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Mathematics in the Joseon farmland tax systems

朝鮮의 田制法과 算學

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The Joseon dynasty (1392–1910) is basically an agricultural country and therefore, the main source of her national revenue is the farmland tax. Thus the farmland tax system becomes the most important state affair. The 4th king Sejong establishes an office for a new law of the tax in 1443 and adopts the farmland tax system in 1444 which is legalized in Gyeongguk Daejeon (1469), the complete code of law of the dynasty. The law was amended in the 19th king Sukjong era. Jo Tae-gu mentioned the new system in his book Juseo Gwan-gyeon (1718) which is also included in Sok Daejeon (1744). Investigating the mathematical structures of the two systems, we show that the systems involve various aspects of mathematics and that the systems are the most precise applications of mathematics in the Joseon dynasty.

Keywords: farmland tax systems in Joseon dynasty, Joseon mathematics, measurement of farmlands, assessment of farmland tax, Gyeongguk Daejeon, Sok Daejeon; 朝鮮의 田制法, 朝鮮算學, 量田法, 農地稅, 經國大典, 續大典.

MSC: 01A13, 01A40, 01A50, 01A80

1 Introduction

The Joseon dynasty (朝鮮, 1392–1910) is an agricultural country as the other countries in East Asia throughout their history. Thus her national income has been mainly made through the farmland products and tax, and hene the tax system of farmland becomes one of the most important subjects of its government affairs through the whole period of the dynasty.

In the beginning of the Joseon dynasty, it adopted the farmland tax system of the preceding Goryeo dynasty (高麗, 918–1392). As the Joseon dynasty becomes stable, the dynasty tries to establish its own code of law. The 4th king Sejong (世宗, r. 1418–1450) is the most prominent king in the whole history of Joseon who creates the

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Korean alphabet, Hangul (한글). He also encourages scholars to study mathematics and astronomy and paves eventually the way for the indigenous development of mathematics and astronomy in the dynasty.

The king ordered to reform the farmland tax system in 1443. Government set up Jeonje Sangjeongso (田制詳定所), the office of the farmland taxation whose head is the king's son, the 7th king Sejo (世祖, r. 1455–1468). Collecting data of years of rich or bad harvest and those of sowing of various seeds and their crops, the office made a proposal for the new farm land tax system in 1444. The new system comprises two basic principles, namely the principle of Jeonbun Yukdeung (田分六等), grading farmlands into six classes and that of Yeonbun Gudeung (年分九等), collecting taxes according to nine classes of the yearly harvest. The law also indicates the amount of tax paid by rice along the size of areas in each grade and the yearly harvest. The detail of the principles will be discussed in the next chapter, in particular the mathematical structures in the system. Government chooses a few areas to practice the system in a couple of years and then it adopts the system which is legalized in Gyeongguk Daejeon (經國大典, 1469), the complete code of law of the Joseon dynasty [1].

Although the system becomes an official law, there arise numerous problems in practicing the law. First because of the so much irregular topography of farmlands in the Korean peninsula, it is almost impossible to measure the lands exactly. Further, the grades are decided by the local officials, sometimes unfair ones and hence they cause a lot of complaints from farmers. Because of the devastating foreign invasions in 1592 and 1636, the records of farmlands were also completely destroyed and hence the government officials began to discuss the reform of the old system.

A new simplified system was proposed to remedy the old system in the late 17th century and it was successfully practiced in Hwanghae province (黃海道) by its governor (觀察使), Yu Jib-il (兪集一, 1653–1724) in 1701 [5]. Eventually the new system was adopted and it was first quoted in the mathematics book, Juseo Gwangyeon (籌書管見, 1718) [3] of Jo Tae-gu (趙泰耉, 1660–1723) and then was legalized in Sok Daejeon (續大典, 1744), the amended complete code of law [2].

The purpose of this paper is to investigate mathematical structures of the two farmland tax systems and then to show that they are mathematically equivalent. We also show that the systems involve various aspects of mathematics which were precisely applied to the systems. For the history of the farmland tax system, we refer to Joseon Wangjo Sillok (朝鮮王朝實錄) [5], the Annals of the Joseon dynasty abbreviated as AJD in this paper.

2 Farmland tax systems in the Joseon dynasty

For the fair taxations of farmland, the measurement of exact area of farmland is foremost essential and it is also very important to grade farmlands according to their productivity. The Goryeo dynasty (918–1392) used the perimeters of farmlands instead of their areas because the measuring areas is almost impossible because of the irregular figures of the farmlands. It also adopted three grades system (三等田) along the productivity. The units for measuring areas in the year 1389 are bu (負), farmland with the perimeter 3 bo 3 ja (3步 3尺) and gyeol (結), that with 33 bo. Since the ancient time, the widths of fingers in a clenched fist were used for a unit of length. In the Goryeo system, the unit 1 ja (尺) of the yardstick for the first class (上田尺) was determined by 10 times of two fingers (二指) in the fist, the one for the second class (中田尺) by 5 times of two fingers and another five times of three fingers (三指) and finally the one for the third class (下田尺) by 10 times of 3 fingers, respectively [5].

The Joseon dynasty adopts the Goryeo system but measuring areas in 1405, it uses a new system, namely 1 bu with the perimeter 3 bo 1 ja 8 chi (寸) and then in 1428, another system of 1 bu with 3 bo 3 ja and 1 gyeol with 35 bo. The Goryeo system assessed the tax along the productivity or fertility and perimeter. In 1430, the king Sejong (世宗, r. 1418–1450) and his officials began to discuss another factors of taxation according to the harvest of the tax year which was inflicted by the natural disasters, e.g. flood (水災), drought (旱災), frost (霜災), hailstone (雹災) and locust infestation (蝗蟲). They also discussed the reform of the three grade system up to the nine grade system. In 1443, king Sejong made a decree as follows. Since the measurement of farmlands by perimeters is not exact, they measure farmlands by their areas with the yardstick jucheok, i.e., Chinese zhouchi (周尺) whose length was fixed in 1437. The old system of areas used pa (把), sok (束), bu (負) or bok (卜) and gyeol (結) with 1 pa = 1 ja^2, 10 pa = 1 sok, 10 sok = 1 bu and 100 bu = 1 gyeol. Instead of the old system, gyeongmubeob, qingmufa (頃畝法) along the Chinese units was employed, where 1 bo (步) = 5 ja (尺), 25 ja^2 = 1 bo^2, 1 mu (畝) = 240 bo^2, 1 gyeong (頃) = 100 mu, 1 ja(字) = 5 頃. The five grade system is introduced. The first grade of dry fields, i.e., not rice fields, is considered as the second grade of the rice fields and the fifth grade of dry fields as the sixth grade. Further, a new system Yeonbun Gudeung (年分九等), the nine classes of tax according to the harvest inflicted by natural disasters in the tax year should be implemented.

With these discussions of a long period of time, the king Sejong ordered to establish Jeonje Sangjeongso (田制詳定所), the office of the farmland taxation in 1443. In the very next day (lunar calendar November 14, 1443) of the establishment of Jeonje Sangjeongso, the king Sejong ordered to collect the data of farmlands, yang-

jeon (量田) in the city Ansan (安山) of Gyeonggi Province. The officers include Ha Yeon (河演, 1376–1453), Jeong In-ji (鄭麟趾, 1396–1478), Lee Sun-ji (李純之, ?–1465), Park Yun-chang (朴允昌), Kim Dam (金淡, 1408–1464). Among them, Lee and Kim are well known astronomers and mathematicians. The yardstick was chosen by the zhouchi (周尺) with the length 5 bo and for measuring the length less than 5 bo, tapeline (量繩) was used. Instead of the perimeter of the fields, they also used the gyeongmubeob. The king Sejong praised their works, in particular those by Lee Sun-ji and Kim Dam and their mathematics. Thus he also ordered to find a way to promote mathematics [5].

One year later, the office proposed a new system of farmland tax system on the lunar calendar November 13, 1444. First, the Jeonje Sangjeongso introduced the system Jeonbun Yukdeung (田分六等), or simply called Yukdeungjeon (六等田), i.e., dividing farmlands into the six classes instead of the Goryeo Samdeungjeon (三等田) based on productivity of farmlands. It also included a new system Yeonbun Gudeung (年分九等), the nine classes of tax according to the harvest inflicted by natural disasters in the tax year.

We now include the details of the new system because it is eventually adopted as a law, and we also need them for further development of this paper.

Using the terms sang(上), jung (中), ha (下) for the top, middle and lower classes, respectively, and combining the three terms twice, the nine classes of Yeonbun Gudeung are arranged from the top year, sangsangnyeon (上上年) to the lowest year hahanyeon (下下年). Regarding the tax rate, let $r_1, r_2, \cdots, r_8, r_9$ denote the tax rates of the year from the top year sangsangnyeon to the lowest year hahanyeon in order, then the tax rate r_k for the k-th year is determined by the formula

$$r_k = (1 - \frac{k-1}{10})r_1.$$

In other words, the tax rates of sangjungnyeon (上中年) to hahanyeon (下下年) are $90\%, 80\%, \cdots, 20\%$ of the tax rate of the sangsangnyeon [5].

For the Yukdeungjeon, the new law also has different units of the yardstick according to the grades, namely 4 ja 7 chi 7 pun (4尺 7寸 7分), i.e., 4.77 ja for the first class, 5.18 ja for the second class, 5.7 ja for the third class, 6.43 ja for the fourth class, 7.55 ja for the fifth class and 9.55 ja for the sixth class in the jucheok in AJD. These lengths will be called yangjeonja (量田尺) in the sequel. Then 1 pa (把) is defined as 1 yangjeonja^2 and then sok (束), bu (負), gyeol (結) are defined as those mentioned above. We note that in the section Yangjeon of Hojeon (戶典) of Gyeongguk Daejeon (經國大典), 1 yangjeonja is given by

4 ja 7 chi 7 pun 5 ri (4尺 7寸 7分 5釐), i.e., 4.775 ja, 5.189 ja, 5.703 ja,

6.434 ja, 7.55 ja, and 9.55 ja

in the jucheok respectively for the first class through the sixth class. The rate for the

farmland tax is given in the section Suse (收稅) of Hojeon [1].

We now discuss the mathematical structures for the Yukdeungjeon. We first state the rates of volumes for grains in the Joseon dynasty. The harvest is measured by the volume of unhusked rice whose unit is called seom, dan (石). And 1 seom is 15 mal, dou (斗). The rate of volumes of unhusked rice and its husked rice is 2 to 1. Tax of farmland was collected by husked rice in the Joseon dynasty and its tax rate of the harvest is $\frac{1}{20}$. In all, the tax of the harvest h seom of unhusked rice is $\frac{h}{40}$ seom, or $\frac{15h}{40} = \frac{3h}{8}$ mal of husked rice. Since the tax was assessed by the harvest of farmlands, the unit of yangjeonja for each class was determined by the harvest of farmland.

The harvest of the first class rice field (一等田) was assumed to be 80 seom in the field with the area 57 mu and in the year of sangsangnyeon (上上年) and the harvests of the remaining classes in the sangsangnyeon were graded to form an arithmetic sequence with the common difference -12. Thus taxes of husked rice for the classified fields with the area 57 mu become an arithmetic sequence with the first term, 30 mal of husked rice with the common difference -4.5 mal, because the tax (y mal) and harvest (x seom) are related by $y = \frac{3x}{8}$.

Instead of fixed areas, say 57 mu, the office Jeonje Sangjeongso fixed amount of tax in each classified farmland by changing their areas according to the harvest as the Goryeo dynasty system. The office also decided the unit amount of the tax by 20 mal instead of 30 mal. Let z_k (y_k, resp.) denote the areas (the amount of the tax for 57 mu, resp.) of classified farmlands for the tax of 20 mal of husked rice from the first class to the sixth class, then they are related by $57 : y_k = z_k : 20$, i.e., $z_k = \frac{1,140}{y_k}$. Thus the areas of classified farmlands for the tax 20 mal become 38 mu, 44.7 mu, 54.2 mu, 69 mu, 95 mu, 152 mu. Here the approximations are taken by truncating z_k to 1 decimal digit. The above areas of classified farmland are called 1 gyeol (結), more precisely the classified 1 gyeol. Thus 1 gyeol should be indicated by their classes, for the areas are different and then the classified bu (負) or bok (卜), sok (束), pa (把) are defined as above. Thus the taxes in the sangsnagnyeon (上上年) for each classified 1 gyeol, 1 bu, 1 sok and 1 pa are 20 mal, 2 doe, sheng (升), 2 hob, ge (合), 2 jak, shao (勺) of husked rice. These rates are all for rice fields.

The tax for the classified dry fields, is simply the half of the tax for the equivalently classified rice fields.

We now discuss the unit of the yangjeonja for the classified farmlands. We recall that 1 mu is 6,000 ja^2 and the classified areas, 1 gyeol become

238,000, 268,200, 325,200, 414,000, 570,000, 912,000 ja^2, respectively, which will be denoted by a_k.

The length of a side of the square with the same area a_k of the classified rice fields is calculated. Truncating $\sqrt{a_k}$ to 1 decimal digit, they are given as follows :

477.4, 517.8, 570.0, 643.4, 754.9, 954.9 ja.

We note that $\sqrt{325,200} \approx 570.26$ but they choose 570.0 instead of 570.2 for the third class. Since the above sides of the classified squares 1 gyeol are too big, so that they take the side of the classified squares 1 pa as the unit of the yangjeonja, i.e., 1 % of the side of the classified square 1 gyeol. In this case, they take an approximation by rounding the numbers instead of truncating which are given as follows:

4.77, 5.18, 5.7, 6.43, 7.55, 9.55 ja.

One can easily check that the units of the yangjeonja of Gyeongguk Daejeon mentioned in the above are obtained by truncating $\sqrt{a_k}$ to 2 decimal digits and rounding them, e.g., the unit of the yangjeonja for the third class is rectified as 5.703 ja. Incidentally, AJD says that these kinds of corrections should be done soon.

The above discussion shows that the officials in Jeonje Sangjeongso were specialists in mathematics. Indeed, they were quite familiar with basic mathematics like proportional relations, arithmetic sequences and extractions of square roots. Further, they understood the approximations, in particular, truncations and roundings [4]. It is interesting that the process of roundings is not discussed in those basic books like Suanxue Qimeng (算學啓蒙, 1299), YangHui Suanfa (楊輝算法) and Xiangming Suanfa (詳明算法, 1373) which were brought into the king Sejong era.

It took almost half a century for the full implementation of the new system in the whole country. The implementation also involves many problems. The fair classifications are clearly difficult and cause many complaints from farmers for the tax are determined by them. The more difficult part is the measurement of the areas of farmlands. The irregular topography of farmlands gives rise to an almost impossible task for officials of measuring areas, even more so for local officials. But it was retained until the end of the 16th century. Because of the foreign invasions in 1592 and 1646, most of records of measurements were destroyed and hence in the second half of the 17th century, the government must have a new record of farmlands. Thus they discussed a reform for the farmland tax and discarded the law of Yeonbun Gudeung. Further, it has been very much awkward for the officials to carry the six yangjeonjas for the six classes of farmlands as discussed in the above. Thus a new system with a unified yangjeonja was introduced.

As discussed above, the tax system of farmlands in the Joseon dynasty was systematized along the harvests of farmlands or its taxes as that in the Goryeo dynasty and hence had the system of classified farmlands. In other words, the tax system was arranged along the fixed amount of harvest, i.e., 20 mal for the classified 1 gyeol or 2 jak for the classified 1 pa. In order to introduce a unit area for all classified

farmlands, one has to fix a unit area and change the amount of tax, or harvest. First the 1 yangjeonja of the first class farmland, i.e., 4.775 ja in the jucheok is taken as the unit. Since the areas of the classified farmland 1 gyeol are 38 mu, 44.7 mu, 54.2 mu, 69 mu, 95 mu, 152 mu, abbreviated as z_k $k = 1, 2, \cdots 6$ in the above, and the unit area is 38 mu for all classified farmlands, we have the amount c_k in the proportional relation $38 : z_k = c_k : 1$, i.e., $c_k = \dfrac{38}{z_k}$ which are as follows :

0.8501. 0.7011, 0.5507, 0.4, 0.25,

where the approximations were taken by truncating to the decimal 4 digits.

The above c_k may be interpreted as the previous classified area 1 pa in the fixed unit of yangjeonja for the second class to the sixth class. Since 1 bu (負) or 1 bok (卜) is 100 pa, one has the following Haebubeob (解負法) or Haebokbeob (解卜法)

100, 85, 70, 55, 40, 25,

where the approximations were taken by the integral part of $100c_k$. We note that the sequence in the Haebokbeob is an arithmetic one with the common difference 15.

The above Haebokbeob was first mentioned in the mathematics book Juseo Gwan-gyeon (籌書管見, 1718) of Jo Tae-gu (趙泰耉, 1660–1723) who served as the minister of Hojo (戶曹判書) many times and eventually became the prime minister, Yeonguijeong (領議政). Most of mathematics books after the Juseo Gwan-gyeon include the Haebokbeob. Jo Tae-gu also added the following remark:

一萬尺一等地 則定百負 二等地 則八十五負 逐等遞降十五負

He says that 10,000 ja^2 of the first class farmland, i.e., 100 bu or 1 gyeol, then the second class of the same area is 85 bu and then the other classes form an arithmetic sequence with the common difference -15 bu.

In the 18th century, the government of the Joseon dynasty needed to amend basic laws like the new system of the farmland tax, it published the amended one, called Sok Daejeon (續大典, 1744). Instead of the arithmetic sequence for 1 gyeol in Juseo Gwan-gyeon quoted above, the following sequence was included in the section Hojeon (戶典) of Sok Daejeon :

8501, 7011, 5507 for the second, third and fourth classes, respectively.

These numbers are precisely the integral parts of $10,000c_k$ and they omit 4,000 and 2,500 for the fifth and sixth classes because they are trivial. Further, Sok Daejeon indicates precisely that the old yangjeonjas of the second to the sixth class will not be used.

This indicates that the authors of Sok Daejeon understand the significant digits.

Beginning with 1 bo, or bu (步) in Chinese, which is 5 chi (尺) of zhouchi, Chinese mu (畝) is defined as 240 bu^2 and then qing (頃) as 100 mu. The Chinese units

of area follow the decimal system but the smallest Chinese unit 1 hu (忽) of area is defined by 6 cun^2 (寸^2) which cannot be the area of a square with the side of a rational number. Thus the system is very much inconvenient because of 6.

But the Joseon government took the 1 yangjeonja which is 4.775 ja (尺) of jucheok and then defined the unit pa (把) as 1 yangjeonja^2 and then the others sok (束), bu (負) = bok (卜) and gyeol (結) following the decimal system. Thus they are much more convenient for a general use. This system begins with a square, bangjeon, i.e., fangtian (方田) with the side of 1 yangjeonja so that the new system is called bangjeonbeob (方田法).

3 Conclusions

The farmland tax is the most important part of the national policy in the Joseon dynasty as in the agricultural countries. For the fair taxation, the king Sejong established the office Jeonje Sangjeongso in 1443 which proposed a new system in 1444 with the basic principles, Jeonbun Yukdeung and Yeonbun Gudeung. Through the king's strong intention to promote mathematics and astronomy, mathematics and astronomy in his era have experienced an unprecedented development in the whole history of Korea. Consequently, his mathematically well trained officials could apply the exact mathematics to make the farmland tax system.

The system was amended in the 18th century. In this case, Joseon government introduced a unified yangjeonja which is about 1 meter and its own units of area along the yangjeonja which begin with the area of square with the side of 1 yangjeonja and follow the decimal system.

We conclude that the farmland tax systems show one of the most precise applications of mathematics in the history of the Joseon dynasty.

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