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Studies on a parallel system with two types of failure

M. A. El-Damcese
Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt

Sharhabeel.Alaidi and M. S. Shama*
Department of Basic Science, Preparatory Year, King Saudi University, KSA

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Abstract. In this paper, we investigate reliability and availability of repairable systems with two types of failure. The first one is to one unit and the second one is to *M* units in parallel structure. Let failure rate and repair rate of [type1, type2] components are assumed to be exponentially distributed. The expressions of availability and reliability characteristics such as the system reliability and the mean time to failure are derived for two systems. We used several cases to analyze graphically the effect of various system parameters on the reliability system and availability system.

Key Words: *Availability, reliability, mean time to system failure, sensitivity analysis*

1. INTRODUCTION

Studying the reliability of machine repair problem is very important in our life because it is widely used in industrial system and manufacturing system. In the traditional systems, the units of the system have only two states up and down. However, in many situations the units of the system can have finite number of states. Most reliability models assume that the up and down times of the components are exponentially distributed. This assumption leads to a Markovian model with constant transition rates. The analysis in such cases is relatively simple and the numerical results can be easily obtained. Redundancy is a very important aspect of system design and reliability in that adding redundancy is one of several methods of improving system reliability. In a simple parallel system at least one of the units must succeed for the system to succeed. Units in parallel are referred to as redundant units.
In this paper, we consider a system consists of a 2-state repairable unit with two types of failures. We also discuss a repairable system with two types of failure in parallel structure.

*Corresponding Author.
E-mail address: *m_shama87@yahoo.com*

Recently El-Damcese and Shama (2015) studied Reliability and availability analysis of a 2-state repairable system with two types of failure. In the past decades; many articles concerning the reliability and availability of standby systems have been published. Among them Galikowsky et al. (1996) analyzed the series systems with cold standby components. Wang and Sivazlian (1989) examined the reliability characteristics of a multiple-server ($M+W$) unit system with exponential failure and exponential repair time distributions. Ke et al. (2011) studied the reliability measures of a repairable system with warm standby switching failures and reboot delay. Ke et al. (2013) analyzed the machine repair problem with unreliable multirepairmen. Wang et al. (2009) considered the single server machine repair problem with working vacation. Wang et al. (2007) introduced the warm-standby machine repair problem with Balking, reneging and standby switching failures. Hsu et al. (2011) examined an availability system with reboot delay, standby switching failures and an unreliable repair facility, which consists of two active components and one warm standby. Jain et al. (2004) studied the degraded model with warm standbys and two repairmen. Wang et al. (2006) compared four different system configurations with warm standby components and standby switching failures. This paper has two objectives for two systems. The first objective is to develop the expressions for availability function, reliability function and mean time to failure using Laplace transform techniques. The second objective is to perform a parametric investigation which presents numerical results to analyze the effects of the various system parameters on the system reliability and system availability.

1.1. Notations

M : The number of units in the initial state
R_1 : The number of repairman in the first service line
R_2 : The number of repairman in the second service line
λ_1 : The failure rate of type 1
λ_2 : The failure rate of type 2
μ_1 : The repair rate of type 1
μ_2 : The repair rate of type 2
$\mu_{1.n}$: Mean repair rate when there are n failed units of type 1
$\mu_{2.n}$: Mean repair rate when there are n failed units of type 2
$P_i(t)$: Probability for i = 0, 1, 2
    0→normal state
    1→failed state due to failure rate of type 1
    2→failed state due to failure rate of type 2
$P_i^*(s)$: Laplace transform of p_i(t).
$P_{i,j}(t)$: Probability that there are i failed units of type 1and j failed units of type 2 in the system at time t where i, j= 0,1,2, ..., M ,$0 \leq i + j \leq M$
s : Laplace transform variable
$P_{i,j}^*(t)$: Laplace transform of $P_{i,j}$(t)
$A(t)$: Availability function of the system (one unit)
$R(t)$: Reliability function of the system(one unit)
$A_Y(t)$: Availability function of the system (M units in parallel structure)

R_Y *(t)* : Reliability function of the system (M units in parallel structure)
MTTF : Mean time to failure

2. FIRST SYSTEM (ONE UNIT)

2.1 Problem description

We consider a system consists of a 2-state repairable unit with two types of failures. Let failure times of type 1 and type 2 are assumed to be exponentially distributed with parameter λ_1 and λ_2 respectively, repair rates of type 1 and type 2 are assumed to be exponentially distributed with parameters μ_1 and μ_2 respectively. We also investigate the sensitivity analysis for the system reliability with changes in a specific value of the system parameters.
Various states probabilities have been evaluated in the form of Laplace transform. The expressions of availability and reliability characteristics will be obtained in addition to we perform sensitivity analysis of system reliability with respected to system parameters.

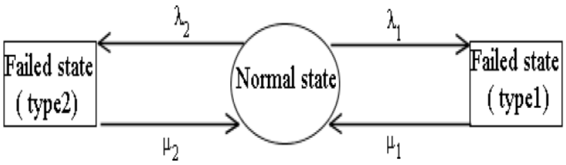

Figure 1. System configuration diagram

2.2 Mathematical formulation of the system

According to System configuration diagram in Figure 1, the difference-differential equations for this stochastic process which is continuous in time and discrete in space are given as follows.

$$\frac{dP_0(t)}{dt} = -[\lambda_1 + \lambda_2]P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t) \quad (1)$$

$$\frac{dP_1(t)}{dt} = -\mu_1 P_1(t) + \lambda_1 P_0(t) \quad (2)$$

$$\frac{dP_2(t)}{dt} = -\mu_2 P_2(t) + \lambda_2 P_0(t) \quad (3)$$

With boundary condition $P_0(0)$ and $P_1(0) = P_2(0) = 0$ using the Laplace transform technique, the solutions of $P_i^*(s)$,i=0, 1, 2 are given

$$P_0^*(s) = \frac{(s+\mu_2)(s+\mu_1)}{(s+\mu_1)(s+\lambda_2+\lambda_1)(s+\mu_2)-\mu_1\lambda_1(s+\mu_2)-\mu_2\lambda_2(s+\mu_1)} \quad (4)$$

$$P_1^*(s) = \frac{\lambda_1(s+\mu_2)}{(s+\mu_1)(s+\lambda_2+\lambda_1)(s+\mu_2)-\mu_2\lambda_1(s+\mu_2)-\mu_2\lambda_2(s+\mu_1)} \quad (5)$$

$$P_1^*(s) = \frac{\lambda_2(s+\mu_1)}{(s+\mu_1)(s+\lambda_2+\lambda_1)(s+\mu_2)-\mu_1\lambda_1(s+\mu_2)-\mu_2\lambda_2(s+\mu_1)} \quad (6)$$

The steady-state probability becomes

$$P_0 = P_0(\infty) = \frac{\mu_1\mu_2}{\mu_1\mu_2+\lambda_1\mu_2+\mu_1\lambda_2} \quad (7)$$

$$P_1 = P_1(\infty) = \frac{\lambda_1\mu_2}{\mu_1\mu_2+\lambda_1\mu_2+\mu_1\lambda_2} \tag{8}$$

$$P_0 = P_0(\infty) = \frac{\mu_1\lambda_2}{\mu_1\mu_2+\lambda_1\mu_2+\mu_1\lambda_2} \tag{9}$$

Thus, the steady-state availability of a two types of failure (these failures are independents) is

$$A_s = P_0 = \frac{\mu_1\mu_2}{\mu_1\mu_2+\lambda_1\mu_2+\mu_1\lambda_2} \tag{10}$$

By taking inverse Laplace transform of equations (4)-(6), we get

$$A(t) = \frac{1}{d}\left[b + e^{-\frac{1}{2}tc}\left[(d-b)\cosh\left(\frac{1}{2}t\sqrt{c^2-4d}\right) + \frac{(2ad-dc-cb)sinh(\frac{1}{2}t\sqrt{c^2-4d})}{\sqrt{c^2-4d}}\right]\right] \tag{11}$$

<u>Special cases</u>:

Case1: when $\lambda_1 = 0$ we find $A(t) = \frac{\lambda_2 e^{-(\lambda_2+\mu_2)t}+\mu_2}{\lambda_2+\mu_2}$ and $A = \frac{\mu_2}{\lambda_2+\mu_2}$

Case2: when $\lambda_2 = 0$ we find $A(t) = \frac{\lambda_1 e^{-(\lambda_1+\mu_1)t}+\mu_1}{\lambda_1+\lambda_2}$ and $A = \frac{\mu_1}{\lambda_1+\mu_1}$

Case3: In this case we assume that failure rates are equal and repair rates are equal, so we substitute $\mu_1 = \mu_2 = \mu$ and $\lambda_2 = \lambda_1 = \lambda$ into equations (4)-(6), we have

$$P_0^*(s) = \frac{(s+\mu)}{s(s+\mu+2\lambda)} = \frac{\mu}{(\mu+2\lambda)s} + \frac{2\lambda}{(\mu+2\lambda)(s+\mu+2\lambda)} \tag{12}$$

$$P_1^*(s) = \frac{\lambda(s+\mu)}{s(s+\mu)(s+\mu+2\lambda)} = \frac{-\lambda}{(\mu+2\lambda)s} + \frac{1}{s+\mu} + \frac{-(\mu+\lambda)}{(\mu+2\lambda)(s+\mu+2\lambda)} \tag{13}$$

$$P_2^*(s) = \frac{\lambda(s+\mu)}{s(s+\mu)(s+\mu+2\lambda)} = \frac{-\lambda}{(\mu+2\lambda)s} + \frac{1}{(s+\mu)} + \frac{-(\mu+\lambda)}{(\mu+2\lambda)(s+\mu+2\lambda)} \tag{14}$$

Inverse Laplace transforms of these equations yield

$$P_0(t) = \frac{\mu}{(\mu+2\lambda)} + \frac{2\lambda}{(\mu+2\lambda)}e^{-(\mu+2\lambda)t} \tag{15}$$

$$P_i(t) = \frac{-\lambda}{(\mu+2\lambda)} + e^{-\mu t} + \frac{-(\mu+\lambda)}{(\mu+2\lambda)}e^{-(\mu+2\lambda)t}, i = 1,2 \tag{16}$$

The availability function of the system can be written as

$$A_s(t) = P_0(t) = \frac{\mu}{(\mu+2\lambda)} + \frac{2\lambda}{(\mu+2\lambda)}e^{-(\mu+2\lambda)t} \tag{17}$$

The steady –state availability can be obtained from this equation

$$A_s = \lim_{t\to\infty} A_s(t) = \frac{\mu}{(\mu+2\lambda)} \tag{18}$$

To obtain the reliability function for this model, we assume that at least one of failed states [type1, type2] is absorbing state and the transition rate from this state equal to zero.

$$R(t) = \frac{1}{d}\left[b + e^{-\frac{1}{2}tc}\left[(d-b)\cosh\left(\frac{1}{2}t\sqrt{c^2-4d}\right) + \frac{(2ad-dc-cb)\sinh(\frac{1}{2}t\sqrt{c^2-4d})}{\sqrt{c^2-4d})}\right]\right] \tag{19}$$

As we know, we have two failed states this lead to three cases of reliability function.
Case 1: Failed state [type1] is absorbing when $\mu_1 = 0$
Case 2: Failed state [type2] is absorbing when $\mu_2 = 0$
Case 3: Failed states [type1, type2] are absorbing when $\mu_1 = \mu_2 = 0$

2.3 Sensitivity analysis

In this section we use numerical example to perform sensitivity analysis for changes in the reliability of the system $R(t)$ from changes in system parameters $\lambda_1, \lambda_2, \mu_1$ and μ_2 in three cases of reliability.

Case 1 When $\mu_1 = 0$
We setting $\lambda_1 = .0007$, $\lambda_2 = .001$, $\mu_1 = 0$, $\mu_2 = .03$, Figure 2 reveals that the sensitivities of λ_2 and μ_2 on the $R(t)$, we observe that the sensitivity of λ_2 reverse the sign from negative to positive nearly at the same certain time when the sensitivity of μ_2 reverse the sign from positive to negative. We also observe that the influence of λ_1 on $R(t)$ be negative from Figure 3.

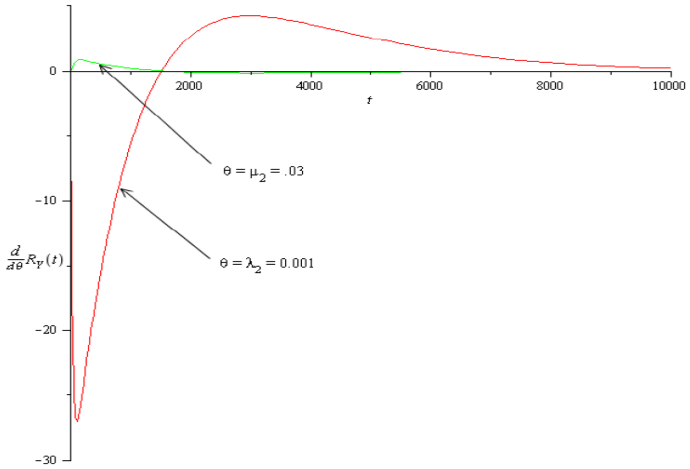

Figure 2. Sensitivity of system reliability with respect to μ_2 and λ_2

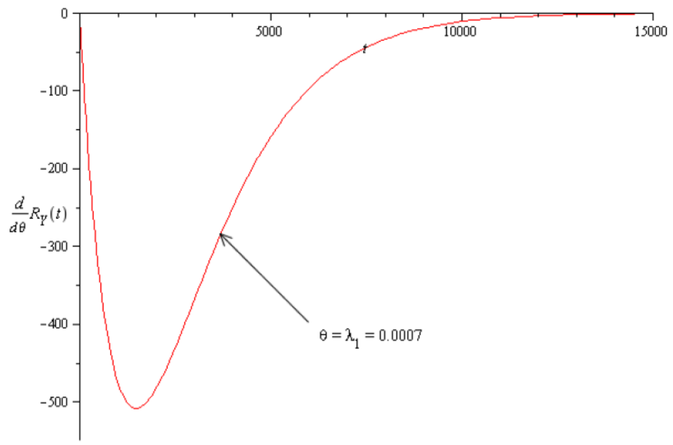

Figure 3. Sensitivity of system reliability with respect to λ_1

<u>Case 2</u> When $\mu_2 = 0$

We setting λ_1=.0007, λ_2=.001, μ_1=.05, μ_2=0, Figure 4 reveals that the sensitivities of λ_1 and μ_1 on the $R(t)$ have the same sensitivities of λ_2 and μ_2 on the $R(t)$ when $\mu_1 = 0$. From Figure 5 the influence of λ_2 on $R(t)$ has the same influence of λ_1 on $R(t)$ when $\mu_1 = 0$.

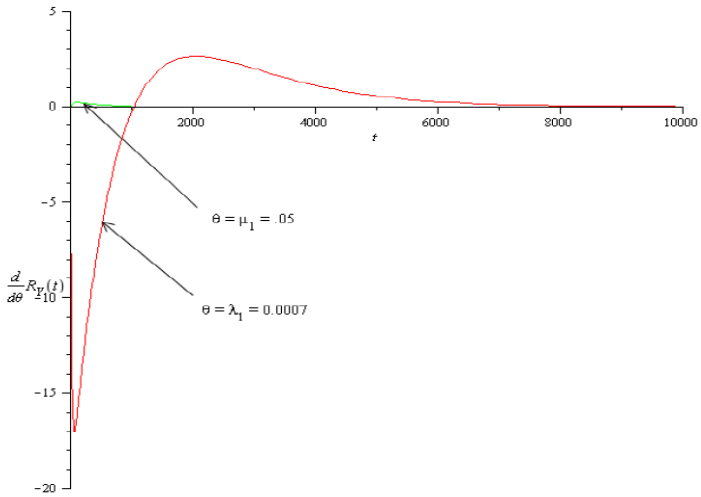

Figure 4. Sensitivity of system reliability with respect to μ_1 and λ_1

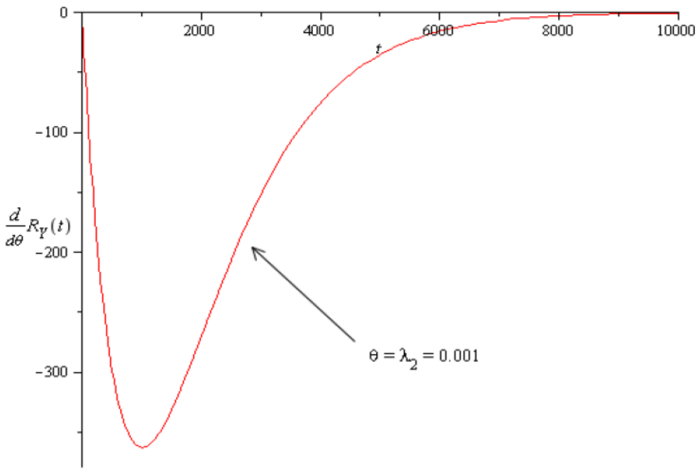

Figure 5. Sensitivity of system reliability with respect to λ_2

<u>Case 3</u> When $\mu_1 = \mu_2 = 0$

We setting λ_1 =.0007, λ_1 =.001, μ_1 =0, μ_2 =0, in Figure 6 it is noticed that λ_1 and λ_2 have the same impact during time.

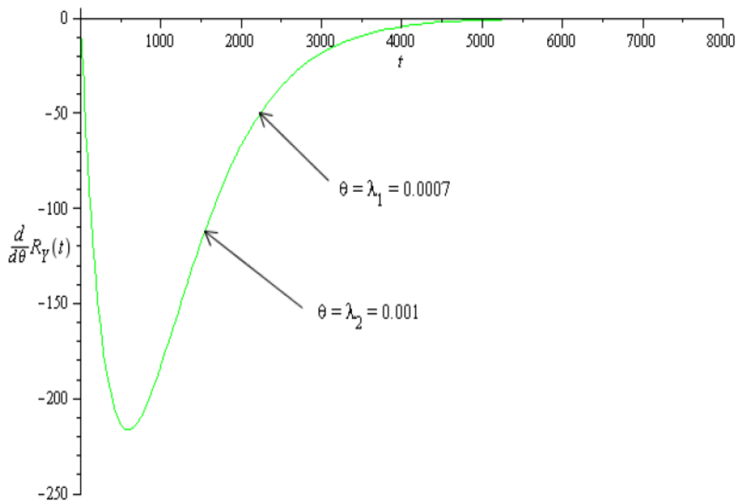

Figure 6. Sensitivity of system reliability with respect to λ_1 and λ_2

3. SECOND SYSTEM (*M* UNITS IN PARALLEL STRUCTURE)

3.1 Problem description

We consider a machining system consisting of *M* identical units operating simultaneously in parallel structure, R_1repairmen in the first service line who repair failed units of type 1 and R_2 repairmen in the second service line who repair failed units of type 2. The assumptions of the model are described as follows. We suppose that the failure rate of type 1 and type 2 occur independently of the states of other units and follow exponential distributions with λ_1, λ_2, respectively.
When an operating unit failed [type 1 or type 2] it is immediately sent to the appropriate service line where it is repaired with time-to-repair which is exponentially distributed with parameter μ_1 or μ_1 according to the type of failure. Moreover, we assume that the secession of failure times and repair times are independently distributed random variable. Let us assume that failed units [type 1 or type 2] arriving at the repairmen form a single waiting line and are repaired in the order of their breakdowns; i.e. according to the first-come, first-served discipline. Suppose that the repairmen in the two service lines can repair only one failed unit at a time and the repair is independent of the failure of the units. Once a unit is repaired, it is as good as new.

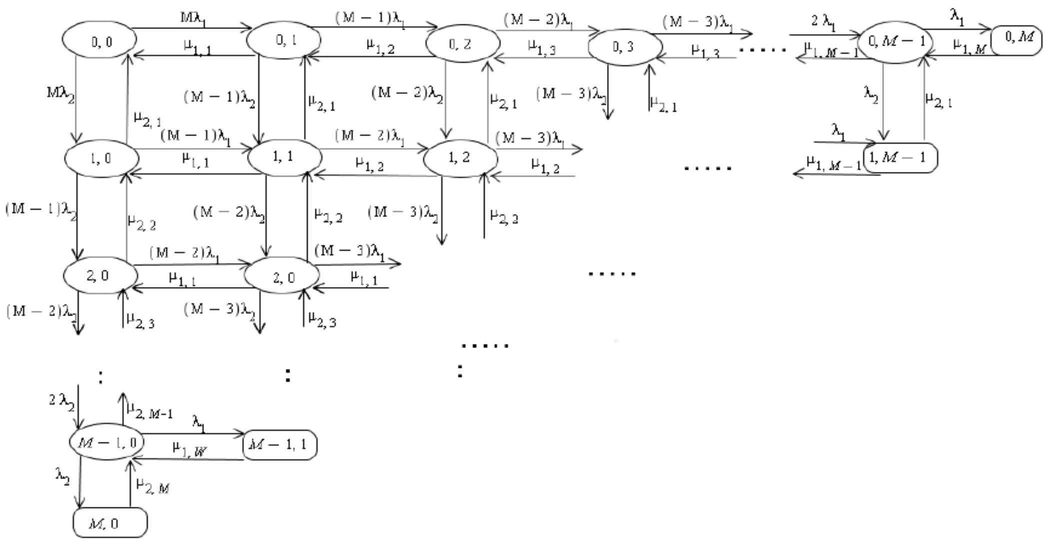

Figure 7. State-transition-rate diagram

3.2 Availability and reliability analysis of the system

The system starts operation at time t = 0 with no failed units. The availability function under the exponential failure time and exponential repair time distributions can be developed through the birth –death process. Let Y be the random variable representing the time to failure of the system.

The mean repair rate $\mu_{1,n}$ is given by

$$\mu_{1,n} = \begin{cases} n\mu_1 \ , & if\ 1 \le n \ \le min(R_1, W) \\ R_1\,\mu_1 \ , & if\ R_1 \ \le n \ \le W \\ 0 & otherwise \end{cases}$$

The mean repair rate $\mu_{2,n}$ is given by:

$$\mu_{2,n} = \begin{cases} n\mu_2 \ , & \text{if } 1 \le \text{n} \ \le \min(R_2, W) \\ R_2\mu_2 \ , & \text{if } R_2 \ \le \text{n} \ \le \text{W} \\ 0 & otherwise \end{cases}$$

The Laplace transforms of $P_{i,j}(t)$ are defined by

$$P_{i,j}^{*}(s) = \int_0^{\infty} e^{-st} P_{i,j}(t)dt \ , \ \ i,j = 0,1,2, \dots, M \ ,0 \le i + j \le M$$

State –transition-rate diagram can be obtained in Figure 7, and it leads to the following Laplace transform expressions for $P_{i,j}^{*}\,(s)$

$$(\text{s} + \text{M}\lambda_1 + \text{M}\lambda_2)P_{0,0}^{*}(\text{s}) - \ \mu_{1,1}P_{0,1}^{*}(\text{s}) - \mu_{2,1}P_{1,0}^{*}(s) = 1 \tag{20}$$

$$(\text{s} + (\text{M} - \text{n})\,\lambda_1 + (\text{M} - \text{n})\lambda_2 + \ \mu_{1,n})P_{0,n}^{*}(s) - \ \mu_{2,1}P_{1,n}^{*}(s) - \ \mu_{1,n+1}P_{0,n+1}^{*}(s)$$
$$- (\text{M} - \text{n} + 1)\lambda_1 P_{0,n-1}^{*}(s) = 0, \ \ 1 \le n \le M - 1 \tag{21}$$

$$\left(\text{s} + \ \mu_{1,M}\right)P_{0,M}^{*}(s) - \ \lambda_1 P_{0,M-1}^{*}(s) = 0 \tag{22}$$

$$(\mathrm{s}+(\mathrm{M}-i)\lambda_1+M-i)\lambda_2+\mu_{2,i})P^*_{i,0}(s)-(M-i+1)\lambda_2 P^*_{i-1,0}(s)-\mu_{1,1}P^*_{i,1}(s)$$
$$-\mu_{2,i+1}P^*_{i+1,0}(s)-0,1\le i\le M-1 \tag{23}$$
$$\left(\mathrm{s}+\mu_{2,M}\right)P^*_{M,0}(s)-\lambda_2 P^*_{M-1,0}(s) \tag{24}$$
$$\left(\mathrm{s}+(\mathrm{M}-\mathrm{n}-i-1)\lambda_1+(M-n-i-1)\lambda_2+\mu_{1,n}+\mu_{2,i+1}\right)P^*_{i+1,n}(s)$$
$$-\mu_{1,n+1}P^*_{i+1,n+1}(s)-\mu_{2,i+2}P^*_{i+2,n}(s)-(\mathrm{M}-\mathrm{n}-\mathrm{i})\lambda_1 P^*_{i+1,n-1}(s)$$
$$-(M-n-1)\lambda_2 P^*_{i+1,n+1}(s)=0,\ 1\le i\le M-3,1\le n\le M-i-2 \tag{25}$$
$$\left(\mathrm{s}+\mathrm{M}\lambda_1+\mu_{2,n}+\mu_{1,M-n}\right)P^*_{n,M-n}(s)-\lambda_2 P^*_{n-1,M-n}(s)-\lambda_1 P^*_{n,M-n-1}(s)=0$$
$$1\le \mathrm{n}\le \mathrm{M}-1 \tag{26}$$

By solving equations $(20)\ -(26)$ and taking inverse Laplace transforms (using maple program) .We obtain the availability function as follows

$$A_Y(t)=L^{-1}\left(\sum_{i+j=0}^{M-1}P^*_{i,j}(s)\right)=\left(\sum_{i+j=0}^{M-1}P_{i,j}\ (t)\right) \tag{27}$$

Where i,j = 0, 1, 2, ..., M −1.

The steady-state availability probability can be obtained from the following relation.

$$A=\lim_{t\to\infty}A(t) \tag{28}$$

As we know that failed states contain units that failed [type1 or type2] so the reliability function of model $(20-26)$ has three cases

Case1: there aren't repair of failure of type1 at failed states.

Case2: there aren't repair of failure of type2 at failed states.

Case3: there aren't repair of failure of type1 and type 2 at failed states.

We study the reliability function in case3, let $P_{i,j}(t)\to\widehat{P_{i,j}}(t)$.the reliability function can be obtained by taking the inverse of Laplace transform as follows.

$$R_Y(t)=\ L^{-1}\left(\sum_{i+j=0}^{M-1}\widehat{P^*_{i,j}}(s)\right)=\left(\sum_{i+j=0}^{M-1}\widehat{P_{i,j}(t)}\right) \tag{29}$$

The mean time to failure *MTTF* can be obtained from the following relation.

$$MTTF=\lim_{s\to0}R^*_Y(s)=\lim_{s\to0}\left\{\sum_{i+j=0}^{M-1}\widehat{P^*_{i,j}}(s)\right\}=\left\{\sum_{i+j=0}^{M-1}\widehat{P^*_{i,j}}(0)\right\} \tag{30}$$

We perform a sensitivity analysis for changes in the reliability of the system $R_Y(t)$ from changes in system parameters $\lambda_1,\lambda_2,\mu_1$ and μ_2 .by differentiating equation (29) with respect to we obtain

$$\frac{\partial R_Y(t)}{\partial\lambda_1}=\frac{\partial}{\partial\lambda_1}\left\{\sum_{i+j=0}^{M-1}\widehat{P_{i,j}}(t)\right\}=\left\{\sum_{i+j=0}^{M-1}\frac{\partial}{\partial\lambda_1}\widehat{P_{i,j}}(t)\right\} \tag{31}$$

We use the same procedure to get $\frac{\partial R_Y(t)}{\partial\lambda_2},\frac{\partial R_Y(t)}{\partial\mu_1},\frac{\partial R_Y(t)}{\partial\mu_2}$.

3.3 Numerical results

In this section, we use MAPLE computer program to provide the numerical results of the effects of various parameters on system reliability and system availability. We choose λ_1

= 0.0007, λ_2= 0.001 and fix $\mu_1 = 0.05$, $\mu_2 = 0.03$. The following cases are analyzed graphically to study the effect of various parameters on system reliability.

Case 1: Fix $R_1 = 1, R_2 = 1, and\ choose\ M = 1,2,3.$

Case 2: Fix $M = 4, R_2 = 1, and\ choose\ R_1 = 1,2,3.$

Case 3: Fix $M = 4, R_1 = 1, and\ choose\ R_2 = 1,2,3.$

It can be observed from Fig.1-6 that the system reliability increases as M increases. It is also noticed from Figures 8-10 that R_1 and R_2 don't effect on system reliability $R_Y(t)$ when number of repairman more than one.

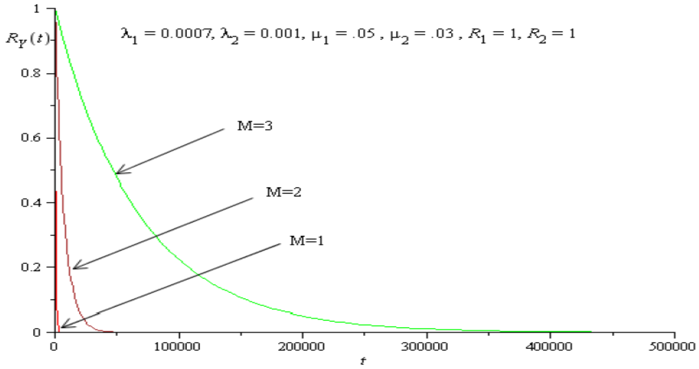

Figure 8. System reliability for different numbers of units

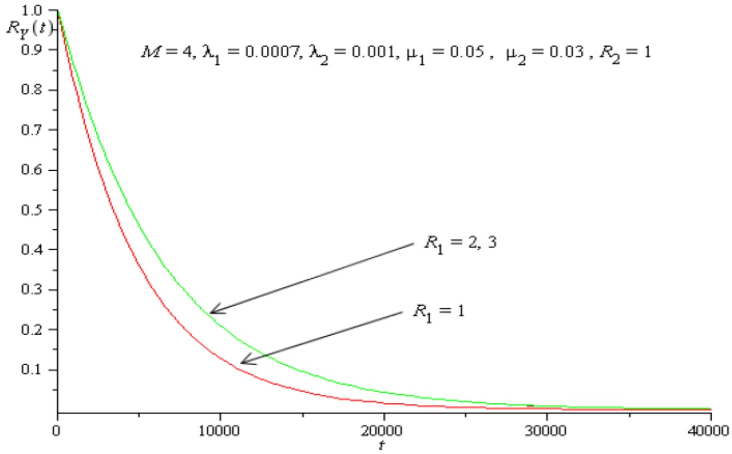

Figure 9. System reliability for different numbers of repairmen in first service line

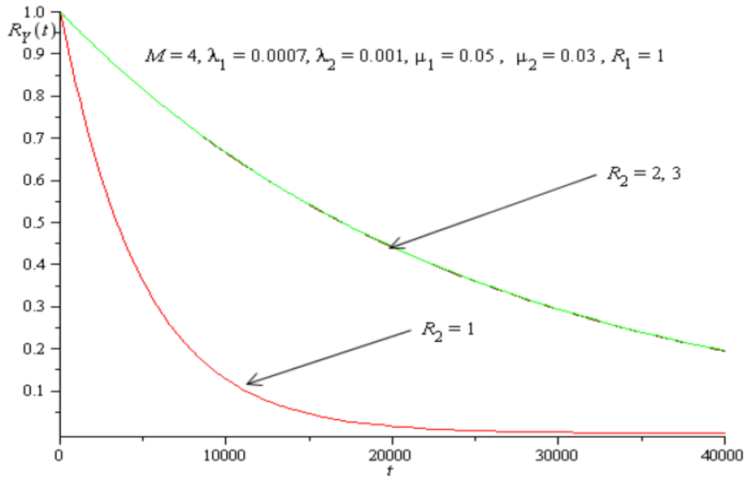

Figure 10. System reliability for different numbers of repairmen in second service line

Next, we study the cross effect of various parameters on *MTTF* . From Figure 11 and Figure 12, we find that the effect of λ_2 on the *MTTF* becomes more significant when λ_1 is smaller and the effect of λ_1 on the *MTTF* becomes more significant when λ_2 is smaller.

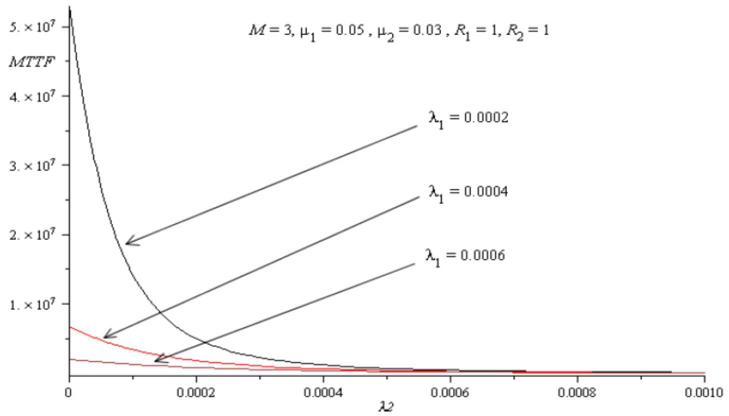

Figure 11. *MTTF* with changes in λ_1

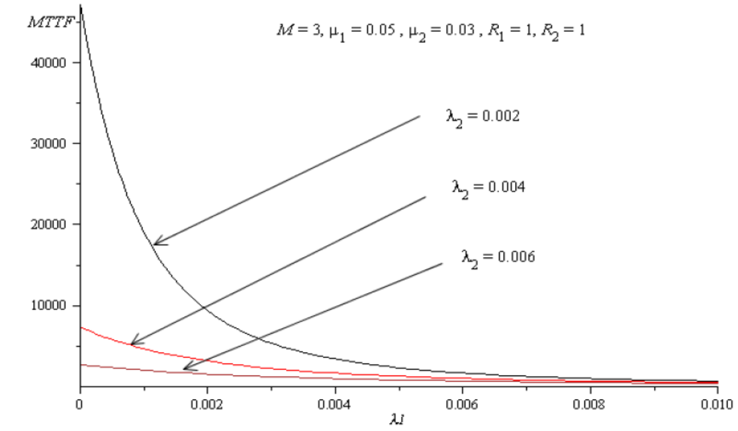

Figure 12. *MTTF* with changes in λ_2

Finally we perform sensitivity analysis for system reliability $R(t)$ with respect to system parameters $\lambda_1, \lambda_2, \mu_1$ and μ_2. In Figure 13 we can easily observe that the order of magnitude of the effect is ($\mu_2 \leq \lambda_1 \leq \lambda_2$) and the sensitivities of μ_2 are almost equal to zero.

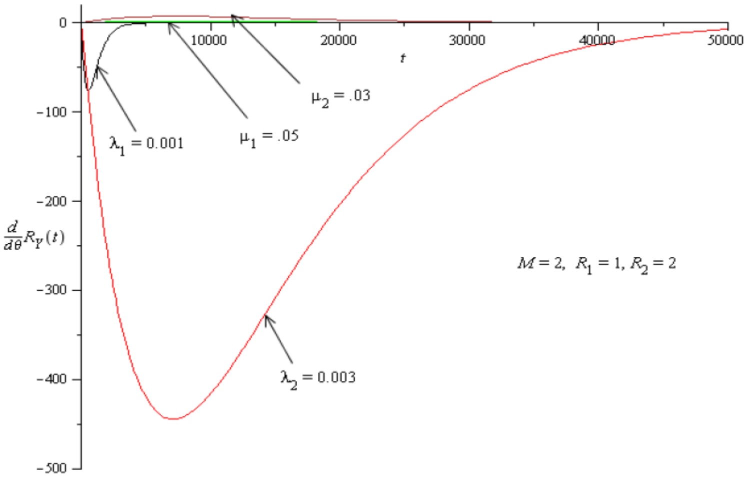

Figure 13. Sensitivity of system reliability with respect to system parameters

4. CONCLUSIONS

In this paper, a mathematical model was constructed for two systems with two types of failure. Availability, reliability and mean time to system failure for the system reliability were obtained and the results were shown graphically by the aid of MAPLE program. .

Results in first system indicate that the *MTTF* and sensitivity analysis for the system reliability depend on which of failed states [type1, type2] is absorbing, moreover in second system indicate that the reliability of the system reliability increases by increasing number of units in parallel system with two types of failures.

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