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Analysis of Contact Force in Eddy-current System Using the Virtual Air-Gap Concept

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Abstract – It is difficult to calculate the magnetic force of an object of magnetic material in contact with other objects using the existing methods, such as Maxwell stress tensor method, magnetic charge method, or magnetizing current method. These methods are applicable for force computation only when the object is surrounded by air. The virtual air-gap concept has been proposed for calculating the contact force. However, its application is limited to magneto-static system. In this paper, we present the virtual air-gap concept for contact surface force in the eddy-current system. Its validity and usefulness are shown by comparison between numerical and experimental examples.

Keywords: Magnetic contact force, Maxwell stress tensor, Finite element analysis, Virtual air-gap.

1. Introduction

The magnetic force is an important design factor for the mechanical structural design of electric power apparatus. Large magnetic force on a part of the system can cause mechanical problems such as deformation, vibration, noise, and even fracture, either in the part, or in neighboring parts. To avoid such mechanical problems, accurate analysis of the magnetic contact force will help in designing and manufacturing of the systems [1-4].

Maxwell stress tensor method, magnetic charge method and magnetizing current method are typical ways to calculate the magnetic force. These methods are effective only when calculating the total magnetic force of an isolated object that is surrounded by air. Since the integration path for calculating the force should be taken in the air, it is difficult to compute the force between contacting objects using such existing methods. To solve this problem, the virtual air-gap concept was introduced, and it has only been used for magnetic systems that have large flux leakage without eddy-current [5].

This paper presents an analysis method for the contact force in two different electromagnet models, the I-model and the E-model, in both magneto-static and eddy-current systems; and its effectiveness is verified by experiment.

2. Maxwell Stress with the Virtual Air-gap

In Fig. 1, there is no air-gap between material 1 and material 2. However, we can imagine an air-gap, which

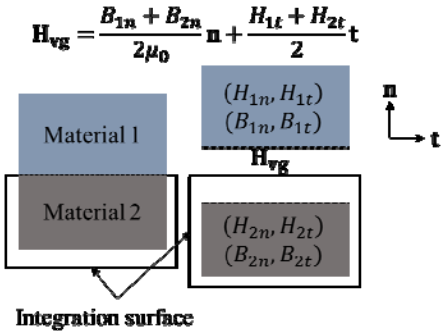

Fig. 1. The magnetic field with virtual air-gap concept continuity, respectively.

exists on the contact surface with distance 0[mm]. That is the basic concept of the virtual air-gap.

The field intensity inside the virtual air-gap is derived as

$$\mathbf{H}_{\text{vg}} = H_n\mathbf{n} + H_t\mathbf{t} = \frac{\mathbf{H}_{\text{vg\_from1}} + \mathbf{H}_{\text{vg\_form2}}}{2} \tag{1}$$

In the above expression, $\mathbf{H}_{\text{vg\_from1}}$ and $\mathbf{H}_{\text{vg\_from2}}$ are, respectively,

$$\mathbf{H}_{\text{vg\_from1}} = \frac{B_{1n}}{\mu_0}\mathbf{n} + H_{1t}\mathbf{t} \tag{2}$$

and

$$\mathbf{H}_{\text{vg\_from2}} = \frac{B_{2n}}{\mu_0}\mathbf{n} + H_{2t}\mathbf{t} \tag{3}$$

where, 1 and 2 are the material number's, $\mathbf{n}$ and $\mathbf{t}$ are a normal and a tangential unit vector, and μ_0 is the permeability in the air. B_{1n} and H_{1t} are the normal component of the magnetic flux density and tangential component of magnetic field intensity in the material 1, respectively. B_{2n} and H_{2t} are the normal component of

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the flux density and tangential component of the field intensity in material 2. So, the field intensity in the virtual air-gap $\mathbf{H}_{\mathrm{vg}}$ is

$$\mathbf{H}_{\mathrm{vg}} = \frac{B_{1n} + B_{2n}}{2\mu_0}\mathbf{n} + \frac{H_{1t} + H_{2t}}{2}\mathbf{t}\,. \tag{4}$$

The virtual air-gap gap field that satisfies the boundary conditions comes from each field in the material 1 and the material 2. In other words, $\mathbf{B_n}$ and $\mathbf{H_t}$ should satisfy the continuity, respectively.

$$\mathbf{B}_{\mathrm{vg}} = \mu_0 \mathbf{H}_{\mathrm{vg}} \tag{5}$$

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H} + \mathbf{B}_{\mathrm{r}}$$

Where,

$$\mathbf{B}_{\mathrm{r}} = \mu_0 \mathbf{M}_0 \tag{6}$$

$$B_n = \mu_0 \mu_r H_n + \mu_0 M_{0n} \quad \text{and} \quad H_t = \frac{B_t - B_{rt}}{\mu_0 \mu_r} \tag{7}$$

where, $\mathbf{B}_{\mathrm{r}}$ is the residual flux density, and $\mathbf{M}_0$ is the magnetization of permanent magnet materials [5-7].

Therefore, the Maxwell stress on the contact surface in magnetic system is

$$\begin{aligned}\boldsymbol{\sigma}_{\mathbf{svg}} &= \frac{1}{\mu_0}\mathbf{B}_{\mathrm{vg}}\left(\mathbf{B}_{\mathrm{vg}} \cdot \mathbf{n}\right) - \frac{1}{2\mu_0}\left|\mathbf{B}_{\mathrm{vg}}\right|^2 \mathbf{n} \\ &= \frac{1}{2\mu_0}\left(B_{vgn}^{\ 2} - B_{vgt}^{\ 2}\right)\mathbf{n} + \frac{1}{\mu_0}B_{vgn}B_{vgt}\mathbf{t}\,.\end{aligned} \tag{8}$$

The above equation is the same form as normal Maxwell stress. However, the magnetic field that is used to calculate stress tensor is not a magnetic field in the materials, but the virtual air-gap field. When the integration surface is with the air, the virtual air-gap field is the same with as the magnetic field at that surface.

3. Numerical Analysis and Experiment of Electromagnet Models

Electromagnet models are divided into I-model and E-model according to flux leakage. In contrast to E-model, the large flux leakage occurs in I-model. Two models are designed, and its numerical analysis and experiment are carried out in two systems. One is magneto-static system and the other is eddy-current system.

3.1 Numerical analysis

In magneto-static system, the input current, 4[A]× 907 [turns], is applied to I-model, and the force is calculated in the range of 0~5[mm]. In E-model analysis, the input current is $0.3[\mathrm{A}]\times 83[\mathrm{turns}]$ and the calculation range is 0~1[mm].

In eddy-current system, the peak value of input current is $2.4[\mathrm{A}]\times 907[\mathrm{turns}]$, and the force is calculated with air-gap from 0 to 5[mm] in I-model. The peak value of input current is $0.42[\mathrm{A}]\times 83[\mathrm{turns}]$ and the force is calculated with air-gap from 0 to 1[mm] in E-model. The frequency of eddy-current system is 1[Hz], which is enough to observe the eddy-current.

It is difficult to consider fringing effect in 2D analysis for I-model. So, the 3D analysis should be carried out. However, there is limitation to analyze I-model when the air-gap becomes smaller due to mesh problem. To solve this problem, 3D analysis results under 1[mm] air-gap is obtained using 2D analysis results because the difference between 2D and 3D model depends on only geometry with the same material properties. The ratio coefficient is calculated using 2D and 3D analysis result near 1[mm] air-gap.

In E-model, the field fringing in z direction is much smaller than the field fringing in x direction. So, we only analyze 2D model.

3.1.1 Numerical analysis method

The force is calculated using Maxwell stress in numerical analysis. When the iron block is contact with the electromagnet, the contact force is calculated using two ways, which are only Maxwell stress and Maxwell stress with the virtual air-gap concept, respectively.

Maxwell stress with the virtual air-gap concept in magneto-static model is written as

$$\begin{aligned}\sigma_{svgx} = &-\frac{1}{2}\left(B_{vgx}H_{vgx} + B_{vgy}H_{vgy}\right)n_x \\ &+\left(n_x H_{vgx} + n_y H_{vgy}\right)\times B_{vgx}\end{aligned} \tag{9.1}$$

$$\begin{aligned}\sigma_{svgy} = &-\frac{1}{2}\left(B_{vgx}H_{vgx} + B_{vgy}H_{vgy}\right)n_y \\ &+\left(n_x H_{vgx} + n_y H_{vgy}\right)\times B_{vgy}\end{aligned} \tag{9.2}$$

where σ_{svg} is Maxwell stress tensor with the virtual air-gap concept. B_{vg} and H_{vg} are B and H in the virtual air-gap. n is normal unit vector. x and y in subscript is a direction of vector.

The maximum intensity of magnetic flux density in eddy-current system is less than 0.3[T]. The magnetic permeability of ferrite is almost linear in this resign. So, time-harmonic analysis is used for the eddy-current system. Time average Maxwell stress with virtual air-gap concept in the eddy-current system is as follows:

$$\begin{aligned}\sigma_{svgx} = \Re\{&-\frac{1}{4}\left(B_{vgx}B_{vgx}^{\ *} + B_{vgy}B_{vgy}^{\ *}\right)n_x \\ &+\frac{1}{2}\left(n_x H_{vgx} + n_y H_{vgy}\right)B_{vgx}^{\ *}\}\end{aligned} \tag{10.1}$$

$$\sigma_{svg\,y} = \Re\{-\frac{1}{4}\left(B_{vg\,x}B_{vg\,x}^{*} + B_{vg\,y}B_{vg\,y}^{*}\right)n_y + \frac{1}{2}\left(n_x H_{vg\,x} + n_y H_{vg\,y}\right)B_{vg\,y}^{*}\} \quad (10.2)$$

where $\Re$ is real function and '$*$' is conjugate operator.

3.1.2 Geometry and material properties of the numerical models

The core in electromagnet is ferrite and the magnetic material, the iron block, is S18. Size and shape of analysis models are shown in Fig. 2. The conductivities of ferrite and S18 are 0[S/m] and 6.17×10^{6}[S/m] and its B-H curve are shown in Fig. 3.

3.2 Experiment

In the experiment of the magneto-static system, the minimum distance to measure the magnetic force exists between the iron block and the electromagnet. When the distance is smaller than 0.05[mm], the iron block contacts partially with the electromagnet due to the inhomogeneity of material surface. So, it is difficult to measure the magnetic force acting on the iron block using the push-pull gauge.

In the eddy-current system, the minimum distance is changed to 0.06[mm] due to the vibration of the iron block.

The input current in each experiment is the same with the numerical analysis.

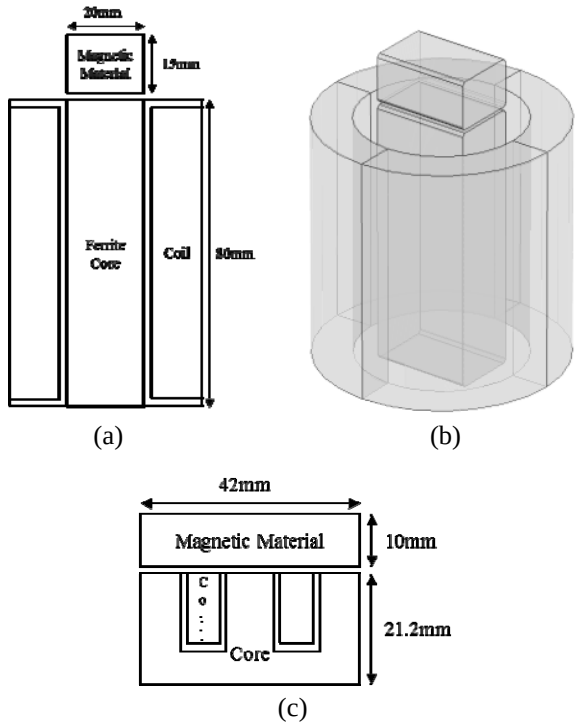


Fig. 2. The (a) I-model for 2D analysis, (b) I-model for 3D analysis and (c) E-model for 2D analysis

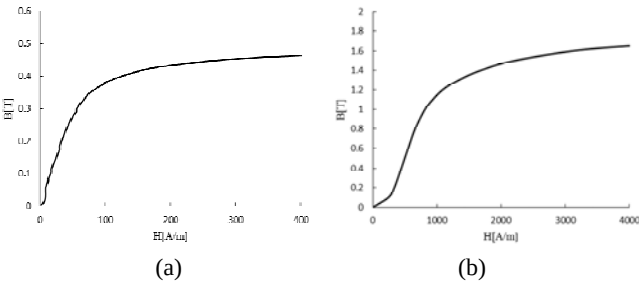


Fig. 3. (a) B-H curve for ferrite and (b) B-H curve for S18

3.2.1 Measuring equipment for magnetic force

The magnetic force is measured using a push-pull gauge, which has a –50~50[N] measuring range, 0.01[N] accuracy, and 0.02[s] sampling time. The jig to hold the electro-magnet and the magnetic material is MC-nylon, which is an inelastic, dielectric and non-magnetic material. The motor controlled gauge stand can move 30~300 [mm/min]. All bolts and nuts are non-magnetic material. The power supply is controllable 0~150[V] with 0.1[V] step and 0~60[Hz] with 1[Hz] step.

3.2.2 Geometry and material properties of the experimental models

In Fig. 5(a), I-model, the iron (S18) block size is $20\times30\times15$[mm], the magnetic core (Ferrite) size is $20\times30\times80$[mm] and the electromagnet is bounded with 907[turns] coil.

Fig. 4. Measuring equipment for magnetic force

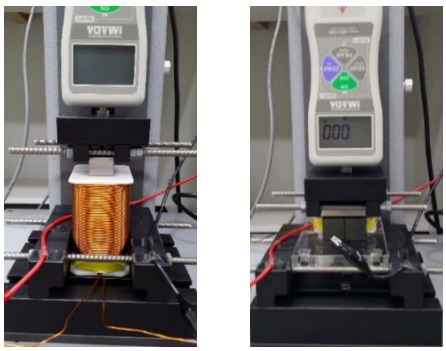


Fig. 5. The (a) I-model and (b) E-model

In Fig. 5(b), E-model, the iron block size is 42× 40× 10[mm], the magnetic core size is 42× 40× 21.2[mm] and the electromagnet is bounded with 83[turns] of coil.

All materials used in experiments and numerical analysis are the same.

3.2.3 Experimental procedures

The experiment in magneto-static system is carried out. The iron block and the electromagnet are fixed at the push-pull gauge and the stand using jig. The push-pull gauge is set zero and enter DC input. The iron block goes down slowly using the electric stand. Regarding the iron block is contact with the electromagnet, when the push force shown in the push-pull gauge. While the iron block rise and lower 0.5[mm/s] velocity, the pull force is recorded in every 0.02 [s]. This process is repeated and calculate the average value at each distance.

The experiment in eddy-current system is as follows. The geometrical setting is the same as for magneto-static experiment. After the zero point is set, the AC input is entered.

It is difficult to measure the force varying with the distance when the force also varies with time. So, the force is measured during 7[s] when the iron block is held at each position. The time average force at each position is calculated using the measured data except for the first and the last second one.

4. Results and Discussions

Compare experimental results and numerical results, we show the numerical models are reflected the experimental models well. And compare the contact force computed by the Maxwell stress with the virtual air-gap concept $\mathbf{F_{vg}}$, the contact force computed by Maxwell stress only $\mathbf{F_c}$ and the force at numerical minimum distance $\mathbf{F_{mn}}$. Through this, we show the validity of virtual air-gap concept.

The distance between the iron block and the electro-magnet is *d*.

4.1 The I-model

The I-model has large flux leakage. So, in this model, the magnetic force slowly increases when the iron block approaches the electromagnet.

4.1.1 The I-model in magneto-static system

In Fig. 6, the magnetic force is 9.30[N] when *d* is 1[mm] in the experiment. The magnetic force is 2.85[N] in 2D model and 11.56[N] in 3D model at the same *d*. The force tendency between 3D nuamerical results and experimental results is similar.

When the distance is near 1[mm], the ratio between model and 3D model is chosen as 4, which is based on the analysis results.

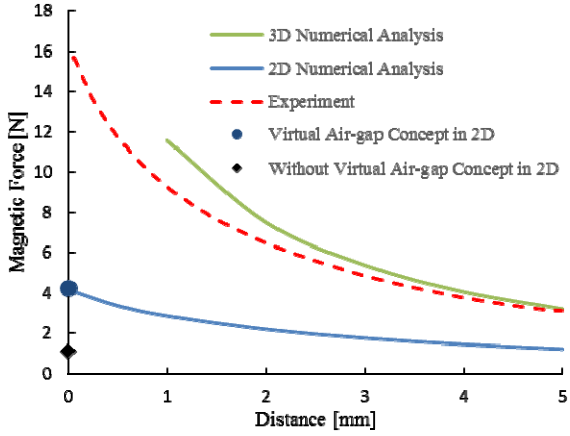


Fig. 6. The magnetic force varying with distance in magneto-static system for the I-model

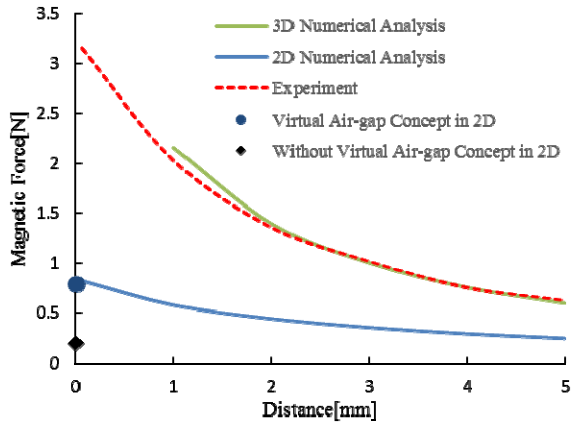


Fig. 7. The magnetic force varying with distance in eddy-current system for the I-model

The 3D analysis results calculated using the 2D analysis results are $\mathbf{F_{vg}}$ =16.17[N], $\mathbf{F_c}$ =4.41[N] and $\mathbf{F_{mn}}$ =16.32 [N]. The force, $\mathbf{F_{me}}$, at minimum distance is 16.33[N] in the experiment.

4.1.2 The I-model in eddy-current system

In Fig.7, the time average magnetic force is 2.03[N] in the experiments when *d* is 1[mm]. The force is 0.58[N] in the 2D model and 2.15[N] in the 3D model at same distance.

The ratio between 2D and 3D analysis is also 4 in eddy-current system. Considering the ratio, the 3D analysis results are $\mathbf{F_{vg}}$ =3.28[N], $\mathbf{F_c}$ =0.79[N], and $\mathbf{F_{mn}}$ =3.28[N]. The magnetic force, $\mathbf{F_{me}}$, at minimum distance is 3.15[N].

4.2 The E-model

The small air-gap makes large field change in the E-model. So, the magnetic force rapidly increases when the iron block approaches the electromagnet.

4.2.1 The E-model in magneto-static system

In Fig. 8, the force at experimental minimum distance, 0.05[mm], is 17.55[N] in the experiments. At the same distance, the magnetic force is 16.39[N] in the numerical analysis result. The magnetic force $\mathbf{F_{mn}}$ is 68.76[N] at the numerical minimum distance, 0.01[mm]. The virtual air-gap force, $\mathbf{F_{vg}}$, is 85.50[N] and the contact force, $\mathbf{F_c}$, without the virtual air-gap is 0.19[N].

4.2.2 The E-model in eddy-current system

In Fig. 9, the force at experimental minimum distance, 0.06[mm], is 9.11[N] in the experiments. At same distance, the magnetic force is 9.28[N] by numerical analysis. Where the numerical minimum distance, the magnetic force $\mathbf{F_{mn}}$ is 66.8[N]. The virtual air-gap force, $\mathbf{F_{vg}}$, is 84.5[N] and the contact force, $\mathbf{F_c}$, is 0.11[N].

4.3 Discussions

The numerical analysis results agree with the experimental ones. The virtual air-gap concept is verified comparing the three forces, which are the magnetic contact force with the virtual air-gap, the magnetic contact force without the virtual air-gap and the magnetic force at numerical minimum distance.

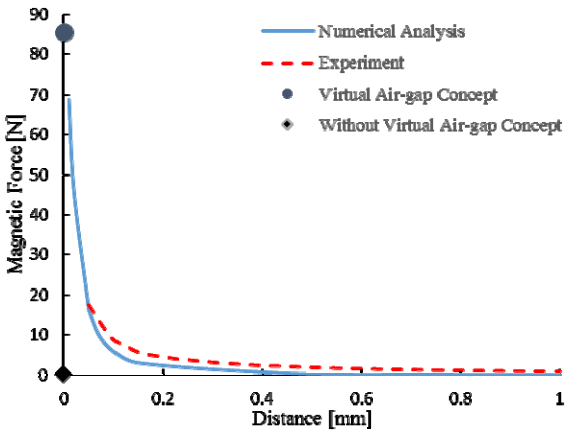


Fig. 8. The magnetic force varying with distance in magneto-static system for the E-model

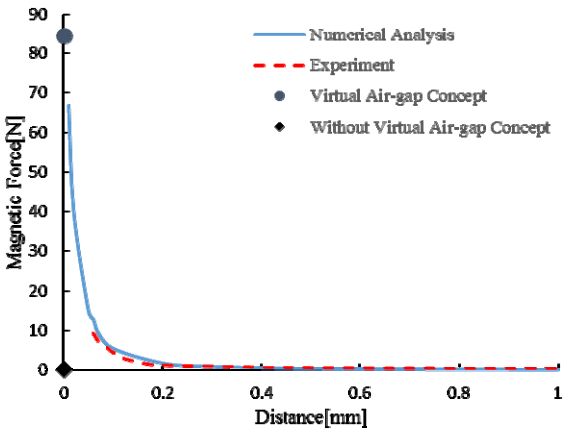


Fig. 9. The magnetic force varying with distance in eddy-current system for the E-model

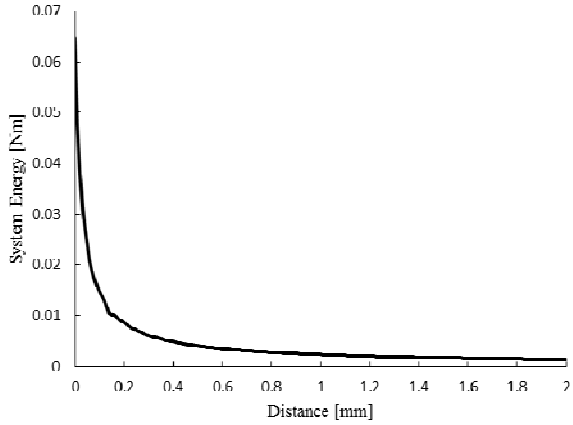


Fig. 10. E-model system energy in eddy-current system according to distance

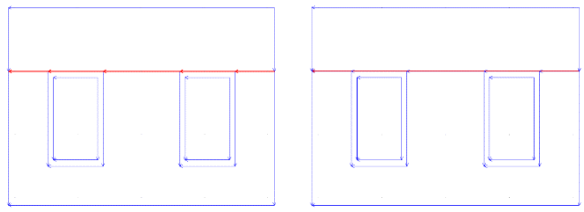
4.3.1 The system energy and the continuity of the force

Force can be defined as the differential of the system energy. If the system energy is differentiable, the force should be continuous. In Fig. 10, the system energy in eddy-current system, E-model, is shown at the gap. The system energy changes smoothly until the iron block is in contact with the electromagnet. The magnetic force is continuous in this model. However, the Maxwell stress without the virtual air-gap concept yields discontinuous results.

4.3.2 Magnetic field and virtual air-gap field

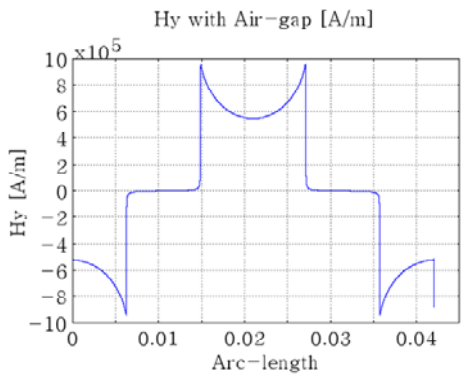
Fig. 11 shows the E-model in eddy-current system, and the red lines in each figure is the surface of the iron block. There is a 0.01[mm] air-gap between the iron block and the electromagnet in Fig. 10(a), while the iron block is in contact with the electromagnet in Fig. 10(b).

Fig. 12(a) shows the normal component of the magnetic field when the air-gap is 0.01[mm]. And Fig. 12(b) shows the magnetic field when the iron block is in contact with the electromagnet. The maximum value of magnetic field in each states are 9×10^5[A/m] and 1.7×10^4[A/m]. The former is about sixty times stronger.

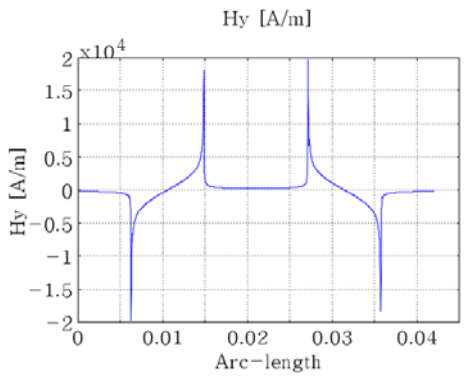


(a) (b)

Fig. 11. The (a) surface of the iron block when the air-gap is 0.01[mm] and (b) the contact surface



(a)



(b)

Fig. 12. Magnetic field on (a) surface of the iron block when the air-gap is 0.01[mm] and (b) the contact surface

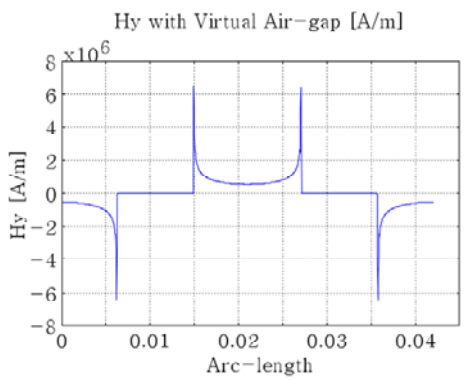


Fig. 13. Virtual air-gap field on contact surface

The magnetic force is normally proportional to the square of the field intensity. Therefore, this field differences make a discontinuous force. This result shows that the magnetic field is not continuous at the boundary of different materials. However, the magnetic force should be continuous in this system.

Fig. 13 shows the virtual air-gap field on the contact surface. Its field tendency is similar to the field when the air-gap is 0.01[mm]. Concretely, the maximum value of magnetic field, 6×10^6[A/m], is sixth times stronger than the magnetic field at 0.01[mm] air-gap. However, when the edge effect is ignored, the virtual air-gap field is only 1.2 times stronger than the magnetic field with minimum gap.

5. Conclusion

In the analysis models, the magnetic force increases as the gap between the electromagnet and the iron block becomes smaller. The force varies continuously when the system energy change smooth. However, the discontinuity occurs in the analysis results obtained using only the existing Maxwell stress tensor when the contact force is compared with the force with the minimum gap. It comes from the Maxwell stress which is only defined in air.

In contrasts, the contact force obtained using the Maxwell stress with virtual air-gap concept increases continuously as the gap becomes smaller.

Therefore, the virtual air-gap concept is a tool that should be used to calculate the contact force for not only magneto-static systems but also eddy-current systems in various shapes.

Acknowledgements

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