East Asian Math. J.
Vol. 31 (2015), No. 3, pp. 301–310
http://dx.doi.org/10.7858/eamj.2015.021

STRONG CONVERGENCE BY PSEUDOCONTRACTIVE MAPPINGS FOR THE NOOR ITERATION SCHEME

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Abstract. In this paper, we establish a strong convergence for the Noor iterative scheme associated with Lipschitz strongly pseudocontractive mappings in real Banach spaces. It's proof-method is very simple by comparing with the previous proofs known.

1. Introduction and preliminaries

Let E be a real Banach space and K a nonempty convex subset of E. Let J denote the normalized duality mapping from E to 2^{E^*} defined by

$$J(x) = \{f^* \in E^* : \langle x, f^* \rangle = ||x||^2 \text{ and } ||f^*|| = ||x||\},$$

where E^* and $\langle \cdot, \cdot \rangle$ denote the dual space of E and the generalized duality pairing, respectively. We shall denote the single-valued duality mapping by j.

Let $T : D(T) \subset E \to E$ be a mapping with a domain $D(T)$ in E.

Definition 1. T is said to be L-Lipschitzian if there exists $L > 1$ such that for all $x, y \in D(T)$

$$\|Tx - Ty\| \leq L \|x - y\|.$$

Definition 2. T is said to be nonexpansive if the following inequality holds:

$$\|Tx - Ty\| \leq \|x - y\| \text{ for all } x,\ y \in D(T).$$

Definition 3. T is said to be *pseudocontractive* if the inequality

$$\|x - y\| \leqslant \| x - y + t((I - T)x - (I - T)y) \|,$$

holds for all $x, y \in K$ and $t > 0$.

Remark 1. As a consequence of a result of Kato [8],
T is *pseudocontractive* if and only if there exists $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y) \rangle \leqslant \|x - y\|^2$$

Received December 1, 2014; Accepted February 13, 2015.
2010 *Mathematics Subject Classification.* Primary 47H10, 47H17: Secondary 54H25.
Key words and phrases. Noor iterative scheme, normalized duality mappings, Lipschitz strongly pseudocontractive mappings, Banach spaces.
This work was supported by Dongeui University (2013AA072).

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(pISSN 1226-6973, eISSN 2287-2833)

for all $x, y \in K$.

Definition 4 [14]. T is said to be k-strongly pseudocontractive if there exists a constant $k > 1$ such that

$$\|x - y\| \leq \|(1 + r)(x - y) - rk(Tx - Ty)\| \tag{1.1}$$

for all $x, y \in D(T)$ and $r > 0$.

Remark 2. From the inequality (1.1) Bogin [2] obtained the following inequality;

$$\langle Tx - Ty, j(x - y)\rangle \leq k||x - y||^2 \text{ for some } k \in (0, 1).$$

Definition 5. (i) T is said to be *accretive* if the inequality

$$||x - y|| \leq ||x - y + s(Tx - Ty)||$$

holds for all $x, y \in D(T)$ and for all $s > 0$.

(ii) T is said to be *strongly accretive* if there exist a constant $k > 0$ and $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty,\ j(x - y)\rangle \geq k||x - y||^2 \text{ for all } x, y \in D(T).$$

The class of pseudocontractions is, perhaps, the most important generalization of the class of nonexpansive mappings because of its strong relationship with the class of accretive mappings as follows;

A mapping $T : E \to E$ is accretive if and only if $I - T$ is pseudocontractive.

In the last few years or so, numerous papers have been published on the iterative approximation of fixed points of Lipschitz *strongly* pseudocontractive mappings using the *Ishikawa iteration scheme* (see for example, [7]). Results which had been known only in *Hilbert spaces* and only for *Lipschitz mappings* have been extended to more general Banach spaces (see for example [3-4, 6, 15] and the references cited therein).

In 1974, Ishikawa [7] introduced an iteration scheme which, in some sense, is more general than that of Mann and whose iterative sequence converges, under this setting, to a fixed point of T. He proved the following result.

Theorem 1.1. *If K is a compact convex subset of a Hilbert space H, $T : K \mapsto K$ is a Lipschitzian pseudocontractive mapping and x_1 is any given point in K, then the sequence $\{x_n\}$ converges strongly to a fixed point of T, where x_n is defined iteratively for each positive integer $n \geq 1$ by*

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n T y_n, \\ y_n &= (1 - \beta_n)x_n + \beta_n T x_n, \end{aligned}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ are sequences of positive numbers satisfying the conditions

$$(i)\ 0 < \alpha_n \leq \beta_n < 1;\ \ (ii)\ \lim_{n\to\infty} \beta_n = 0;\ \ (iii)\ \sum_{n\geq 1} \alpha_n\beta_n = \infty.$$

Let E be a real normed space and K be a nonempty closed convex subset of E. Let $T : K \to K$ be a mapping.

Algorithm NRH [10, 11]**.** For a given $x_0 \in K$, compute the sequence $\{x_n\}_{n\geq 1}$ defined by the iterative schemes

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n, \\ y_n &= (1-\beta_n)x_n + \beta_n T z_n, \\ z_n &= (1-\gamma_n)x_n + \gamma_n T x_n,\ n \geq 1, \end{aligned}$$

which is called a three-step iterative process, where $\{\alpha_n\}_{n\geq 1}$, $\{\beta_n\}_{n\geq 1}$ and $\{\gamma_n\}_{n\geq 1}$ are real sequences in [0,1] satisfying some certain conditions.

If $\gamma_n = 0$ and $\beta_n = 0$, then Algorithm NRH reduces to:
Algorithm M. For a given $x_0 \in K$, compute the sequence $\{x_n\}_{n\geq 1}$ defined by the iterative scheme

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, n \geq 1,$$

which is called the Mann iterative process [9].

For $\gamma_n = 0$, Algorithm NRH becomes:
Algorithm I. Let K be a nonempty convex subset of E and let $T : K \to K$ be a mapping. For any given $x_0 \in K$, compute the sequence $\{x_n\}_{n\geq 1}$ defined by the iterative schemes

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n, \\ y_n &= (1-\beta_n)x_n + \beta_n T x_n,\ n \geq 1, \end{aligned}$$

which is called the two-step Ishikawa iterative process [7].

In [3], Chidume extended the results of Schu [15] from Hilbert spaces to the much more general class of real Banach spaces and the approximation of the fixed points of strongly pseudocontractive mappings.

In [6], Haiyun and Yuting gave the answer of the question raised by Chidume [3] and proved: If X is a real Banach space with a uniformly convex dual X^*, K is a nonempty bounded closed convex subset of X, and $T : K \to K$ is a continuous strongly pseudocontractive mapping, then the iterative sequence due to Ishikawa iteration scheme converges strongly to the unique fixed point of T.

In [10], Noor et al. proved the following result:

Theorem 1.2. *Let E be a real uniformly smooth Banach space and K be a nonempty closed convex subset of E. Let T be strongly pseudocontractive self* *mapping of K with $T(K)$ bounded. Let $\{x_n\}_{n\geq 1}$ be the sequence defined by*

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n, \\ y_n &= (1-\beta_n)x_n + \beta_n T z_n, \\ z_n &= (1-\gamma_n)x_n + \gamma_n T x_n,\ n \geq 0, \end{aligned}$$

where $\{\alpha_n\}_{n\geq 1}$, $\{\beta_n\}_{n\geq 1}$ and $\{\gamma_n\}_{n\geq 1}$ are real sequences in $[0,1]$ satisfying the conditions:

$$\lim_{n\to\infty} \alpha_n = 0 = \lim_{n\to\infty} \beta_n = \lim_{n\to\infty} \gamma_n \text{ and } \sum_{n=1}^{\infty} \alpha_n = \infty.$$

Then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique fixed point of T.

In [12, 13], Rafiq proved the generalization of the results of Noor et al. [10, 11] in the form of the following result:

Theorem 1.3. *Let E be a real Banach space and K a nonempty closed convex subset of E. Let T be a uniformly continuous and strongly pseudocontractive self mapping of K with $T(K)$ bounded. Let $\{x_n\}_{n\geq 1}$ be the sequence defined by*

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n, \\ y_n &= (1-\beta_n)x_n + \beta_n T z_n, \\ z_n &= (1-\gamma_n)x_n + \gamma_n T x_n,\ n \geq 1, \end{aligned}$$

where $\{\alpha_n\}_{n\geq 1}$, $\{\beta_n\}_{n\geq 1}$ and $\{\gamma_n\}_{n\geq 1}$ are real sequences in $[0,1]$ satisfying the conditions:

$$\lim_{n\to\infty} \alpha_n = 0 = \lim_{n\to\infty} \beta_n \text{ and } \sum_{n=1}^{\infty} \alpha_n = \infty.$$

Then the sequence $\{x_n\}_{n\geq 1}$ converges strongly to the unique fixed point of T.

In this paper, we establish a strong convergence by the Noor iterative scheme associated with Lipschitz strongly pseudocontractive mappings in real Banach spaces. We also generalize the results of Schu [15] from Hilbert spaces to more general Banach spaces and improve the results of Chidume [3] and Haiyun and Yuting [6].

2. Main Results

The following results will be needed.

Lemma 2.1. *[17] Let $J : E \to 2^E$ be the normalized duality mapping. Then for any $x, y \in E$, we have*

$$||x+y||^2 \leq ||x||^2 + 2\langle y, j(x+y)\rangle \quad \text{for } j(x+y) \in J(x+y).$$

Lemma 2.2. *[12] Let nonnegative real sequences $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ satisfy*

$$a_{n+1} \leq (1+c_n)a_n + b_n \quad (n \in \mathbb{N}), \quad \Sigma b_n < \infty, \quad \Sigma c_n < \infty, \text{ then}$$

(a) $\lim_{n\to\infty} a_n$ *exists,*

(b) If $\underline{\lim}_{n\to\infty} a_n = 0$*, then* $\lim_{n\to\infty} a_n = 0$.

Lemma 2.3. *[1] Let* $\{a_n\}_{n=0}^{\infty}$ *and* $\{b_n\}_{n=0}^{\infty}$ *be sequences of nonnegative real numbers and* $0 \le q < 1$*, so that*

$$a_{n+1} \le q a_n + b_n, \text{ for } n \in \mathbb{N}.$$

(a) If $\lim_{n\to\infty} b_n = 0$*, then* $\lim_{n\to\infty} a_n = 0$.
(b) If $\sum_{n=1}^{\infty} b_n < \infty$*, then* $\sum_{n=1}^{\infty} a_n < \infty$.

Now we prove our main results.

Theorem 2.4. *Let* K *be a nonempty closed convex subset of a real Banach space* E *and* $T : K \to K$ *a L-Lipschitz k-strongly pseudocontractive mapping. Let* $\{\alpha_n\}_{n\ge 1}, \{\beta_n\}_{n\ge 1}$ *and* $\{\gamma_n\}_{n\ge 1}$ *be real sequences in* $[0,1]$ *such that*

(i) $\sum_{n\ge 1}(1-\alpha_n) < \infty$,
(ii) $\alpha_n \le \beta_n$ *for* $n \in \mathbb{N}$.

For given $x_1 \in K$*, let* $\{x_n\}_{n\ge 1}$ *be iteratively defined by*

$$\begin{aligned} z_n &= \gamma_n x_n + (1-\gamma_n)Tx_n, \\ y_n &= \beta_n x_n + (1-\beta_n)Tz_n, \\ x_{n+1} &= \alpha_n x_n + (1-\alpha_n)Ty_n, \ n \ge 1. \end{aligned} \tag{2.1}$$

Then the sequence $\{x_n\}_{n\ge 1}$ *converges strongly to the unique fixed point of* T.

Proof. The existence of a fixed point of the mapping T follows from Deimling [5]. And it is shown in [6] that the set of fixed points for a strongly pseudocontraitive mapping is a singleton.

Let p be the unique fixed point of T.

By (i) $\lim_{n\to\infty} \alpha_n = 1$, so there exists $n_0 \in \mathbb{N}$ such that for $n \ge n_0$,

$$1 - \alpha_n \le \min\{\frac{1}{1+k}, \frac{\eta - 1}{2k\eta}\}, \text{ where } \eta > 1. \tag{2.2}$$

Consider

$$\begin{aligned} ||x_{n+1} - p||^2 &= \langle x_{n+1} - p, j(x_{n+1} - p)\rangle \\ &= \langle \alpha_n x_n + (1-\alpha_n)Ty_n - p, j(x_{n+1} - p)\rangle \\ &= \langle \alpha_n (x_n - p) + (1-\alpha_n)(Ty_n - p), j(x_{n+1} - p)\rangle \\ &= \alpha_n \langle x_n - p, j(x_{n+1} - p)\rangle + (1-\alpha_n)\langle Ty_n - p, j(x_{n+1} - p)\rangle \\ &\le \alpha_n \|x_n - p\| \|x_{n+1} - p\| + (1-\alpha_n)\langle Tx_{n+1} - p, j(x_{n+1} - p)\rangle \\ &\quad + (1-\alpha_n)\langle Ty_n - Tx_{n+1}, j(x_{n+1} - p)\rangle \\ &\le \alpha_n \|x_n - p\| \|x_{n+1} - p\| + k(1-\alpha_n)||x_{n+1} - p||^2 \\ &\quad + (1-\alpha_n)\|Ty_n - Tx_{n+1}\| \|x_{n+1} - p\|, \end{aligned}$$

which implies that

$$||x_{n+1}-p|| \leq \frac{\alpha_n}{1-k(1-\alpha_n)}\|x_n-p\| + \frac{1-\alpha_n}{1-k(1-\alpha_n)}\|Ty_n - Tx_{n+1}\|. \quad (2.3)$$

On the other hand, it can be easily seen that

$$\frac{\alpha_n}{1-k(1-\alpha_n)} < 1, \quad (2.4)$$

and by (2.2),

$$\frac{1-\alpha_n}{1-k(1-\alpha_n)} \leq 1 \text{ and } \frac{1}{1-2k(1-\alpha_n)} \leq \eta. \quad (2.5)$$

Hence from (2.3), we have

$$||x_{n+1}-p|| \leq \|x_n-p\| + L\|y_n - x_{n+1}\|, \quad (2.6)$$

where

$$\begin{aligned} \|y_n - x_{n+1}\| &\leq \|y_n - x_n\| + \|x_n - x_{n+1}\| \quad (2.7) \\ &= (1-\beta_n)\|x_n - Tz_n\| + (1-\alpha_n)\|x_n - Ty_n\| \\ &\leq (1-\beta_n)(\|x_n-p\| + \|Tz_n - p\|) \\ &\quad +(1-\alpha_n)(\|x_n-p\| + \|Ty_n-p\|) \\ &\leq (1-\beta_n)(\|x_n-p\| + L\|z_n-p\|) \\ &\quad +(1-\alpha_n)(\|x_n-p\| + L\|y_n-p\|), \end{aligned}$$

$$\begin{aligned} \|y_n-p\| &= \|\beta_n x_n + (1-\beta_n)Tz_n - p\| \quad (2.8) \\ &= \|\beta_n(x_n-p) + (1-\beta_n)(Tz_n-p)\| \\ &\leq \beta_n\|x_n-p\| + (1-\beta_n)\|Tz_n-p\| \\ &\leq \beta_n\|x_n-p\| + (1-\beta_n)L\|z_n-p\|, \end{aligned}$$

and

$$\begin{aligned} \|z_n-p\| &= \|\gamma_n x_n + (1-\gamma_n)Tx_n - p\| \quad (2.9) \\ &= \|\gamma_n(x_n-p) + (1-\gamma_n)(Tx_n-p)\| \\ &\leq \gamma_n\|x_n-p\| + (1-\gamma_n)\|Tx_n-p\| \\ &\leq \gamma_n\|x_n-p\| + (1-\gamma_n)L\|x_n-p\| \\ &= (L-(L-1)\gamma_n)\|x_n-p\| \\ &\leq L\|x_n-p\|. \end{aligned}$$

Substituting (2.9) in (2.8), yields

$$\begin{aligned} \|y_n-p\| &\leq (L^2 - (L^2-1)\beta_n)\|x_n-p\| \quad (2.10) \\ &\leq L^2\|x_n-p\|, \end{aligned}$$

and consequently from (2.7), we obtain

$$\|y_n - x_{n+1}\| \leq \big((1-\beta_n)(1+L^2) + (1-\alpha_n)(1+L^3)\big)\|x_n-p\|.$$

Hence from (2.6), we obtain

$$||x_{n+1} - p|| \leq \left(1 + L\left((1-\beta_n)(1+L^2) + (1-\alpha_n)(1+L^3)\right)\right)\|x_n - p\|.$$

So, from the above discussion, by using the conditions (i), (ii) and Lemma 2, we can conclude that the sequence $\{x_n - p\}_{n\geq 1}$ is bounded. Since T is Lipschitzian, so $\{Tx_n - p\}_{n\geq 1}$ is also bounded.

Moreover, by (2.9) $\{z_n - p\}_{n\geq 1}$ is bounded. Thus $\{Tz_n - p\}_{n\geq 1}$ is also bounded.

Also by (2.9)

$$\begin{aligned}
\|x_n - y_n\| &= (1-\beta)_n ||x_n - Tz_n|| \\
&\leq (1-\beta)_n \left(||x_n - p|| + ||Tz_n - p||\right) \\
&\leq (1-\beta)_n \left(||x_n - p|| + L||z_n - p||\right) \\
&\leq (1-\beta)_n \left(1 + L^2\right) ||x_n - p|| \\
&\leq \left(1 + L^2\right) ||x_n - p||.
\end{aligned}$$

So $\{x_n - y_n\}_{n\geq 1}$ is bounded.

On the other hand, since

$$\|y_n - p\| \leq \|y_n - x_n\| + ||x_n - p||,$$

$\{y_n - p\}_{n\geq 1}$ is bounded. Therefore $\{Ty_n - p\}_{n\geq 1}$ is also bounded.

Put $M = \max\left\{\sup_{n\geq 1}\|x_n - p\|,\ \sup_{n\geq 1}\|Tx_n - p\|,\ \sup_{n\geq 1}\|Ty_n - p\|\right\}$.

Now from Lemma 1 for all $n \geq 1$, we obtain

$$\begin{aligned}
\|x_{n+1} - p\|^2 &= ||\alpha_n x_n + (1-\alpha_n)Ty_n - p||^2 \\
&= ||\alpha_n(x_n - p) + (1-\alpha_n)(Ty_n - p)||^2 \\
&\leq \alpha_n^2 ||x_n - p||^2 + 2(1-\alpha_n)\langle Ty_n - p, j(x_{n+1} - p)\rangle \\
&= \alpha_n^2 ||x_n - p||^2 + 2(1-\alpha_n)\langle Tx_{n+1} - p,\ j(x_{n+1} - p)\rangle \\
&\quad +2(1-\alpha_n)\langle Ty_n - Tx_{n+1}, j(x_{n+1} - p)\rangle \\
&\leq \alpha_n^2 ||x_n - p||^2 + 2k(1-\alpha_n)||x_{n+1} - p||^2 \\
&\quad +2(1-\alpha_n)\|Ty_n - Tx_{n+1}\|\,\|x_{n+1} - p\| \\
&\leq \alpha_n^2 ||x_n - p||^2 + 2k(1-\alpha_n)||x_{n+1} - p||^2 \\
&\quad +4M^2(1-\alpha_n),
\end{aligned}$$

which implies that

$$\|x_{n+1} - p\|^2 \leq \frac{\alpha_n^2}{1 - 2k(1-\alpha_n)}||x_n - p||^2 + \frac{4M^2(1-\alpha_n)}{1-2k(1-\alpha_n)}. \tag{2.11}$$

By (ii), $\lim_{n\to\infty} \beta_n = 1$, thus there exists $n_0 \in \mathbb{N}$ such that for $n \geq n_0$,

$$\beta_n \leq \frac{\theta}{\eta}; 0 < \theta < 1. \tag{2.12}$$

Hence from condition (ii), (2.5), (2.11) and (2.12) we get

$$\begin{aligned} \|x_{n+1}-p\|^2 &\leq \alpha_n^2\eta\|x_n-p\|^2+4\eta M^2(1-\alpha_n) \\ &\leq \theta\|x_n-p\|^2+4\eta M^2(1-\alpha_n). \end{aligned} \tag{2.13}$$

For all $n \geq 1$, put

$$\begin{aligned} a_n &= \|x_n-p\|, \\ b_n &= 4\eta M^2(1-\alpha_n), \\ q &= \theta, \end{aligned}$$

then according to Lemma 3, we obtain from (2.13) that

$$\lim_{n\to\infty}\|x_n-p\|=0,$$

which completes the proof. □

Corollary 2.5. *Let K be a nonempty closed convex subset of a real Hilbert space E and $T: K \to K$ a Lipschitz strongly pseudocontractive mapping. Let $\{\alpha_n\}_{n\geq1}, \{\beta_n\}_{n\geq1}$ and $\{\gamma_n\}_{n\geq1}$ be sequences in $[0,1]$ such that (i) $\sum_{n\geq1}(1-\alpha_n) < \infty$ and (ii) $\alpha_n \leq \beta_n$ for $n \in \mathbb{N}$. For given $x_1 \in K$, let $\{x_n\}_{n\geq1}$ be iteratively defined by*

$$\begin{aligned} z_n &= \gamma_n x_n+(1-\gamma_n)Tx_n, \\ y_n &= \beta_n x_n+(1-\beta_n)Tz_n, \\ x_{n+1} &= \alpha_n x_n+(1-\alpha_n)Ty_n,\ n\geq1. \end{aligned}$$

Then the sequence $\{x_n\}_{n\geq1}$ converges strongly to the unique fixed point of T.

The proof of the following result is the same as the Proof of Theorem 4.

Theorem 2.6. *Let K be a nonempty closed convex subset of a real Banach space E and $T, S, H: K \to K$ Lipschitz strongly pseudocontractive mappings such that $F(T)\cap F(S)\cap F(H)$ is nonempty. Let $\{\alpha_n\}_{n\geq1}, \{\beta_n\}_{n\geq1}$ and $\{\gamma_n\}_{n\geq1}$ be sequences in $[0,1]$ such that (i) $\sum_{n\geq1}(1-\alpha_n) < \infty$ and (ii) $\alpha_n \leq \beta_n$ for $n \in \mathbb{N}$. For given $x_1 \in K$, let $\{x_n\}_{n\geq1}$ be iteratively defined by*

$$\begin{aligned} z_n &= \gamma_n x_n+(1-\gamma_n)Tx_n, \\ y_n &= \beta_n x_n+(1-\beta_n)Tz_n, \\ x_{n+1} &= \alpha_n x_n+(1-\alpha_n)Ty_n,\ n\geq1. \end{aligned}$$

Then the sequence $\{x_n\}_{n\geq1}$ converges strongly to the common fixed point of T, S and H.

Corollary 2.7. *Let K be a nonempty closed convex subset of a real Hilbert space E and $T, S, H: K \to K$ Lipschitz strongly pseudocontractive mappings such that $F(T)\cap F(S)\cap F(H)$ is nonempty. Let $\{\alpha_n\}_{n\geq1}, \{\beta_n\}_{n\geq1}$ and $\{\gamma_n\}_{n\geq1}$*

be sequences in $[0,1]$ *such that (i)* $\sum_{n\geq 1}(1-\alpha_n)<\infty$*, (ii)* $\alpha_n \leq \beta_n$ *for* $n \in \mathbb{N}$*. For given* $x_1 \in K$*, let* $\{x_n\}_{n\geq 1}$ *be iteratively defined by*

$$\begin{aligned} z_n &= \gamma_n x_n + (1-\gamma_n)Hx_n, \\ y_n &= \beta_n x_n + (1-\beta_n)Sz_n, \\ x_{n+1} &= \alpha_n x_n + (1-\alpha_n)Ty_n,\ n \geq 1. \end{aligned}$$

Then the sequence $\{x_n\}_{n\geq 1}$ *converges strongly to the common fixed point of* T, S *and* H*.*

Remark 3. It is worth to mention that,

1. The results of Chidume[3] and Haiyun and Yuting [6] depend upon the geometry of the Banach space, where as in our case we do not need such geometry.
2. We remove the boundedness assumption on K introduced both in [3] and [6].
3. We remove the assumption $T(K)$ on the mapping T in [10-13].

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