's struggles in connection with his dispositional characteristics related to critical thinking, we borrowed the strategies of *line-by-line coding* and *focused coding* from the grounded theory approach (Glaser & Strauss, 1967). We re-assessed each utterance made by Aaron to look for indications of these dispositional characteristics and for the information it gave on the nature of his struggles. Then, we read within and across categories to develop conclusions, and we rechecked our conclusions against Aaron's own words. During these processes, we looked for instances when Aaron appeared to demonstrate *eagerness*, which we considered to be expressed as an inclination towards inquisitive learning, enthusiasm for engaging in complex tasks and dedication to reason; *flexibility*, which we considered to be demonstrated by the ability to adapt the knowledge, skills and strategies used in mathematics to the new context of statistics; and *willingness*, which we considered to be the willingness to try to distinguish between critical thinking skills and the use of the skills or knowledge that are useful in a statistical context and those that are not. The data provide evidence of how Aaron understood the mathematical concepts that underlie statistical expressions and how he conceived of the statistics concepts being taught in the class that he was enrolled in.

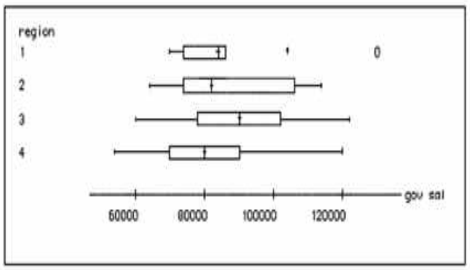
Ⅳ. RESULTS AND DISCUSSION

The data analysis intended to reveal the nature of the challenges that Aaron was facing in learning statistics. The findings discussed in this section rely mostly on Aaron's narratives produced during the interview. The analysis and discussion do not focus on whether Aaron's (or other students') responses to the survey items were correct. The excerpts exemplify how Aaron's critical thinking disposition specific to mathematics hinders his learning of statistics. We present three main findings: (1) Aaron showed a strong inclination towards inquisitive learning, (2) he showed strength in internalizing mathematical concepts, and (3) he showed an inability to transfer his disposition that supports critical thinking in mathematics to statistics. Each of the three findings will be explained in connection with the three characteristics of a subject-specific critical thinking disposition - *eagerness*, *flexibility* and *willingness*. All excerpts are from the conversation between the interviewer (I) and Aaron (A).

Strong inclination towards inquisitive learning

During the interview, Aaron revealed a learning inclination towards clarifying confusion and inquisitive learning. For some of the survey questions he was confused by, he would not begin solving the problem until I (the interviewer and first author of this paper) clarified the question for him. This characteristic appears in his answers to items 5 and 6, which are shown in Figure 1. (All survey items in this section are retyped so that the items fit the format of the paper.)

The following boxplots display the distributions of the 1993 governors' salaries according to the states' geographic region of the country. Region 1 is the Northeast, 2 the Midwest, 3 the South, and 4 the West.



5. Which region has the state with the highest governor's salary?

6. which region has the state with the highest median governor's salary?

[Fig. 1] Survey Items 5 and 6

Excerpt 1 shows part of the conversation about item 5.

Excerpt 1

A: ... I didn't really understand this, the highest governor's salary. Are they saying ... when they said the highest governor's salary, does that mean the highest ever reached?

I: Something like that.

A: Which state does –

I: One of these regions [overlapping voices]. Which region does, which region includes the state that has the highest governor's salary, is the question.

A: Okay.

I: Is this clearer now?

A: Maybe. Is this, what does this circle represent?

I: Which one?

A: This data point outside the ... outside, okay. Well, I put two, because I was just looking at, I wasn't counting this.

The questions being discussed in Excerpt 1 included box plots with outliers indicated by small circles. Unlike most of the other students, who tried to provide answers immediately (whether correct or not), Aaron wanted to clarify unclear phrasing and figures first, and was reluctant to begin working on the task until his confusion was resolved (lines 1–4 and 16–18). Aaron's comments in this excerpt indicate his dispositional characteristic of having a strong inclination towards inquisitive learning. Two other students who were confused by the inconsistent outlier notation did not hesitate to answer the questions in the survey. During the interview, these two students expressed curiosity about the circle notation: one stated "in [number] five, the one part I wasn't sure about these two points. And, what I thought the little zeros here meant was probably an outlier, and so they were excluding that from the rest" and another stated, "the highest, in this case, if this is a five-number summary and this is the high end of the scale, three goes higher." Although these two students were confused by the inconsistent circle notation, this did not stop them from answering the question. The other five students answered the questions during the survey and did not mention any issue with the notations during the interview. Thus, the other students' behavior contrasts with that of Aaron, who did not attempt to answer the questions until his confusion was clarified.

Aaron's comments on survey item 7 further reveal his propensity towards dispelling confusion. Item 7 is shown in Figure 2.

7. A variable is defined as a characteristic of an individual. A variable can take different values for different individuals.

You are interested in the number of siblings that entering students of ... have. You are to take a random sample of 10 freshmen to give descriptive statistics. Let X be the variable that represents the number of siblings of those 10 students. Circle any of the following that could be considered as a variable.

μ $\quad \bar{x}$ $\quad s$ $\quad \sigma$

[Fig. 2] Survey Item 7

Towards the end of the interview, Aaron mentioned item 7, as shown in Excerpt 2.

Excerpt 2

And then, there was another question where, you said, like, which of these can be considered variables, or something? Well, I never understood, he [the professor] never grasped <u>what variables were considered in stats</u>. So, I guess, when you don't have that basic, basic stuff, it's every thing that come safter, you just struggle to try to put pieces together, all at the same time.

Aaron's comments in lines 5 – 9 of Excerpt 2 reveal his discontent with his inability to dispel his confusion. The comments further demonstrate that Aaron was engaged in metacognition about the need to acquire basic relevant concepts before gaining connected understanding of a new concept.

Aaron comments in Excerpts 1 and 2 illuminate the two characteristics of eagerness and willingness in his critical thinking disposition specific to learning statistics. His efforts to dispel confusion indicate that he makes conscious efforts to be engaged in complex tasks and metacognitive thinking on the need to acquire basic relevant concepts to connect to his understanding of a new concept. This means that Aaron's *eagerness* is characterized by a strong inclination towards inquisitive learning. Aaron's metacognitive thinking further demonstrates his *willingness*. For example, although Aaron is willing to distinguish the use of the term *variable* in statistics (as in random variables) from the use of the term in mathematics, other of Aaron's comments (e.g. lines 3 –5 in Excerpt 6-A) indicated that his willingness did not result in successful transfer of his critical thinking disposition to the statistics domain.

Strength in internalizing mathematical concepts

The interview data provide evidence that Aaron has an exceptionally strong mathematical understanding of the concepts that underlie statistical expressions. For instance, this is shown in Aaron's claims in Excerpt 3 about item 9, a true/false question about measures of center and dispersion, shown in Figure 3.

9. True or False?

a. The sample median is sensitive to outliers.

b. The sample mean is not affected by some observations.

c. The mean and median both describe the center of the distribution.

d. If a density curve has more than one peak, then there is more than one mean.

[Fig. 3] Survey Item 9

Excerpt 3

The mean is affected by all observations. I mean, unless you had a number exactly on the mean, you're having multiple ... If you change observations, the mean's going to change, no matter what, unless you, unless you change means equally on both sides.

Aaron's explanation in Excerpt 3 of how changing some data points influences the mean shows that he understands the underlying mathematical concept of *mean*. Out of the eight participants, seven answered this question correctly. However, only two (Adam and Aaron) explained the dynamics of how all observations affect the sample mean. The others simply restated their answers when they were asked to explain their understanding. Another example appears in Aaron's answer to item 11, shown in Figure 4.

11. Suppose that X is a random variable that follows a normal distribution. The z-score is found by the computation: $z=(X-\mu)/\sigma$.

a. The z-score follows a normal distribution with center 0 and standard deviation 1. If you only computed $x-\mu$, but forgot to divide it by σ what kind of distribution does this follow? What is the center and standard deviation of $X-\mu$?

b. If you did $x/\sigma-\mu$ instead of $(x-\mu)/\sigma$, would you still obtain the same z score? Why or why not?

[Fig. 4] Survey Item 11

This item asks participants to describe the distribution of $X-\mu$ in terms of its mean and standard deviation in comparison with the distribution of $(X-\mu)/\sigma$ where X follows a normal distribution. Excerpt 4 shows the exchange between the interviewer and Aaron regarding item 11.

Excerpt 4

I: Yeah. And then, we still have a distribution by subtracting. How would you describe that distribution, in terms of the center and the standard -

A: Oh, well, the center would still be zero. But the standard deviation would be, because you forgot to divide.

I: Can you say that again one more time?

A: I said, if we subtracted, our center would be zero. Because you moved it. But our standard deviation would be, instead of being one.

I: Okay.

A: Because we didn't divide.

I: Okay. Then, B? [unintelligible].

A: Yeah, I was assuming that was -

I: Do you understand the question?

A: Okay. So, if you did x divided by σ first, and then subtracted μ.

I: Right.

A: [overlapping voices] Yes, it matters. Because you have to subtract μ divided by σ, because if you divided, if you do the shift first by μ, you're centering it at zero. But if you divide X by σ first, then subtract μ, your center would actually be [unintelligible], because you're going to decrease your center when you divide by σ, and then you're going to shift it ... So, you'd have to actually subtract μ divided by σ, which would actually be that first one.

Aaron's claims underlined in Excerpt 4 show that Aaron understands the role that each of the operations in the expression $(x-\mu)/\sigma$, plays in obtaining the standardized value. Out of the eight students in the original study, only two participants (Judy and Aaron) gave valid explanations of item 11. The other participants either did not understand the question or had difficulty understanding $X-\mu$ as a probability distribution.

Excerpts 3 and 4 together demonstrate that Aaron understood, better than the other students, the dynamics that algebraic operations create within the basic expressions in descriptive statistics. This is an indication that Aaron had a stronger foundation for the mathematical concepts that underlie the statistical expressions taught in class compared to most of the other participants, who were unable to explain the operations of the statistical expressions. Aaron probably would not have been able to provide the explanations in Excerpts 3 and 4 if he did not possess a probing inquisitiveness and a zealous dedication to reason in mathematics. Yet Aaron seems to have struggled in this class, as shown in the following comment:

> ... this is my first statistics class, and I feel like we just didn't get any of the foundational stuff. Like, this is the most lost I've ever been in a class. ... I've got like a B [as a grade in the course], right now, but I don't feel like I know what's going on. Like, it's me really struggling to get a B.

In this comment, Aaron shows his dissatisfaction about the challenges he has faced in trying to understand the "foundational" concepts. This comment, along with his explanations in Excerpts 3 and 4, further shows Aaron's dispositional characteristic of eagerness: his engagement with the conceptual tasks of mathematics and his enthusiasmabout learning the mathematical foundations that underlie statistics concepts.

Transferring a disposition that supports critical thinking in mathematics to statistics

Aaron's comments in Excerpts 1-4 demonstrate that he has problem-solving knowledge and skills well-developed for learning mathematics. Of the three characteristics we are considering, his comments expressed the *eagerness* and *willingness* of his critical thinking disposition specific to learning statistics. Next we look into Aaron's *flexibility* in shifting his problem-solving knowledge and skills from mathematics to statistics. This is shown in his verbal description of survey item 8 (Figure 5).

> 8. In a university, 75% of the students are male and 25% are female. 5% of the male students and 15% of the female students own a car. For each statement, determine whether it is true:
>
> a. We can conclude that 20% of the students in the university own a car.
>
> b. We can conclude that the number of male students who own a car is equal to the number of female students who own a car.

[Fig. 5] Survey Item 8

In regard to item 8, the interview conversation with Aaron proceeded as in Excerpt 5.

Excerpt 5

I: ... let's do number eight. And A, you said, no.

A: I said no, then, yes.

I: And why did you say that?

A: Well, I said, if we have 5% of 75%, you get 15% of 25%. I thought we were coming up with the same number.

I: Were you thinking about B?

A: Yeah. A, I was just saying, that sounded foolish to me because I didn't see why they said 20% of the students.

I: It didn't make sense at all?

…

A: I was just looking at it, and I said okay, and then I didn't feel like spending too much time thinking about it, and I said 75% -

I: How can you convince someone that it's wrong if it is wrong?

A: If I had an example, I could show someone that it's definitely wrong.

I: Using these four numbers (75%, 25%, 5%, and 15%). You have 75, 25, these numbers?

A: Yeah.

…

I: If you have part, why, what do you say, part of a person? We're talking about the whole population, here. And we have, we're assuming that 75% are male, 25% are female. So, we have controlled for everyone, there. And then, [unintelligible] 75% for male, we know that only 5% own a car. In here, 15% own a car. So, we know, for each individual in the population, not for each individual. We know, we know exactly how many people, we know exactly the proportion of the people who own a car, among the male, and the same with the female. Okay? We're not taking a sample, here. Would you say that it's between 5 and 15, or would you say it's below 5, or would you say it's above 15?

A: <u>I would say it's between 5 and 15. Probably around 7%?</u>

Aaron's response to the interviewer's question in lines 24 - 25, "I would say it's between 5 and 15. Probably around 7%?", suggests that he acknowledges the question as a problem of finding a weighted average. Out of the eight participants, five students solved this problem correctly. Only Aaron gave a number "7%" (underlined) without doing actual calculation. This implies that Aaron grasped the mathematical concept of a weighted average.

In Excerpt 1, we saw from Aaron's comments such as, "Are they saying ... when they said the highest governor's salary, does that mean the highest ever reached?" and "what does this circle represent?" his probing inquisitiveness and *eagerness* for reliable information. Aaron's comments in Excerpt 5 (underlined) together with Excerpt 4 provide evidence that Aaron had developed insights into mathematics concepts that underlie probability and statistics more than most of the other participants. This shows Aaron's keenness of mind in relation to learning mathematics concepts. However, Aaron's verbal description of item 7 (see Figure 2) suggests that he struggled to adopt a term commonly used in mathematics in the context of statistics.

For this item, although the explanations of six participants (all except Aaron and Judy) were invalid, they did show signs of adopting the term "variable" in the context of their statistics learning; to describe their understanding of the term, they used terms such as "sample," "population," "vary" and "change." For example, Judy gave the following verbal description:

> ... when it said a variable is defined as a characteristic of an individual, I took that to mean the sample, and so, I said, the x-bar ($\overline{x}$) and the s, rather than mu (μ)and sigma (σ), which I think refer to population.

Not only was Judy promptly able to apply a new definition in this question, but her explanation was valid. In Aaron's case, although he used the term "change" in his explanation, he confined the use of the term "variable" to the mathematics context, as shown in Excerpts 6-A and 6-B.

Excerpt 6-A

I: What is a variable, in your understanding?

…

A: I don't know. I've always just seen variables as, like, things that could change, kind of. I'm thinking like algebra.

Aaron's comments imply that he did not develop the flexibility to adopt a new definition of the term "variable" to answer a question embedded in a new context. The continued conversation reveals that Aaron did not identify statistics symbols such as and as representing variables, as shown in Excerpt 6-B.

Excerpt 6-B

I: Yeah, that's a good point. So, what are the things there, then, that could change?

A: Based on what, though?

I: You have a population. Okay? But the population is a population that's fixed.

A: Okay. So, μ can't change, but $\overline{x}$ and x could change.

I: Okay.

A: Given different samples, and then, μ and σ can't change, because the population, overall, will always be the population.

I: But why did you pick $\overline{x}$ and μ, in the beginning, there?

A: Because I didn't understand that at all. I didn't know what we were looking at, as what was changing and what wasn't changing.

Excerpts 5, 6-A and 6-B contrast distinct aspects of Aaron's skill in applying his mathematics knowledge in different contexts. The question in Excerpt 5 has a probabilistic context that involves percent. Aaron's comment in this excerpt, "I would say it's between 5 and 15. Probably around 7%", is an example that shows Aaron had the mathematical knowledge of weighted averages and was able to use that knowledge in solving problems in a probabilistic context. In contrast, Aaron's statement, "I've always just seen variables as, like, things that could change. I'm thinking like algebra," in Excerpt 6-A, shows that Aaron knows the definition of a variable in mathematics, but initially failed to apply this mathematics knowledge of a variable in the statistics context. When Aaron was interrupted by an explanation from the interviewer, "You have a population. Okay? But the population is a population that's fixed," he was able to quickly transfer his understanding of a variable to a statistical context. Excerpts 5, 6-A and 6-B together show that Aaron does not have enough *flexibility* to shift his problem-solving knowledge and skills from mathematics to statistics. The implication is that when one has knowledge of a concept learned in a specific domain, it is not as natural to apply that knowledge in the context of a different domain as it is to apply it in the context of the same domain.

V. CONCLUSION AND IMPLICATIONS

This article, which discusses Aaron's subject-specific critical thinking disposition in terms of the three characteristics of *eagerness*, *flexibility* and *willingness*, makes two major contributions to the learning and teaching of statistics. The first lies in the creation and application of the constructs of eagerness, flexibility and willingness. In this study, we synthesized the literature regarding critical thinking in epistemology, created these new constructs to describe a subject-specific critical thinking disposition for statistics, and used the constructs as analytical generalizations of subject-specific critical thinking disposition. By showing the utility of the constructs in exploring the characteristics of one student, Aaron, in terms of his subject-specific critical thinking disposition, the constructs serve as theoretical explanations of the observed phenomena that need further confirmation.

The second contribution is the study's finding that a strong mathematics student, with well-developed knowledge and certain characteristics of a critical thinking disposition (for example, eagerness to engage in conceptually challenging tasks in mathematics), might struggle to transfer his knowledge and critical thinking abilities to his study of statistics. Drawing on the three characteristics – *eagerness, flexibility and willingness* – we further discuss these findings in detail in the following paragraphs.

We showed evidence that Aaron's critical thinking disposition is characterized by a strong inquisitive learning inclination, which is an indication of an "eagerness" to immerse himself in conceptually challenging mathematical tasks. However, such an inclination could run counter to developing an eagerness towards undertaking complicated statistical tasks because the focus of statistical tasks is different from that of mathematics tasks; it requires understanding the context of the tasks, determining which statistical tests are appropriate, conducting related computations, and interpreting outcomes. Aaron's eagerness, in contrast, was for understanding the mathematical concepts that underlie statistical expressions needed to run tests. Current statistics curricula focus on statistical literacy, reasoning and thinking (Ben-Zvi & Garfield, 2004), and they emphasize understanding context, analyzing data and using software; in such curricula, the underlying mathematical concepts are relatively less emphasized. Without explicit instruction about which kinds of critical thinking skills are emphasized in statistics as opposed to mathematics, students like Aaron face challenges in learning statistics.

We also noted that a disposition that supports subject-specific critical thinking requires flexibility in applying problem-solving knowledge and strategies to new contexts. Aaron had trouble applying problem-solving knowledge and strategies developed within mathematics to problems that required the same knowledge or strategies in the new context of statistics. Aaron compared his mathematics knowledge metacognitively to his statistics knowledge when he stated in Excerpt 2, "I never grasped what variables were considered in stats". Aaron acknowledged use of variables as a critical thinking skill used in both mathematics and statistics, and he tried to discern the boundary between information about variables in mathematics that was still valid in statistics and information that was no longer useful. However, until the interviewer explained how statistics symbols could be identified as representing variables, Aaron failed to transfer the definition of a variable in the mathematical context to a statistical context. In addition, we found in Excerpt 5 that Aaron was able to apply strategies commonly used in mathematics to problems in a probabilistic context. At the same time, Excerpts 6-A and 6-B showed that he did not successfully transfer knowledge commonly used in mathematics to a statistical context. This is evidence that Aaron had limited *flexibility*.

Aaron's *willingness* to try to distinguish between critical thinking skills that are useful in a statistical context and those that are not was difficult to observe in our data. However, Aaron's demonstration of met acognitive thinking in Excerpt 2 ("I never grasped what variables were considered, in stats"), suggests that he may have taken the first step towards such *willingness* when he recognized that certain knowledge and problem-solving strategies and skills learnt in mathematics were not applicable in the same way in the study of statistics. As a next step, a future study could explore the possibility of training students to become *willing* to develop necessary subject-specific critical thinking skills.

Implications

This study investigated the nature of the challenges a student successful in mathematics had in transferring that success to his first statistics course. The data were not initially collected to investigate the correlation between the first two findings: an inquisitive learning disposition and understanding of mathematical concepts. However, the data revealed that Aaron differed from other participants in both regards. As the discussion of Excerpts 1 and 2 claimed, Aaron seems to have a strong inquisitive learning inclination, which led to his understanding of the mathematical concepts. The first finding further seems to explain the third finding, which was that Aaron failed to transfer his problem-solving knowledge and skills learned in mathematics to statistics. The first and third findings, taken together, could imply that an inquisitive inclination of a student does not necessarily support learning in current statistics curricula. In mathematics courses or textbooks, students are often taught the underlying concepts or at least given access to them when a formula is presented. In contrast, statistics students are not expected to develop insights into formulas, which are often not explained when they are given to the students.

The findings from Aaron in this study indicate that transferring knowledge and problem-solving skills from mathematics to statistics is not automatic and that students can get confused as to how much insight they are expected to develop into the statistics expressions they learn. As mentioned, current curricula in statistics emphasize statistical literacy, reasoning and thinking (Ben-Zvi & Garfield, 2004). While Garfield and her colleagues (2010) emphasize statistical literacy, reasoning and thinking at the college level in their *Guidelines for Assessment and Instruction in Statistics Education* (GAISE), and suggest "using formulas that enhance the understanding of concepts" (p. 18), students are still expected to accept certain statistical expressions without a full explanation of the underlying mathematical concepts in such curricula. When statistical expressions are given without full explanations (which may be inevitable for some statistics expressions), the instructors should draw clear boundaries between the aspects that are assumed and those that are not.

The results of this study suggest that the concept of a disposition that supports developing subject-specific critical thinking is useful for understanding such challenges. The study's results further support, in the contextsof mathematics and statistics, Ennis's (1989) assertion that students need instruction in order to successfullytransfer a subject-specific critical thinking disposition to another subject.

The challenges demonstrated in the case of Aaron further suggest pedagogical implications for high school and college-level elementary statistics courses. In many small to mid-sized post-secondary institutions in the United States, statistics courses are offered in the department of mathematics, and many mathematics instructors teach both mathematics and statistics. It is important that the instructors understand the differences of the two disciplines, and, by extension, how students' critical thinking disposition that supports their learning of mathematics could hinder their learning of statistics. The study results also suggest that in order to ease the process of transferring a critical thinking disposition from mathematics to statistics, instructors should strive to be clear aboutthe extent to which students are expected to know the mathematics concepts that underlie statistics expressions.

Limitations and future studies

The qualitative methodology used in this study, like qualitative research in general, sought to describe particular phenomena by providing a situational perspective. The study's goal was to achieve the richness and depth of exploration and description that only qualitative approaches can provide (Myers, 2000). Nevertheless, this approach may limit the extent to which the study's implications can be generally applied.

In order to generalize the findings of this study to a larger mathematics and statistics education community, it is essential to conduct larger-scale research. Future studies could expand the scope of this study by investigating which subject-specific critical thinking skills in mathematics hinder student learning in statistics. In addition, if, as Ennis (1989) claimed, transfer is likely only when practice and instruction are provided, then it is important to explore how instruction might be designed to make this transfer likely in statistics.

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Challenges Faced by a Mathematically Strong Student in Transferring his Success in Mathematics to Statistics: A Case Study

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This study qualitatively investigates the nature of the challenges that a student who is highly successful in mathematics faces in learning college-level elementary statistics. The study draws on the constructs of eagerness, flexibility and willingness to characterize the necessary disposition for critical thinking that is essential in learning statistics. The case study is based on data collected through a survey assessment and a follow-up interview with a mathematics major enrolled in an elementary college statistics course at the time of the study. The qualitative analysis relies on the student's verbal descriptions of the challenges he was experiencing in the course. The findings suggest that while his strong inclination towards inquisitive learning and strong understanding of mathematical concepts supported this student's mathematics learning, the same characteristics might have been causing him difficulties in learning college-level elementary statistics

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