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Exposed Bilinear Forms of $\mathcal{L}(^2d_*(1,w)^2)$

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Abstract. First we present the explicit formula for the norm of a (continuous) linear functional of $\mathcal{L}(^2d_*(1,w)^2)^*$. Using this formula and results of [16] and [17], we show that every extreme bilinear form of the unit ball of $\mathcal{L}(^2d_*(1,w)^2)$ is exposed.

1. Introduction

We write B_E for the closed unit ball of a real Banach space E and the dual space of E is denoted by E^*. $x \in B_E$ is called an *extreme point* of B_E if $y, z \in B_E$ with $x = \frac{1}{2}(y+z)$ implies $x = y = z$. $x \in B_E$ is called an *exposed point* of B_E if there is a $f \in E^*$ so that $f(x) = 1 = \|f\|$ and $f(y) < 1$ for every $y \in B_E \setminus \{x\}$. It is easy to see that every exposed point of B_E is an extreme point. We denote by $expB_E$ and $extB_E$ the sets of exposed and extreme of B_E, respectively. For $n \geq 2$, we denote by $\mathcal{L}(^nE)$ the Banach space of all continuous n-linear forms on E endowed with the norm $\|T\| = \sup_{\|x_k\|=1, 1\leq k\leq n} |T(x_1, \cdots, x_n)|$. $\mathcal{L}_s(^nE)$ denotes the subspace of all continuous symmetric n-linear forms on E. A mapping $P : E \to \mathbb{R}$ is a continuous n-homogeneous polynomial if there exists $T \in \mathcal{L}_s(^nE)$ such that $P(x) = T(x, \cdots, x)$ for every $x \in E$. We denote by $\mathcal{P}(^nE)$ the Banach space of all continuous n-homogeneous polynomials from E into $\mathbb{R}$ endowed with the norm $\|P\| = \sup_{\|x\|=1} |P(x)|$. For more details about the theory of multilinear mappings and polynomials on a Banach space, we refer to [7]. We will denote by $T((x_1, y_1), (x_2, y_2)) = ax_1x_2 + by_1y_2 + \frac{c}{2}(x_1y_2 + x_2y_1)$ and $P(x, y) = ax^2 + by^2 + cxy$ a symmetric bilinear form and a 2-homogeneous polynomial on a real Banach space of dimension 2 respectively. For $1 \leq p \leq \infty$, we let $l_p^2 = \mathbb{R}^2$ with the l_p-norm. Note that in ([6], Theorem 1, remark after Theorem 1, and Theorem 2) the following

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results are proved: (i) $expB_{\mathcal{P}(^2l_1^2)} = extB_{\mathcal{P}(^2l_1^2)} \backslash \{\pm(x^2 - y^2 \pm 2xy)\}$;

(ii) $expB_{\mathcal{P}(^2l_\infty^2)} = extB_{\mathcal{P}(^2l_\infty^2)} \backslash \{\pm(\frac{1}{2}x^2 - \frac{1}{2}y^2 \pm xy)\}$.

The author [11] characterized $expB_{\mathcal{P}(^2l_p^2)}$ as follows: (i) If $1 < p < 2$, then $expB_{\mathcal{P}(^2l_p^2)} = extB_{\mathcal{P}(^2l_p^2)}$;

(ii) If $2 < p < \infty$, then $expB_{\mathcal{P}(^2l_p^2)} = extB_{\mathcal{P}(^2l_p^2)} \backslash \{\pm x^2, \pm y^2)\}$.

We refer to ([1–6, 8–22] and references therein) for some recent work about extremal properties of multilinear mappings and homogeneous polynomials on some classical Banach spaces.

We denote the 2-dimensional real predual of the Lorentz sequence space with a positive weight $0 < w < 1$ by

$$d_*(1,w)^2 := \{(x,y) \in \mathbb{R}^2 : \|(x,y)\|_{d_*} := \max\{|x|,\ |y|,\ \frac{|x|+|y|}{1+w}\ \}.$$

Very recently, the author [16] characterizes the extreme points of the unit ball of $\mathcal{L}(^2d_*(1,w)^2)$. The author [17] also proves that every extreme symmetric bilinear form of the unit ball of $\mathcal{L}_s(^2d_*(1,w)^2)$ is exposed.

In this paper we first present the explicit formula for the norm of a (continuous) linear functional of $\mathcal{L}(^2d_*(1,w)^2)^*$. Using the explicit formula for the norm of a (continuous) linear functional of $\mathcal{L}(^2d_*(1,w)^2)^*$ and results of [16] and [17], we prove that every extreme bilinear form of the unit ball of $\mathcal{L}(^2d_*(1,w)^2)$ is exposed.

2. The Results

If $T \in \mathcal{L}(^2d_*(1,w)^2)$, then $T((x_1,y_1),\ (x_2,y_2)) = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1$ for some reals a, b, c, d.

Theorem 2.1.([16], Theorem 2.1) *Let* $T((x_1,y_1),\ (x_2,y_2)) = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1 \in \mathcal{L}(^2d_*(1,w)^2)$. *Then there exists (unique)* $T^{'}((x_1,y_1),\ (x_2,y_2)) = a^*x_1x_2 + b^*y_1y_2 + c^*x_1y_2 + d^*x_2y_1 \in \mathcal{L}(^2d_*(1,w)^2)$ *such that* $a^*, b^*, c^*, d^* \in \{\pm a, \pm b, \pm c, \pm d\}$ *with* $a^* \geq b^* \geq 0, c^* \geq |d^*|$ *and* $\|T\| = \|T^{'}\|$ *and that* T *is extreme if and only if* $T^{'}$ *is extreme.*

Theorem 2.2.([16], Theorem 2.2) *Let* $T((x_1,y_1),\ (x_2,y_2)) = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1 \in \mathcal{L}(^2d_*(1,w)^2)$ *with* $a \geq |b|, c \geq |d|$. *Then*

$$\|T\| = \max\{a+bw^2+(c+d)w, a-bw^2+(c-d)w, (a+b)w+c+dw^2, (a-b)w+c-dw^2\}.$$

Theorem 2.3.([16], Theorem 2.4) *Let* $S((x_1,y_1),\ (x_2,y_2)) = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1 \in \mathcal{L}(^2d_*(1,w)^2)$ *with* $a \geq b \geq 0, c \geq |d|$. *Then*

(a) *Let* $w < \sqrt{2}-1$. *S is extreme if and only if*

$$S \in \{x_1x_2, x_1y_2, \frac{1}{1+w}(x_1x_2+x_1y_2), \frac{1}{(1+w)^2}(x_1x_2+y_1y_2+x_1y_2+x_2y_1),$$
$$\frac{1}{1+w^2}(x_1x_2+y_1y_2+wx_1y_2-wx_2y_1), \frac{1}{1+w^2}(wx_1x_2+wy_1y_2+x_1y_2-x_2y_1),$$
$$\frac{1}{1+2w-w^2}(x_1x_2+y_1y_2+x_1y_2-x_2y_1),$$
$$\frac{1}{(1+w)^2(1-w)}(x_1x_2+y_1y_2+(1-w-w^2)x_1y_2-wx_2y_1),$$
$$\frac{1}{(1+w)^2(1-w)}((1-w-w^2)x_1x_2+wy_1y_2+x_1y_2-x_2y_1)\}.$$

(b) *Let* $w = \sqrt{2}-1$. *Then S is extreme if and only if*

$$S \in \{x_1x_2, x_1y_2, \frac{1}{\sqrt{2}}(x_1x_2+x_1y_2), \frac{1}{2}(x_1x_2+y_1y_2+x_1y_2+x_2y_1),$$
$$\frac{\sqrt{2}}{4}((\sqrt{2}+1)(x_1x_2+y_1y_2)+x_1y_2-x_2y_1),$$
$$\frac{\sqrt{2}}{4}(x_1x_2+y_1y_2+(\sqrt{2}+1)(x_1y_2-x_2y_1))\}.$$

(c) *Let* $w > \sqrt{2}-1$. *Then S is extreme if and only if*

$$S \in \{x_1x_2, x_1y_2, \frac{1}{1+w}(x_1x_2+x_1y_2), \frac{1}{(1+w)^2}(x_1x_2+y_1y_2+x_1y_2+x_2y_1),$$
$$\frac{1}{1+2w-w^2}(x_1x_2+y_1y_2+x_1y_2-x_2y_1),$$
$$\frac{1}{1+w^2}(\frac{1-w}{1+w}(x_1x_2+y_1y_2)+x_1y_2-x_2y_1),$$
$$\frac{1}{1+w^2}(x_1x_2+y_1y_2+\frac{1-w}{1+w}(x_1y_2-x_2y_1)),$$
$$\frac{1}{2+2w}(x_1x_2+y_1y_2+(2+w)x_1y_2-\frac{1}{w}x_2y_1),$$
$$\frac{1}{2+2w}((2+w)x_1x_2+\frac{1}{w}y_1y_2+x_1y_2-x_2y_1)\}.$$

Theorem 2.4.([16], Theorem 2.5) $T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ *if and only if there exist* $n \in \mathbb{N}$ *and* $S((x_1,y_1),(x_2,y_2))$ $= ax_1x_2+by_1y_2+cx_1y_2+dx_2y_1 \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ *with* $a \geq |b|, c \geq |d|$ *such that* $T((x_1,y_1),(x_2,y_2)) := S((u_1^{(n)},v_1^{(n)}),(u_2^{(n)},v_2^{(n)}))\circ\cdots\circ((u_1^{(1)},v_1^{(1)}),(u_2^{(1)},v_2^{(1)}))$, *where*

$$\text{for } j=1,\ldots,n, ((u_1^{(j)},v_1^{(j)}),(u_2^{(j)},v_2^{(j)}))\in\{((\pm x_1,\pm y_1),(\pm x_2,\pm y_2)), ((\pm x_2,\pm y_2),(\pm x_1,\pm y_1)),((\pm x_1,\pm y_1),(\pm y_2,\pm x_2)),((\pm y_2,\pm x_2), (\pm x_1,\pm y_1)),((\pm y_1,\pm x_1),(\pm x_2,\pm y_2)),((\pm x_2,\pm y_2),(\pm y_1,\pm x_1)), ((\pm y_2,\pm x_2),(\pm y_1,\pm x_1)),((\pm y_1,\pm x_1),(\pm y_2,\pm x_2))\}.$$

Theorem 2.5. *Let* $f\in\mathcal{L}(^2d_*(1,w)^2)^*$ *and* $\alpha=f(x_1x_2),\beta=f(y_1y_2)$, $\delta=f(x_1y_2),\gamma=f(x_2y_1)$.

(a) *Let* $w<\sqrt{2}-1$. *Then*

$$\begin{aligned}\|f\| \;=\; & \max\{|\alpha|,|\beta|,|\delta|,|\gamma|,\frac{1}{1+w}|\alpha+\gamma|,\frac{1}{1+w}|\alpha+\delta|,\\ & \frac{1}{1+w}|\beta+\gamma|,\frac{1}{1+w}|\beta+\delta|,\frac{1}{(1+w)^2}(|\alpha+\beta|+|\delta+\gamma|),\\ & \frac{1}{(1+w)^2}(|\alpha-\beta|+|\delta-\gamma|),\frac{1}{1+2w-w^2}(|\alpha+\beta|+|\delta-\gamma|),\\ & \frac{1}{1+2w-w^2}(|\alpha-\beta|+|\delta+\gamma|),\frac{1}{1+w^2}(|\alpha+\beta|+w|\delta-\gamma|),\\ & \frac{1}{1+w^2}(|\alpha-\beta|+w|\delta+\gamma|),\frac{1}{1+w^2}(|\delta+\gamma|+w|\alpha-\beta|),\\ & \frac{1}{1+w^2}(|\delta-\gamma|+w|\alpha+\beta|),\\ & \frac{1}{(1+w)^2(1-w)}(|\alpha+\beta|+|(1-w-w^2)\delta-w\gamma|)\\ & \frac{1}{(1+w)^2(1-w)}(|\alpha-\beta|+|(1-w-w^2)\delta+w\gamma|),\\ & \frac{1}{(1+w)^2(1-w)}(|\alpha+\beta|+|(1-w-w^2)\gamma-w\delta|)\\ & \frac{1}{(1+w)^2(1-w)}(|\alpha-\beta|+|(1-w-w^2)\gamma+w\delta|),\\ & \frac{1}{(1+w)^2(1-w)}(|\delta+\gamma|+|(1-w-w^2)\alpha-w\beta|),\\ & \frac{1}{(1+w)^2(1-w)}(|\delta-\gamma|+|(1-w-w^2)\alpha+w\beta|),\\ & \frac{1}{(1+w)^2(1-w)}(|\delta+\gamma|+|(1-w-w^2)\beta-w\alpha|),\\ & \frac{1}{(1+w)^2(1-w)}(|\delta-\gamma|+|(1-w-w^2)\beta+w\alpha|)\}.\end{aligned}$$

(b) *Let* $w = \sqrt{2}-1$. *Then*

$$\begin{aligned}\|f\| &= \max\{|\alpha|,|\beta|,|\delta|,|\gamma|,\frac{1}{\sqrt{2}}|\alpha+\gamma|,\frac{1}{\sqrt{2}}|\alpha+\delta|,\frac{1}{\sqrt{2}}|\beta+\gamma|,\\ &\quad \frac{1}{\sqrt{2}}|\beta+\delta|,\frac{\sqrt{2}}{4}((\sqrt{2}+1)|\alpha+\beta|+|\delta-\gamma|),\\ &\quad \frac{\sqrt{2}}{4}((\sqrt{2}+1)|\delta+\gamma|+|\alpha-\beta|),\frac{\sqrt{2}}{4}((\sqrt{2}+1)|\alpha-\beta|+|\delta+\gamma|),\\ &\quad \frac{\sqrt{2}}{4}(|\alpha+\beta|+(\sqrt{2}+1)|\delta-\gamma|)\}.\end{aligned}$$

(c) *Let* $\sqrt{2}-1 < w$. *Then*

$$\begin{aligned}\|f\| &= \max\{|\alpha|,|\beta|,|\delta|,|\gamma|,\frac{1}{1+w}|\alpha+\gamma|,\frac{1}{1+w}|\alpha+\delta|,\\ &\quad \frac{1}{1+w}|\beta+\gamma|,\frac{1}{1+w}|\beta+\delta|,\frac{1}{(1+w)^2}(|\alpha+\beta|+|\delta+\gamma|),\\ &\quad \frac{1}{(1+w)^2}(|\alpha-\beta|+|\delta-\gamma|),\frac{1}{1+2w-w^2}(|\alpha+\beta|+|\delta-\gamma|),\\ &\quad \frac{1}{1+2w-w^2}(|\alpha-\beta|+|\delta+\gamma|),\frac{1}{1+w^2}(\frac{1-w}{1+w}|\alpha+\beta|+|\delta-\gamma|),\\ &\quad \frac{1}{1+w^2}(|\alpha-\beta|+\frac{1-w}{1+w}|\delta+\gamma|),\frac{1}{2+2w}(|\alpha+\beta|+|(2+w)\delta-\frac{1}{w}\gamma|),\\ &\quad \frac{1}{2+2w}(|\alpha+\beta|+|(2+w)\gamma-\frac{1}{w}\delta|),\frac{1}{2+2w}(|(2+w)\alpha-\frac{1}{w}\beta|+|\delta+\gamma|),\\ &\quad \frac{1}{2+2w}(|(2+w)\beta-\frac{1}{w}\alpha|+|\delta+\gamma|)\}.\end{aligned}$$

Proof. It follows from Theorems 2.3–4 since

$$\|f\| = \sup\{|f(T)| : T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}\}. \qquad \square$$

Theorem 2.6.([17], Theorem 2.3) *Let* E *be a real Banach space such that* $extB_E$ *is finite. Suppose that* $x \in extB_E$ *satisfies that there exists an* $f \in E^*$ *such that* $f(x) = 1 = \|f\|$ *and* $|f(y)| < 1$ *for every* $y \in extB_E \backslash \{x\}$. *Then* $x \in expB_E$.

Using Theorems 2.1–6, we classify the exposed bilinear forms of the unit ball of $\mathcal{L}(^2d_*(1,w)^2)$.

Theorem 2.7. $expB_{\mathcal{L}(^2d_*(1,w)^2)} = extB_{\mathcal{L}(^2d_*(1,w)^2)}$.

Proof. Let $L = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1 \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$. By Theorems 2.1 and 2.5, we may assume that $a \geq |b|, c \geq |d|$. Now we can use Theorem 2.3.

Case 1: $w < \sqrt{2}-1$

Claim: x_1x_2 is exposed

Let $f \in \mathcal{L}(^2d_*(1,w)^2)^*$ and $\alpha = f(x_1x_2), \beta = f(y_1y_2), \delta = f(x_1y_2), \gamma = f(x_2y_1)$. Let $\alpha = 1, \beta = 0 = \delta = \gamma$. By Theorem 2.5(a), $f(x_1x_2) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ with $T \neq x_1x_2$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{1+w}(x_1x_2 + x_1y_2)$ is exposed

Let $\alpha = \frac{1+w}{2} = \delta, \beta = 0 = \gamma$. By Theorem 2.5(a), $f(\frac{1}{1+w}(x_1x_2 + x_1y_2)) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ with $T \neq \frac{1}{1+w}(x_1x_2 + x_1y_2)$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{(1+w)^2}(x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1)$ is exposed

Let $\alpha = \frac{1+w^2}{2}, \beta = \frac{1+w^2}{2} - \epsilon, \delta = w + \frac{\epsilon}{2} = \gamma$ for a sufficiently small $\epsilon > 0$. By Theorem 2.5(a), $f(\frac{1}{(1+w)^2}(x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1)) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ with $T \neq \frac{1}{(1+w)^2}(x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1)$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{1+w^2}(x_1x_2 + y_1y_2 + wx_1y_2 - wx_2y_1)$ is exposed

Let $\alpha = \frac{1}{2} = \beta, \delta = \frac{w}{2}, \gamma = -\frac{w}{2}$. By Theorem 2.5(a), $f(\frac{1}{1+w^2}(x_1x_2 + y_1y_2 + wx_1y_2 - wx_2y_1)) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ with $T \neq \frac{1}{1+w^2}(x_1x_2 + y_1y_2 + wx_1y_2 - wx_2y_1)$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{1+2w-w^2}(x_1x_2 + y_1y_2 + x_1y_2 - x_2y_1)$ is exposed

Let $w < \delta < \frac{1-w^2}{2}$ and $\alpha = \frac{1+2w-w^2-2\delta}{2}, \beta = \alpha, \gamma = -\delta$. By Theorem 2.5(a), $f(\frac{1}{1+2w-w^2}(x_1x_2 + y_1y_2 + x_1y_2 - x_2y_1)) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ with $T \neq \frac{1}{1+2w-w^2}(x_1x_2 + y_1y_2 + x_1y_2 - x_2y_1)$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{(1+w)^2(1-w)}(x_1x_2 + y_1y_2 + (1 - w - w^2)x_1y_2 - wx_2y_1)$ is exposed

Let $\alpha = \frac{1+\epsilon(-1+w+w^2)}{2} = \beta, \delta = w + \epsilon, \gamma = 0.$ for a sufficiently small $\epsilon > 0$. By Theorem 2.5(a), $f(\frac{1}{(1+w)^2(1-w)}(x_1x_2 + y_1y_2 + (1-w-w^2)x_1y_2 - wx_2y_1)) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ with $T \neq \frac{1}{(1+w)^2(1-w)}(x_1x_2 + y_1y_2 + (1 - w - w^2)x_1y_2 - wx_2y_1)$. By Theorem 2.6, it is exposed.

Case 2: $w = \sqrt{2} - 1$

By the similar argument as Case 1, $\pm x_1x_2, \pm\frac{1}{\sqrt{2}}(x_1x_2+x_1y_2), \pm\frac{1}{2}[x_1x_2+y_1y_2+x_1y_2 + x_2y_1]$ are exposed. It is enough to show that $\frac{\sqrt{2}}{4}[x_1x_2 + y_1y_2 + (\sqrt{2} + 1)(x_1y_2 - x_2y_1)]$ is exposed. Let $\alpha = 0 = \beta, \delta = 2 - \sqrt{2} = -\gamma$. By Theorem 2.5(b), $f(\frac{\sqrt{2}}{4}[x_1x_2 - y_1y_2 + (\sqrt{2} + 1)(x_1y_2 + x_2y_1)]) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ with $T \neq \frac{\sqrt{2}}{4}[x_1x_2 - y_1y_2 + (\sqrt{2}+1)(x_1y_2 + x_2y_1)]$. By Theorem 2.6, it is exposed.

Case 3: $\sqrt{2} - 1 < w$

By the similar argument as Case 1, $\pm x_1x_2, \pm\frac{1}{1+w}(x_1x_2 + x_1y_2), \pm\frac{1}{(1+w)^2}[x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1], \pm\frac{1}{1+2w-w^2}[x_1x_2 + y_1y_2 + x_1y_2 - x_2y_1)]$ are exposed.

Claim: $\frac{1}{1+w^2}[x_1x_2 + y_1y_2 + \frac{1-w}{1+w}(x_1y_2 - x_2y_1)]$ is exposed

Let $\alpha = \frac{1+w^2}{2} = -\beta, \delta = \gamma$. By Theorem 2.5(c), $f(\frac{1}{1+w^2}[x_1x_2 + y_1y_2 +$

$\frac{1-w}{1+w}(x_1y_2 - x_2y_1)]) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ with $T \neq \frac{1}{1+w^2}[x_1x_2 + y_1y_2 + \frac{1-w}{1+w}(x_1y_2 - x_2y_1)]$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{2+2w}[(2+w)x_1x_2 + \frac{1}{w}y_1y_2 + (x_1y_2 - x_2y_1)]$ is exposed

Let $\alpha = 1-\epsilon, \beta = w^2, \delta = \frac{\epsilon(2+w)}{2} = -\gamma$ for a sufficiently small $\epsilon > 0$. By Theorem 2.5(c), $f(\frac{1}{2+2w}[(2+w)x_1x_2 + \frac{1}{w}y_1y_2 + (x_1y_2 - x_2y_1)]) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in extB_{\mathcal{L}(^2d_*(1,w)^2)}$ with $T \neq \frac{1}{2+2w}[(2+w)x_1x_2 - \frac{1}{w}y_1y_2 + (x_1y_2 + x_2y_1)]$. By Theorem 2.6, it is exposed. □

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