

# Energy Efficiency Resource Allocation for MIMO Cognitive Radio with Multiple Antenna Spectrum Sensing

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## Abstract

The energy-efficient design of sensing-based spectrum sharing of a multi-input and multi-output (MIMO) cognitive radio (CR) system with imperfect multiple antenna spectrum sensing is investigated in this study. Optimal resource allocation strategies, including sensing time and power allocation schemes, are studied to maximize the energy efficiency (EE) of the secondary base station under the transmit power and interference power constraints. EE problem is formulated as a nonlinear stochastic fractional programming of a nonconvex optimal problem. The EE problem is transformed into its equivalent nonlinear parametric programming and solved by one-dimension search algorithm. To reduce searching complexity, the search range was founded by demonstration. Furthermore, simulation results confirms that an optimal sensing time exists to maximize EE, and shows that EE is affected by the spectrum detection factors and corresponding constraints.

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**Keywords:** Resource management, energy efficiency, MIMO, CR, multiple antenna spectrum sensing

## 1. Introduction

Cognitive radio (CR) [1] is an important technology for efficiently increasing the utilization of available radio spectrum. The operation for CR systems has three types: i) opportunistic spectrum access (OSA) [2], where the secondary user (SU) can only access the licensed bands detected as idle; ii) spectrum sharing (SS) [3], where the SU is allowed to coexist with the primary user (PU) as long as the quality of service (QoS) of PU is protected; iii) sensing-based spectrum sharing (SBSS) [4] [5], where the SU first senses the status of the PU before choosing the appropriate transmit power according to the decision.

Multiple input and multiple output (MIMO) technology [6][7] is used in cognitive radio to achieve high capacity, increase diversity, and reduce interference suppression. In [8], opportunistic spectrum access in MIMO CR network was considered wherein detection operation and transmit power were optimized for maximum throughput under transmission power and detection probability constraints. In [9], spectrum sharing in MIMO CR network was considered, wherein beamforming and power allocation were optimized for maximum throughput. Interference power constraint was imposed to avoid interference to PU. The sensing-based spectrum sharing model should also be studied in MIMO CR network. In [10], an algorithm that finds the optimal beamforming and power allocation for maximum throughput was proposed in sensing-based spectrum sharing. However, multiple antennas were only employed at the SU transmitter. The work was extended to the MIMO CR network in [11]. In [11], sensing-based spectrum sharing in MIMO CR network was studied by exploiting the non-cooperative game to optimize the detection operation and power allocation for the maximum opportunistic throughput. All the aforementioned studies only considered the optimal algorithms to obtain maximum throughput.

With the tremendous increasing demand of ubiquitous multimedia communications and energy saving, energy efficiency (EE) has become a key issue for cognitive radio networks in recent research [12]–[16]. In [14], EE was maximized in spectrum sharing CR network, and the PU outage probability constraint was considered to protect PU. In [15], the EE optimization problem in CR MIMO broadcasting channels was transformed into an equivalent one-dimension problem with a quasi-concave objective function. However, the above studies performed the optimization problem based on the perfect channel state information (CSI) without considering the status of PU. In [16], SU first detected the status of PU, then solved the EE optimization problem by considering the stop-and-wait and channel handoff schemes to avoid interference to PU in opportunistic spectrum access CR network. Similar to a single-antenna opportunistic spectrum access case, investigating the maximum EE problem in sensing-based spectrum sharing for MIMO CR networks is meaningful.

In the present study, the EE problem in sensing-based spectrum sharing for MIMO CR network is studied with imperfect spectrum sensing. The optimal resource allocation, including sensing time and transmission power, is designed to maximize EE for secondary base station (SBS) in downlink transmission.

The main contributions of this study are summarized as follows:

The optimal sensing time and power allocation strategy for the maximum EE in sensing-based spectrum sharing MIMO CR system is first studied. The transmission power and interference power constraints are imposed to protect the transmission of PU.

- 1) The algorithm based on singular-value decomposition (SVD) of the channel between the SBS and each SU are used to avoid interference between different SUs. This approach indicates that the MIMO channel between SBS and each SU is decomposed into independent spatial subchannels and each SU could receive the independent data from BS.
- 2) Formulated as a nonlinear stochastic fractional programming, the EE problem is transformed into the equivalent nonlinear parametric programming. The optimal sensing time and power allocation strategies for the maximum EE should be solved using one-dimension search algorithm. Furthermore, it is demonstrated that the research range in the search algorithm could be found.

The rest of this study is organized as follows. Section 2 provides the system model of a MIMO CR network under sensing-based spectrum sharing. In Section 3, the optimal resource allocation strategy to maximize the EE problem is derived. The simulation results are given in Section 4. Section 5 presents the conclusions of the study.

## 2. System Model

In MIMO cognitive radio system, one central secondary base station with  $N$  antennas transmits to  $K$  independent SUs in one frequency band, as shown in Fig. 1. The frequency band is licensed to a primary user with a single antenna. The  $k$ th SU is equipped with  $n_k$  receive antennas and the total receive antennas is defined as  $N_r = \sum_{k \in K} n_k$ . The MIMO channel matrix from the SBS to the  $k$ th SU, from the primary receiver to the SBS, and from the primary transmitter to the  $k$ th SU are denoted as  $\mathbf{H}_k \in C^{n_k \times N}$ ,  $\mathbf{G}_{pb} \in C^{N \times 1}$ , and  $\mathbf{g}_k \in C^{n_k \times 1}$ . SBS has the channel state information (CSI) of PU and each SU, whereas each SU has its CSI.

The frame structure of this system consists of sensing time and transmission slot. In the sensing time slot, the SBS uses the multiple antenna spectrum sensing to decide whether PU works or not. In the transmission slot, the SBS then transmits independent data to each SU based on the sensing results. The assumptions are the channel between SBS and each SU, as well as the channel between SBS and the PU, are quasi-static. The slot structure of CR network is synchronized with that of the primary network.

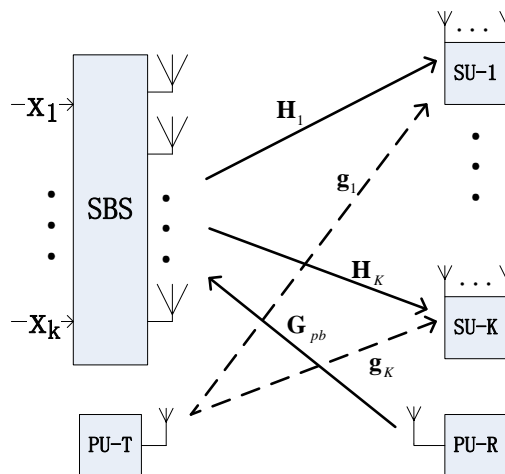


Fig. 1. System model for MIMO CR network

## 2.1 Spectrum sensing

In the sensing slot, the SBS detects the status of PU by the multiple antenna sensing technique. The sample rate of each antenna in the SBS is assumed as  $L$ . The observed signals at  $N$  antennas are a complex matrix,  $\mathbf{Y} = [\mathbf{y}(1), \dots, \mathbf{y}(L)] \in \mathbb{C}^{N \times K}$ . When the primary user is inactive  $H_0$  or active  $H_1$ , the discrete received signal at the BS can be represented as follows:

$$\begin{aligned} H_0 : \mathbf{y}(l) &= \mathbf{N}(l) \\ H_1 : \mathbf{y}(l) &= \mathbf{G}_{pb} s(l) + \mathbf{N}(l) \end{aligned} \quad (1)$$

where  $\mathbf{y}(l) \in \mathbb{C}^{N \times 1}$  denote the  $l$ th received signal samples at SBS,  $\mathbf{N}(l) \in \mathbb{C}^{N \times 1}$  is noise vector satisfying independent, and identically distributed (i.i.d) circularly symmetric complex Gaussian such as  $CN(\mathbf{0}, \sigma_n^2 \mathbf{I}_N)$  and  $s(l)$  is the transmission signal of PU with variance  $\sigma_s^2$ . For simplicity, PU is assumed to be either idle or busy for the whole slot.

Under hypothesis  $H_0$  and  $H_1$ , the received signals at SBS has a Gaussian distribution,

$$\begin{aligned} H_0 : \mathbf{Y} &\sim CN(\mathbf{0}, \sigma_n^2 \mathbf{I}_N) \\ H_1 : \mathbf{Y} &\sim CN(\mathbf{0}, \sigma_s^2 \mathbf{G}_{pb} \mathbf{G}_{pb}^H + \sigma_n^2 \mathbf{I}_N) \end{aligned} \quad (2)$$

During the sensing slot, the probability density function (PDF) of the sample matrix  $\mathbf{Y}$  under the hypothesis  $H_0$  and  $H_1$  is obtained at SBS. Accordingly, the final decision on whether PU is active will be dependent on the detection outcomes on all the  $N$  antennas. Specifically, the SBS makes the decision based on the Logarithm of Likelihood Ratio (LLR) function:

$$LLR = \ln \frac{f(\mathbf{Y}; H_1, \mathbf{G}, \sigma_s^2, \sigma_n^2)}{f(\mathbf{Y}; H_0, \sigma_n^2)} = \frac{\|\mathbf{G}_{pb}^H \mathbf{Y}\|^2}{(\frac{\sigma_n^2}{\sigma_s^2} + \|\mathbf{G}_{pb}\|^2) \sigma_n^2} - f_s \tau \ln \left( \frac{\sigma_n^2}{\sigma_s^2} \|\mathbf{G}_{pb}\|^2 + 1 \right) \quad (3)$$

where  $f(\mathbf{Y}; H_0, \sigma_n^2)$  and  $f(\mathbf{Y}; H_1, \mathbf{G}, \sigma_s^2, \sigma_n^2)$  are the PDF of the sample matrix under the different hypothesis  $H_0$  and  $H_1$ .

Sampling frequency is represented by  $f_s$  and  $\varepsilon$  is the decision threshold. The probability of false alarm and detection can be expressed as [17]:

$$\begin{aligned} p_f &= \frac{\Gamma(f_s \tau, \frac{\varepsilon}{\|\mathbf{G}_{pb}\|^2 \sigma_n^2})}{\Gamma(f_s \tau)} \\ p_d &= \frac{\Gamma(f_s \tau, \frac{\varepsilon}{\|\mathbf{G}_{pb}\|^2 (\|\mathbf{G}_{pb}\|^2 \sigma_s^2 + \sigma_n^2)})}{\Gamma(f_s \tau)} \end{aligned} \quad (4)$$

## 2.2 Pre-processing and post-processing

The SBS transmits independent data to  $K$  independent SUs using a pre-coding matrix  $\mathbf{P} = [\mathbf{P}_1, \dots, \mathbf{P}_K]$  at SBS and some post-processing matrix at each SU to avoid the interference between each SU.

The  $k$ th received signal vector at the  $k$ th SU is denoted as:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{P}_k x_k + \mathbf{H}_k \sum_{i=1, i \neq k}^K \mathbf{P}_i x_i + \mathbf{N}_k \quad (5)$$

where  $\mathbf{P}_k$  represents the pre-coding matrix associated with the  $k$ th SU,  $\mathbf{X} = [x_1, \dots, x_K]$  is the transmission data vector at SBS, and  $\mathbf{N}_k$  is the noise vector on the  $k$ th channel. The

assumption is the noise vector on each channel from SBS to each SU is an independently and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and variance  $\sigma_n^2$ .

The  $k$ th SU processes the received signal  $\mathbf{y}_k$  by the received filter vector  $\mathbf{m}_k$ . The received filter output vector  $\hat{\mathbf{y}}_k$  at the  $k$ th SU can be written as:

$$\hat{\mathbf{y}}_k = \mathbf{m}_k \mathbf{H}_k \mathbf{P}_k x_k + \mathbf{m}_k \mathbf{H}_k \sum_{i=1, i \neq k}^K \mathbf{P}_i x_i + \mathbf{m}_k \mathbf{N}_k \quad (6)$$

To avoid the interference between each SU and to enable the  $k$ th SU to receive one single independent signal  $x_k$ , the following function should be satisfied:

$$\mathbf{H}_k \mathbf{P}_i = 0, i \neq k, \forall i, k \in \{1, \dots, K\} \quad (7)$$

To satisfy the condition in Equation (7), the matrix is defined as:

$$\bar{\mathbf{H}}_k = [\mathbf{H}_1^T, \dots, \mathbf{H}_{k-1}^T, \mathbf{H}_{k+1}^T, \dots, \mathbf{H}_K^T]^T \quad (8)$$

Then the singular value decomposition (SVD) of  $\bar{\mathbf{H}}_k$  in the CR system can be expressed as:

$$\bar{\mathbf{H}}_k = \bar{\mathbf{U}}_k \bar{\Lambda}_k [\bar{\mathbf{V}}_k^{(1)} \bar{\mathbf{V}}_k^{(0)}]^H \quad (9)$$

where  $\bar{\mathbf{U}}_k$  denotes the unitary matrixes,  $[\bar{\mathbf{V}}_k^{(1)} \bar{\mathbf{V}}_k^{(0)}]$  denotes the right singular vector,  $\bar{\mathbf{V}}_k^{(1)}$  corresponds to non-zero singular values,  $\bar{\mathbf{V}}_k^{(0)}$  is the orthogonal basis for the null space of  $\bar{\mathbf{H}}_k$  that correspond to zero singular values.

The orthogonal basis for the null space of  $\bar{\mathbf{H}}_k$  has the properties as follows.

$$\begin{aligned} \bar{\mathbf{H}}_k \bar{\mathbf{V}}_k^{(0)} &= [\mathbf{H}_1^T, \dots, \mathbf{H}_{k-1}^T, \mathbf{H}_{k+1}^T, \dots, \mathbf{H}_K^T]^T \bar{\mathbf{V}}_k^{(0)} \\ &= \mathbf{0} \end{aligned} \quad (10)$$

If the pre-coding matrix  $\mathbf{P}_k$  would contain  $\bar{\mathbf{V}}_k^{(0)}$ , Equation (7) should be satisfied and the interference between each SU should be avoided.

To decouple the  $k$ th block channel into parallel sub-channel, the SVD of  $\mathbf{H}_k \bar{\mathbf{V}}_k^{(0)}$  should be obtained as  $\mathbf{H}_k \bar{\mathbf{V}}_k^{(0)} = \hat{\mathbf{U}}_k \hat{\Lambda}_k \hat{\mathbf{V}}_k^H$ .

Assuming the pre-coding matrix  $\mathbf{P}_k = \bar{\mathbf{V}}_k^{(0)} \hat{\mathbf{V}}_k^H$ ,  $\mathbf{P}_k \in C^{N \times n_k}$  and the receive filter vector  $\mathbf{m}_k = \hat{\mathbf{U}}_k^H$ , the received filter output vector becomes:

$$\hat{\mathbf{y}}_k = \hat{\Lambda}_k x_k + \hat{\mathbf{U}}_k^H \mathbf{N}_k \quad (11)$$

where  $\hat{\Lambda}_k = \text{diag}\{h_{k,1}, \dots, h_{k,n_k}\}$  is a diagonal matrix of size  $n_k$ .

### 3. Optimal EE for SBSS Model

In this section, the system model is analyzed and the problems of energy efficient design are formulated. In the sensing-based spectrum sharing MIMO CR system, BS adapts transmit power matrix based on the outcome of the detection. If PU is detected to be inactive, BS will transmit to SUs with high power matrix  $\mathbf{\Omega}^{(0)}$ ; if PU is detected to be active, BS will transmit to SUs with low power matrix  $\mathbf{\Omega}^{(1)}$ . Diagonal matrices are composed of  $\mathbf{\Omega}^{(0)}$  and  $\mathbf{\Omega}^{(1)}$ :

$$\mathbf{\Omega}^{(0)} = \text{diag}\{\mathbf{\Sigma}_1^{(0)}, \dots, \mathbf{\Sigma}_K^{(0)}\}$$

$$\mathbf{\Omega}^{(1)} = \text{diag}\{\mathbf{\Sigma}_1^{(1)}, \dots, \mathbf{\Sigma}_K^{(1)}\}$$

where  $\Sigma_k^{(0)} = \text{diag}\{P_{k,1}^{(0)}, \dots, P_{k,n_k}^{(0)}\}$  and  $\Sigma_k^{(1)} = \text{diag}\{P_{k,1}^{(1)}, \dots, P_{k,n_k}^{(1)}\}$  are the diagonal matrices of size  $n_k$  and represent associated high power and low power matrices from BS to the  $k$ th SU.

The four scenarios for sensing the state of PU are the following.

If PU is inactive and is detected to be inactive, then SBS will transmit data to SUs with high power matrix  $\Omega^{(0)}$ . The probability of this scenario is  $\alpha_0 = P(H_0)(1-p_f)$ , where  $P(H_0)$  denotes the probability that the licensed band is idle. The instantaneous transmission capacity is:

$$R_{00} = \sum_{k=1}^K \sum_{i=1}^{n_k} \log_2 \left( 1 + \frac{|h_{k,i}|^2 P_{k,i}^{(0)}}{\sigma_n^2} \right) \quad (12)$$

If PU is inactive and is detected to be active, then the false alarm happens. SBS will transmit to SUs with low power matrix  $\Omega^{(1)}$ . The probability of this scenario is  $\alpha_1 = P(H_0)p_f$ . The instantaneous transmission capacity is:

$$R_{01} = \sum_{k=1}^K \sum_{i=1}^{n_k} \log_2 \left( 1 + \frac{|h_{k,i}|^2 P_{k,i}^{(1)}}{\sigma_n^2} \right) \quad (13)$$

If PU is active and is detected to be inactive, then misdetection happens. SBS will transmit to SUs with high power matrix  $\Omega^{(0)}$ . The probability of this scenario is  $\beta_0 = (1-P(H_0))(1-p_d)$ . The instantaneous transmission capacity is:

$$R_{10} = \sum_{k=1}^K \sum_{i=1}^{n_k} \log_2 \left( 1 + \frac{|h_{k,i}|^2 P_{k,i}^{(0)}}{\varphi_k + \sigma_n^2} \right) \quad (14)$$

where  $\varphi_k = \|\mathbf{g}_k\|^2 P_p$  is the interference of PU to the  $k$ th SU and  $P_p$  is transmission power of PU.

If PU is active and is detected to be active, SBS will transmit to SUs with low power matrix  $\Omega^{(1)}$ . The probability of this scenario is  $\beta_1 = (1-P(H_0))p_d$ . The instantaneous transmission capacity is:

$$R_{11} = \sum_{k=1}^K \sum_{i=1}^{n_k} \log_2 \left( 1 + \frac{|h_{k,i}|^2 P_{k,i}^{(1)}}{\varphi_k + \sigma_n^2} \right) \quad (15)$$

Then, the average throughput of SBS in MIMO cognitive radio system can be expressed as:

$$R(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)}) = \frac{T-\tau}{T} (\alpha_0 R_{00} + \alpha_1 R_{01} + \beta_0 R_{10} + \beta_1 R_{11}) \quad (16)$$

In the sensing slot, the energy consumption for sensing the status of PU at SBS is:

$$E_s(\tau) = \tau P_{cs} \quad (17)$$

where  $P_{cs}$  is the power consumption of sensing.

In the transmission slot, the energy consumption for transmission at SBS is:

$$E_t(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)}) = (T-\tau) \sum_{k=1}^K \sum_{i=1}^{n_k} (\alpha_0 + \beta_0) P_{k,i}^{(0)} + (\alpha_1 + \beta_1) P_{k,i}^{(1)} \quad (18)$$

Then the EE for the sensing-based spectrum sharing MIMO cognitive radio system with the metric "bit per joule" is:

$$U_{EE}(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)}) = \frac{R(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)})}{E_t(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)}) + E_s(\tau) + E_c} \quad (19)$$

where  $E_c$  is the circuit power consumption derived from signal processing, battery backup, and others.

$P_s$  denotes the maximum average transmission power of SBS. The transmission power constraint can be expressed as:

$$\frac{T-\tau}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} (\alpha_0 + \beta_0) P_{k,i}^{(0)} + (\alpha_1 + \beta_1) P_{k,i}^{(1)} \leq P_s \quad (20)$$

When PU is active, SU makes correct and wrong detection decisions. PU may then suffer from the interference of the CR system. The interference power constraint can be defined as:

$$\frac{T-\tau}{T} \sum_{k=1}^K \beta_0 \|\mathbf{G}_{bp} \mathbf{P}_k \boldsymbol{\Sigma}_k^{(0)}\|^2 + \beta_1 \|\mathbf{G}_{bp} \mathbf{P}_k \boldsymbol{\Sigma}_k^{(1)}\|^2 \leq \Gamma \quad (21)$$

where  $\Gamma$  is the maximum tolerable interference power at the PU.

Vector  $\mathbf{G}_k = [g_{k,1}, \dots, g_{k,n_k}]$  denotes  $\mathbf{G}_{bp} \mathbf{P}_k$ . The interference power constraint can be rewritten as:

$$\frac{T-\tau}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} \beta_0 g_{k,i} P_{k,i}^{(0)} + \beta_1 g_{k,i} P_{k,i}^{(1)} \leq \Gamma \quad (22)$$

Accordingly, the EE resource allocation problem of the sensing-based spectrum sharing MIMO CR system can be written as:

$$\max_{\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)}} U_{EE}(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)}) \quad (23)$$

subject to: (20), (22),  $0 \leq \tau \leq T$ ,  $P_{k,i}^{(0)} \geq 0$ ,  $P_{k,i}^{(1)} \geq 0$

The objective function is not convex with respect to the sensing time  $\tau$ . Therefore, convex optimization techniques cannot be directly applied. Since  $0 \leq \tau \leq T$ , the optimal sensing time can be obtained through one-dimensional exhaustive search:

$$\tau_{opt} = \operatorname{argmax}_{\tau} U_{EE}(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)})$$

**Lemma 1.** Function  $U_{EE}(P_{k,i}^{(0)}, P_{k,i}^{(1)})$  is strictly quasi-concave in  $P_{k,i}^{(0)}$ ,  $P_{k,i}^{(1)}$ , respectively.

**Proof:** See Appendix A.

Therefore, a unique globally optimal power allocation exists for the strictly quasi-concave function  $U_{EE}(P_{k,i}^{(0)}, P_{k,i}^{(1)})$ .

Problem (23) can be associated with the following function problem  $F(q)$  for fixed  $\hat{\tau}$ :

$$\max_{\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)}} R(\hat{\tau}, P_{k,i}^{(0)}, P_{k,i}^{(1)}) - q(E_i(\hat{\tau}, P_{k,i}^{(0)}, P_{k,i}^{(1)}) + E_s(\hat{\tau}) + E_c) \quad (24)$$

where  $q \in R$  is a parameter.

**Theorem 1.**  $(P_{k,i}^{(0)*}, P_{k,i}^{(1)*})$  is the optimal solution of (23) associated with the maximum value  $q^*$  if and only if  $F(q^*) = F(q^*, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) = 0$ .

**Proof:** See Appendix B.

The Lagrangian function of  $F(q)$  with respect to the transmission power  $(P_{k,i}^{(0)}, P_{k,i}^{(1)})$  for given sensing time  $\hat{\tau}$  is derived as:

$$\begin{aligned}
L(P_{k,i}^{(0)}, P_{k,i}^{(1)}, \lambda, \mu) &= \frac{T-\hat{\tau}}{T} (\alpha_0 R_{00} + \alpha_1 R_{01} + \beta_0 R_{10} + \beta_1 R_{11}) - q(E_t(\hat{\tau}, P_{k,i}^{(0)}, P_{k,i}^{(1)}) + E_s(\hat{\tau}) + E_c) \\
&\quad - \lambda \left( \frac{T-\hat{\tau}}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} (\alpha_0 + \beta_0) P_{k,i}^{(0)} + (\alpha_1 + \beta_1) P_{k,i}^{(1)} - P_s \right) \\
&\quad - \mu \left( \frac{T-\hat{\tau}}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} \beta_0 g_{k,i} P_{k,i}^{(0)} + \beta_1 g_{k,i} P_{k,i}^{(1)} - \Gamma \right)
\end{aligned} \tag{25}$$

where  $\lambda$  and  $\mu$  are the Lagrangian multipliers. The dual objective function can be expressed as:

$$\begin{aligned}
g(\lambda, \mu) &= \max_{P_{k,i}^{(0)}, P_{k,i}^{(1)}} L(P_{k,i}^{(0)}, P_{k,i}^{(1)}, \lambda, \mu) \\
&= \max_{P_{k,i}^{(0)}, P_{k,i}^{(1)}} \tilde{g}(\lambda, \mu) - q(E_s(\hat{\tau}) + E_c) + \lambda P_s + \mu \Gamma
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
\tilde{g}(\lambda, \mu) &= \frac{T-\hat{\tau}}{T} (\alpha_0 R_{00} + \alpha_1 R_{01} + \beta_0 R_{10} + \beta_1 R_{11}) - qE_t(\hat{\tau}, P_{k,i}^{(0)}, P_{k,i}^{(1)}) \\
&\quad - \lambda \frac{T-\hat{\tau}}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} (\alpha_0 + \beta_0) P_{k,i}^{(0)} + (\alpha_1 + \beta_1) P_{k,i}^{(1)} \\
&\quad - \mu \frac{T-\hat{\tau}}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} \beta_0 g_{k,i} P_{k,i}^{(0)} + \beta_1 g_{k,i} P_{k,i}^{(1)}
\end{aligned}$$

The Lagrange dual optimization problem is given by  $\min_{\lambda \geq 0, \mu \geq 0} \tilde{g}(\lambda, \mu)$ .

For given sensing time  $\hat{\tau}$ , the joint optimization problem is convex with respect to the transmission power  $P_{k,i}^{(0)}$ ,  $P_{k,i}^{(1)}$ , respectively. Transmission power  $P_{k,i}^{(0)}$  and  $P_{k,i}^{(1)}$  are independent of each other in the joint optimization problem. Therefore, the problem can be solved by using dual decomposition method. The joint optimization problem can be decomposed into two optimization subproblems P1 and P2:

$$\begin{aligned}
P0: \max_{P_{k,i}^{(0)} \geq 0} & \frac{T-\hat{\tau}}{T} (\alpha_0 R_{00} + \beta_0 R_{10}) - \left( q + \frac{\lambda}{T} \right) (T-\hat{\tau}) \sum_{k=1}^K \sum_{i=1}^{n_k} (\alpha_0 + \beta_0) P_{k,i}^{(0)} \\
& - \mu \frac{T-\hat{\tau}}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} \beta_0 g_{k,i} P_{k,i}^{(0)}
\end{aligned} \tag{27}$$

$$\begin{aligned}
P1: \max_{P_{k,i}^{(1)} \geq 0} & \frac{T-\hat{\tau}}{T} (\alpha_1 R_{01} + \beta_1 R_{11}) - \left( q + \frac{\lambda}{T} \right) (T-\hat{\tau}) \sum_{k=1}^K \sum_{i=1}^{n_k} (\alpha_1 + \beta_1) P_{k,i}^{(1)} \\
& - \mu \frac{T-\hat{\tau}}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} \beta_1 g_{k,i} P_{k,i}^{(1)}
\end{aligned} \tag{28}$$

By writing the Lagrangian function  $L_{P0}(P_{k,i}^{(0)}, \lambda, \mu, \rho)$  of the optimization subproblem P1, the Karush-Kuhn-Tucker (KKT) conditions is given as follows.

$$\begin{aligned}
\frac{\partial L_{P0}(P_{k,i}^{(0)}, \lambda, \mu, \rho)}{\partial P_{k,i}^{(0)}} &= 0, \\
\rho &\geq 0, P_{k,i}^{(0)} \geq 0, \\
\rho P_{k,i}^{(0)} &= 0, \quad \forall k, i \in \{1, \dots, K\}
\end{aligned} \tag{29}$$

where  $\rho$  is the Lagrangian multiplier.



The optimal transmission power when PU is detected to be inactive can be obtained as:

$$P_{k,i}^{(0)} = \left[ \frac{B_{k,i}^{(0)} + \sqrt{\Delta_{k,i}^{(0)}}}{2A_{k,i}^{(0)}} \right]^+ \tag{30}$$

where

$$\begin{aligned} A_{k,i}^{(0)} &= [(qT + \lambda)(\alpha_0 + \beta_0) + \mu\beta_0 g_{k,i}] |h_{k,i}|^4 \ln 2 \\ B_{k,i}^{(0)} &= (\alpha_0 + \beta_0) |h_{k,i}|^4 - [(qT + \lambda)(\alpha_0 + \beta_0) + \mu\beta_0 g_{k,i}] (\varphi + 2\sigma_n^2) |h_{k,i}|^2 \ln 2 \\ \Delta_{k,i}^{(0)} &= (B_{k,i}^{(0)})^2 - 4A_{k,i}^{(0)} \delta_{k,i}^{(0)} \\ \delta_{k,i}^{(0)} &= [(qT + \lambda)(\alpha_0 + \beta_0) + \mu\beta_0 g_{k,i}] \sigma_n^2 (\varphi + \sigma_n^2) \ln 2 - [\alpha_0 (\varphi + \sigma_n^2) + \beta_0 \sigma_n^2] |h_{k,i}|^2 \end{aligned}$$

The optimal transmission power when PU is detected to be active can be obtained by the same method as follows:

$$P_{k,i}^{(1)} = \left[ \frac{B_{k,i}^{(1)} + \sqrt{\Delta_{k,i}^{(1)}}}{2A_{k,i}^{(1)}} \right]^+ \tag{31}$$

where

$$\begin{aligned} A_{k,i}^{(1)} &= [(qT + \lambda)(\alpha_1 + \beta_1) + \mu\beta_1 g_{k,i}] |h_{k,i}|^4 \ln 2 \\ B_{k,i}^{(1)} &= (\alpha_1 + \beta_1) |h_{k,i}|^4 - [(qT + \lambda)(\alpha_1 + \beta_1) + \mu\beta_1 g_{k,i}] (\varphi + 2\sigma_n^2) |h_{k,i}|^2 \ln 2 \\ \Delta_{k,i}^{(1)} &= (B_{k,i}^{(1)})^2 - 4A_{k,i}^{(1)} \delta_{k,i}^{(1)} \\ \delta_{k,i}^{(1)} &= [(qT + \lambda)(\alpha_1 + \beta_1) + \mu\beta_1 g_{k,i}] \sigma_n^2 (\varphi + \sigma_n^2) \ln 2 - [\alpha_1 (\varphi + \sigma_n^2) + \beta_1 \sigma_n^2] |h_{k,i}|^2 \end{aligned}$$

According to Theorem 1, obtaining the optimal  $(P_{k,i}^{(0)*}, P_{k,i}^{(1)*})$  is equivalent to finding the root for the equation,  $F(q^*, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) = 0$ . For fixed  $\lambda$  and  $\mu$ , the bi-section search method can be used to solve the problem. The search range  $[q_{\min}, q_{\max}]$  in the bisection search should be identified.

**Proposition 1:** Assuming  $h_{K,n_k} = \max\{h_{k,i}\}, \forall k \in \{1, \dots, K\}, i \in \{1, \dots, n_k\}$ ,  $q$  exists in the range  $[0, q_{\max}]$  to make  $F(q^*, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) = 0$  satisfied, where  $q_{\max} = \max\{Q_0^*, Q_1^*\}$ ,

$$Q_0^* = \frac{\alpha_0 |h_{K,n_k}|^2}{(\alpha_0 + \beta_0)T\sigma_n^2} + \frac{\beta_0 |h_{K,n_k}|^2}{(\alpha_0 + \beta_0)T(\varphi + \sigma_n^2)} \quad \text{and} \quad Q_1^* = \frac{\alpha_1 |h_{K,n_k}|^2}{(\alpha_1 + \beta_1)T\sigma_n^2} + \frac{\beta_1 |h_{K,n_k}|^2}{(\alpha_1 + \beta_1)T(\varphi + \sigma_n^2)} .$$

**Proof:** See Appendix C.

The optimal sensing time and power allocation strategy can be obtained by the algorithm in Appendix D.

### 4. Numerical Simulations

The optimal system performance of sensing-based spectrum sharing MIMO CR network is numerically evaluated in this section. The channel power gains are assumed to be exponentially distributed random variables with unit mean. The noise variance is set to 1. The frame duration is chosen to be  $T=100$  ms, and the transmit power of PU is assumed to be 10 dB. The circuit power and the sensing power are set to be 0.4 W and 0.2 W, respectively.

In Fig. 2, the EE of secondary base station versus sensing time is presented for several values of the probability  $P(H_0)$  that the frequency band is idle. The secondary base station with six antennas transmits to three SUs. Each SU is equipped with two antennas. The

probability of false alarm  $P_f$  is set to 0.001. The maximum average transmit power and interference power is assumed as  $P_s = 5\text{dB}$ ,  $\Gamma = -10\text{dB}$ . **Fig. 2** shows the EE of base station is a convex function of the sensing time. The optimal sensing time for the maximum EE depends on the active status of PU. The more inactive the PU, the smaller the optimal sensing time is. Furthermore, the EE clearly increases with the probability that PU is inactive because of the quasi-concave relationship between the EE and the transmit power in mathematics. In physics, when the probability of PU's inactive  $P(H_0)$  is 0.9, the SBS has much more opportunistic to use high power to transmit compared with other cases when the probability of PU's inactive is lower, then the EE would increase. Thus, EE increases with sensing time, and probability  $P(H_0)$  increases before reaching the maximum point.

As shown in **Fig. 3**, the EE of the secondary base station versus the average transmit power constraint  $P_s$  is shown for various values of the number of SU and for the different probabilities of the false alarm  $P_f$ . The secondary base station is equipped with eight antennas, and each SU is equipped with two antennas. The probability that PU is idle is assumed to be 0.6 (the same condition in [3],[4]), and the maximum average interference power is assumed to be -10 dB. **Fig. 3** shows that the EE increases with the number of SU and the average transmit power constraint. EE is slightly higher when the probability of false alarm is 0.003 than when the probability of false alarm is 0.001 under low values of the maximum average transmit power  $P_s$ . As less transmit power is allocated during the transmitter slot, the order of the spectrum sensing results will become smaller for the maximum EE.

In **Fig. 4**, the EE of the secondary base station versus the average interference power constraint  $\Gamma$  is presented for different values of the probabilities that the frequency band is idle under different sensing scenarios. The sensing scenarios include two cases: (1) perfect sensing when the SBS correctly senses the status of PU,  $P_d=1$ ,  $P_f=0$ ; (2) imperfect sensing due to the limitation of sensing technology results in missed detection and false alarm,  $P_d < 1$ . The secondary base station is equipped with six antennas, and three SUs are equipped with two antennas. The maximum average transmit power  $P_s$  is set to 5 dB, and the probability of false alarm  $P_f$  is 0.001. The condition is similar to that in **Fig. 2**. Therefore, sensing time is assumed to be the optimal sensing time, which is 3 ms. Owing to the reality of false alarm and misdetection, EE in perfect sensing scenario is always higher than that in imperfect sensing scenario. EE increases slowly when the interference power constraint threshold becomes higher in value. This condition is reasonable because the interference power constraint is not the main limit for the maximum EE when the interference power constraint threshold is a higher power.

In **Fig. 5**, optimal sensing time versus the probability that the frequency band is idle is presented for various values of the probabilities of false alarm  $P_f$ . The condition is similar to that in **Fig. 4**. The optimal sensing time decreases when PU is more inactive. As the probability of false alarm increases, optimal sensing time also decreases. This finding indicates the more rigorous the detection, the smaller the transmission rate will be.

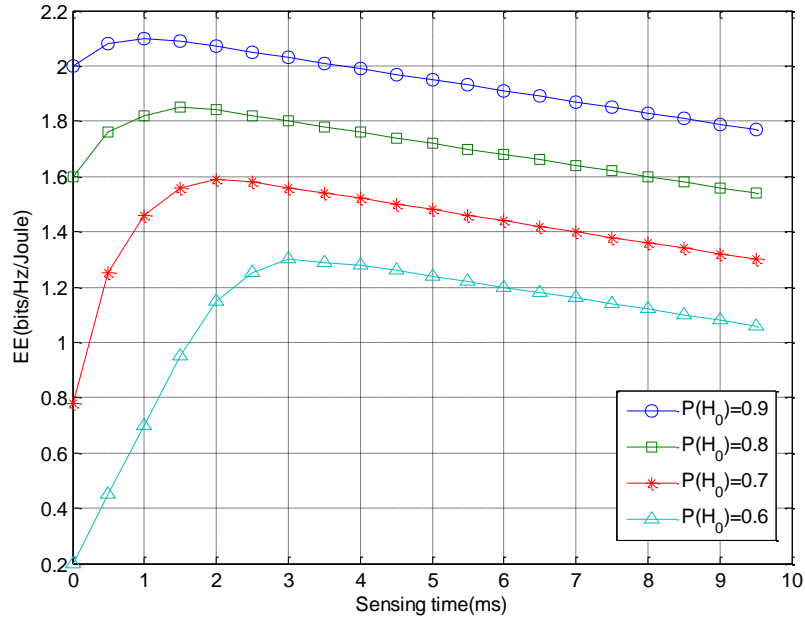


Fig. 2. EE versus sensing time for different values of  $P(H_0)$

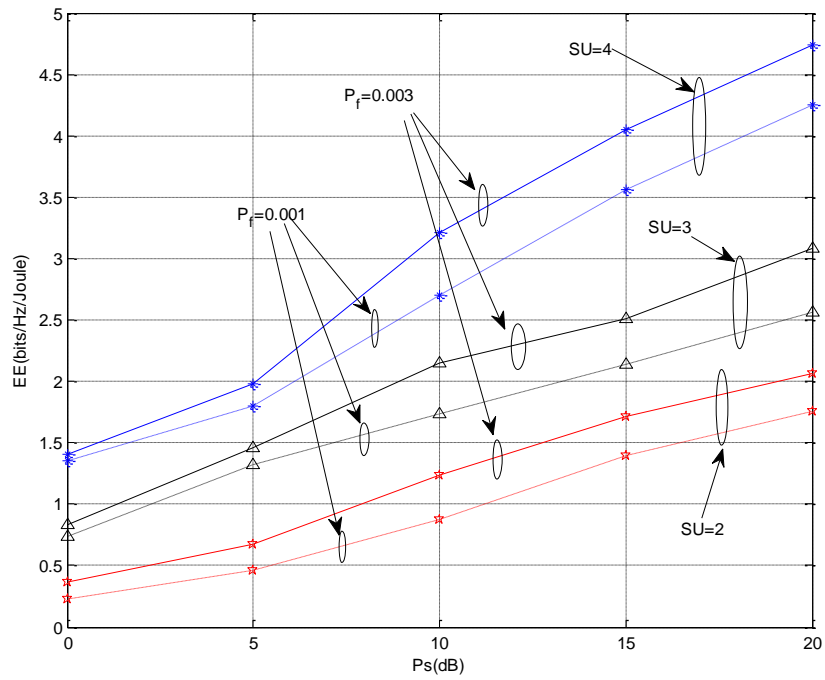


Fig. 3. EE versus transmit power constraint for several numbers of SU and different values  $P_f$

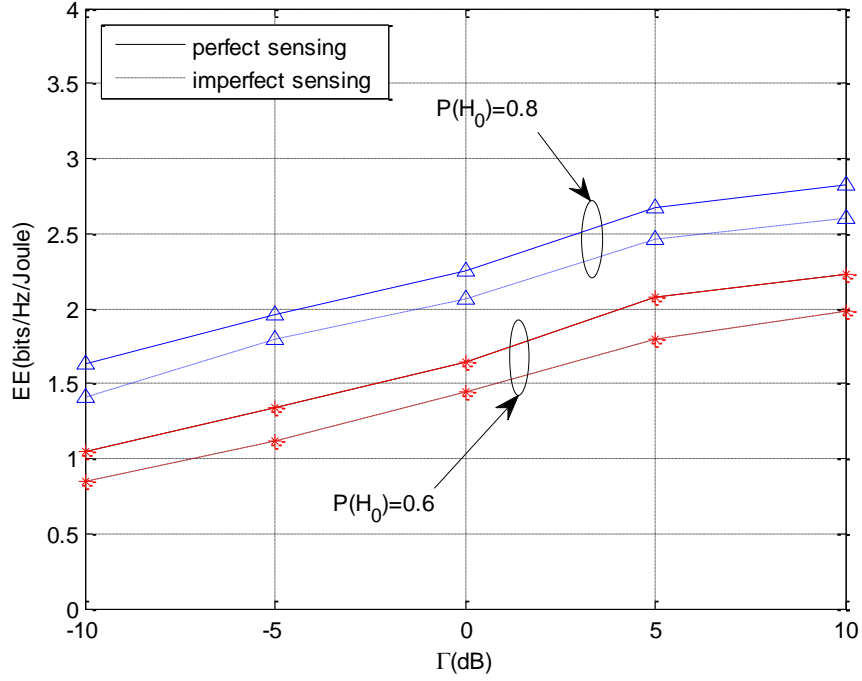


Fig. 4. EE versus the interference power constraint for different values of  $P(H_0)$  under different sensing scenarios

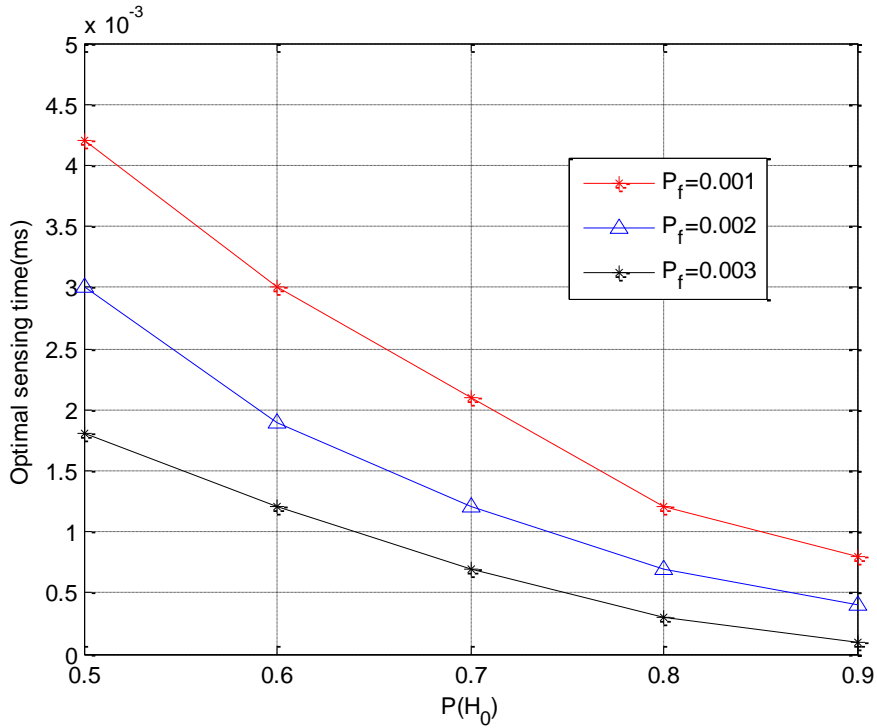
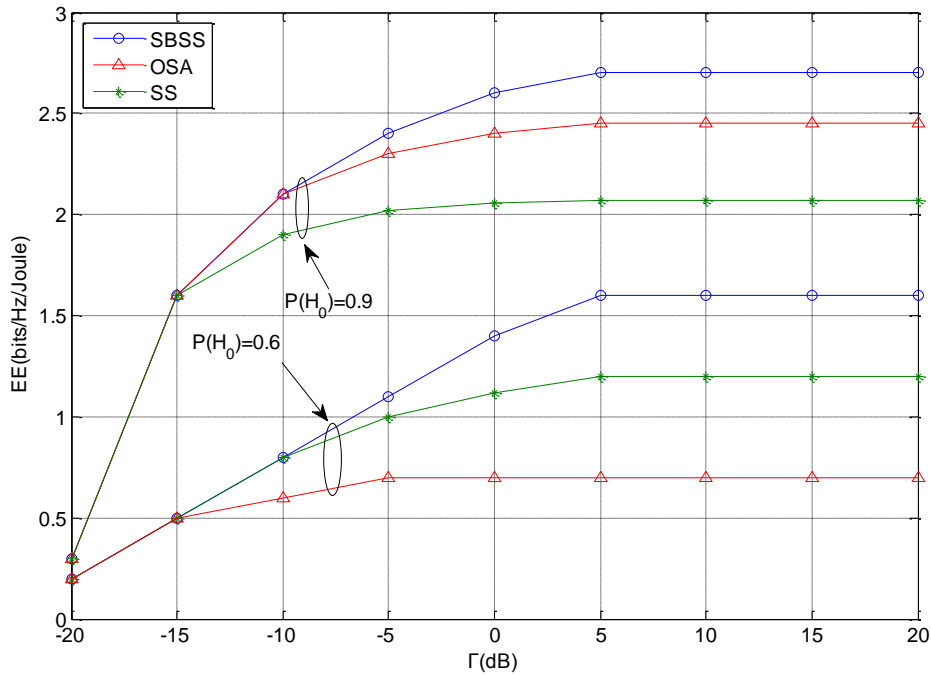


Fig. 5. Optimal sensing time versus  $P(H_0)$  for different values of  $P_f$



**Fig. 6.** The energy efficiency versus the interference power constraint  $\Gamma$  for various values of the probability  $P(H_0)$

In **Fig. 6**, the EE of the secondary base station versus the interference power constraint threshold  $\Gamma$  is presented for various values of the probability  $P(H_0)$  that the frequency band is idle under the sensing-based spectrum sharing, conventional spectrum sharing and opportunistic spectrum access model. The optimal sensing time is 3 ms and the probability of false alarm  $P_f$  is set to 0.001. The maximum average transmit power is assumed as  $P_s = 5\text{dB}$ . The secondary base station equipped with eight antennas, which transmit data to four SUs. Each SU is equipped with two antennas. EE under the three conditions increases with the interference constraint and the transmission constraint, respectively. However, EE does not increase when the interference power constraint threshold  $\Gamma$  becomes larger because the interference power constraint would not be the dominating constraint for the EE when it becomes larger. For the same probability  $P(H_0)$ , the EE of the secondary base station in the sensing-based spectrum sharing is always higher than those of the conventional spectrum sharing and the opportunistic spectrum access. This finding is attributed to the facts that the secondary base station in sensing-based spectrum sharing model could adjust transmission power according to the detection results. When probability  $P(H_0)$  is 0.6, the EE in OSA model is very small because there is less opportunity to access the licensed spectrum. However, when the probability  $P(H_0)$  is 0.9, the EE of the secondary base station in the SBSS model is slightly larger than that in OSA model and much larger than that in SS model. This finding is attributed to the facts that when the probability  $P(H_0)$  is higher, the case in the SBSS model is almost same as that in the OSA model.

## Conclusion

The optimization problem of sensing time and power allocation for maximizing the EE of the secondary base station in sensing-based spectrum sharing MIMO CR networks is investigated in this study. The multiple antenna spectrum sensing technology is employed to detect the status of PU accurately. The corresponding pre-coding matrix at the secondary base station has been used to avoid inner interference among SUs. The transmission power constraint and the interference power constraint limit the transmission power of the secondary base station and protect the quality of service of PUs. The EE problem is formulated as a nonlinear stochastic fractional programming, which is a nonconvex optimal problem. The EE problem is transformed into the equivalent nonlinear parametric programming and solved by the one-dimension search algorithm. To reduce search complexity, the search range was founded by demonstration. Simulation results revealed the EE can be enhanced via spectrum sensing and corresponding constraints adjustment.

## APPENDIX A

Denote the upper contour sets of  $U_{EE}(P_{k,i}^{(0)}, P_{k,i}^{(1)})$  as:

$$S_\rho = \{P_{k,i}^{(0)} \geq 0, P_{k,i}^{(1)} \geq 0 \mid U_{EE}(P_{k,i}^{(0)}, P_{k,i}^{(1)}) \geq \rho\}$$

According to [1],  $U_{EE}(P_{k,i}^{(0)}, P_{k,i}^{(1)})$  is strictly quasi-concave in  $P_{k,i}^{(0)}, P_{k,i}^{(1)}$ , respectively, if and only if  $S_\rho$  is strictly convex for any real number  $\rho$ .

When  $\rho < 0$ , no points exist on the contour  $U_{EE}(P_{k,i}^{(0)}, P_{k,i}^{(1)}) = \rho$ . When  $\rho = 0$ , only  $P_{k,i}^{(0)} = 0$  and  $P_{k,i}^{(1)} = 0$  is on the contour  $U_{EE}(P_{k,i}^{(0)}, P_{k,i}^{(1)}) = \rho$ . When  $\rho < 0$ ,  $S_\rho$  is equivalent to:

$$S_\rho = \{P_{k,i}^{(0)} \geq 0, P_{k,i}^{(1)} \geq 0 \mid \rho(E_t(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)}) + E_s(\tau) + E_c) - R(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)}) \leq 0\}$$

Proving that  $\rho(E_t(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)}) + E_s(\tau) + E_c) - R(\tau, P_{k,i}^{(0)}, P_{k,i}^{(1)})$  is strictly convex in  $P_{k,i}^{(0)}, P_{k,i}^{(1)}$ , is easy. Hence,  $S_\rho$  is strictly convex and  $U_{EE}(P_{k,i}^{(0)}, P_{k,i}^{(1)})$  is strictly quasi-concave in  $P_{k,i}^{(0)}, P_{k,i}^{(1)}$ .

## APPENDIX B

If  $(P_{k,i}^{(0)*}, P_{k,i}^{(1)*})$  is the optimal solution of (3) associated with the maximum value  $q^*$ , then:

$$q^* = \frac{R(\hat{\tau}, P_{k,i}^{(0)*}, P_{k,i}^{(1)*})}{E_t(\hat{\tau}, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) + E_s(\hat{\tau}) + E_c}$$

This equation implies that

$$R(\hat{\tau}, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) - q^*(E_t(\hat{\tau}, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) + E_s(\hat{\tau}) + E_c) = 0$$

$$R(\hat{\tau}, P_{k,i}^{(0)}, P_{k,i}^{(1)}) - q^*(E_t(\hat{\tau}, P_{k,i}^{(0)}, P_{k,i}^{(1)}) + E_s(\hat{\tau}) + E_c) \leq 0$$

Then conclude  $F(q^*, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) = 0$ .

If  $F(q^*) = F(q^*, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) = 0$ , then  $F(q^*, P_{k,i}^{(0)}, P_{k,i}^{(1)}) \leq F(q^*, P_{k,i}^{(0)*}, P_{k,i}^{(1)*})$ .

Then:

$$\frac{R(\hat{\tau}, P_{k,i}^{(0)*}, P_{k,i}^{(1)*})}{E_t(\hat{\tau}, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) + E_s(\hat{\tau}) + E_c} = q^*; \frac{R(\hat{\tau}, P_{k,i}^{(0)}, P_{k,i}^{(1)})}{E_t(\hat{\tau}, P_{k,i}^{(0)}, P_{k,i}^{(1)}) + E_s(\hat{\tau}) + E_c} \leq q^*$$

Thus  $(P_{k,i}^{(0)*}, P_{k,i}^{(1)*})$  is the optimal solution of (3) with the maximum value  $q^*$ .

### APPENDIX C

If  $q = 0$ ,  $F(q, P_{k,i}^{(0)}, P_{k,i}^{(1)}) > 0$ ; if  $q > 0$ , there may exist  $F(q, P_{k,i}^{(0)}, P_{k,i}^{(1)}) < 0$ . Furthermore, if a point  $q$  exists satisfying the following inequality,

$$\alpha_0 R_{00} + \alpha_1 R_{01} + \beta_0 R_{10} + \beta_1 R_{11} - qT \sum_{k=1}^K \sum_{i=1}^{n_k} (\alpha_0 + \beta_0) P_{k,i}^{(0)} + (\alpha_1 + \beta_1) P_{k,i}^{(1)} < 0$$

Meanwhile, the above inequality could decompose two subproblems as follows:

$$\begin{aligned} & \sum_{k=1}^K \sum_{i=1}^{n_k} \alpha_0 \log_2 \left( 1 + \frac{|h_{k,i}|^2 P_{k,i}^{(0)}}{\sigma_n^2} \right) + \beta_0 \log_2 \left( 1 + \frac{|h_{k,i}|^2 P_{k,i}^{(0)}}{\varphi + \sigma_n^2} \right) - qT(\alpha_0 + \beta_0) P_{k,i}^{(0)} < 0 \\ & \sum_{k=1}^K \sum_{i=1}^{n_k} \alpha_1 \log_2 \left( 1 + \frac{|h_{k,i}|^2 P_{k,i}^{(1)}}{\sigma_n^2} \right) + \beta_1 \log_2 \left( 1 + \frac{|h_{k,i}|^2 P_{k,i}^{(1)}}{\varphi + \sigma_n^2} \right) - qT(\alpha_1 + \beta_1) P_{k,i}^{(1)} < 0 \end{aligned}$$

Assuming that  $h_{K,n_k} = \max\{h_{k,i}\}, \forall k \in \{1, \dots, K\}, i \in \{1, \dots, n_k\}$ , the first subproblem must be established if the following equality is satisfied.

$$\frac{\alpha_0 \log_2 \left( 1 + \frac{|h_{K,n_k}|^2 P_{k,i}^{(0)}}{\sigma_n^2} \right)}{T(\alpha_0 + \beta_0) P_{k,i}^{(0)}} + \frac{\beta_0 \log_2 \left( 1 + \frac{|h_{K,n_k}|^2 P_{k,i}^{(0)}}{\varphi + \sigma_n^2} \right)}{T(\alpha_0 + \beta_0) P_{k,i}^{(0)}} < q$$

Then, the minimum value  $q$  can be obtained as follows:

$$\begin{aligned} & \max_{P_{k,i}^{(0)}} \frac{\alpha_0 \log_2 \left( 1 + \frac{|h_{K,n_k}|^2 P_{k,i}^{(0)}}{\sigma_n^2} \right)}{T(\alpha_0 + \beta_0) P_{k,i}^{(0)}} + \frac{\beta_0 \log_2 \left( 1 + \frac{|h_{K,n_k}|^2 P_{k,i}^{(0)}}{\varphi + \sigma_n^2} \right)}{T(\alpha_0 + \beta_0) P_{k,i}^{(0)}} \\ & = \lim_{P_{k,i}^{(0)} \rightarrow 0} \frac{\alpha_0 \log_2 \left( 1 + \frac{|h_{K,n_k}|^2 P_{k,i}^{(0)}}{\sigma_n^2} \right)}{T(\alpha_0 + \beta_0) P_{k,i}^{(0)}} + \frac{\beta_0 \log_2 \left( 1 + \frac{|h_{K,n_k}|^2 P_{k,i}^{(0)}}{\varphi + \sigma_n^2} \right)}{T(\alpha_0 + \beta_0) P_{k,i}^{(0)}} \\ & = \lim_{P_{k,i}^{(0)} \rightarrow 0} \frac{\alpha_0 |h_{K,n_k}|^2}{T(\alpha_0 + \beta_0)(\sigma_n^2 + |h_{K,n_k}|^2 P_{k,i}^{(0)})} + \frac{\beta_0 |h_{K,n_k}|^2}{T(\alpha_0 + \beta_0)(\varphi + \sigma_n^2 + |h_{K,n_k}|^2 P_{k,i}^{(0)})} \\ & = \frac{\alpha_0 |h_{K,n_k}|^2}{T(\alpha_0 + \beta_0)\sigma_n^2} + \frac{\beta_0 |h_{K,n_k}|^2}{T(\alpha_0 + \beta_0)(\varphi + \sigma_n^2)} \end{aligned}$$

For the second subproblem, the minimum value can be obtained by  $\frac{\alpha_1 |h_{K,n_k}|^2}{T(\alpha_1 + \beta_1)\sigma_n^2} + \frac{\beta_1 |h_{K,n_k}|^2}{T(\alpha_1 + \beta_1)(\varphi + \sigma_n^2)}$  using the same method. If  $q_{\max} = \max\{Q_0^*, Q_1^*\}$ , where

$Q_0^* = \frac{\alpha_0 |h_{K,n_k}|^2}{(\alpha_0 + \beta_0)T\sigma_n^2} + \frac{\beta_0 |h_{K,n_k}|^2}{(\alpha_0 + \beta_0)T(\varphi + \sigma_n^2)}$  and  $Q_1^* = \frac{\alpha_1 |h_{K,n_k}|^2}{(\alpha_1 + \beta_1)T\sigma_n^2} + \frac{\beta_1 |h_{K,n_k}|^2}{(\alpha_1 + \beta_1)T(\varphi + \sigma_n^2)}$ , the equality  $F(P_{k,i}^{(0)}, P_{k,i}^{(1)}) < 0$  must be satisfied. Thus, a point  $q \in [0, q_{\max}]$  must exist, satisfying  $F(q^*, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) = 0$ .

## APPENDIX D

<b>Algorithm.</b> The optimal sensing time and power allocation strategy in MIMO CR system
For $\hat{t} = 0:T$
1. Find $h_{K,n_k} = \max\{h_{k,i}\}$ , compute $Q_0^*, Q_1^*, q_{\max} = \max\{Q_0^*, Q_1^*\}$ .
2. Initialization $q_j = \frac{a+b}{2}$ , $a=0$ , $b=q_{\max}$ , $j=1$ .
3. Repeat <ul style="list-style-type: none"> <li>I. Initialization <math>\lambda_k, k=1</math>.</li> <li>II. Repeat           <ul style="list-style-type: none"> <li>i. Initialization <math>\mu_l, l=1</math>.</li> <li>ii. Calculate <math>P_{k,i}^{(0)}, P_{k,i}^{(1)}</math>.</li> <li>iii. Calculate the subgradient <math>\tilde{g}(\mu^*) _{\mu_l}</math> by <math>\Gamma - \frac{T-\hat{t}}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} \beta_0 g_{k,i} P_{k,i}^{(0)} + \beta_1 g_{k,i} P_{k,i}^{(1)}</math></li> <li>iv. Update <math>\mu_{l+1}</math> by <math>\mu_l - \nu(\Gamma - \frac{T-\hat{t}}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} \beta_0 g_{k,i} P_{k,i}^{(0)} + \beta_1 g_{k,i} P_{k,i}^{(1)})</math>.</li> <li>v. Stop when <math> \mu_{l+1} - \mu_l  \leq \mathcal{G}</math>.</li> </ul> </li> <li>III. Calculate the subgradient <math>\tilde{g}(\lambda, \mu^*)</math> for the given <math>\mu</math> by           <math display="block">P_S - \frac{T-\hat{t}}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} (\alpha_0 + \beta_0) P_{k,i}^{(0)} + (\alpha_1 + \beta_1) P_{k,i}^{(1)}</math> </li> <li>IV. Update <math>\lambda_{k+1}</math> by <math>\lambda_k - \nu(P_S - \frac{T-\hat{t}}{T} \sum_{k=1}^K \sum_{i=1}^{n_k} (\alpha_0 + \beta_0) P_{k,i}^{(0)} + (\alpha_1 + \beta_1) P_{k,i}^{(1)})</math>.</li> <li>V. Stop when <math> \lambda_{k+1} - \lambda_k  \leq \mathcal{G}</math>.</li> </ul>
4. Calculate $F(q_j, P_{k,i}^{(0)*}, P_{k,i}^{(1)*})$ . If $F(q_j, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) > 0$ , $a = \frac{a+b}{2}$ ; If $F(q_j, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) < 0$ , $b = \frac{a+b}{2}$ ; else, stop.
5. Update $q_{j+1} = \frac{a+b}{2}$
6. Stop when $ F(q_{j+1}, P_{k,i}^{(0)*}, P_{k,i}^{(1)*}) - F(q_j, P_{k,i}^{(0)*}, P_{k,i}^{(1)*})  \leq \mathcal{G}$

## References

- [1] J.M.III and J.G.Q.Maguire, "Cognitive radios: making software radio more personal," *IEEE Personal Commun. Mag.*, vol. 6, no.4, pp.13-18, Aug.1999. [Article \(CrossRef Link\)](#)
- [2] Q. Chen, Y. Liang, M. Motani, and W-C. Wong, "A two-level mac protocol strategy for opportunistic spectrum access in cognitive radio networks," *IEEE Transactions on Vehicular Technology*, vol.60, no.5, pp.2164-2180, June 2011. [Article \(CrossRef Link\)](#)
- [3] X. Kang, Y. Liang, A. Nallanathan, H.K. Garg and R. Zhang, "Optimal power allocation for OFDM-based cognitive radio with new primary transmission protect criteria," *IEEE Transactions on Wireless Communications*, vol.9, no.6, pp. 2066-2075, February 2010. [Article \(CrossRef Link\)](#)
- [4] S. Stotas and A. Nallanathan, "Optimal sensing time and power allocation in multiband cognitive radio networks," *IEEE Transactions on Communications*, vol.59, no.1, pp.226-235, January



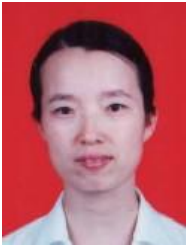
2011. [Article \(CrossRef Link\)](#)
- [5] B. Ning, S. Yang, L. Liu, and Y. Lu, "Resource allocation for OFDM cognitive radio within enhanced primary transmission protection," *IEEE Communications Letters*, vol.18, no.11, pp. 2027-2030, November 2014. [Article \(CrossRef Link\)](#)
  - [6] Chengwen Xing, Shuo Li, Zesong Fei, Jingming Kuang, "How to understand linear minimum mean-square-error transceiver design for multiple-input –multiple-output systems from quadratic matrix programming," *IET Communications*, 2013, vol.7, iss.12, pp.1231-1242, August 2013. [Article \(CrossRef Link\)](#)
  - [7] Chengwen Xing, Shaodan Ma, and Yiqing Zhou, "Matrix-Monotonic Optimization for MIMO Systems," *IEEE Transactions on Signal Processing*, vol.63, no.2, pp. 334-348, January 2015. [Article \(CrossRef Link\)](#)
  - [8] Farzad Moghimi, Ranjan K.Mallik and Robert Schober, "Sensing time and power optimization in MIMO cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol.11, no.9, pp. 3398-3408, September 2012. [Article \(CrossRef Link\)](#)
  - [9] Rui Zhang and Ying-Chang Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE Journal of selected topics in Signal Processing*, vol.2, no.1, pp. 88–102, February 2008. [Article \(CrossRef Link\)](#)
  - [10] Siyoung Choi, Hyunsung Park and Taewon Huang, "Optimal beamforming and power allocation for sensing-based spectrum sharing in Cognitive radio networks," *IEEE Transactions on Vehicular Technology*, vol.63, no.1, pp. 412–417, January 2014. [Article \(CrossRef Link\)](#)
  - [11] Xiaoge Huang, Baltasar Beferull-Lozano, Carmen Botella, "Non-convex power allocation games in MIMO cognitive radio networks," in *Proc. of IEEE 14<sup>th</sup> workshop on Signal Processing Advances in Wireless Communications*, pp. 145-149, 2013. [Article \(CrossRef Link\)](#)
  - [12] Guowang Miao, "Energy-Efficient uplink multi-user MIMO," *IEEE Transactions on Wireless Communications*, vol.12, no.5, pp. 2302–2313, May 2013. [Article \(CrossRef Link\)](#)
  - [13] Cong Xiong, Geoffrey Ye Li, Shunqing Zhang, Yan Chen and Shugong Xu, "Energy-Efficient resource allocation in OFDMA networks," *IEEE Transactions on Communications*, vol.60, no.12, pp. 3767–3778, December 2012. [Article \(CrossRef Link\)](#)
  - [14] Liang Wang, Min Sheng, Xijun Wang, Yan Zhang and Xiao Ma, "Mean energy efficiency maximization in cognitive radio channels with PU outage constraint," *IEEE Communications Letters*, vol.19, no.2, pp. 287–290, February 2015. [Article \(CrossRef Link\)](#)
  - [15] Junling Mao, Gang Xie, Jinchun Gao and Yuanan Liu, "Energy efficiency optimization for cognitive radio MIMO broadcast channels," *IEEE Communications Letters*, vol.17, no.2, pp. 337–340, February 2013. [Article \(CrossRef Link\)](#)
  - [16] Jing Zhang, Fu-chun Zheng, Xi-Qi Gao and Hong-Bo Zhu, "Sensing-energy efficiency tradeoff for cognitive radio networks," *IET Communications*, vol.8, no.18, pp.3414-3423, December 2014. [Article \(CrossRef Link\)](#)
  - [17] Abbas Taherpour, M. Nasiri-Kenari and Saeed Gazor, "Multiple antenna spectrum sensing in cognitive radios," *IEEE Transactions on Wireless Communications*, vol.9, no.2, pp. 814–823, February 2010. [Article \(CrossRef Link\)](#)



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