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NON-EXISTENCE FOR SCREEN QUASI-CONFORMAL IRROTATIONAL HALF LIGHTLIKE SUBMANIFOLDS OF A SEMI-RIEMANNIAN SPACE FORM ADMITTING A SEMI-SYMMETRIC NON-METRIC CONNECTION

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Abstract. We study screen quasi-conformal irrotational half lightlike submanifolds M of a semi-Riemannian space form $\bar{M}(c)$ equipped with a semi-symmetric non-metric connection subject such that the structure vector field of $\bar{M}(c)$ belongs to the screen distribution $S(TM)$. The main result is a non-existence theorem for such half lightlike submanifolds.

1. Introduction

The theory of lightlike submanifolds is used in mathematical physics, in particular, in general relativity since lightlike submanifolds produce models of different types of horizons. Lightlike submanifolds are also studied in the theory of electromagnetism [2]. As for any semi-Riemannian manifold there is a natural existence of lightlike subspaces, Duggal-Bejancu published their work [2] on the general theory of lightlike submanifolds to fill a gap in the study of submanifolds. Since then there has been very active study on lightlike geometry of submanifolds (see up-to date results in two books [3, 4]).

Ageshe-Chafle [1] introduced the notion of a semi-symmetric non-metric connection on a Riemannian manifold. Although now we have lightlike version of a large variety of Riemannian submanifolds, the theory of lightlike submanifolds of semi-Riemannian manifolds equipped with semi-symmetric non-metric connections is few known. Yasar, *et. al.* [8] studied lightlike hypersurfaces of such a semi-Riemannian manifold. Recently Jin [5] and Jin-Lee [6] studied r-lightlike and half lightlike submanifolds of such a semi-Riemannian manifold.

The objective of this paper is to study screen quasi-conformal irrotational half lightlike submanifolds M of a semi-Riemannian space form $\bar{M}(c)$ equipped with a semi-symmetric non-metric connection subject such that the structure

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vector field of $\bar{M}(c)$ belongs to $S(TM)$. The main result is a non-existence theorem for such half lightlike submanifolds.

2. Semi-symmetric non-metric connection

Let $(\bar{M}, \bar{g})$ be a semi-Riemannian manifold. A connection $\bar{\nabla}$ on $\bar{M}$ is called a *semi-symmetric non-metric connection* [1] if $\bar{\nabla}$ and its torsion tensor $\bar{T}$ satisfy

$$(\bar{\nabla}_X \bar{g})(Y, Z) = -\pi(Y)\bar{g}(X, Z) - \pi(Z)\bar{g}(X, Y), \tag{2.1}$$
$$\bar{T}(X, Y) = \pi(Y)X - \pi(X)Y, \tag{2.2}$$

for any vector fields X, Y and Z on $\bar{M}$, where π is a 1-form associated with a smooth non-vanishing vector field ζ, which is called the *structure vector field* of $\bar{M}$, by $\pi(X) = \bar{g}(X, \zeta)$.

A submanifold (M, g) of codimension 2 is called *half lightlike submanifold* if the radical distribution $Rad(TM) = TM \cap TM^{\perp}$ is a subbundle of the tangent bundle TM and the normal bundle $TM^{\perp}$ of rank 1. In this case, there exists complementary non-degenerate distributions $S(TM)$ and $S(TM^{\perp})$ of $Rad(TM)$ in TM and $TM^{\perp}$ respectively, which are called the *screen* and *co-screen distributions* on M respectively, such that

$$TM = Rad(TM) \oplus_{orth} S(TM), \ TM^{\perp} = Rad(TM) \oplus_{orth} S(TM^{\perp}), \tag{2.3}$$

where $\oplus_{orth}$ denotes the orthogonal direct sum. We denote such a half lightlike submanifold by $M = (M, g, S(TM))$. Denote by $F(M)$ the algebra of smooth functions on M, by $\Gamma(E)$ the $F(M)$ module of smooth sections of a vector bundle E over M and by $(*\,.\,*)_i$ the i-th equation of $(*\,.\,*)$. We use same notations for any others. Choose $L \in \Gamma(S(TM^{\perp}))$ as a unit spacelike vector field, *i.e.*, $\bar{g}(L, L) = 1$, without loss of generality. We call L the *canonical normal vector field* of M. Consider the orthogonal complementary distribution $S(TM)^{\perp}$ to $S(TM)$ in $T\bar{M}$. Certainly $Rad(TM)$ and $S(TM^{\perp})$ are subbundles of $S(TM)^{\perp}$. As $S(TM^{\perp})$ is non-degenerate, we have

$$S(TM)^{\perp} = S(TM^{\perp}) \oplus_{orth} S(TM^{\perp})^{\perp},$$

where $S(TM^{\perp})^{\perp}$ is the orthogonal complementary to $S(TM^{\perp})$ in $S(TM)^{\perp}$. It is well-known [2, 6] that, for any null section ξ of $Rad(TM)$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a uniquely defined lightlike vector bundle $ltr(TM)$ and a null vector field N of $ltr(TM)$ on $\mathcal{U}$ [2] satisfying

$$\bar{g}(\xi, N) = 1, \ \bar{g}(N, N) = \bar{g}(N, X) = \bar{g}(N, L) = 0, \ \forall\, X \in \Gamma(S(TM)).$$

We call N, $ltr(TM)$ and $tr(TM) = S(TM^{\perp}) \oplus_{orth} ltr(TM)$ the *lightlike transversal vector field, lightlike transversal vector bundle* and *transversal vector bundle* of M with respect to $S(TM)$ respectively [6]. Therefore $T\bar{M}$ is decomposed as

$$\begin{aligned} T\bar{M} &= TM \oplus tr(TM) = \{Rad(TM) \oplus tr(TM)\} \oplus_{orth} S(TM) \quad (2.4)\\ &= \{Rad(TM) \oplus ltr(TM)\} \oplus_{orth} S(TM) \oplus_{orth} S(TM^{\perp}). \end{aligned}$$

In the entire discussion, we shall assume that ζ is unit spacelike vector field and it belongs to $S(TM)$. In the sequel, we take X, Y, Z, $W \in \Gamma(TM)$ unless otherwise specified. Let P be the projection morphism of TM on $S(TM)$ with respect to the decomposition $(2.3)_1$. Then the local Gauss and Weingartan formulas of M and $S(TM)$ are given respectively by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X,Y)N + D(X,Y)L, \tag{2.5}$$

$$\bar{\nabla}_X N = -A_N X + \tau(X)N + \rho(X)L, \tag{2.6}$$

$$\bar{\nabla}_X L = -A_L X + \phi(X)N; \tag{2.7}$$

$$\nabla_X PY = \nabla^*_X PY + C(X,PY)\xi, \tag{2.8}$$

$$\nabla_X \xi = -A^*_\xi X - \tau(X)\xi, \tag{2.9}$$

where ∇ and ∇^* are induced linear connections on TM and $S(TM)$ respectively, B and D are called the *local lightlike* and *screen second fundamental forms* of M respectively, C is called the *local second fundamental form* on $S(TM)$. A_N, A^*_ξ and A_L are linear operators on TM and τ, ρ, ϕ and σ are 1-forms on TM. We say that $h(X,Y) = B(X,Y)N + D(X,Y)L$ is the *global second fundamental form tensor* of M. Using (2.1), (2.2) and (2.5), we have

$$\begin{aligned}(\nabla_X g)(Y,Z) = B(X,Y)\eta(Z) + B(X,Z)\eta(Y)& \\ - \pi(Y)g(X,Z) - \pi(Z)g(X,Y)&,\end{aligned} \tag{2.10}$$

$$T(X,Y) = \pi(Y)X - \pi(X)Y \tag{2.11}$$

and B and D are symmetric on TM, where T is the torsion tensor with respect to the induced connection ∇ and η is a 1-form on TM such that

$$\eta(X) = \bar{g}(X,N).$$

From the facts that $B(X,Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$ and $D(X,Y) = \bar{g}(\bar{\nabla}_X Y, L)$, we know that B and D are independent of the choice of $S(TM)$ and satisfy

$$B(X,\xi) = 0, \qquad D(X,\xi) = -\phi(X). \tag{2.12}$$

The above second fundamental forms are related to their shape operators by

$$g(A^*_\xi X, Y) = B(X,Y), \qquad \bar{g}(A^*_\xi X, N) = 0, \tag{2.13}$$

$$g(A_L X, Y) = D(X,Y) + \phi(X)\eta(Y), \qquad \bar{g}(A_L X, N) = \rho(X), \tag{2.14}$$

$$g(A_N X, PY) = C(X,PY) - \eta(X)\pi(PY), \quad \bar{g}(A_N X, N) = 0. \tag{2.15}$$

From (2.13) and (2.15), we show that A^*_ξ is $S(TM)$-valued self-adjoint and

$$A^*_\xi \xi = 0. \tag{2.16}$$

Denote by $\bar{R}$, R and R^* the curvature tensors of the semi-symmetric non-metric connection $\bar{\nabla}$ on $\bar{M}$, the induced connection ∇ on M and the induced connection ∇^* on $S(TM)$ respectively. Using the Gauss-Weingarten formulas, we obtain the Gauss-Codazzi equations for M and $S(TM)$:

$$\begin{aligned}\bar{R}(X,Y)Z = R(X,Y)Z &+ B(X,Z)A_N Y - B(Y,Z)A_N X \\ &+ D(X,Z)A_L Y - D(Y,Z)A_L X \\ &+\{(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) \\ &\quad + B(Y,Z)[\tau(X) - \pi(X)] - B(X,Z)[\tau(Y) - \pi(Y)] \\ &\quad + D(Y,Z)\phi(X) - D(X,Z)\phi(Y)\}N \\ &+\{(\nabla_X D)(Y,Z) - (\nabla_Y D)(X,Z) + B(Y,Z)\rho(X) \\ &\quad - B(X,Z)\rho(Y) - D(Y,Z)\pi(X) + D(X,Z)\pi(Y)\}L,\end{aligned} \tag{2.17}$$

$$\begin{aligned}\bar{R}(X,Y)N = -\nabla_X(A_N Y) &+ \nabla_Y(A_N X) + A_N[X,Y] \\ &+ \tau(X)A_N Y - \tau(Y)A_N X + \rho(X)A_L Y - \rho(Y)A_L X \\ &+\{B(Y,A_N X) - B(X,A_N Y) + 2d\tau(X,Y) \\ &\quad + \phi(X)\rho(Y) - \phi(Y)\rho(X)\}N \\ &+\{D(Y,A_N X) - D(X,A_N Y) + 2d\rho(X,Y) \\ &\quad + \rho(X)\tau(Y) - \rho(Y)\tau(X)\}L,\end{aligned} \tag{2.18}$$

$$\begin{aligned}\bar{R}(X,Y)L = -\nabla_X(A_L Y) &+ \nabla_Y(A_L X) + A_L[X,Y] \\ &+ \phi(X)A_N Y - \phi(Y)A_N X \\ &+\{B(Y,A_L X) - B(X,A_L Y) + 2d\phi(X,Y) \\ &\quad + \tau(X)\phi(Y) - \tau(Y)\phi(X)\}N \\ &+\{D(Y,A_L X) - D(X,A_L Y) + \rho(X)\phi(Y) - \rho(Y)\phi(X)\}L,\end{aligned} \tag{2.19}$$

$$\begin{aligned}R(X,Y)PZ = R^*(X,Y)PZ &+ C(X,PZ)A^*_\xi Y - C(Y,PZ)A_\xi X \\ &+\{(\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) \\ &\quad + C(X,PZ)[\tau(Y) + \pi(Y)] - C(Y,PZ)[\tau(X) + \pi(X)]\}\xi\end{aligned} \tag{2.20}$$

$$\begin{aligned}R(X,Y)\xi = -\nabla^*_X(A^*_\xi Y) &+ \nabla^*_Y(A^*_\xi X) + A^*_\xi[X,Y] \\ &+ \tau(Y)A^*_\xi X - \tau(X)A^*_\xi Y \\ &+ \{C(Y,A^*_\xi X) - C(X,A^*_\xi Y) - 2d\tau(X,Y) \\ &\quad + \rho(X)\phi(Y) - \rho(Y)\phi(X)\}\xi.\end{aligned} \tag{2.21}$$

A semi-Riemannian manifold $\bar{M}$ of constant curvature c is called a *semi-Riemannian space form* and denote it by $\bar{M}(c)$. In this case, for any vector fields X, Y and Z of $\bar{M}$, the curvature tensor $\bar{R}$ of $\bar{M}(c)$ is given by

$$\bar{R}(X,Y)Z = c\{\bar{g}(Y,Z)X - \bar{g}(X,Z)Y\}. \tag{2.22}$$

Taking the scalar product with ξ and L to (2.22) by turns, we get

$$\bar{g}(\bar{R}(X,Y)Z,\, \xi) = 0, \qquad \bar{g}(\bar{R}(X,Y)Z,\, L) = 0,$$

for any $X,\, Y,\, Z \in \Gamma(TM)$. From these results and (2.17), we obtain

$$\bar{R}(X,Y)Z = R(X,Y)Z + B(X,Z)A_{_N}Y - B(Y,Z)A_{_N}X + D(X,Z)A_{_L}Y - D(Y,Z)A_{_L}X. \tag{2.23}$$

3. Non-existence theorem

Definition 1. A half lightlike submanifold M of a semi-Riemannian manifold $\bar{M}$ is said to be *irrotational* if $\bar{\nabla}_X\xi \in \Gamma(TM)$ for any $X \in \Gamma(TM)$.

From (2.5) and (2.12), we show that the above definition is equivalent to the condition: $D(X,\xi) = 0 = \phi(X)$ for all $X \in \Gamma(TM)$.

Lemma 3.1. *Let M be an irrotational half lightlike submanifold of a semi-Riemannian manifold $\bar{M}$ equipped with a semi-symmetric non-metric connection subject such that ζ belongs to $S(TM)$. Then ζ is conjugate to any vector field X on M, i.e., ζ is satisfied $h(X,\zeta) = 0$.*

Proof. Taking the scalar product with ξ to (2.18) and N to (2.21) by turns and using (2.12) and the facts that $\bar{R}(X,Y)\xi = R(X,Y)\xi$ and $\phi = 0$, we obtain

$$\begin{aligned}\bar{g}(\bar{R}(X,Y)\xi,\, N) &= B(X,A_{_N}Y) - B(Y,A_{_N}X) - 2d\tau(X,Y)\\ &= C(Y,A^*_\xi X) - C(X,A^*_\xi Y) - 2d\tau(X,Y).\end{aligned}$$

From these two representations, we obtain

$$B(X,A_{_N}Y) - B(Y,A_{_N}X) = C(Y,A^*_\xi X) - C(X,A^*_\xi Y).$$

Using $(2.13)_1$ and $(2.15)_1$, we have

$$\pi(A^*_\xi X)\eta(Y) = \pi(A^*_\xi Y)\eta(X).$$

Replacing Y by ξ to this equation and using (2.13) and (2.16), we have

$$B(X,\zeta) = \pi(A^*_\xi X) = 0. \tag{3.1}$$

Taking the scalar product with L to (2.18) and N to (2.19) with $\phi = 0$, we get

$$\begin{aligned}&\bar{g}(\bar{R}(X,Y)N,\, L)\\ &= \bar{g}(\nabla_X(A_{_L}Y) - \nabla_Y(A_{_L}X) - A_{_L}[X,Y],\; N)\\ &= D(Y,A_{_N}X) - D(X,A_{_N}Y) + 2d\rho(X,Y) + \rho(X)\tau(Y) - \rho(Y)\tau(X).\end{aligned}$$

Using these two representations and $(2.14)_2$, we show that

$$\begin{aligned}&D(Y,A_{_N}X) - D(X,A_{_N}Y) + 2d\rho(X,Y) + \rho(X)\tau(Y) - \rho(Y)\tau(X)\\ &= \bar{g}(\nabla_X(A_{_L}Y),\, N) - \bar{g}(\nabla_Y(A_{_L}X),\, N) - \rho([X,Y]).\end{aligned}$$

As D is symmetric and $\phi = 0$, $A_{_L}$ is self-adjoint operator. Applying $\bar{\nabla}_X$ to $\bar{g}(A_{_L}Y,N) = \rho(Y)$ and using (2.1), (2.5), (2.6) and $(2.14)_2$, we have

$$\bar{g}(\nabla_X(A_{_L}Y),\, N) = X(\rho(Y)) + \pi(A_{_L}Y)\eta(X) + g(A_{_L}Y,A_{_N}X) - \tau(X)\rho(Y).$$

Substituting this equation into the last equation and using $(2.14)_1$, we have

$$\pi(A_{_L}X)\eta(Y) = \pi(A_{_L}Y)\eta(X).$$

Replacing Y by ξ to this equation, we have

$$\pi(A_L X) = \pi(A_L \xi)\eta(X).$$

Taking $X = \xi$ and $Y = \zeta$ to $(2.14)_1$, we get $\pi(A_L \xi) = 0$. Therefore we have

$$D(X, \zeta) = \pi(A_L X) = 0. \tag{3.2}$$

From (3.1) and (3.2), we show that $h(X, \zeta) = 0$ for all $X \in \Gamma(TM)$.

Definition 2. A half lightlike submanifold M of a semi-Riemannian manifold $\bar{M}$ admitting a semi-symmetric non-metric connection is called *screen quasi-conformal* if the second fundamental forms B and C satisfy

$$C(X, PY) = \varphi B(X, Y) + \eta(X)\pi(PY),$$

where φ is a non-vanishing function on a coordinate neighborhood $\mathcal{U}$ in M.

Due to (2.13) and (2.15), we show that M is screen quasi-conformal if and only if the shape operators A_N and A^*_ξ are related by

$$A_N = \varphi A^*_\xi. \tag{3.3}$$

Theorem 4.2. *There exist no irrotational and screen quasi-conformal half lightlike submanifolds M of a semi-Riemannian space form $\bar{M}(c)$ admitting a semi-symmetric non-metric connection such that ζ belongs to $S(TM)$.*

Proof. Taking the scalar product with PZ to (2.18) and (2.21), we get

$$\begin{aligned}\bar{g}(\bar{R}(X, Y)N,\, PZ) &= g(-\nabla_X(A_N Y) + \nabla_Y(A_N X) + A_N[X, Y],\, PZ) \\ &\quad + \tau(X)g(A_N Y, PZ) - \tau(Y)g(A_N X, PZ) \\ &\quad + \rho(X)g(A_L Y, PZ) - \rho(Y)g(A_L X, PZ),\end{aligned} \tag{3.4}$$

$$\begin{aligned}g(R(X, Y)\xi,\, PZ) &= g(-\nabla^*_X(A^*_\xi Y) + \nabla^*_Y(A^*_\xi X) + A^*_\xi[X, Y],\, PZ) \\ &\quad + \tau(Y)g(A^*_\xi X, PZ) - \tau(X)g(A^*_\xi Y, PZ)\end{aligned} \tag{3.5}$$

respectively. Applying ∇_X to (3.3), we have

$$\nabla_X(A_N Y) = X[\varphi]A^*_\xi Y + \varphi\nabla_X(A^*_\xi Y).$$

Substituting this equation into (3.4) and using (2.13), (2.14), (2.23), (3.2) and (3.5), we get

$$\begin{aligned}&\bar{g}(\bar{R}(X, Y)N, PZ) - \varphi\bar{g}(\bar{R}(X, Y)\xi, PZ) \\ &= \{Y[\varphi] - 2\varphi\tau(Y)\}B(X, PZ) - \rho(Y)D(X, PZ) \\ &- \{X[\varphi] - 2\varphi\tau(X)\}B(Y, PZ) + \rho(X)D(Y, PZ).\end{aligned}$$

Substituting (2.22) into the last equation and using (2.12) with $\phi = 0$, we get

$$\begin{aligned}&c\{\eta(Y)g(X, Z) - \eta(X)g(Y, Z)\} \\ &= \{Y[\varphi] - 2\varphi\tau(Y)\}B(X, Z) - \rho(Y)D(X, Z) \\ &- \{X[\varphi] - 2\varphi\tau(X)\}B(Y, Z) + \rho(X)D(Y, Z).\end{aligned} \tag{3.6}$$

Taking $X = Z = \zeta$ and $Y = \xi$ to this and using (3.1) and (3.2), we have $c = 0$.

As $c = 0$, the last equation (3.6) is reduced to

$$\begin{aligned} &\rho(X)D(Y,Z) - \rho(Y)D(X,Z) \\ &= \{X[\varphi] - 2\varphi\tau(X)\}B(Y,Z) - \{Y[\varphi] - 2\varphi\tau(Y)\}B(X,Z). \end{aligned} \tag{3.7}$$

Taking the scalar product with ξ to (2.17) and using (2.22), we have

$$\begin{aligned} &(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) \\ &= B(Y,Z)\{\pi(X) - \tau(X)\} - B(X,Z)\{\pi(Y) - \tau(Y)\}. \end{aligned} \tag{3.8}$$

Applying $\bar{\nabla}_X$ to $\eta(Y) = \bar{g}(Y,N)$ and using (2.1), (2.5) and (2.6), we have

$$X(\eta(Y)) = -\pi(Y)\eta(X) + \bar{g}(\nabla_X Y, N) - g(A_N X, Y) + \tau(X)\eta(Y).$$

Substituting this equation into the right term of the following relation

$$2d\eta(X,Y) = X(\eta(Y)) - Y(\eta(X)) - \eta([X,Y])$$

and using (2.11), (3.2) and the fact A^*_ξ is self-adjoint, we get

$$2d\eta(X,Y) = \tau(X)\eta(Y) - \tau(Y)\eta(X). \tag{3.9}$$

Taking the scalar product with N to (2.17) and (2.20) by turns and using $(2.14)_2$, $(2.15)_2$ and the fact $\bar{R} = 0$, we get

$$\begin{aligned} &\bar{g}(R(X,Y)Z,N) = \rho(X)D(Y,Z) - \rho(Y)D(X,Z), \\ &\bar{g}(R(X,Y)PZ,N) = (\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) \\ &\qquad + C(X,PZ)\{\pi(Y) + \tau(Y)\} - C(Y,PZ)\{\pi(X) + \tau(X)\}. \end{aligned}$$

From the last two equations and (3.9), we get

$$\begin{aligned} &\{X[\varphi] - 2\varphi\tau(X)\}B(Y,Z) - \{Y[\varphi] - 2\varphi\tau(Y)\}B(X,Z) \\ &= (\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) + C(X,PZ)\{\pi(Y) + \tau(Y)\} \\ &\quad - C(Y,PZ)\{\pi(X) + \tau(X)\}. \end{aligned} \tag{3.10}$$

Applying ∇_X to $C(Y,PZ) = \varphi B(Y,PZ) + \eta(Y)\pi(PZ)$, we have

$$\begin{aligned} &(\nabla_X C)(Y,PZ) = X[\varphi]B(Y,PZ) + \varphi(\nabla_X B)(Y,PZ) \\ &+ \{X(\eta(Y)) - \eta(\nabla_X Y)\}\pi(PZ) + \eta(Y)\{X(\pi(PZ)) - \pi(\nabla^*_X PZ)\}. \end{aligned}$$

Substituting this into (3.10) and using (2.11), (3.8), and (3.9), we obtain

$$\eta(X)\{Y(\pi(PZ)) - \pi(\nabla^*_Y PZ)\} = \eta(Y)\{X(\pi(PZ)) - \pi(\nabla^*_X PZ)\}. \tag{3.11}$$

Applying ∇_X to $\pi(PZ) = g(\zeta, PZ)$ and using (2.8), we have

$$\begin{aligned} &X(\pi(PZ)) - \pi(\nabla^*_X PZ) \\ &= -\, g(X,PZ) - \pi(X)\pi(PZ) + fB(X,PZ) + g(\nabla_X \zeta, PZ). \end{aligned}$$

Substituting this equation into (3.11), we obtain

$$\begin{aligned} &\eta(Y)\{g(X,PZ) + \pi(X)\pi(PZ) - g(\nabla_X \zeta, PZ)\} \\ &= \eta(X)\{g(Y,PZ) + \pi(Y)\pi(PZ) - g(\nabla_Y \zeta, PZ)\}. \end{aligned} \tag{3.12}$$

Applying $\bar{\nabla}_X$ to $g(\zeta, \zeta) = 1$ and using (2.1) and (2.5), we have

$$g(\nabla_X \zeta, \zeta) = \pi(X). \tag{3.13}$$

Taking $X = Z = \zeta$ and $Y = \xi$ to (3.12) and using (3.1) and (3.13), we get $1 = 0$. It is a contradiction. Thus there exist no irrotational screen quasi-conformal half lightlike submanifolds M of a semi-Riemannian space form $\bar{M}(c)$ admitting a semi-symmetric non-metric connection such that ζ belongs to $S(TM)$.

References

[1] N.S. Ageshe and M.R. Chafle, *A semi-symmetric non-metric connection on a Riemannian manifold*, Indian J. Pure Appl. Math., 23(6), 1992, 399-409.

[2] K.L. Duggal and A. Bejancu, *Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications*, Kluwer Acad. Publishers, Dordrecht, 1996.

[3] K.L. Duggal and D.H. Jin, *Null curves and Hypersurfaces of Semi-Riemannian Manifolds*, World Scientific, 2007.

[4] K.L. Duggal and B. Sahin, *Differential geometry of lightlike submanifolds*, Frontiers in Mathematics, Birkhäuser, 2010.

[5] D.H. Jin, *Lightlike submanifolds of a semi-Riemannian manifold with a semi-symmetric non-metric connection*, J. Korean Soc Math. Edu. Ser. B: Pure Appl. Math., 19(3), 2012, 211-228.

[6] D.H. Jin and J.W. Lee, *A classification of half lightlike submanifolds of a semi-Riemannian manifold with a semi-symmetric non-metric connection*, Bull. Korean Math. Soc. 50(3), 2013, 705-717.

[7] B. O'Neill, *Semi-Riemannian Geometry with Applications to Relativity*, Academic Press, 1983.

[8] E. Yasar, A.C. Cöken and A. Yücesan, *Lightlike hypersurfaces in semi-Riemannian manifold with semi-symmetric non-metric connection*, Math. Scand. 102, 2008, 253-264.

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