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Reliability analysis of an embedded system with multiple vacations and standby

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Abstract. This investigation deals with reliability and sensitivity analysis of a repairable embedded system with standby wherein repairman takes multiple vacations. The hardware system consists of 'M' operating and 'S' standby components. The repairman can leave for multiple vacations of random length during its idle time. Whenever any operating unit fails, it is immediately replaced by a standby unit if available. Moreover, governing equations of an embedded system are constructed using appropriate birth-death rates. The vacation and repair time of repairman are exponentially distributed. The matrix method is used to find the steady-state probabilities of the number of failed components in the embedded system as well as other performance measures. Reliability indexes are presented. Further, numerical experiments are carried out for various system characteristics to examine the effects of different parameter. Using a special class of neuro-fuzzy systems i.e. Adaptive Network-based Fuzzy Interference Systems (ANFIS), we also approximate various performance measures. Finally, the conclusions and future research directions are provided.

Key Words: *Markov process, matrix method, mean time to failure, multiple vacations, reliability, repairable embedded system*

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1. **INTRODUCTION**

Reliability has become an even greater concern in recent years because high-tech industrial processes with increasing levels of sophistication comprise most engineering systems today. Embedded system is dedicated to specific tasks, design engineers can optimize it to reduce the size and cost of the product and increase the reliability and its performance. Now-a-days, embedded systems are used in day to day life. The reliability of an embedded system has become an important aspect of planning, designing and operation due to their wide applicability in many areas such as transportation, banking, health care system, nuclear reactors, industry, aircraft control and elevator etc. The main aim of reliability analysis is to measure the probability that designed equipment will work its intended function in the hands of the customers. In the field of reliability a large number of researchers have paid their attention in the estimation as well as prediction of the reliability of an embedded system. Subramanian and Anantharaman (1995) gave the reliability analysis of a complex standby redundant system. Rajamanickam and Chandrasekar (1997) studied the reliability measures for two unit systems with a dependent structure for failure and repair times. Kuo et al. (2001) presented the framework for modeling software reliability using various testing-effort and fault detection rates. They have provided an efficient parametric decomposition method for software reliability modeling, which considers both testing efforts and fault detection rates. Four models have been investigated by Wattanapongsakorn and Levitan (2004) for optimizing the reliability of embedded systems considering both software and hardware reliability under cost constraints. Azaron et al. (2005) studied the reliability function of a class of time dependent systems with standby redundancy. El-Damcese (2009) examined a warm standby system subject to common cause failures with time varying failure and repair rates. The Markov state transition diagram has been used by him to analyze reliability and availability of the system. Wu et al. (2010) discussed various characteristics of the embedded software and analyzed the difference between embedded system and ordinary software. According to them, embedded software are real time and embedde. Recently, safety reliability for automotive embedded system was analyzed by Pattanaik and Chandrasekaran (2013). They have discussed reliability principles which assist system improvement for reducing the high unreliability. Sharma (2015) dealt with the reliability analysis of repairable system under N-policy, setup and imperfect coverage. Runge-Kutta (RK) method was used by her to elaborate various reliability performance measures of the system.

Vacation models have been studied by many prominent researchers with several combinations wherein the repairman takes a vacation when there are no failed components present in the system. Also, the repairman comes back to the normal level if there is at least one failed units waiting for the service otherwise takes another vacation. The vacation policies were discussed by Ke and Wang (2007) for machine repair problem consisting of M operating units with two type spare machines and multi repairmen. Moreover, a direct search algorithm was used to determine the optimal values of the number of two types of spares and the number of servers while maintaining a minimum specified level of system availability. Hu et al. (2008) investigated the reliability characteristics of an n-dissimilar-component series repairable system with multiple vacations. Moreover, Yuan and Cui (2013) focused on reliability analysis for K-out-of-N system wherein repairmen taking multiple vacations. A K-out-of-N repairable system with repairman who takes multiple vacations was studied by Wen-qing et al. (2013). They derived various performance measures such as the steady-state availability of the system, mean time to the first failure, failure frequency of the system, the probability for the repairman being busy, the steady-state unavailability of the repair facility, the expected number of failed components and the expected waiting time of failed components.

Standby support is most important factor for the smooth running of any embedded system. The provision of taking standby may be helpful in improving the system reliability/availability by introducing redundancy as additional equipment's, assemblies, devices, *etc.*. The bi-level control of degraded machining system with warm standbys was studied by Jain et al. (2004) including the concept of vacations. Jain and Bhargava (2009) considered machine repair system with mixed standbys and unreliable server in their studied. They have obtained the system performance measures using matrix method. Further, Kumar and Jain (2013) applied threshold policies for multi component machining system with warm standbys. Haggag (2014) dealt with a K-out of-N repairable system and standby with repair facility. He obtained system characteristics using a continuous-time discrete-state Markov process. Recently, 2-state repairable complex system with two types of failure was examined by EL-Damcese and Shama (2015). They investigated the sensitivity analysis for the system reliability with changes in a specific value of the system parameters.

In decision-making processes whenever the sources of uncertainty are involved, fuzzy approaches are used. In conventional reliability models, the probabilistic approach seems to be inadequate due to built-in uncertainties in data; therefore the theory of fuzzy reliability can play important role to tackle these types of difficulties. ANFIS is a soft computing approach which has facilitated the task of computation for many complex systems. Wang (2001) analyzed ranking engineering design concepts using a fuzzy outranking preference model. Verdegay et al. (2008) focused on heuristics as a fundamental constituent of soft computing. The application of fuzzy logic was suggested by Pando et al. (2013) for hardware/software partitioning in embedded systems. Recently, Kaushik et al. (2014) has been focused on availability analysis for an embedded system considering both the software and hardware reliability using fuzzy approach.

Now, we cite an example of an embedded system based electricity generation and transmission (ESEGT) of wherein all features occur simultaneously under consideration. In ESEGT, there are 'SG_i' (i=1,2,…,M) operating synchronous generator and 'SB_i' (i=1,2,…,S) standby synchronous generator which generate the electricity for load. If and only if 'K' no of synchronous generator is failed, the ESEGT failed which means the power transmission to load is stopped completely. There is one repairman who immediately repairs the failed synchronous generator. On the other hand, if there is no failed synchronous generator available in the ESEGT then repairman takes multiple vacations.

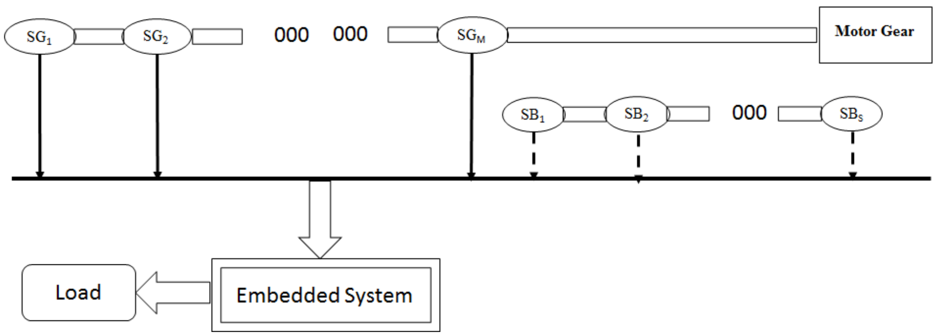

Figure 1. Embedded system based electricity generation and transmission.

In this investigation, we analyze reliability analysis of an embedded system with standby and multiple vacations. In our study, a new modelling approach is suggested for determining the reliability indexes of an embedded system. If there is no failed component present in an embedded system then repairman may go for a vacation. After returning from a vacation; repairman finds there is no failed unit present in the system, he may go for another vacation. On the similar pattern, repairman may take multiple vacations till no failed component is present in the system. As any working unit fails, it is immediately detected and located. A failed unit is replaced by a standby unit if we have one. Moreover, ANFIS results are also determined. The reminder of the paper is structured as follows. Section 2 presents mathematical formulation of an embedded system by stating the requisite assumptions. The transient analysis of an embedded system using appropriate birth-death rates are done in section 3. Next, we employ the matrix method for the solution purpose in section 4. Further, some performance characteristics are obtained in section 5. Also, we provide the illustration of the suggested problem. In section 6, we describe numerical results for validation purpose. Finally, the paper comes to end with the insights of future prospects and concluding remarks in section 7.

2. MODEL DESCRIPTION

Consider repairable embedded system with standby wherein a repairman takes multiple vacations. The components of an embedded system may fail due to hardware failure. The following assumptions and notations are used to formulate the model:

- An embedded system consists of M-operating subsystem and S-standby components under the care of one repairman. The system functions successfully with at least K-components.
- There is single repairman and he is always available to repair the failed components. The failed components are repaired in first come first served (FCFS) manner.
- In case of failure of an operating unit, it is replaced by standby; if available. When all standbys are used, the system may also work till K operating components are

functioning well. The failure rate of remaining components increases and the system is called to work in short mode.

- The life time, vacation time and repair time of operating components, standbys and repairman are exponentially distributed.
- The switch over times to replace the failed components by standbys and standbys by repaired components is negligible.
- Whenever there are L=M+S-K+1 (K=1, 2, …,M) components in the system, the repairman is in breakdown state.

For formulating the mathematical model, the following notations are used:

M Total number of operating components in the embedded system
S Total number of standby components in the embedded system
λ_i Mean failure rate of i[th] (i=1, 2, …, M) operating subsystem component
λ_s Mean failure rate of operating components which supervises the system and system works in degraded i.e. short mode
λ_C Mean failure rate of fault coverage
α Mean failure rate of standby components
μ_i Mean repair rate of i[th] (i=1, 2, …, L) failed components
θ Vacation rate of repairman
E(F) Expected number of failed components in the system
E(O) Expected number of operating components in the system
E(S) Expected number of standby components in the system

The state of the repairman at time t is denoted by variable $\omega(t)$ as follows

$$\omega(t) = \begin{cases} 0, & \text{when repairman is on vacation.} \\ 1, & \text{when repairman is turned on and busy in rendering repair of failed components.} \end{cases}$$

Define the probabilities as

$P_{0,n}(t)$: Probability of n failed components at time t in the system when the repairman is on vacation where n=0,1,…,L-1, L.

$P_{1,n}(t)$: Probability of n failed components at time t in the system when the repairman is in busy state rendering repair of failed components where n=0,1,…,L-1, L.

The Laplace transform of probabilities are defined as:

$$\tilde{P}_{0,n}(s) = L\{P_{0,n}(t)\} = \int_0^\infty e^{-st} P_{0,n}(t)dt, \qquad \tilde{P}_{1,n}(s) = L\{P_{1,n}(t)\} = \int_0^\infty e^{-st} P_{1,n}(t)dt$$

3. TRANSIENT ANALYSIS

The effective mean failure rates which depend upon the states of the repairman are defined as

$$\Lambda_i = \begin{cases} M\lambda_M + (S-i)\alpha, & 0 \le i \le S \\ (M+S-i)\lambda_{M+S-i}, & S+1 \le i \le L-1 \end{cases} \tag{1}$$

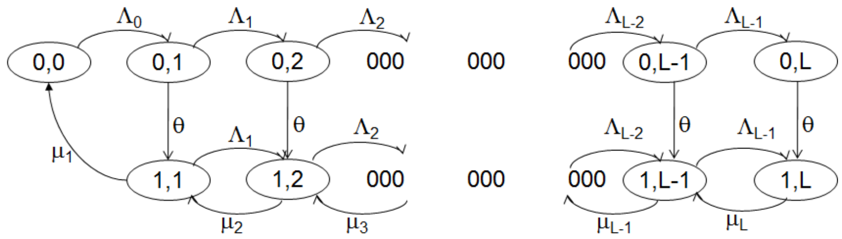

Figure 2. State transition diagram for repairable embedded system with multiple vacation and standby

The differential difference equations are constructed by taking appropriate transition rates as shown in Figure 2.

Case I: When the repairman is on multiple vacation

$$P'_{0,0}(t) = -(\Lambda_0 + \lambda_S + \lambda_C)P_{0,0}(t) + \mu_1 P_{1,1}(t) \tag{2}$$

$$P'_{0,i}(t) = -(\Lambda_i + \lambda_S + \lambda_C + \theta)P_{0,i}(t) + \Lambda_{i-1}P_{0,i-1,}(t), \quad 1 \le i \le L-1 \tag{3}$$

$$P'_{0,L}(t) = -\theta P_{0,L}(t) + \Lambda_{L-1}P_{0,L-1,}(t) \tag{4}$$

Case II: When the repairman is busy state rendering repair of failed components

$$P'_{1,1}(t) = -(\Lambda_1 + \lambda_S + \lambda_C + \mu_1)P_{1,1}(t) + \theta P_{0,1}(t) + \mu_2 P_{1,2}(t) \tag{5}$$

$$P'_{1,i}(t) = -(\Lambda_i + \lambda_S + \lambda_C + \mu_i)P_{1,i}(t) + \Lambda_{i-1}P_{1,i-1}(t) + \theta P_{0,i}(t) + \mu_{i+1}P_{1,i+1}(t), \quad 2 \le i \le L-1 \tag{6}$$

$$P'_{1,L}(t) = -\mu_L P_{1,L}(t) + \Lambda_{L-1}P_{1,L-1}(t) + \theta P_{0,L}(t) \tag{7}$$

The initial conditions are given as:

$$P_{0,n}(0) = \begin{cases} 1, & n = 0 \\ 0, & 1 \le n \le L \end{cases}$$

4. MATRIX METHOD

To provide the solution of set of differential equations (2)-(7), we employ very powerful approach namely 'matrix method'. Using this method, we put differential difference equations governing the model in the form of block matrix. Then, using the initial conditions, the Laplace transforms of equations (2)-(7) yields

$$(s+\Lambda_0+\lambda_S+\lambda_C)\,\tilde{P}_{0,0}(s)-\mu_1\tilde{P}_{1,1}(s)=P_{0,0}(0) \tag{8}$$

$$(s+\Lambda_i+\lambda_S+\lambda_C+\theta)\,\tilde{P}_{0,i}(s)-\Lambda_{i-1}\tilde{P}_{0,i-1,}(s)=P_{0,i}(0), \qquad 1\le i\le L-1 \tag{9}$$

$$(s+\theta)\,\tilde{P}_{0,L}(s)-\Lambda_{L-1}\tilde{P}_{0,L-1,}(s)=P_{0,L}(0) \tag{10}$$

$$(s+\Lambda_1+\lambda_S+\lambda_C+\mu_1)\,\tilde{P}_{1,1}(s)-\theta\tilde{P}_{0,1}(s)-\mu_2\,\tilde{P}_{1,2}(s)=P_{1,1}(0) \tag{11}$$

$$(s+\Lambda_i+\lambda_S+\lambda_C+\mu_i)\,\tilde{P}_{1,i}(s)-\Lambda_{i-1}\tilde{P}_{1,i-1}(s)-\theta\tilde{P}_{0,i}(0)-\mu_{i+1}\,\tilde{P}_{1,i+1}(0)=P_{1,i}(0), \quad 2\le i\le L-1 \tag{12}$$

$$(s+\mu_L)\,\tilde{P}_{1,L}(s)-\Lambda_{L-1}\tilde{P}_{1,L-1}(s)-\theta\tilde{P}_{0,L}(s)=P_{1,L}(0) \tag{13}$$

Equations (8)-(13) can be put in matrix form as:

$$Q(s)\tilde{P}(s)=P(0) \tag{14}$$

where

$$Q(s)=\begin{bmatrix} A_0 & B_0 \\ C_1 & D_1 \end{bmatrix}_{(2L+1)\times(2L+1)}$$

$$A_0=\begin{bmatrix} -\begin{pmatrix} s+\Lambda_0+ \\ \lambda_S+\lambda_C \end{pmatrix} & 0 & 0 & \cdots & 0 & 0 \\ \Lambda_0 & -\begin{pmatrix} s+\Lambda_1+\lambda_S \\ +\lambda_C+\theta \end{pmatrix} & 0 & \cdots & 0 & 0 \\ 0 & \Lambda_1 & -\begin{pmatrix} s+\Lambda_2+\lambda_S \\ +\lambda_C+\theta \end{pmatrix} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & -\begin{pmatrix} s+\Lambda_{L-1}+\lambda_S \\ +\lambda_C+\theta \end{pmatrix} & 0 \\ 0 & 0 & 0 & \cdots & \Lambda_{L-1} & -(s+\theta) \end{bmatrix}_{(L+1)\times(L+1)};$$

$$B_0=\begin{bmatrix} \mu_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{(L+1)\times L}; \quad C_1=\begin{bmatrix} 0 & \theta & 0 & \cdots & 0 \\ 0 & 0 & \theta & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & 0 & \theta \end{bmatrix}_{L\times(L+1)};$$

$$D_1 = \begin{bmatrix} -\begin{pmatrix} s+\Lambda_1+\lambda_S \\ +\lambda_C+\mu_1 \end{pmatrix} & \mu_2 & 0 & \dots & 0 & 0 \\ \Lambda_1 & -\begin{pmatrix} s+\Lambda_2+\lambda_S \\ +\lambda_C+\mu_2 \end{pmatrix} & \mu_3 & \dots & 0 & 0 \\ 0 & \Lambda_2 & -\begin{pmatrix} s+\Lambda_3+\lambda_S \\ +\lambda_C+\mu_3 \end{pmatrix} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -\begin{pmatrix} s+\Lambda_{L-1}+\lambda_S \\ +\lambda_C+\mu_{L-1} \end{pmatrix} & \mu_L \\ 0 & 0 & 0 & \dots & \Lambda_{L-1} & -(s+\mu_L) \end{bmatrix}_{L\times L};$$

Also,

$\widetilde{\mathbf{P}}(s) = [\widetilde{\mathbf{P}}_{0,n}(s), \widetilde{\mathbf{P}}_{1,n}(s)]^T$ is a column vector of order $(2L+1)\times 1$ and

$\widetilde{\mathbf{P}}_{0,n}(s) = [\widetilde{P}_{0,0}(s), \widetilde{P}_{0,1}(s), \dots, \widetilde{P}_{0,L-1}(s), \widetilde{P}_{0,L}(s)]^T_{(L+1)\times 1}$,

$\widetilde{\mathbf{P}}_{1,n}(s) = [\widetilde{P}_{1,0}(s), \widetilde{P}_{1,1}(s), \dots, \widetilde{P}_{1,L}(s), \widetilde{P}_{1,L}(s)]^T_{L\times 1}$.

$\mathbf{P}(0) = [1,0,0,\dots,0,0,0,0,\dots,0,0,0,0,\dots,0]_{(2L+1)\times 1}$ is an initial vector.

To compute probabilities $\widetilde{\mathrm{P}}_{\mathrm{i,n}}(\mathrm{s})$, (i=0, 1), we apply Cramer's rule on matrix $\mathbf{Q}(s)$ and obtain

$$\widetilde{P}_{i,n}(s) = \frac{|Q_{n+1}(s)|}{|Q(s)|},\ \ i=0,1,\ \ n=0,1,2,\dots,L \tag{15}$$

where $Q_{n+1}(s)$ is obtained by replacing $(n+1)^{th}$ column of Q(s) with initial vector P(0).
Now, we proceed to calculate the characteristic roots of matrix Q(s). It is noted that s=0 is one of the root. Let *s* = (-d), so that we have

Q(-d) = (Q-dI) (16)

Now equation (14) converts into

$$Q(-d)\widetilde{\mathbf{P}}(s) = (Q-dI)\widetilde{\mathbf{P}}(s) = \mathbf{P}(0) \tag{17}$$

Suppose that other roots in which *l* are real roots and m are complex roots in pairs are denoted by:

$d_1, d_2, \dots, d_l$ and $(d_{l+1}, \overline{d}_{l+1}), (d_{l+2}, \overline{d}_{l+2}), \dots, (d_{l+m}, \overline{d}_{l+m})$, respectively.

Thus, we get

$$|Q(s)| = s\left[\prod_{j=1}^{l}(s+d_j)\right]\left[\prod_{j=1}^{m}(s+d_{l+j})(s+\overline{d}_{l+j})\right] \tag{18}$$

Applying Cramer's rule on equation (15) and using equation (16), we get

$$\widetilde{P}_{i,n}(s) = \frac{|Q_{n+1}(s)|}{s\left[\prod_{j=1}^{l}(s+d_j)\right]\left[\prod_{j=1}^{m}(s+d_{l+j})(s+\overline{d}_{l+j})\right]},\ i=0,\ 1,\ n=0,1,2,\dots,L \tag{19}$$

Using partial fractions, we expand equation (19), we get

$$\tilde{P}_{i,n}(s)=\frac{a_0}{s}+\frac{a_1}{s+d_1}+...+\frac{a_l}{s+d_l}+\frac{b_1 s+c_1}{(s+d_{l+1})(s+\bar{d}_{l+1})}+...+\frac{b_m s+c_m}{(s+d_{l+m})(s+\bar{d}_{l+m})} \quad (20)$$

Here a_0 and a_p (p=1,2, …, l) are real numbers calculated as:

$$a_0=\frac{|Q_{n+1}(s)|}{\left(\prod_{j=1}^{l} d_j\right)\left(\prod_{j=1}^{m} d_{l+j}\bar{d}_{l+j}\right)} \quad (21)$$

$$a_p=\frac{|Q_{n+1}(-d_p)|}{(-d_p)\left[\prod_{\substack{j=1\\ j\neq p}}^{l}(d_j-d_p)\right]\left[\prod_{\substack{j=1\\ j\neq p}}^{m}(-d_p+d_{l+j})(-d_p+\bar{d}_{l+j})\right]} \quad (22)$$

Let complex characteristic root d_{l+p} is a combination of real part u_p and imaginary part v_p. Then

$$b_p(-d_{l+p})+c_p=\frac{|Q_{n+1}(-d_{l+p})|}{(-d_{l+m})\left[\prod_{\substack{j=1\\ j\neq p}}^{l}(d_j-d_{l+p})\right]\left[\prod_{\substack{j=1\\ j\neq p}}^{m}(-d_{l+p}+d_{l+j})(-d_{l+p}+\bar{d}_{l+p})\right]},\ \text{p=1,2, …,m} \quad (23)$$

On taking inverse Laplace transform of equation (23), we get

$$P_{i,n}(t)=a_0+\sum_{p=1}^{l}a_p e^{-d_p t}+\sum_{p=1}^{m}\left[b_p e^{-u_p t}\cos(v_p t)+\frac{c_p-b_p u_p}{v_p}e^{-u_p t}\sin(v_p t)\right], \quad (24)$$

$$i=0,1,\ n=0,1,2,....,L$$

where a_0, a_p, b_p, c_p, u_p and v_p all are real numbers.

5. PERFORMANCE MEASURES

To evaluate the overall performance of an embedded system under consideration, we establish some characteristics which are as follows:

- The expected number of failed components in the system is given by

$$E(F)=\sum_{i=0}^{1}\sum_{n=i}^{L} n P_{i,n} \quad (25)$$

- The expected number of operating components in the system is

$$E(O)=M-\sum_{i=0}^{1}\sum_{n=S+1}^{L}(n-S)P_{i,n} \quad (26)$$

- The expected number of spare components in the system acting as standbys is

$$E(S)=\sum_{i=0}^{1}\sum_{n=i}^{S}(S-n)P_{i,n} \tag{27}$$

- The probability of the repairman being on vacation at time t

$$P_V(t)=\sum_{n=0}^{L}P_{0,n}(t) \tag{28}$$

-The probability of the repairman being busy at time t

$$P_B(t)=\sum_{n=1}^{L}P_{1,n}(t) \tag{29}$$

- Reliability of an embedded system at time t is given by

$$R(t)=1-\sum_{n=0}^{L-1}P_{0,n}(t)+\sum_{n=1}^{L-1}P_{1,n}(t) \tag{30}$$

- MTTF of the system at time t is obtained as

$$\int_0^{\infty}R(t)\,dt=1-\sum_{n=0}^{L-1}\tilde{P}_{0,n}(0)+\sum_{n=1}^{L-1}\tilde{P}_{1,n}(0) \tag{31}$$

Illustrations:

Extensive numerical experiment has been performed by taking an illustration. The matrix method described in previous section has been employed to develop the computer program in MATLAB software to explore the effect of various descriptors on the performance indices. In this section, we present numerical example of ESEGT discussed in the 'introduction' section. The tractability of numerical results shows that our model can be easily used to represent the real time congestion systems. For illustration purpose, we assume that in ESEGT there are M=8 operating synchronous generator and S=4 standby synchronous generator. The ESEGT failed when K=6. There is one repairman who immediately repairs the failed synchronous generators with rates $\mu_1=0.5$, $\mu_2=0.6$, $\mu_3=0.7$, $\mu_4=0.8$, $\mu_5=0.9$, $\mu_6=0.9$ and $\mu_7=0.9$. On the other hand, if there is no failed synchronous generator available in the ESEGT then repairman takes multiple vacations with rate $\theta=0.9$. For the considered problem, we obtain various performance indices given in equations (25)-(31) as:

E(F)=0.6949; E(O)=7.8510; E(S)=0.1985; $P_V(t)=0.0992$; $P_B(t)=0.8686$; R(t)=0.83867.

6. NUMERICAL RESULTS

In this section, we first compute the performance measures for given model using the analytical results. Moreover, we compare the analytical results with the neuro-fuzzy results by building ANFIS in MATLAB 8.0. Whenever classical analytical tools are difficult to use, soft computing approach provides flexible information processing capabilities for handling real life ambiguous situations. ANFIS is a very powerful and useful soft computing approach for solving complex problems. For computation purpose, we set default parameters as $\lambda_s=0.3, \lambda_c=0.03, \lambda_8=0.05, \lambda_7=0.01, \lambda_6=0.04, \mu_1=$

$0.5, \mu_2 = 0.6, \mu_3 = 0.7, \mu_4 = 0.8, \mu_5 = 0.9, \mu_6 = 0.9, \mu_7 = 0.9, \alpha = 0.2, \theta = 0.9, t = 1.5, S = 4, M = 8, K = 6, L = 7$.

The performance measures are obtained by varying parameters namely failure rate of operating and standby components, repair rate of operating components and vacation rate. These parameters are treated as the linguistic variables in the contexts of the fuzzy systems. These parameters are also considered as the input parameters while building the respective inference system. The gaussian function is used for describing the membership functions for these input parameters. The shapes of the corresponding membership functions are displayed in figures 5(a)-8(a). In figures 5(b)-8(b), a comparative study of analytical results and neuro-fuzzy results has been done. Table 1 represents the linguistic values of the membership functions.

It is clear from Tables 2-3 that E(O) decreases while E(S), $P_v(t)$, $P_B(t)$ increase on increasing λ_8. It is also noted from these tables that the reverse trend has been observed for increasing values of μ_1.

Table 1. Linguistic values of the membership functions for various input parameters

Input Variables	Number of membership functions	Linguistic Values
α	5	Very low Low Average High Very high
θ	5	Very low Low Average High Very high
μ_1	5	Very low Low Average High Very high
λ_8	5	Low Below average Average Above average High

Table 2. Various performance measures by varying (i) t and (ii) λ_8

t	λ_8	E(O)	E(S)	$P_v(t)$	$P_B(t)$
1.47	0.1	7.9278	0.0961	0.0480	0.0420
	0.2	7.9085	0.1219	0.0609	0.0533
	0.3	7.8843	0.1542	0.0771	0.0674
	0.4	7.8538	0.1948	0.0974	0.0852
	0.5	7.8158	0.2455	0.1227	0.1074
	0.6	7.7683	0.3089	0.1544	0.1351
1.48	0.1	7.9217	0.1043	0.0521	0.0456
	0.2	7.9006	0.1324	0.0662	0.0579
	0.3	7.8740	0.1678	0.0839	0.0734
	0.4	7.8408	0.2122	0.1061	0.0928
	0.5	7.7990	0.2678	0.1339	0.1171
	0.6	7.9278	0.3374	0.1687	0.1476

Table 3. Various performance measures by varying (i) t and (ii) μ_1

t	μ_1	E(O)	E(S)	$P_v(t)$	$P_B(t)$
1.47	0.3	7.8069	0.2573	0.1286	0.1125
	0.4	7.8334	0.2220	0.1110	0.09716
	0.5	7.8592	0.1876	0.09384	0.08211
	0.6	7.8843	0.1542	0.0771	0.0674
	0.7	7.9086	0.1217	0.0608	0.0532
	0.8	7.9324	0.0900	0.0450	0.0394
1.48	0.3	7.9264	0.0980	0.0490	0.0428
	0.4	7.9006	0.1324	0.0662	0.0579
	0.5	7.8740	0.1678	0.0839	0.0734
	0.6	7.8468	0.2042	0.1021	0.0893
	0.7	7.8187	0.2416	0.1208	0.1057
	0.8	7.7899	0.0900	0.1400	0.1225

From Figures 3(a)-3(d), we observe that the number of failed components (E(F)) increase (decrease) as we increase the values of $\lambda_8, \alpha, \theta$ and t(μ_1). It can be easily seen from Figures 4(a)-4(d) that reliability (R(t)) of an embedded system decreases with respect to failure rates, vacation rate and time. On the other hand, reliability increases incredibly with the increment in repair rate. Moreover, we plot various graphs of reliability for the different values of $\alpha, \theta, \mu_1, \lambda_8$, respectively in Figures 5(b)-8(b) wherein black continuous (grey discrete) lines are used to represent the analytical (ANFIS) results. We observe from these figures that reliability of an embedded system increases on increasing the values of 1 but it decreases with an increase in α, θ and λ_8. The shapes of the corresponding membership functions for figures 5(b)-8(b) are shown in Figures 5(a)-8(a). It can be seen that the results determined by soft computing approach (ANFIS) are at par with the numerical results obtained analytically.

Finally, we conclude that

- The expected number of failed components in the system increases with an increment in failure rates of both operating and standby components whereas it decreases on increasing the repair rate. This result profoundly matches with many real life situations.
- It can be seen easily from the above observation that the expected number of operating components increases on increasing repair rate and vacation rate.
- Adaptive Network-based Fuzzy Interference Systems (ANFIS) provide an easy and fast solution and can be easily implemented to real time system.

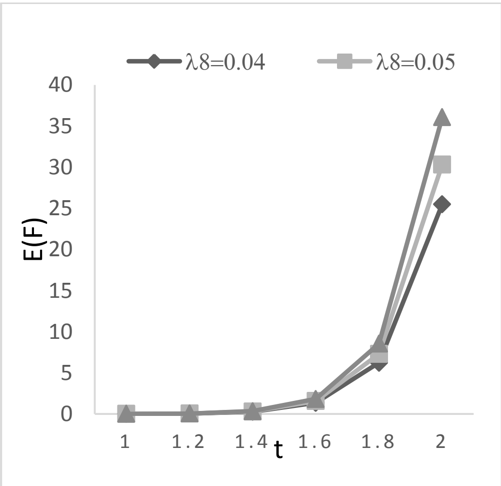

Figure 3. (a): Failed components vs time by varying λ_8

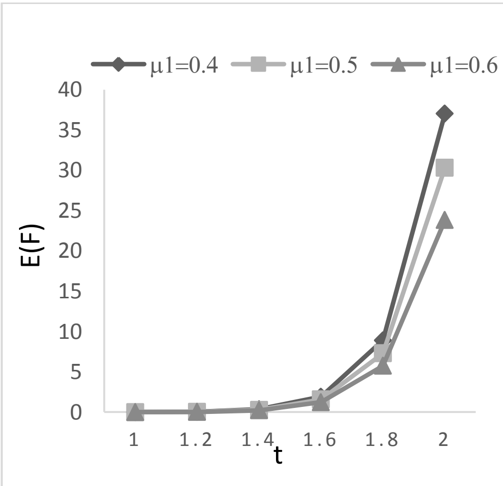

Figure 3. (b): Failed components vs time by varying μ_1

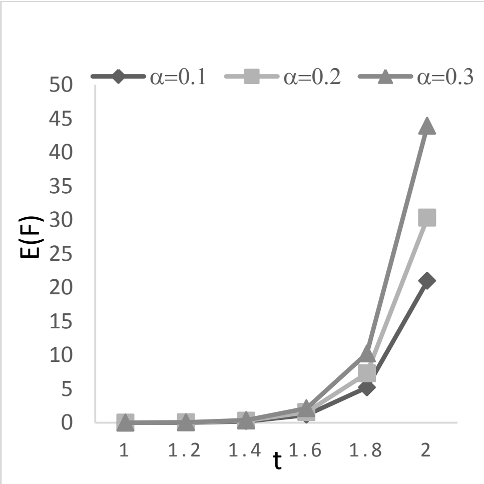

Figure 3. (c): Failed components vs time by varying α

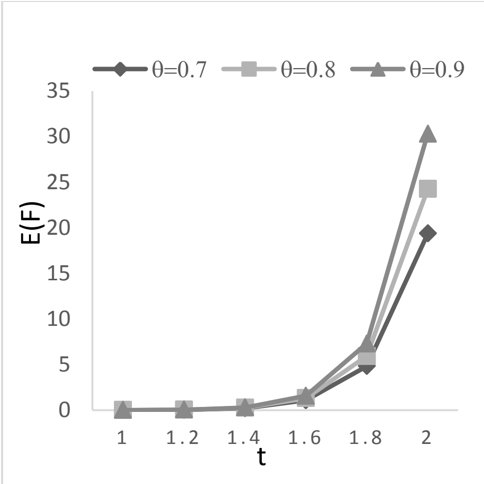

Figure 3. (d): Failed components vs time by varying θ

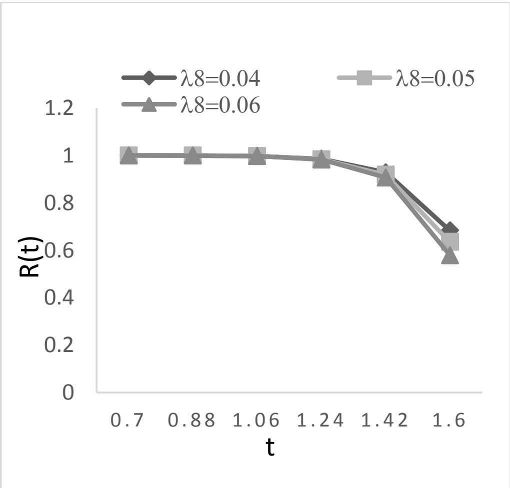

Figure 4. (a): Reliability vs time by varying λ_8

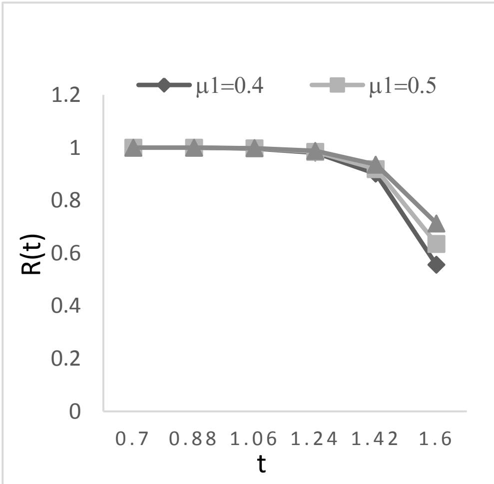

Figure 4. (b): Reliability vs time by varying μ_1

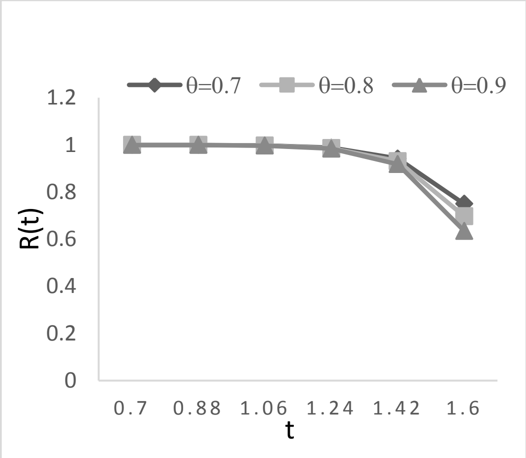

Figure 4. (c): Reliability vs time by varying α

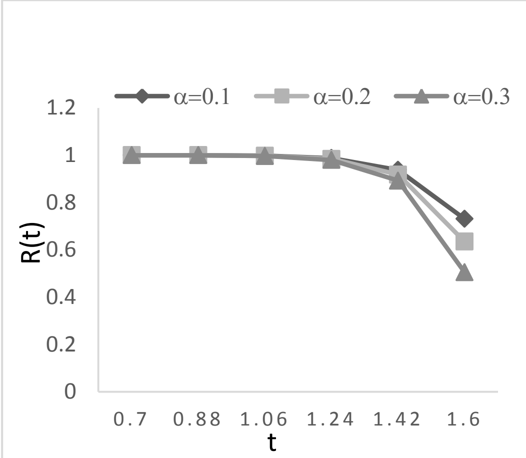

Figure 4. (d): Reliability vs time by varying θ

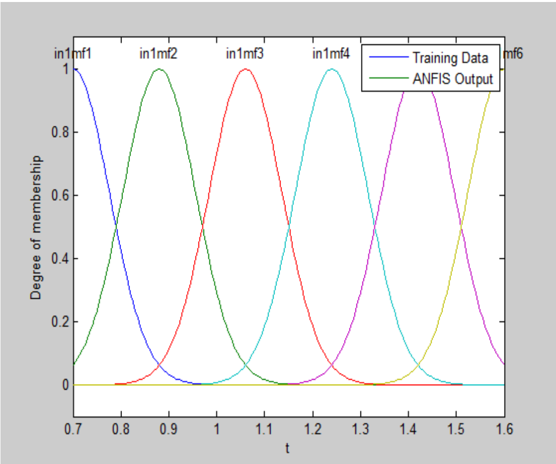

Figure 5. (a): Membership functions for input parameters α

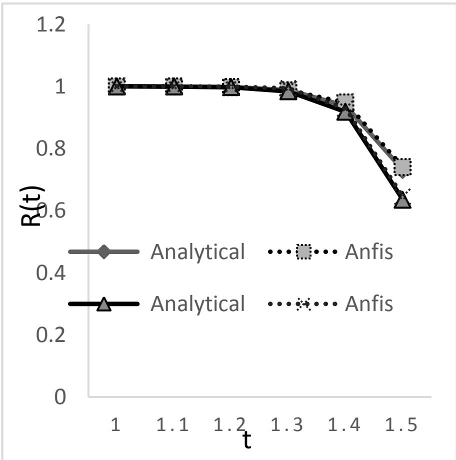

Figure 5. (b): R(t) by varying t for different values of α

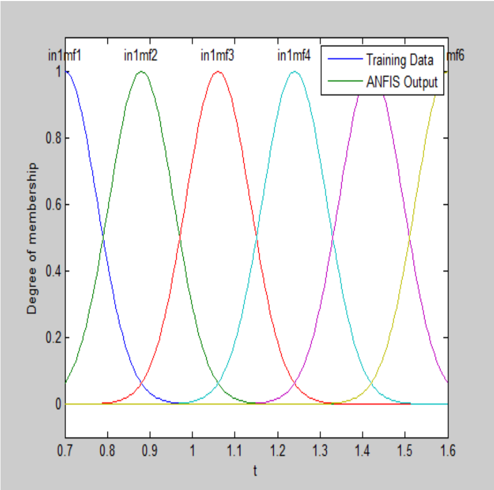

Figure 6. (a): Membership functions for input parameters θ

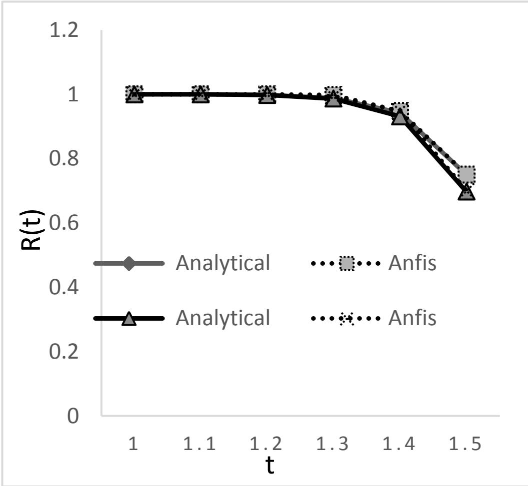

Figure 6. (b): R(t) by varying t for different values of θ

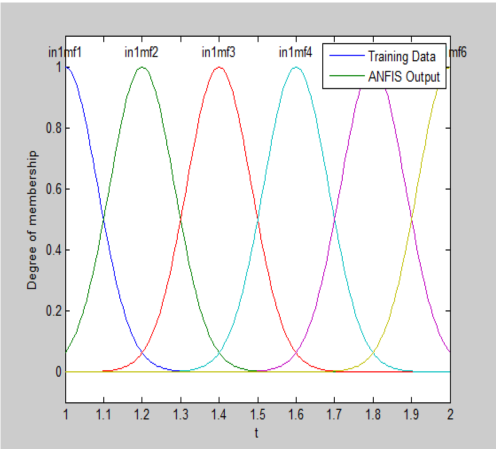

Figure 7. (a): Membership functions for input parameters μ_1

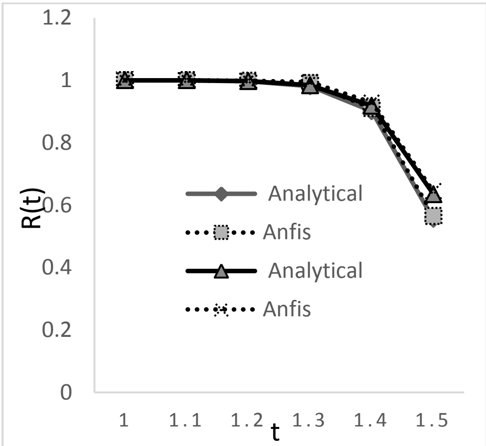

Figure 7. (b): R(t) by varying t for different values of μ_1

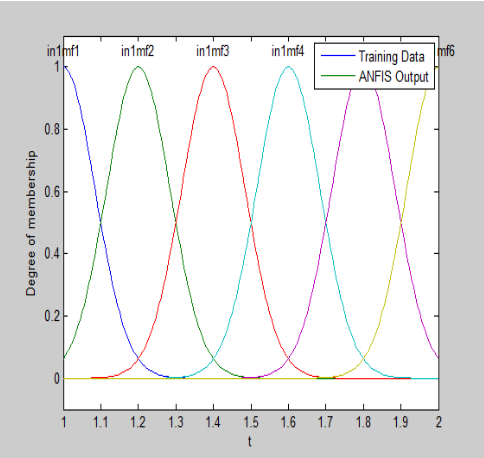

Figure 8. (a): Membership functions for input parameters λ_8

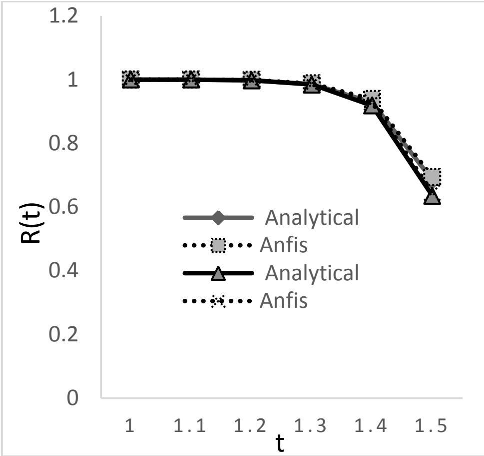

Figure 8. (b): R(t) by varying t for different values of λ_8

7. CONCLUSION

In this paper, an embedded system with vacation has been considered wherein repairman can go for multiple vacations of random length. The concept of standby is also taken into consideration which is helpful for the smooth functioning of an embedded system. The considered model is helpful for the system designer and developer for lowering down the burden on the system and in making proper utilization of its resources. The incorporation of degraded failure rates leads our model to deal with embedded engineering systems including electronic/electrical, computer and communication systems, etc.. The transient reliability and other performance indices obtained may be helpful to improve the system availability. Also, the analytical results obtained are also compared using a soft computing approach namely ANFIS. Our investigation in the present study facilitates an insight to the system designers and developers to produce more reliable embedded computer systems by judging correct measure of fault generation. The present investigation can be further extended by taking the concepts of 'R' and unreliable repairmen.

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