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<Research Letter>

Optimality of the Sole Sourcing under Random Yield

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불확실한 수율하에서 단일소싱의 최적성

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Though the supplier diversification is considered as a vital tool to mitigate the risk due to supply chain disruptions, there are results which show the optimality of the sole sourcing. This paper further generalizes the results to show that the sole sourcing is optimal under very mild conditions. Discussion on why the sole sourcing is optimal is given with the insight on the value of supplier diversification.

Keywords: Supplier diversification, Random yield, EOQ, Inventory control

1. Introduction

Supplier diversification has been advocated by many researchers as a vital tool to mitigate the supply risks due to unexpected supply chain disruptions. The random yield has been used to model the situation where the received number of (non-defective) units is random and so it cannot be known when the order is issued (Yano and Lee, 1995). If the supplier's yield is random, it seems quite plausible to resort to the multiple sourcing since it can lead to the reduction in the variability of the received quantities.

However, when an EOQ-like inventory control system is used, Fadıloğlu *et al*. (2008) show that the sole sourcing is optimal when the supplier's yield is binomial. The result is generalized to include the cases with more general cost structure by Tajbakhsh *et al*. (2010), and Yan and Wang (2013) give a further generalization which shows that the sole sourcing is optimal under general random yield.

Those results raise a fundamental question : When and under what conditions is the sole sourcing an effective choice? The answer to the question is intimately related to the value of supplier diversification. In particular, the drivers which make the supplier diversification effectual should be carefully evaluated.

The key factors which should be considered when the sourcing decision is made include the following:

(1) Pooling effects
- Variance pooling effect: the variance of the actual yield decreases due to the negative correlations among the suppliers' yields.
- Fixed-cost pooling effect: the fixed cost incurred when using multiple suppliers is less than the sum of the fixed costs charged by each supplier.

(2) Economies of scale
- Economy of scale in the procurement cost : the marginal procurement cost is nonincreasing in the ordering quantity (so the ordering cost function is concave).
- Economy of scale in the random yield : the mean yield is a convex function and the variance of the yield is a concave function in the ordering quantity, respectively.

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Certainly, the above list does not subsume all the factors relevant to the supplier diversification. For example, the capacity of the supplier clearly does matter. The list contains only the elements which are pertinent to the main results given in this paper.

Pooling effects are supportive of the supplier diversification, while economies of scale advocate the sole sourcing. Since economies of scale in the random yield is a new concept, it deserves more explanation. The convexity of the mean yield means that the expected marginal yield is nondecreasing in the order quantity. Similarly, the concavity of the variance of the random yield means that its marginal variance is nonincreasing. As a whole, economies of scale in the random yield is almost equivalent to the improvement of the quality in the supplier's production process as the lot size increases. This may come from the learning effect, the incentive for the supplier to invest in the production process, and the development of close supplier-buyer relationship, to name a few.

In terms of the above-mentioned factors, the results given in Yan and Wang (2013) can be summarized as follows. When there are no pooling effects (variance pooling and fixed-cost pooling), the sole sourcing is optimal and it is also optimal in the other cases where the variance of the yield is a special type of concave functions of the order quantity. They assume that the procurement cost is the sum of the fixed cost and the variable cost which is linear in the order quantity. Though their results are quite general, further generalizations can be made. Specifically, this paper proves the following :

(1) If there is no fixed-cost pooling effect, the sole sourcing is always optimal irrespective of the variance pooling effect and the ordering cost structure. Especially, it is optimal for any type of ordering cost functions even though the yields are negatively correlated.
(2) When the ordering cost function and the random yield exhibit economies of scale (that is, economies of scale prevail), the sole sourcing is optimal if there is no variance pooling effect.

The results are proper generalizations of the previous results. Also the proof of the results is much simpler and elegant, which is helpful to gain an insight on the reason why the sole sourcing is optimal. Managerial implications of the results are discussed, which can serve as a guideline when making sourcing decisions.

The random yield has been one of the major issues in operations management. The following is only a partial list of the previous researches related to the random yield. A comprehensive survey on the lot sizing problem under random yield can be found in Yano and Lee (1995). Henig and Gerchak (1990) consider the periodic review system under random yield. Anupindi and Akella (1993), Gerchak and Parlar (1990), Gürler and Parlar (1997), and Parlar and Wang (1993) consider the problem of allocating orders to two unreliable suppliers. Surveys on the multiple supplier inventory models can be found in Minner (2003) and Tajbakhsh *et al*. (2007). For more recent results, see Federgruen and Yang (2011) and Feng (2010), where many references on the topic can be found.

The rest of the paper is structured as follows. In Section 2, we describe the model. The optimality of the sole sourcing when there is no fixed-cost pooling effect is proved in Section 3. Section 4 proves the optimality of the sole sourcing when economies of scale prevail. Finally, Section 5 closes the paper with discussion on the results.

2. Model Description

Let λ be a constant demand rate and h be a unit inventory holding cost charged when we carry a unit of item for a unit time period. An EOQ-type inventory control scheme is used where an order is placed and received immediately upon the depletion of the stock hold.

There are n suppliers each of which differs in its cost structure and its yield characteristic. The yield (number of good units received) is a random variable which depends on the order quantity. For each supplier $i \in N$, let $R_i(y_i)$ be the random variable denoting the yield of supplier i when an order of size $y_i \geq 0$ is placed on the supplier, where N is the set of n suppliers. Let the mean and the variance of the supplier i's yield (both as functions of the order size) be denoted as $\mu_i(y_i)$ and $v_i(y_i)$, respectively, for each $i \in N$.

The ordering cost function when we order exclusively from a subset S of suppliers is given by

$$C(\mathbf{y}|S) = K(S) + \sum_{i \in S} D_i(y_i) \tag{1}$$

where $K(S)$ is the fixed cost incurred when we order from the suppliers in S. The structure of the fixed cost $K(S)$ is completely arbitrary except the requirement that $K_i = K(\{i\}) \leq K(S)$ for all $i \in S$. The function $D_i : R_+ \rightarrow R_+$ is the (individual) ordering cost function corresponding to each supplier $i \in N$, each of which specific form is also completely arbitrary unless stated otherwise. When there is no fixed-cost pooling effect, we assume $K(S) = 0$ in the ordering cost function given in (1), so the total ordering cost is separable.

The expected cycle length (CL) and the expected cycle cost (CC) when we order exclusively from the suppliers in the set S (that is, $y_i > 0$, for only $i \in S$) are as follows :

$$CL(\mathbf{y}|S) = \sum_{i \in S} \mu_i(y_i)/\lambda \tag{2}$$

$$CC(\mathbf{y}|S) = K(S) + \sum_{i \in S} D_i(y_i) + (h/2\lambda) E[(\sum_{i \in S} R_i(y_i))^2] \tag{3}$$

where $\mathbf{y}$ is an $|S|$-dimensional vector each of which component is the order quantity $y_i > 0$, $i \in S$.

By the renewal-reward theorem, the corresponding expected cost rate (CR) is

$$CR(\mathbf{y}|S) = \frac{CC(\mathbf{y}|S)}{CL(\mathbf{y}|S)} \tag{4}$$

Then the problem of choosing the subset of suppliers and the corresponding order quantities can be formulated as follows:

$$\min_{S \subseteq N} \min_{y_i \geq 0,\, i \in S} CR(\mathbf{y}|S) \tag{5}$$

Remark 1 : In the above formulation, we assume that the payment is made for the units ordered. This assumption is made for simplicity of presentation. Later, we show that the results hold under more general payment schemes.

3. The Optimality of Sole Sourcing: No Fixed-Cost Pooling Effect

In this section, we prove the optimality of sole sourcing when there is no fixed-cost pooling effect. Recall that in this case we assume $K(S) = 0$ for each subset S of N. No specific form of the ordering cost function is assumed and the yields of the suppliers can be negatively correlated.

By expanding the expectation in (3), we have the following :

$$\begin{aligned} \mathrm{E}(\textstyle\sum_{i \in N} R_i(y_i))^2 = {} & \textstyle\sum_{i \in N} \mathrm{E}R_i(y_i)^2 \\ & + \textstyle\sum_{i \neq j \in N} \mathrm{E}R_i(y_i) R_j(y_j) \end{aligned} \tag{6}$$

Note that since $R_i(y_i)$, $i \in N$ is a nonnegative random variable for all $y_i \geq 0$, $\mathrm{E}[R_i(y_i) R_j(y_j)] \geq 0$, for all y_i, $y_j \geq 0$, i, $j \in N$. Then, the problem of finding a subset of suppliers and the order quantities (5) can be rewritten as follows :

$$\begin{aligned} \min_{y_i \geq 0,\, i \in N} \{ & \lambda \textstyle\sum_{i \in N} D_i(y_i) + (h/2) \\ & E(\textstyle\sum_{i \in N} R_i(y_i))^2 \} / \textstyle\sum_{i \in N} \mu_i(y_i) \end{aligned} \tag{7}$$

By using (6), it is easy to see that the following problem (8) gives a lower bound to the problem (7) :

$$\begin{aligned} \min_{y_i \geq 0,\, i \in N} \textstyle\sum_{i \in N} \{ & \lambda D_i(y_i) + (h/2) \\ & ER_i(y_i)^2 / (\textstyle\sum_{i \in N} \mu_i(y_i) \end{aligned} \tag{8}$$

Note that the numerator in (8) is a separable function of the variables $y_i \geq 0$, $i \in N$. Also note that if an optimal solution to (8) turns out to the ordering from a single supplier, then using that supplier only is also optimal to the original problem (7).

Before proving the optimality of the sole sourcing, we need the following lemma used in Yan and Wang (2013).

Lemma 1 : Let $a_i, b_i > 0$, $i \in S$, and $a_t/b_t = \min_{i \in S} a_i/b_i$, where S is a nonempty set of indices. Then $\sum_{i \in S} a_i / \sum_{i \in S} b_i \geq a_t/b_t$.

Using the Lemma 1, we can prove that the sole sourcing is always optimal when there is no fixed-cost pooling effect.

Theorem 1 : Sole sourcing is optimal whenever there is no fixed-cost pooling effect.

Proof : Let an optimal solution to the problem (7) be $y_i > 0$, for $i \in S \subseteq N$ and $y_i = 0$, otherwise. Without loss of generality, we assume the set S has at least two elements. Then by using (6) and Lemma 1 (and also the previous discussion),

$$\begin{aligned} & \{\lambda \textstyle\sum_{i \in S} D_i(y_i) + (h/2)\mathrm{E}(\sum_{i \in S} R_i(y_i))^2\} / \textstyle\sum_{i \in S} \mu_i(y_i) \\ & \geq \textstyle\sum_{i \in S} \{\lambda D_i(y_i) + (h/2)\mathrm{E}R_i(y_i)^2\} / \textstyle\sum_{i \in S} \mu_i(y_i) \\ & \geq \min_{i \in S} \{\lambda D_i(y_i) + (h/2)\mathrm{E}R_i(y_i)^2\} / \mu_i(y_i) \end{aligned}$$

Hence if we let $t = \arg min_{i \in S} \{\lambda D_i(y_i) + (h/2)\mathrm{E}R_i(y_i)^2\} / \mu_i(y_i)$, then ordering only from the supplier t is also optimal, which completes the proof. □

Note that the above Theorem 1 does not imply that the multiple sourcing is never optimal. There may be an alternative optimal solution where multiple sourcing is used.

When the sole sourcing strategy is adopted, the supplier which should be chosen can be characterized as in the following corollary.

Corollary 1 : It is optimal to choose any supplier $t \in N$ which satisfies the following :

$$\begin{aligned} t = \arg min_{i \in N} \{ & \min_{y_i \geq 0} \{\lambda D_i(y_i) \\ & + (h/2)\mathrm{E}R_i(y_i)^2\} / \mu_i(y_i)] \end{aligned} \tag{9}$$

Proof : By Theorem 1, we know that it is sufficient to consider the sole sourcing only. Hence it is optimal to choose a supplier which gives the minimum cost rate among the suppliers. □

Example : One of the most widely studied cases is the binomial random yield (Fadıloğlu *et al*., 2008; Tajbakhsh *et al*., 2010). In this case, the random yield follows a binomial distribution with parameter p which denotes the probability of a unit being good. When the order size is y, the mean and the variance of the random yield are py and pqy, respectively, where q =1-p. First, consider the case where there is a single supplier. Let us assume that the ordering cost is given by K+cy, for some positive numbers K and c. Then the cost rate when the order size is $y > 0$ is as follows :

$$(\lambda K/p)/y+(hp/2)y+\lambda c/p+hq/2 \quad (10)$$

Note in (10), the cost parameters are adjusted to reflect the probability of a unit of item being good (p). Since (10) is of the same form as the cost function in the usual EOQ case, it is easy to see that the optimal order quantity is $(1/p)\sqrt{2\lambda K/h}$ and the corresponding minimum cost rate is $\lambda c/p+hq/2+\sqrt{2\lambda Kh}$. Hence if there are n suppliers, it is optimal to choose the supplier t such that $t=\arg min_{i\in N}[\lambda c_i/p_i+h(1-p_i)/2+\sqrt{2\lambda K_i h}]$.

Yan and Wang (2013) show that sole sourcing is optimal if there are nonnegative correlations among the random yields and the cost function is of the form $D_i(y_i)=K_i+c_i y_i$, where K_i and c_i are positive fixed cost and variable cost, respectively, for $i\in N$. Theorem 1 states that sole sourcing is always optimal *even if there is a negative correlation between random yields and the form of the ordering cost functions is arbitrary*. Hence Theorem 1 gives more general result than that given in Yan and Wang (2013).

Remark 2 : If there is a fixed-cost pooling effect, the optimality of sole sourcing cannot be guaranteed in general. Such an example can be found in Yan and Wang (2013).

Now we consider the case where payment is made for good units only. In this case, the procurement cost in itself is a random variable. However, by replacing $D_i(y_i)$ with $\mathrm{E}D_i(R_i(y_i))$ in the proof of Theorem 1, we can easily conclude that the single sourcing is still optimal.

Corollary 2 : Sole sourcing is optimal even when the payment is made for good units only.

Remark 3 : Federgruen and Yang (2011) considers the case where the expected price paid to a supplier is given by $c^o y+c^e\mu(y)$ when y units of an item are ordered. This type of cost structure includes, as special cases, settings where the payment is made for every ordered items ($c^e=0$) and those where the payment is made for good items only ($c^o=0$). It is easy to see that Corollary 2 also holds in this case.

4. The Optimality of the Sole Sourcing : Economies of Scale

Now we consider the case where the ordering cost as well as the random yield exhibits the economies of scale when there is no variance-pooling effect. We present the result under the assumption that the random yields are pairwise uncorrelated. The result can be easily generalized to the case where there are nonnegative correlations among the random yields. The functions $D_i: R_+\to R_+$, $i\in N$, are nondecreasing concave functions in the order quantity. Also the mean yield $\mu_i(\cdot)$ (the variance of the yield $v_i(\cdot)$) is a nondecreasing convex (concave) function of the order quantity, for all $i\in N$.

Let us define for each supplier $i\in N$, the function $\eta_i: R_+\to R_+$ as follows :

$$\eta_i(x_i)=\min\{y_i|\mu_i(y_i)\geq x_i\} \quad (11)$$

Those functions are well defined since each function $\mu_i(y_i)$ is convex (so it is continuous in the interior of its domain), for each $i\in N$. One can interpret the value $\eta_i(x_i)$ as the minimum number of units which should be ordered to obtain the long-run average number x_i of good units, for $x_i\geq 0$, $i\in N$. If each function $\mu_i(y_i)$ is strictly increasing, then $\eta_i(x_i)$ is nothing but its inverse function, for $i\in N$.

To prove the main result of this section, we need the following lemma.

Lemma 2 : If a function $\mu: R_+\to R$ is convex and nondecreasing, then the function $\eta: R\to R_+$, defined by $\eta(x)=\min\{y|\mu(y)\geq x\}$, is concave and nondecreasing.

Proof : Let x and z be real numbers, and choose any real number $\theta\in[0,1]$. Let $\eta(x)=y$ and $\eta(z)=w$. Then by the definition of the function η, $\mu(y)=x$ and $\mu(w)=z$. Since the function μ is convex, we have $\mu(\theta y+(1-\theta)w)\leq\theta\mu(y)+(1-\theta)\mu(w)=\theta x+(1-\theta)z$. So it follows that $\eta(\theta x+(1-\theta)z)\geq\theta y+(1-\theta)w=\theta\eta(x)+(1-\theta)\eta(z)$. □

By noting that $\mathrm{E}(\sum_{i\in S}R_i(y_i))^2=\sum_{i\in S}v_i(y_i)+(\sum_{i\in S}\mu_i(y_i))^2$, the cost rate (4) can be rearranged as follows :

$$CR(\mathbf{y}|S)=\frac{\lambda\{K(S)+\sum_{i\in S}D_i(y_i)\}+(h/2)\sum_{i\in S}v_i(y_i)}{\sum_{i\in S}\mu_i(y_i)}+(h/2)\sum_{i\in S}\mu_i(y_i) \quad (12)$$

Now to make the analysis of the problem easier, let us introduce the following function :

$$AC(x|S)=\min\{\sum_{i\in S}(\lambda D_i(\eta_i(x_i))+(h/2)v_i(\eta_i(x_i)))|\sum_{i\in S}x_i=x,\ x_i\geq 0,\ i\in S\} \quad (13)$$

where $x\geq 0$.

By Lemma 2, it can be easily seen that the optimization problem in (13) is a problem of minimizing a concave function on the convex (polyhedral) region. Hence there is an optimal solution which is an extreme point of the feasible region, and the only one component of it can be positive and has value equal to x.

Using (13), the problem (5) can be restated as follows :

$$\min_{x>0}\min_{S\subseteq N}[\{\lambda K(S)+AC(x|S)\}/x+(h/2)x]. \quad (14)$$

It can be easily seen that if the set S and the value of x are fixed, then the problem (14) is equivalent to the problem in (13).

Theorem 2 : Suppose the ordering cost function and the means and variances of the suppliers' random yields exhibit economies of scale. Then the sole sourcing is optimal if there is no variance pooling effect.

Proof : Suppose an optimal solution to (14) corresponds to ordering exclusively from the suppliers in a subset $S^* \subseteq N$. Without loss of generality, we can assume the set S has at least two elements. Let $x^* = \sum_{i\in S^*} x_i > 0$, where $y_i^* = \eta_i(x_i^*)$, $i\in S$, is the optimal order quantity. Consider the problem in (13) with $S = S^*$ and $x = x^*$. Then by the above observation, there is an optimal solution with $x_t = x^*$ for some $t\in S$. By recalling the assumption $K_t \le K(S)$, we can see that the solution is also optimal to the problem (14). □

By the examination of the proof of Theorem 1, we can easily identify the conditions under which the multiple sourcing is not optimal.

Corollary 3 : Under the assumptions of Theorem 2, the multiple sourcing cannot be optimal either if $K_i < K(S)$ for all $i\in S$, $S\subseteq N$ or if all the functions D_i, $i\in N$ are strictly concave.

Remark 4 : When there are nonnegative correlations among the random yields, the problem (14) gives a lower bound on the original problem. Hence if the problem (14) has an optimal solution which exclusively orders from a single supplier, then it will be an optimal solution to the original problem. Thus the optimality of sole sourcing holds when the random yields are nonnegatively correlated.

Remark 5 : If the payment is made for good units only, the result of Theorem 2 can be shown to be valid if $\mathrm{E}D_i(R(y_i)) = D_i(\mu_i(y_i))$. For example, if $D_i(y_i) = L_i + c_i y_i$, for some nonnegative numbers L_i and c_i, $i\in N$, then $\mathrm{E}D_i(R(y_i)) = D_i(\mu_i(y_i))$, for all $y_i \ge 0$ and so the sole sourcing is optimal. More generally, when $D_i(y_i) = L_i + c_i^o y + c_i^e R_i(y_i)$, the result still remains valid (see Remark 3).

5. Discussion and Concluding Remarks

This paper proves the optimality of sole sourcing either when the fixed-cost pooling effect does not exist or when the economies of scale prevail and there is no variance pooling effect. So when the economies of scale prevail, the multiple sourcing may be beneficial only when there are negative correlations among the random yields. In addition, if there is no fixed-cost pooling effect, the sole sourcing is always optimal irrespective of the ordering cost structure and the existence of the variance pooling. Hence, the results given in this paper show that the sole sourcing is an (sometimes the only) effective choice in many cases. The reason why the supplier diversification is of little value in those case is explained in the following.

As shown in Section 3, when there is no fixed-cost pooling effect, the sole sourcing is always optimal. This result seems to be counter-intuitive since the variance pooling effect does not play any role in this case. However, if we think of the assumptions made in an EOQ model, we can notice that a supply disruption occurred in any cycle does not turn out to be serious. This is mainly due to the assumption that a new order can be placed and received whenever necessary. Hence even if complete disruption of the supply occurs, the problem can be resolved easily by initiating a new order with the expense of the additional ordering cost. Hence under an EOQ-type inventory control, the major issue is not the risk but the cost. The importance of the economy of scale given in Section 4 can also be understood in the same way.

Arguably, the assumptions made in an EOQ type inventory control are very unrealistic. Especially the assumption that the lead time is (close to) zero cannot be validated in many practical situations. The supplier diversification will be very beneficial if the lead time is not so short and the loss caused from the supply chain disruption is large enough.

However, even though the EOQ-like inventory control policy is problematic to be applied in practice, the results given in this paper open a new way of evaluating the value of suppler diversification. Especially when the order can be fulfilled sufficiently quickly, it may be a good strategy to stick to a single supplier. If a purchaser has developed quite a close relationship with a supplier for a long time, we can expect that many supply problems can be resolved in a sufficiently short time. On the other hand, single sourcing can help in developing such a close relationship and can encourage the supplier to invest in the production process to improve the yield. Hence considering the long term performance, single sourcing may be a good strategy in many practical cases.

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