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Case studies for solving the Saint-Venant equations using the method of characteristics: pipeline hydraulic transients and discharge propagation

Regina Mambeli Barros[1], Geraldo Lúcio Tiago Filho[1], Ivan Felipe Silva dos Santos and Fernando das Graças Braga da Silva[1]

[1] IRN/ UNIFEI, Researcher from National Reference Center in SHP-CERPCH; Natural Resources Institute – IRN; Federal University of Itajubá, UNIFEI, Av.BPS, 1303, Itajubá-MG, CEP: 37500-903 Av.BPS, 1303, Itajubá-MG, Brazil, CEP: 37500-903, phone number +55 35 36291224, remambeli@hotmail.com, Tiago_unifei@hotmail.com, ivanfelipedeice@hotmail.com, ffbraga.silva@gmail.com

Abstract

This study aims to present a hydraulic transitory study as MOC applications for solving the Saint-Venant equations in two case studies: 1) in a penstock of a small hydropower system as a simple pipeline in the case of valve-closure in the downstream boundary with a reservoir in the upstream boundary; and 2) for discharge propagation into a channel by velocity and depth of the flow channel along space evaluation. The proposed methodology by Chaudry [5] concerning the development of hydrodynamic models was used. The obtained results for first and second case study has been confirmed that MOC numerical approach is useful for several engineering purposes, including cases of hydraulic transients and discharge propagation in hydraulic systems.

Keywords: Method of characteristics, Hydraulic transient, Discharge propagation, Saint-Venant equations Numerical Approach, Hydraulic Systems.

1. Introduction

Hydraulic transient events occur during a change in state from one steady or equilibrium condition to another, according to Afshar *et al.* [2]. Examples include after sudden valve opening or closure, starting or stopping of pumps or turbines, mechanical failure of an item, rapid changes in demand condition, etc. [1]. The main components of these disturbances are pressure and flow changes that bring about propagation of pressure waves throughout the system, and velocity of this wave may exceed 1000 m/s, which may lead to severe damages [1, 2]. Since within any pipeline system head and flow distribution in the system is predicted at different operating conditions, the modeling of these phenomena is possible and is pursued in this paper.

Various numerical approaches have been introduced for calculation of the pipeline transients, including method of characteristics (MOC), finite volume method (FVM), finite element method (FEM), wave characteristics method (WCM), and finite difference method (FDM). Among these methods, MOC is the most commonly used method due to its simplicity and its superior performance as compared with other methods [2].

The water hammer effects caused by closure of spherical valves against discharge were studied by Karadžić *et al.* [8], in which the case study analyzed a Perućica high-head hydropower plant (HPP) in Montenegro. In this HPP case study, safety spherical valves (inlet turbine valves) have been refurbished on the first two Pelton turbines unit. According to the authors (op. cit.), the spherical valve boundary condition was incorporated into the MOC algorithm. As a result, we found that flow conditions do not have a significant impact on the spherical valve closure time for the cases investigated by Karadžić *et al.* [8], and the developed numerical models show reasonable agreement with measured results.

In spite of the lack of experiments for quantitative validation, the purpose of the present paper is to present computational results that are expected to be instructive for the optimum design of the SHPs to mitigate the potential damage caused by valve-induced closing-time water hammers.

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Corresponding author: Regina Mambeli Barros, Professor, remambeli@hotmail.com

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2. Methodology

2.1. The Saint-Venant Equations

We use the methodology proposed by Chaudry [5] concerning the development of hydrodynamic models, in which runoff is regarded as a phenomenon using the laws of physics, namely conservation of mass (assuming space), conservation of momentum etc.

Equations (1) and (2) , e.g., mass and momentum conservation equations, have been called the Saint-Venant equations. They are partial differential equations with few explicit solutions, as recommended by Chaudhry [5].

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q'.T \tag{1}$$

where T is the surface width [L]; A is the cross-sectional area [L^2]; Q is the discharge rate [L^3/T]; and $q'.T = q(m^3/s/m)$ is the unitary side entrance (meters), as developed by Chaudhry [5].

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=q'\frac{Tv}{A}+g\left(S_f-S_0\right)=0 \tag{2}$$

where v is the speed; S0 is the slope of the water line (slope of the bottom); and Sf is the slope of the energy line.

By assuming one-dimensional flow and based on the continuity and momentum equations that describe the general behavior of fluids in a closed pipe in terms of two variables (i.e., y: piezometric head, and v: fluid velocity), the analyses of most hydraulic transients in pressurized systems may be carried out. Wave propagation velocity or celerity, c, friction f, and pipe diameter D are pipe parameters that can be considered constant through time, despite the fact that they are spatial functions, as recommended by Izquierdo and Iglesias [7].

The alternatives for solving such equations for both the discharge and depth of water variations along both the flow (x) and over time (t), according to Chaudhry [5] are the following:

I) To simplify the equations.

II) To use numerical methods (by replacement of derived by differences).

III) To make changes.

2.1.1. Methods of characteristics (MOC)

This method aims to transform the two partial differential equations (with two independent terms $\partial v/\partial x$ and $\partial v/\partial t$) into ordinary equations that have more convenient properties for numerical calculation and also allow explicit solutions [5]. The particle trajectories within the wave can be observed in eq. 3 to 6, according to Chaudhry [5]:

$$\text{I}: \frac{dx}{dt}=v+\sqrt{\frac{gA}{T}} \tag{3}$$

$$\text{II}: \frac{dv}{dt}+\sqrt{\frac{gT}{A}}\frac{dy}{dt}+\frac{q'T}{A}\left(v-\sqrt{\frac{gA}{T}}\right)+g\left(S_f-S_0\right)=0 \tag{4}$$

$$\text{III}: \frac{dx}{dt}=v-\sqrt{\frac{gA}{T}} \tag{5}$$

$$\text{IV}: \frac{dv}{dt}-\sqrt{\frac{gT}{A}}\frac{dy}{dt}+\frac{q'T}{A}\left(v+\sqrt{\frac{gA}{T}}\right)+g\left(S_f-S_0\right)=0 \tag{6}$$

By observing eq. (3) to (6), the two partial differential equations have expanded to four ordinary differential equations simplifying the work of resolution as can be seen [5] in fig. 1.

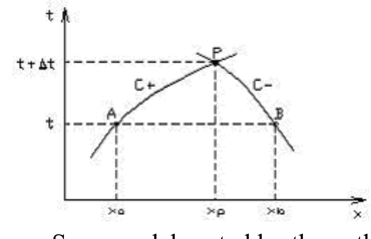

Fig. 1 Characteristics equations. Source: elaborated by the authors as based on Chaudhry [5]

If c^+ and c^- intersect in P point, the eq. (4) and (6) can be solved simultaneously (P as intersection of c^+ and c^-).

2.1.2. Solving strategy of the four equations

When the numerical solution of the differential equation is obtained, it is possible to replace the derivative by these approaches. Thus, the approach gives the following:

$$\frac{dx}{dt} = v + \sqrt{\frac{gA}{T}}$$
$$\frac{x_p - x_A}{t_p - t_A} = \left(v + \sqrt{\frac{gA}{T}} \right)_A \rightarrow \text{Explicit Method (progressing to the major point, from A to D) or}$$
$$\frac{x_p - x_A}{t_p - t_A} \cong \frac{(\ )_A + (\ )_p}{2} \rightarrow \text{As an average (Implicit Method)} \quad (7)$$

$$\frac{dx}{dt} = v - \sqrt{\frac{gA}{T}}$$
$$\frac{x_p - x_B}{t_p - t_B} = \left(v + \sqrt{\frac{gA}{T}} \right)_B \rightarrow \text{explicit approaching, or}$$
$$\frac{x_p - x_B}{t_p - t_B} \cong \frac{(\ )_B + (\ )_p}{2} \rightarrow \text{implicit approaching} \quad (8)$$

The Implicit MOC was proposed by Afshar and Rohani [1] and aims to alleviate the shortcomings and limitations of the mostly used conventional MOC. It allows for any arbitrary combination of devices in the pipeline system. The Implicit MOC was used to solve two example problems of transient flow caused by failure of a pump system and closure of a valve, and the results were presented and compared with those of the explicit MOC.

The approximation of an implicit differential equation is stable—and sometimes unconditionally stable—while the explicit is unstable unless Δt is very small and a consistent width Δx has been chosen, and according to the Courant condition, as recommended by Tucci [10]:

$$\frac{\Delta x}{\Delta t} \le \frac{1}{v_0 + c_0} \quad \text{and} \quad \frac{T.S_0.v_0}{y_0} \ge 171$$

The simultaneous resolution of these equations provides coordinates of P as shown in Figure 2a. The solution to the explicit scheme is straightforward, as recommended by Chaudhry [5]. Equations (3) to (6) can be written as follows in eq. (9) to (12):

$$\text{I}: \frac{dx}{dt} = v + c; \quad (9)$$

$$\text{II}: \frac{dv}{dt} + \frac{g}{c}\frac{dy}{dt} = q.(S_0 - S_f) + \ldots \quad (10)$$

$$\text{III}: \frac{dx}{dt} = v - c; \quad (11)$$

$$\text{IV}: \frac{dv}{dt} - \frac{g}{c}\frac{dy}{dt} = q.(S_0 - S_f) + \ldots \quad (12)$$

$$x_p - x_A = (v + c)(t_p - t_R)$$
$$v_p - v_R + \frac{g}{c_R}(y_p - y_A) = g\left(S_0 - S_f\big|_A\right)(t_p - t_A)$$
$$x_p - x_B = (v + c)(t_p - t_B)$$
$$v_p - v_B + \frac{g}{c_B}(y_p - y_B) = g\left(S_0 - S_f\big|_B\right)(t_p - t_B)$$

Since P is known, the equations I and IV (eq. 2 and 15) must be searched with the coordinates x_A and x_B, and the equations II and IV (equations 10 and 11) are numerically solved in order to obtain v_p and y_p into the all grid points, as recommended by Chaudhry [5]. The contouring points have no negative feature (c^+ and c^-) so that the channel correctly ends. This calculation is applied only to "interior points" but not for points where the contour has only one characteristic curve. At x=0 there is only the negative curve (c^-) and at x=L there is only a positive curve (c^+) [5].

In these contouring points, the characteristic equation for v and y (II and IV, i.e., eq.10 and 12) must be supplemented by another equation from the boundary condition (Figure 2b). Generally, at x=0 the hydrograph is used as an input boundary condition, and at the extreme downstream (x=L) a relationship between flow and elevation (curve-key) is also a boundary condition [5]. This sequence of calculations is described by Streeter and Wylie [9] and was recommended by Chaudhry [5].

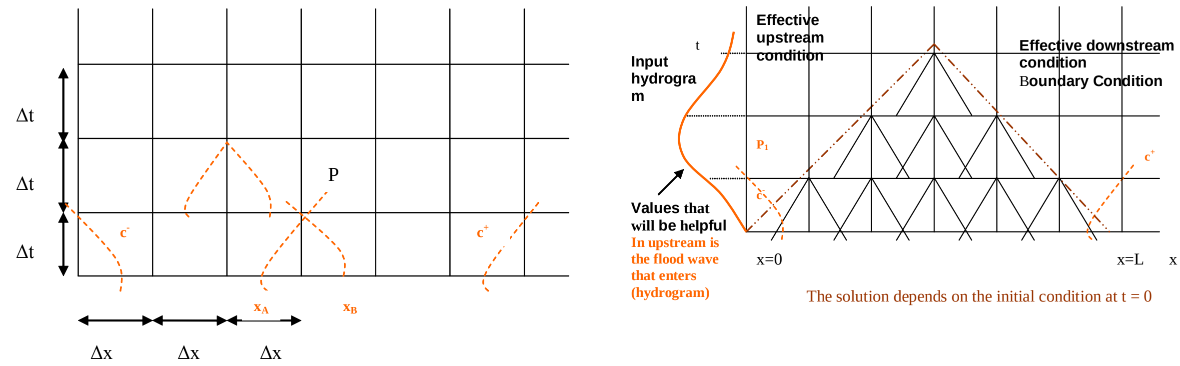

Fig. 2 MOC: **a)** Solving strategy of the characteristic equations with a regular grid and **b)** Boundary conditions in the characteristic equations solving strategy with a regular grid. Source: adapted from Chaudhry [5]

A hydraulic transitory study in a small hydropower system by the MOC was studied also by Barros, Tiago Filho, and Silva [3]. Also, Barros *et al.* [4] studied MOC in order to surge tank dimensioning for Small Hydro Power (SHP) plant design, especially regarding its surge tank sizing. For this purpose, the authors (*op. cit.*) used the criteria for maximum allowable pressure, as studied in the warmer hammer. We also use the maximum permissible overspeed, i.e., the last one, in the case of load rejection, according to Brazilian Electric Power, Eletrobras [6].

3. Data of cases studies

This study aims to present the hydraulic transitory study as MOC applications for solving the Saint-Venant equations in two case studies as follows:

3.1 Valve Closure in a Small Hydropower System

A spreadsheet in *Microsoft® Excel®* for modelling water hammer as proposed in Chaudhry [5] and presented by Streeter and Wylie [9] was developed in order to conduct a simple case study that aims at valve-closing at the end of downstream, and considers a constant level upstream reservoir (Figure 3). The valve-closing equation was specified by $C_dA_v/(C_dA_v)0 = (1\text{-}t / tc)_m$ where t_c was the closure-time whose value ranged from 4.0 to 12.0 s; m=3.2; L=2,152.50 meters; D= 4 meters; f=0.019, 13.82 m^3/s for turbine discharge and H_0=182.00 meters. For these calculations, Δx was equal to 706.20 meters and $\Delta t = 0.15$ s were used. Equations (4) and (6) have been solved by the MOC using a numerical grid, as recommended by Streeter and Wylie [6] and Chaudhry [5].

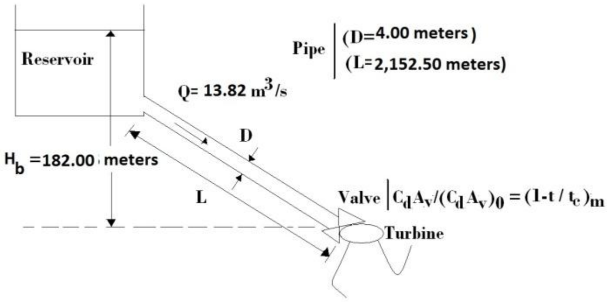

Fig. 3 Schematic representation of system of the valve closure in a small hydropower case study

3.2 Discharge Propagation into a Channel

For this study case, an entry hydrogram was inserted into a rectangular channel with a width of 6.1 meters, length of 3,048 meters, slope of 0.0016, and a uniform steady flow with nominal depth of 2.44 meters. The nominal height of the uniform flow is y_n = 6.0 ft, and the input hydrograph is presented in Figure 4. The Chézy coefficient is C = 100, and a *Microsoft® Excell®* spreadsheet calculated the velocity and depth of flow into the channel at each interval of space. The characteristic curve of the discharge in the downstream extremity is $Q = 158.(y - 3.25)^{3/2}$. Also in this case, eq. (4) and (6) have been solved by the MOC using a numerical grid (Streeter and Wylie [6] and Chaudhry [5]).

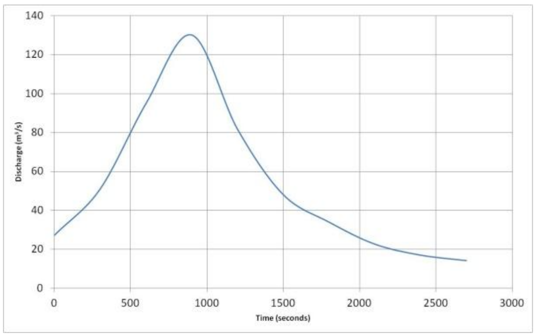

Fig. input hydrograph in a discharge propagation into a channel case study

4. Results and discussion

4.1. Water hammer results

The pressures over the valve (mca) for the various valve closure-times (between 4s and 12s) are presented in the graph in Figure 5. Table 1 shows the pressure over values for the first two peak pressures, pi, observed in Figure 5, referring to the times of 1.65s and 3.15s (Table 1).

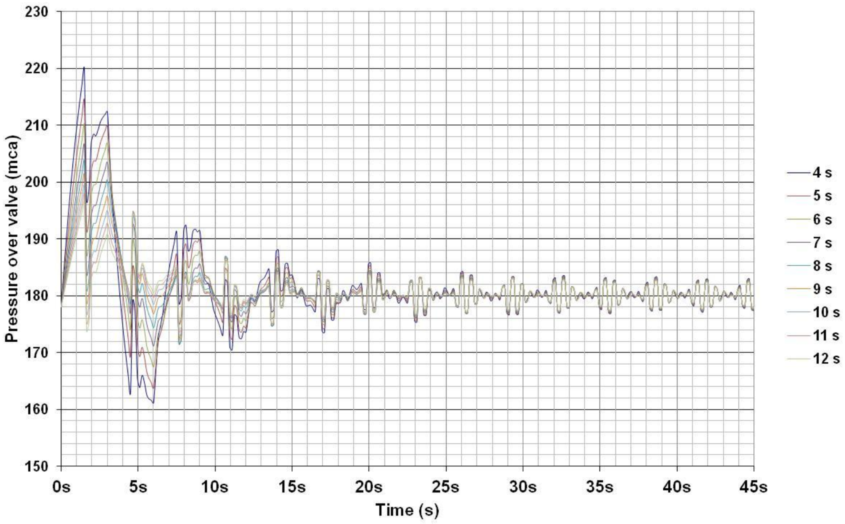

Fig 5. Variation of pressure over the valve

For instance, in the calculation of pressure and depression for valve closure-time, t = 4s:

- Overpressure: ($^{+}h_s$) resulted in a pressure with value of pi equal to 219.97 mca, in time equal to 1.65 s after valve-cloruse (elapsed time);
- Depression: ($^{-}h_s$) resulted in a pressure with value of pi equal to 161.10 mca, in elapsed time equal to 4 s.

The calculation of pressure and depression for valve closure-time, t = 12s:

- Overpressure: ($^{+}$hs) resulted in a pressure with value of pi equal to 196.42 mca, in elapsed time equal to 1.65 s after valve-cloruse;
- Depression: ($^{-}h_s$) resulted in a pressure with value of pi equal to 180.57 mca, in elapsed time equal to 4.35 s.

Table 1: Pressure over a valve as a function of the valve closure time

		Time of valve closure (s)								
		4	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
	1.65	219.97	214.41	210.00	206.49	203.67	201.35	199.43	197.81	196.42
Elapsed time (s)	3.15	212.39	209.96	206.76	203.47	200.38	197.57	195.07	192.84	190.86
Reduce the pressure in relation to t_c = 4.0 s			2.53%	4.53%	6.13%	7.41%	8.46%	9.34%	10.07%	10.70%
			1,14%	2,65%	4,20%	5,66%	6,98%	8,16%	9,21%	10,14%

The smaller valve closure-time (between 4s to 12s) calculations result in more pressure over the valve as can be observed from the Table 1. This is especially true for the first two peaks at 1.65 s and 3.15 s after valve-closure. At these times after valve-closing (i.e., 1.65 s and 3.15 s and t_c equal to 4 s) pressure values of 219.97 mca and 212.39 mca were obtained. These values for a valve closing-time of 12s would be 196.42 mca and 190.86 mca, and represent a decrease in relation to the valve closure-time of 4s of respectively 10.70% and 10.14%. The behavior of discharge values is much milder for longer periods of valve-closing as can be seen from the graphs in Figures 6 to 8.

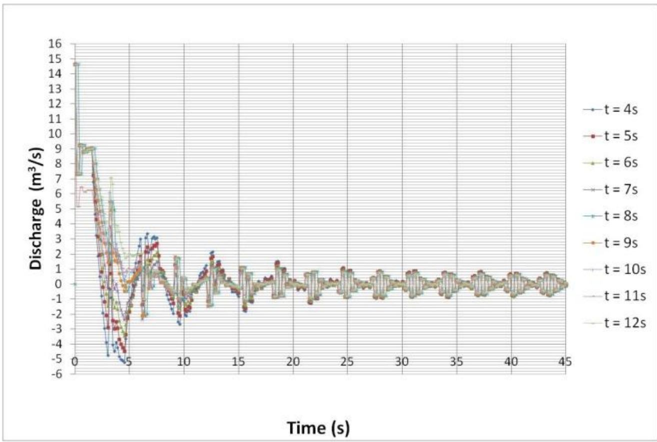

Fig. 6 Discharge at input of pipe in terms of closing times between 4.0 s and 12.0 s

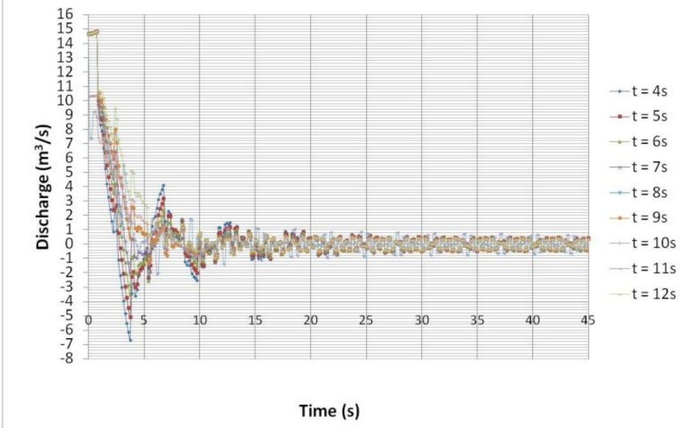

Fig. 7 Discharge in the middle of pipe in terms of closing times between 4.0 s and 12.0 s

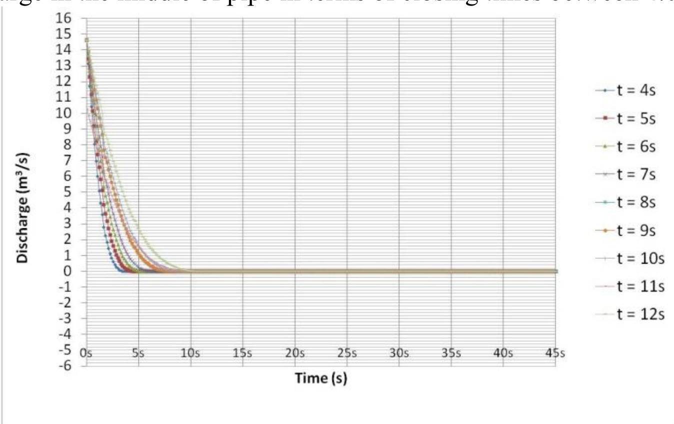

Fig. 8 Discharge at the end of pipe in terms of closing times between 4.0 s and 12.0 s

Negative values of discharge for smaller valve closure-times—for instance, for the closure-time t_c equal to 4s (fig. 6)—were presented at the input of pipe. These values were as large as –5.08 m^3/s at 4.35s elapsed after valve-closure. For the closing time of 12s, the smaller value was -12.45 m^3/s (at 12.45 s elapsed after valve-closure). In the middle of the tube (fig. 7) for a closing time of 4s, the minimum flow rate was -6.67 m^3/s (at 3.75s elapsed after valve-closure), and for the closing time of 12s the value was -0.94 m^3/s (at 11.4s elapsed after valve-closure).

4.2. Discharge Propagation into a Channel

Figures 9 to 11 present simulation results for the first time yP (depth), vP (velocity), and discharge at each interval of space.

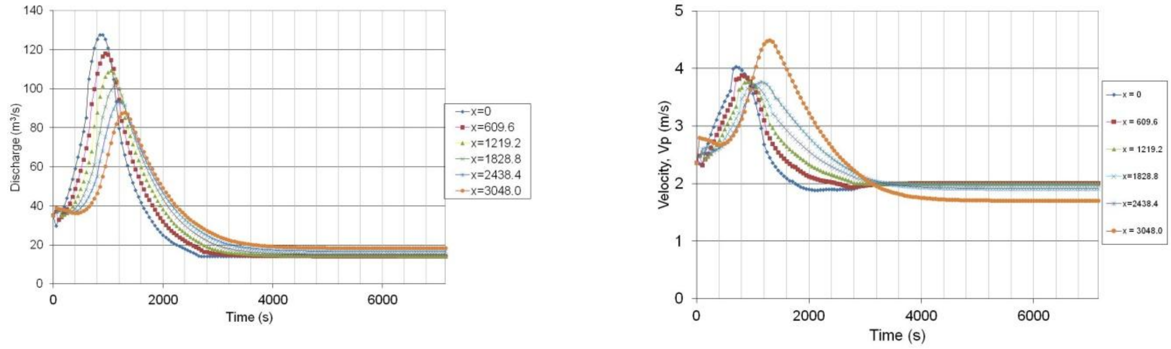

Fig. 9 results for discharge propagation into a channel: a) discharge (left); and b) velocity (right)

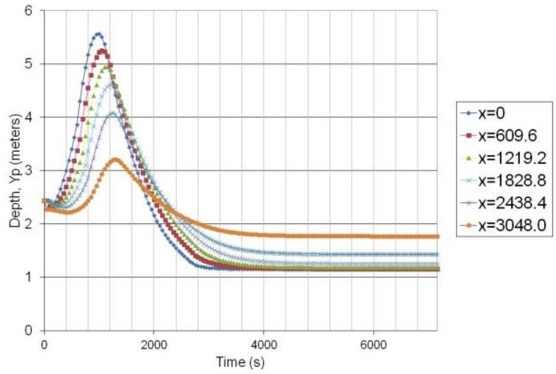

Fig. 10 results for depth as a result to the discharge propagation into a channel

Values of discharge, velocity (v_p), and depth (y_p) at x=0 and an elapsed time of 850s were 127.70m^3/s, 3.87m/s, and 5.36m; for x=0 and an elapsed time of 1,230s, they were 87.92m^3/s, 4.49m/s, and 3.21m.

5. Conclusions

A method of characteristics (MOC) was used in this paper to simulate the following: the response of a pipe system upstream from power plants in the case of valve closure; and discharge propagation into a channel. The first case study was a fictional Small Hydro Power (SHP) Plant that exhibited valve-closure at the end of the downstream, and also exhibited a constant level reservoir at the extreme upstream. The valve-closure value ranged from 4.0 to 12.0. As a result, the measurement of peak pressure over the valve is seen to be reduced within increasing time value, i.e., 4s to 12s. Our simulations also showed that the behavior of the discharge values is much milder for longer periods of valve-closing. The benefits that are obtained on reducing the peak pressure and the minimum (negative) discharge are greatly justified, since both could reach a value of zero flow at the exit of the tube with lower possibility of damages on the pipe by pressure values.

Therefore, the MOC numerical approach has been useful for several engineering purposes, including cases of hydraulic transients and discharge propagation in hydraulic systems. We suggest that a validation of both systems at actual and laboratory scales will help to produce more realistic results.

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Nomenclature

A_c	Cross-section area [L^2]	T	surface width [L]
V	speed [L/T]	H_0	Head
S_0	the slope of the water line (slope of the bottom)	Q	the discharge rate [L^3/T]
		t_c	valve closure-time [T]
S_f	the slope of the energy line	C	Chézy coefficient
q'.T=q(m^3/s/m)	the unitary side entrance [m]	^+h_s	Overpressure in pipe
y_P	depth in channel [L]	^-h_s	Depression in pipe
v_P	velocity depth in channel [L/T]		

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