J. Soc. Korea Ind. Syst. Eng Vol. 38, No. 2 : 40-46, June 2015
http://dx.doi.org/10.11627/jkise.2015.38.2.40
ISSN : 2005-0461(print)
ISSN : 2287-7975(online)

An Approximation Approach for Solving a Continuous Review Inventory System Considering Service Cost

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서비스 비용을 고려한 연속적 재고관리시스템 해결을 위한 근사법

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The modular assembly system can make it possible for the variety of products to be assembled in a short lead time. In this system, necessary components are assembled to optional components tailor to customers' orders. Budget for inventory investments composed of inventory and purchasing costs are practically limited and the purchasing cost is often paid when an order is arrived. Service cost is assumed to be proportional to service level and it is included in budget constraint. We develop a heuristic procedure to find a good solution for a continuous review inventory system of the modular assembly system with a budget constraint. A regression analysis using a quadratic function based on the exponential function is applied to the cumulative density function of a normal distribution. With the regression result, an efficient heuristics is proposed by using an approximation for some complex functions that are composed of exponential functions only. A simple problem is introduced to illustrate the proposed heuristics.

Keywords : Continuous Review Inventory System, Supply Chain Management, Non-linear Optimization

1. Introduction

To respond customer's various orders quickly, the modular assembly system is an attractive way to make the variety of products in a short lead time. In the modular assembly system, a semi-finished product (or necessary component) is ready in downstream of the assembly line and it is assembled to optional components tailor to customer's order.

A continuous review inventory system has two decision variables, constant order size (Q) and reorder point (r). In this system, the inventory position keeps reviewed and places order Q when it reaches r. There are some researches on the continuous review inventory system with storage space restriction. Zhao et al. [10] provides an optimal solution procedure for a single item problem. Based on the procedure for a single item problem, they propose an undominated solution procedure for multi-items. Hariga [6] proposes an EOQ (Economic Order Quantity) based heuristic. This heuristic has 0.19% average cost difference from optimal cost by the sensitivity analysis. Zhao et al. [11] propose an optimal solution procedure using renewal process for a single item problem with a resource shortage cost. They suggest a near-optimal solution procedure for a multi-item problem. The storage space restriction constraint is similar to the budget constraint because the storage space limit can be considered as the budget limit. However, the researches on storage space restriction deal with a simple constraint : the total amount of required space for an inventory is just smaller than a storage space.

Received 23 March 2015; Finally Revised 22 April 2015;
Accepted 22 April 2015
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Bera et al.[1] study on a multi-item continuous review inventory problem with a budget constraint in which both demand and lead time are random. This problem is transformed into a min-max problem using fuzzy chance-programming technique, and then a pareto-optimal solution procedure is proposed.

A higher service level requires the cost increments of labor, facilities, and other related services. This service cost is proportional to the service level. For example, the service cost can be linearly dependent on the service level. Wang and Hu [8, 9] formulate continuous review inventory problem of one necessary and many optional components under the budget constraint and service cost. They find an optimal solution using Newton-Raphson and Hadley-Whitin methods. However, their approach is very complicated because it solves complex equations as a whole.

Lee and Lee [7] propose an efficient optimal solution approach based on the bisection search and the normalization of bivariate normal distribution for Wang and Hu's problem. They decoupled equations for necessary components with those of optional components with the help of normalization of demand distribution, which is a bivariate normal distribution. With this, the order quantity is eliminated and each equation for safety factor (z) is solved by the bisection search. This approach, however, still has a complex equation to be solved.

As an alternative, in this paper, we use a regression analysis to solve Wang and Hu's problem [8, 9]. Equations are composed of exponential functions, such as the loss integral, probability density function (PDF), and cumulative density function (CDF) of normal distribution. The existing regression analysis is to approximate the loss integral function only. However, the existing regression approximations are not efficient because they cannot be combined with other exponential functions in the equations.

To sidestep this difficulty, a quadratic function based on exponential functions is proposed to approximate CDF of a normal distribution. Since the approximated function is based on an exponential function, the approximated function can be combined with other exponential functions in the equations. As a result, a simple quadratic formula can be used to solve, at least approximately if not exactly, the equations.

When the purchasing costs for components are paid at the time an order is received, the total inventory investment and service cost are not supposed to be over the allowed budget limit. The following assumptions are used :

- (Q, r) inventory policy is applied for all components. That is, the inventory for each component is continuously reviewed and the amount of Q is ordered whenever the inventory level hits the reorder point r.
- Q and r are continuous variables.
- Customers can order a necessary component with one or more optional components. If a chosen optional component is not available, it is backlogged.
- The customer's willingness to buy a necessary component is not affected by the inventory status of the optional components. The joint probability distribution of X and Y_j is a bivariate normal distribution.

The rest of this paper is organized as follows. Section 2 presents notation and the problem formulation. Regression analysis using exponential function is done for some complex functions and then the proposed solution procedures are proposed in Section 3. Section 4 presents an illustrative example, and Section 5 concludes the study.

2. Model Formulation

The notations used for the model formulation are as follows :

A : fixed ordering cost,
h : yearly inventory holding cost per unit,
p : penalty cost for shortages per unit,
κ : service cost rate,
D : yearly expected demand,
C : unit price.

Other notations are as follows :

W: the maximum available budget limit,
Z : the random variable of the standard normal distribution, $N(0, 1)$,
γ : the smallest acceptable probability that total investment is within budget,
X : the random variable of the lead time demand for the necessary component v during the lead time demand, $X \sim N(\mu_v, \sigma_v^2)$
Y_j: the random variable of the lead time demand for the optional component j, $Y_j \sim N(\mu_j, \sigma_j^2)$

$L_v(r_v)$: the expected shortages for the necessary item *v*, the loss function, $L_v(r_v)=\int_{r_v}^{\infty}(x-r_v)f_x(x)dx$,

$L_{oj}(r_{oj})$: the expected shortages for the optional item *j*, the loss function,

$$L_{oj}(r_{oj})=\int_{r_{oj}}^{\infty}(y_j-r_{oj})f_{Y_j|X}(y_j|x=r_v)dy_j,$$

$f(x,y_j)$: the joint probability distribution function of *X* and Y_j, a bivariate normal distribution,

$$f(x,y_j)=f_x(x)\times f_{Y_j|X}(y_j;\mu,\Sigma_j|x)=$$
$$\frac{1}{\sqrt{2\pi}\sigma_v}exp\left\{-\frac{1}{2\sigma_v^2}(x-\mu_v)^2\right\}\times\frac{1}{\sqrt{2\pi}\sigma_j\sqrt{1-\rho j_j^2}}$$
$$\times\exp\left\{-\frac{1}{2\sigma_j^2(1-\rho_j^2)}\left(y_j-\left(\mu_j+\rho_j\frac{\sigma_j}{\sigma_v}(x-\mu_v)\right)\right)^2\right\},$$

where $\mu=(\mu_0,\mu_j)$, $\Sigma_j=\begin{bmatrix}\sigma_v^2 & \sigma_{oj}\\ \sigma_{oj} & \sigma_j^2\end{bmatrix}$, $\rho_j=\frac{\sigma_{oj}}{\sigma_v\sigma_j}$(the correlation coefficient between *X* and Y_j).

$F(x)$: the cumulative density function of *X*, $F(x)=\int_0^x f(x)dx$.

Note that subscript *v* stands for the necessary component and *oj* for the *j*-th optional component.

The decision variables are the order quantity *Q* and reorder point *r* for necessary component and optional components. With the assumption, the problem (F1) can be formulated as follows :

$$\mathrm{F1:Min}\quad OBJ(Q_v,r_v,Q_{oj},r_{oj})=A_v\frac{D_v}{Q_v}+C_vD_v+$$
$$h_v\left(\frac{Q_v}{2}+r_v-\mu_v\right)+p_v\frac{D_vL_v(r_v)}{Q_v}+\Sigma_j\left[A_{oj}\frac{D_{oj}}{Q_{oj}}+C_{oj}D_{oj}+\right.$$
$$\left.h_{oj}\left(\frac{Q_{oj}}{2}+r_{oj}-\mu_{oj}\right)+p_{oj}\frac{D_{oj}L_{oj}(r_{oj})}{Q_{oj}}\right]\tag{1}$$

subject to

$$P(C_v(r_v-X+Q_v)+\Sigma_jC_k(r_{oj}-Y_{oj}+Q_{oj})+$$
$$\kappa_vF_x(r_v)+\kappa_{oj}F_{Y_j|X}(r_{oj}|x=r_v)\le W)\ge\gamma\tag{2}$$

$$Q_v,r_v,Q_{oj},r_{oj}\ge 0,\ \forall j\tag{3}$$

Equation (1) is the objective function that minimizes annual inventory cost, holding cost, shortage cost, and purchasing cost. Since Eq. (2) is the budget constraint, the sum of the purchasing cost and the service cost should not be over the allowed budget. Equation (3) is the nonnegative constraints for each decision variable. By using the normal probability distribution, the constraint of Eq. (2) can be rewritten as

$$C_v(Q_v+r_v)+\Sigma_jC_{oj}(Q_{oj}+r_{oj})+\kappa_vF_X(r_v)+$$
$$\kappa_{oj}F_{Y_j|X}(r_{oj}|x=r_v)\le W+\mu_Y+z_{1-\gamma}\sigma_Y\tag{4}$$

Here, by letting $Y=C_vX+\Sigma_jC_{oj}Y_j$, then $Y\sim N(\mu_Y,\sigma_Y^2)$, where $\mu_Y=C_v\mu_v+\Sigma_jC_{oj}\mu_{oj}$, $\sigma_Y^2=C_v^2\sigma_v^2+\Sigma_jC_{oj}^2\sigma_{oj}^2$, and $\mu_{oj}=\mu_j+\rho_j\frac{\sigma_j}{\sigma_v}(r_v-\mu_v)$, $\sigma_{oj}^2=\sigma_j^2(1-\rho_j^2)$.

3. Solution Methodology

The Lagrangian function for the problem, F1, is given as

$$J(Q_v,r_v,Q_{oj},r_{oj},\lambda)=$$
$$OBJ(Q_v,r_v,Q_{oj},r_{oj})+\lambda(C_v(Q_v+r_v)+\Sigma_jC_{oj}(Q_{oj}+r_{oj})+$$
$$\kappa_vF_X(r_v)+\kappa_{oj}F_{Y_j|X}(r_{oj}|x=r_v)-W-\mu_Y-z_{1-\gamma}\sigma_Y).$$

By letting $G(z)$ = 1-$F(z)$ and equating the partial derivatives of Lagrange function $J(Q_v,r_v,Q_{oj},r_{oj},\lambda)$ with respect to $Q_v,r_v,Q_{oj},r_{oj},\lambda$ to zero yields

$$\frac{\partial_J}{\partial Q_{oj}}=-\frac{A_{oj}D_{oj}}{Q_{oj}^2}+\frac{h_{oj}}{2}-\frac{p_{oj}D_{oj}}{Q_{oj}^2}\sigma_j\sqrt{1-\rho_j^2}L(z_{oj})\tag{5}$$
$$+\lambda C_{oj}=0$$

$$\frac{\partial_J}{\partial r_{oj}}=h_{oj}-\frac{p_{oj}D_{oj}}{Q_{oj}}G(z_{oj})+\lambda C_{oj}$$
$$+\lambda\kappa_{oj}\frac{1}{\sqrt{2\pi}\sigma_j\sqrt{1-\rho_j^2}}exp\left(-\frac{z_{oj}^2}{2}\right)=0\tag{6}$$

$$\frac{\partial_J}{\partial Q_v}=-\frac{A_vD_v}{Q_v^2}+\frac{h_v}{2}-\frac{p_vD_v}{Q_v^2}\sigma_vL(z_v)+\lambda C_v=0\tag{7}$$

$$\frac{\partial_J}{\partial r_v}=h_v-\frac{p_vD_v}{Q_v}G(z_v)+\lambda C_v$$
$$+\lambda\kappa_v\frac{1}{\sqrt{2\pi}\sigma_v}exp\left(-\frac{z_v^2}{2}\right)=0\tag{8}$$

$$C_v(Q_v+r_v)+\Sigma_jC_{oj}(Q_{oj}+r_{oj})+\kappa_vF_X(z_v)+$$
$$\kappa_{oj}F_{Y_j|X}(z_{oj})-W-\mu_Y-z_{1-\gamma}\sigma_Y=0\tag{9}$$

Rearranging (5) and (7) yields

$$Q_{oj} = \sqrt{\frac{2D_{oj}[A_{oj} + p_{oj}\sigma_j\sqrt{1-\rho_j^2}\,L(z_{oj})]}{h_{oj} + 2\lambda C_{oj}}}, \quad (10)$$

$$Q_v = \sqrt{\frac{2D_v[A_v + p_v\sigma_v L(z_v)]}{h_v + 2\lambda C_v}}. \quad (11)$$

There are some researches on approximation for the normalized loss integral $L(z)$, a type of the integral of exponential function. The normalized loss integral is approximated to get a solution in a more efficient way. Herron [5] and Byrkett [2] use exponential approximation as follows :

$$L(z) = ae^{-bz},$$

where a = 0.4400, b = 0.5760 if $0 \le z \le 1$ and a = 1.5792, b = 2.6879 if $1 \le z \le 3$.

Das [3, 4] uses quadratic approximation as follows :

$$L(z) = a[G(Z)]^2 + bG(z) + c,$$

where a = 0.79838, b = 0.39694, c = -0.0000044 if $0 \le z \le 2.57$ Das [3] and a = 0.98265, b = 0.37377, c = 0.0000055 if $1 \le z \le 3$ Das [4]

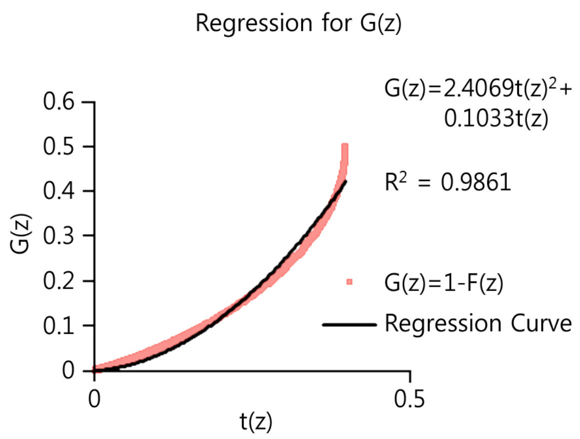
Based on the experiment using the data set of 56 inventory items from two empirical inventory system, the percentage increase in total costs including the holding cost, the ordering cost, and the shortage cost for the exponential approximation and quadratic approximation is less than 1.5% ,0.1%, respectively [3]. The quadratic approximation looks more accurate than the exponential approximation.

Since the service cost is considered in the budget constraint of this paper, Das's approximation is not proper. Equations (6) and (8) have $G(z)$, $L(z)$ as well as the exponential function of $\exp\left(-\frac{z^2}{2}\right)$. Therefore, the quadratic function of $\exp\left(-\frac{z^2}{2}\right)$ will be better for the approximation of $G(z)$. $G(z)$ can be approximated as follows :

$$G(z) \cong a_1 t(z)^2 + b_1 t(z), \quad (12)$$

$$\text{where } t(z) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{z^2}{2}\right).$$

<Figure 1> shows that regression curve that has R^2 = 0.9861.



<Figure 1> Regression Analysis for $G(z)$

The proposed approximation model fits well except when z ($t(z)$) approaches to 0 (0.5). That is, at z = 0, the real value of $G(z)$ is 0.5 and the approximated value by the regression function is 0.4243. Hence, other approximation model is needed when z approaches to 0. Approximated line crosses the real value at $t(z$ = 0.26) = 0.3857 and $t(z$ = 1.12) = 0.21. Out of two possibilities, $t(z$ = 0.26) = 0.3857 is selected because z = 0.26 is close to z = 0. Therefore, linear approximation is used at $0 \le z \le 0.26$.

With the above discussion, the proposed approximation is

$$G(z) \cong 2.4069t(z)^2 + 0.1033t(z)$$

$$\text{if } 0.26 \le z \le 3;$$

$$G(z) \cong 7.7382t(z) - 2.5871$$

$$\text{if } 0 \le z \le 0.26;$$

Inserting $G(z) \cong a_1 t(z)^2 + b_1 t(z)$ into Eq. (6) and rearranging it, we have

$$\frac{p_{oj}D_{oj}}{Q_{oj}} a_1 t(z_{oj})^2 - \left(\lambda\kappa_{oj}\frac{1}{o_j\sqrt{1-\rho_j^2}} - \frac{p_{oj}D_{oj}}{Q_{oj}} b_1\right) t(z_{oj}) - (h_{oj} + \lambda C_{oj}) = 0 \quad (13)$$

By letting $\beta_1 = \frac{p_{oj}D_{oj}}{Q_{oj}} a_1$, $\beta_2 = \lambda\kappa_{oj}\frac{1}{\sigma_j\sqrt{1-\rho_j^2}} - \frac{p_{oj}D_{oj}}{Q_{oj}} b_1$, $\beta_3 = h_{oj} + \lambda C_{oj}$, the solution for Eq. (13) will be

$$t(z_{oj}) = \frac{\beta_2 \pm \sqrt{\beta_2^2 + 4\beta_1\beta_3}}{2\beta_1}$$

Note that $\beta_1 > 0$ and $\beta_3 > 0$. The left term of the above equation, $t(z)$, should be larger than or equal to 0 because $t(z_{oj}) = \exp\left(-\frac{z_{oj}^2}{2}\right)$ cannot have negative value. Therefore, the right term should be larger than or equal to 0. In addition, $\beta_2^2 + 4\beta_1\beta_3 > 0$ is always true because $\beta_2^2 > 0$ and $4\beta_1\beta_3 \geq 0$. Hence, there exist solutions for the above equation. Further, $\beta_2 - \sqrt{\beta_2^2 + 4\beta_1\beta_3} < 0$ is always true for the following reason. If $\beta_2 \geq 0$, then $\sqrt{\beta_2^2 + 4\beta_1\beta_3} > \beta_2$. If $\beta_2 < 0$, then $-\sqrt{\beta_2^2 + 4\beta_1\beta_3} < 0$. However, $\frac{\beta_2 - \sqrt{\beta_2^2 + 4\beta_1\beta_3}}{2\beta_1}$ cannot be a solution for Eq. (13). Therefore, the solution for Eq. (14) is

$$t(z_{oj}) = \frac{\beta_2 + \sqrt{\beta_2^2 + 4\beta_1\beta_3}}{2\beta_1} \tag{14}$$

where $z_{oj} = \frac{r_{oj} - B}{A}$, $A \equiv \sigma_j\sqrt{1-\rho_j^2}$, $B \equiv \mu_j + \rho_j\frac{\sigma_j}{\sigma_v}(r_v - \mu_v)$

Inserting $G_0(z_v) \cong a_1 t(z_v)^2 + b_1 t(z_v)$ into Eq. (8) and rearranging it in terms of $t(z_v)$ yields

$$\frac{p_v D_v}{Q_v} a_1 t(z_v)^2 - \left(\lambda\kappa_v \frac{1}{\sigma_v} - \frac{p_v D_v}{Q_v} b_1\right) t(z_{oj}). \\ -(h_v + \lambda C_v) = 0 \tag{15}$$

By letting $\alpha_1 = \frac{p_v D_v}{Q_v} a_1$, $\alpha_2 = \lambda\kappa_v \frac{1}{\sigma_v} - \frac{p_v D_v}{Q_v} b_1$, $\alpha_3 = (h_v + \lambda C_v)$, the solution for Eq. (15) is

$$t(z_v) = \frac{\alpha_2 \pm \sqrt{\alpha_2^2 + 4\alpha_1\alpha_3}}{2\alpha_1},$$

where $z_v = \frac{r_v - \mu_v}{\sigma_v}$.

By the same token, $t(z_v) = \frac{\alpha_2 - \sqrt{\alpha_2^2 + 4\alpha_1\alpha_3}}{2\alpha_1}$ cannot be a solution. Therefore,

$$t(z_v) = \frac{\alpha_2 - \sqrt{\alpha_2^2 + 4\alpha_1\alpha_3}}{2\alpha_1}. \tag{16}$$

With all these, we propose the solving algorithm as follows :

Step 1 : Find λ_1, λ_2 such that $g(\lambda_1) > 0, g(\lambda_2) < 0$.

Step 2 : For each λ_1, λ_2,

a) compute $Q_{oj} = \sqrt{\frac{2D_{oj}A_{oj}}{h_{oj}}}$. Find z_{oj} from (14). Insert z_{oj} into (10), and compute Q_{oj}. Repeat this until Q_{oj} and z_{oj} converge.

b) compute $Q_v = \sqrt{\frac{2D_v A_v}{h_v}}$. Find z_v from (16). Insert z_v into (11), and compute Q_v. Repeat this until Q_v and z_v converge.

Step 3 : Let $\lambda_{new} = \frac{\lambda_1 + \lambda_2}{2}$ and find Q_v, r_v, Q_{oj}, r_{oj} from Step 2. If $g(\lambda_{new}) > 0$, then $\lambda_1 = \lambda_{new}$; otherwise, $\lambda_2 = \lambda_{new}$.

Step 4 : If $(g(\lambda_1), -g(\lambda_2)) < \epsilon$, where ϵ is a predetermined small value, then stop; otherwise return to Step 3.

4. An Illustrative Example

A simple example with one necessary component and two optional components [1] is given to show the result of the proposed algorithm. Parameters for the examples are listed in <Table 1>.

<Table 1> Data for Numerical Example

	Necessary	Optional
A	700	40, 20
C	150	3,2
D	10000	4000, 6000
h	6	0.7, 0.4
p	8	1.0, 0.7
ρ	-	0.5,0.8
μ	300	100,170
σ	40	15,20
κ	4000	200,150

Initial solutions for the example are shown in <Table 2>. Let W = 150,000, $\epsilon = W \times 0.3\% = 450$, $\gamma = 0.9031$ from Eq. (2). The budget constraint is violated when λ = 0 as $g(\lambda) \geq 0$, whereas the constraint is satisfied when λ = 0.0625 as $g(\lambda) < 0$.

<Table 2> λ, T_1, T_2, $g(\lambda)$, and Total Cost by Iteration

Iteration	λ	Q_v, r_v	Q_{01}, r_{01}	Q_{02}, r_{02}	$g(\lambda)$	Total Cost
1	0.0000	1547.8,347.1	681.8,124.0	778.3,205.4	104555.3	1534500.2
2	0.0625	765.7,340.8	551.5,121.2	611.4,201.4	-14582.6	1536844.2
3	0.0313	969.1,344.0	606.2,122.6	679.9,203.4	16776.2	1535433.9
4	0.0469	849.4,342.3	576.9,121.9	642.9,202.4	-1626.5	1536141.0
5	0.0391	903.3,343.1	590.9,122.2	660.6,202.9	6674.4	1535786.0
6	0.0429	875.3,342.7	583.8,122.0	655.9,202.6	2375.7	1535964..2
7	0.0449	862.1,342.5	580.3,121.9	647.2,202.5	332.3	1536053.8
8	0.0459	855.7,342.4	578.5,121.9	645.0,202.4	-658.0	1536097.4
9	0.0454	858.9,342.5	579.5,121.9	646.1,202.5	-165.2	1536075.6

<Table 3> Solutions by the Proposed Heuristics and Wang and Hu's Algorithm

	Proposed Heuristics		Wang and Hu	
	Necessary	Optional	Necessary	Optional
Q	858.9	579.5,646.1	862.3	579.6,647.5
r	342.5	121.9,202.5	340.3	122.8,203.4
λ	.0454		.0450	
Total Cost	1536075.6		1536061.0	

Values of variables and total cost for 9 iterations are shown in <Table 2>. The bisection search is used to find an optimal λ which makes g(λ) close to 0. The solutions by the proposed heuristics and Wang and Hu's algorithm is shown in <Table 3> when $\epsilon = 100$. The solution by the proposed heuristics is close to the solution by Wang and Hu's algorithm.

5. Conclusions

The modular assembly system is an attractive system in that the system produces various products in a short lead time. In this paper, we present a heuristic procedure to find a near optimal solution for continuous review inventory system for modular assembly system with a budget constraint. A higher service level requires the increment of costs of labor, facilities, and other related services. The service cost is assumed to be proportional to the service level and this cost is included in the budget constraint.

In order to find a solution for this problem, complex equations are required to solve at the same time. A regression analysis using a quadratic function based on exponential function makes these equations quadratic equations and provides a simple solution approach.

The advantages of the proposed method are as follows :

- The proposed method is proper for the problem contains exponential functions. The existing approximating method for the loss integral, L(z), cannot be used for this problem. The service cost assumed to be proportional to the service level is considered in this research and the service level is represented by CDF of a normal distribution. The differentiation of the service cost generates an exponential function.
- The proposed method is simple and efficient. The closed form of solution cannot be found for equations (6) and (8). Approximation of G(z) by the exponential function of z generates a quadratic equation. Usually, the quadratic equation has two solutions and we show that only one solution exist in this problem. By using an illustrative example, the proposed method finds a near optimal solution.

In an illustrative example, we compare the solution by the proposed heuristic with optimal solution by Wang and Hu's approach. The values of decision variables such as order quantity and reorder points of necessary component and optional components are very close and total costs are only 0.001% different.

Acknowledgement

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (NRF-2010-0022163).

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