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Latitudinal Distribution of Sunspots and Duration of Solar Cycles

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Abstract: We study an association between the duration of solar activity and characteristics of the latitude distribution of sunspots by means of center-of-latitude (COL) of sunspots observed during the period from 1878 to 2008 spanning solar cycles 12 to 23. We first calculate COL by taking the area-weighted mean latitude of sunspots for each calendar month to determine the latitudinal distribution of COL of sunspots appearing in the long and short cycles separately. The data set for the long solar cycles consists of the solar cycles 12, 13, 14, 20, and 23. The short solar cycles include the solar cycles 15, 16, 17, 18, 19, 21, and 22. We then fit a double Gaussian function to compare properties of the latitudinal distribution resulting from the two data sets. Our main findings are as follows: (1) The main component of the double Gaussian function does not show any significant change in the central position and in the full-width-at-half-maximum (FWHM), except in the amplitude. They are all centered at $\sim 11°$ with FWHM of $\sim 5°$. (2) The secondary component of the double Gaussian function at higher latitudes seems to differ in that even though their width remains fixed at $\sim 4°$, their central position peaks at $\sim 22.1°$ for the short cycles and at $\sim 20.7°$ for the long cycles with quite small errors. (3) No significant correlation could be established between the duration of an individual cycle and the parameters of the double Gaussian. Finally, we conclude by briefly discussing the implications of these findings on the issue of the cycle 4 concerning a lost cycle.

Key words: Sun: Sunspots: Data analysis

1. Introduction

The most conspicuous conclusion from the analysis of sunspot records is that solar activity is variable in time, given that sunspot observations provide the longest running direct records of solar activity. Long-term observations of sunspots indicate that the solar activity cycle is not exactly periodic, but that an individual cycle is quite different from others with its own amplitude and duration (Petrovay 2010; Gopalswamy et al. 2012; Cho et al. 2014a). Solar activity as a time series shows in general various periodicities, such as, of $\sim$ 11 years (Schwabe 1843; Maunder 1904), of $\sim 80 - 90$ years (Gleissberg 1971; Pulkkinen et al. 1999), of $\sim$ 1.3 years (Howe et al. 2000; Krivova & Solanki 2002; Obridko & Shelting 2007; Kim & Chang 2011; Cho et al. 2014b), of $\sim$ 154 days (Rieger et al. 1984), of $\sim$ 51, 78, 104, and 129 days (Bai & Sturrock 1991, 1993), which makes it difficult to predict the amplitude and the duration of solar activity to a serviceable level. This is partly why space weather forecast is challenging from both practical and scientific points of view, and why many empirical relations based on freshly formulated parameters have been proposed routinely.

In addition to the variation of sunspot records over time, the latitudinal location of sunspots with time provides invaluable information about the physical processes that create the solar magnetic flux and drive its evolution. Although sunspots seem to emerge randomly at any latitude lower than $\sim 40°$, sunspots are scattered around a mean latitude drifting equatorward as a solar cycle proceeds, which forms a well-known butterfly diagram. The drift rate slows down as the centroid of sunspots approaches the equator (Li et al. 2001, 2002; Hathaway et al. 2003; Li 2010). Furthermore, the centroid does not migrate monotonically toward the equator, which result in a sequence of few 'pulses of activity' or 'active latitude' that the spot activity consists of (Ternullo 1997; Norton & Gilman 2004; Solanki et al. 2008). In this context, an interesting observation is that the drift rate at sunspot cycle maximum shows a highly significant anticorrelation with the duration of each cycle and is significantly correlated with the amplitude of the following cycle (Hathaway et al. 2003).

Explorations of the butterfly diagram reveals that the two wings of the butterfly diagram are different from each other (Li et al. 2002; Li 2009), which may imply the activity on the Sun is governed by more than one single process. To further investigate the solar North-South asymmetry, many solar activity indices have been thoroughly examined (Waldmeier 1971; Roy 1977; Swinson et al. 1986; Yi 1992; Carbonell et al. 1993; Oliver & Ballester 1994; Meunier et al. 1997; Temmer et al. 2002, 2006; Hathaway et al. 2003; Gigolashvili et al. 2005; Zaatri et al. 2006; Zolotova & Ponyavin 2006, 2007; Donner & Thiel 2007; Chang 2008, 2009; Javaraiah 2008; Li 2010; Li et al. 2010). Since it was established, the North-South asymmetry is also considered as one of the most interesting fea-

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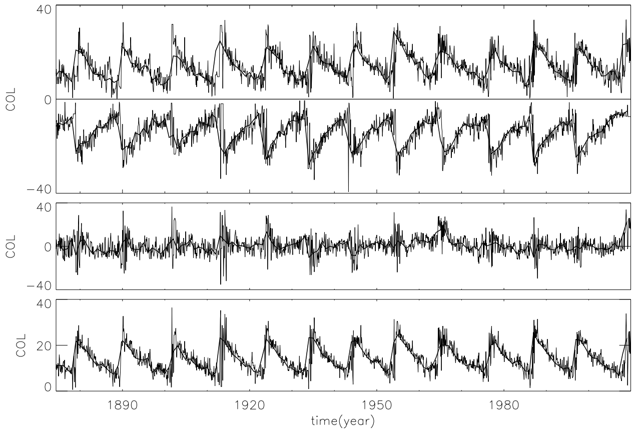

Figure 1. 'Center-of-latitude (COL)' as a function of time during the period from September in 1878 to January in 2008. Thick and thin curves represent the yearly averaged COL and the monthly averaged COL, respectively. In the upper panel, we separately plot COL for the sunspots appearing in the northern and southern hemispheres. In the middle panel, the averaged COL for sunspots appearing in both hemispheres is plotted taking into account its sign. In the bottom panel, we plot the averaged COL for sunspots appearing in two hemispheres but the absolute value of latitude is considered.

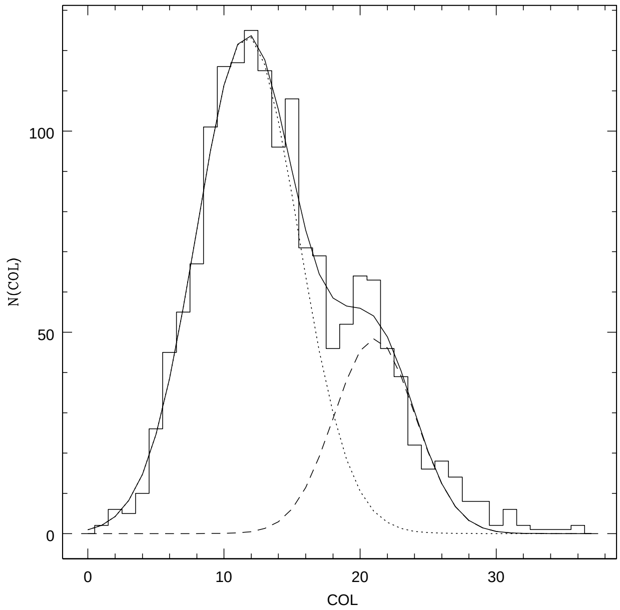

Figure 2. Histogram of COL and their best fits using the double Gaussian function. In this histogram, COL of sunspots appearing during the period from 1878 to 2008 is counted using the absolute value of latitudes of sunspots. Solid curve represents the best fit of the double Gaussian function. Dotted and dashed curves represent the main and secondary Gaussian functions, respectively.

tures in relating to solar activity (Knaack et al. 2005; Javaraiah 2007, 2015; Chang 2011, 2012; Cho et al. 2011; Pesnell 2012; Chowdhury et al. 2013; Pandey et al. 2015).

A latitude at which sunspots of a particular cycle appear seems to follow a well-defined probability density function having maximum probability at $\sim 10^\circ - 15^\circ$ (e.g., Li et al. 2003; Chang 2012). It is further claimed that a correlation can be found between characteristics of solar activity and parameters of the distribution (Li et al. 2002; Miletskii & Ivanov 2009; Chang 2011). The bimodal attribute of the sunspot latitudinal distribution recently suggested by Chang (2012) characterizes more appropriately the aspect of the active latitude mentioned earlier (Ternullo 1997, 2001, 2007, 2010, 2013; Solanki et al. 2008). Interestingly, Chang (2012) has also demonstrated that when the northern (southern) hemisphere is active the width of the secondary component of the double Gaussian function in the northern (southern) hemisphere is about twice as wide as that in the southern (northern) hemisphere.

In this paper, we attempt to address a question of whether the duration of a solar cycle is associated with properties of the latitudinal distribution of sunspots, as in an earlier study. Chang (2011) has shown that the characteristic width of the latitudinal distribution of sunspots is correlated with the cycle amplitude. Upon answering to this question we compare the distributions resulting from subsets based on the duration of solar cycles. One thing that must be kept in mind when analyzing statistically the butterfly diagram is that the butterfly diagram takes no account of the sunspot lifetime, nor the spatial size. Since all sunspots are given equal weight, regardless of their temporal and spatial extention, the butterfly diagram is dominated by small and numerous sunspots which are randomly distributed over wider ranges than large ones (Ternullo 2007; Cho & Chang 2011). By replotting the butterfly diagram by weighting the sunspot with its area, one may observe intriguing features of solar magnetic activities more clearly (see, e.g., Chang 2011). For example, latitudinal migration of center-of-latitude (COL) occur asynchronously in the northern and southern hemispheres as reported earlier (Swinson et al. 1986; Zolotova & Ponyavin 2006, 2007; Li et al. 2010). Furthermore, COL is not monotonically decreasing as commonly assumed. Short plateaus between solar minima are seen commonly at a latitude of $\sim \pm 10^\circ$, which can be found in plots of similar works (Antalova & Gnevushev 1983; Li et al. 2003; Ternullo 2007). In fact, these small humps appear around every solar maxima over time. They may well be related to the latitudes of higher activity, that is, 'active latitude' (e.g., Solanki et al. 2008).

This paper begins with descriptions of data and the distribution of COL in Section 2. We present results obtained by subsampling sunspot data in terms of the duration of solar cycles in Section 3. Finally, we summarize and conclude by briefly discussing the implications of these findings on the issue concerning a 'lost cycle' in Section 4.

2. LATITUDINAL DISTRIBUTION OF COL

We have used for the present analysis the daily sunspot areas and latitudes downloaded from the

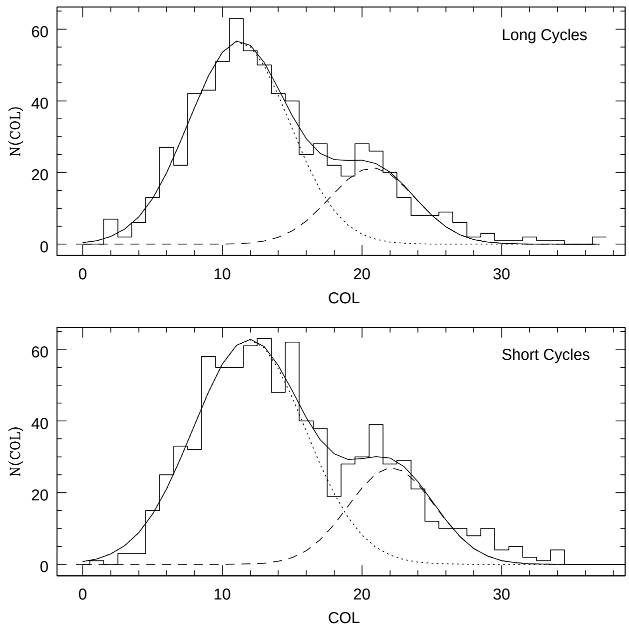

Figure 3. Histograms of COL and their best fits using the double Gaussian function. In the upper and lower panels, histograms are obtained by sunspots appearing in the long and short cycles, respectively. The long cycles include the solar cycles 12, 13, 14, 20, and 23. The short solar cycles include the solar cycles 15, 16, 17, 18, 19, 21, and 22. Solid, dashed, and dotted curves represent the observed histogram, the best fit of the double Gaussian function, and the individual Gaussian function, respectively, as in Figure 2.

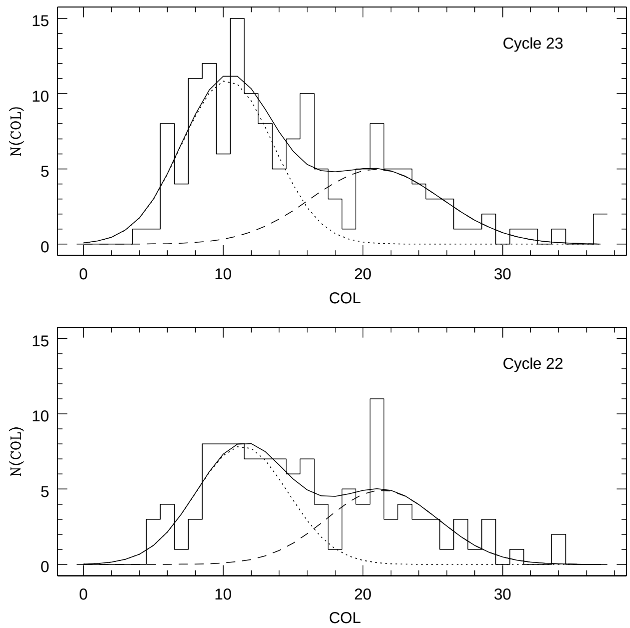

Figure 4. Similar to Figure 3, except that the upper and lower panels result from the solar cycles 23 and 22, respectively. Note that they are one of the longest and the shortest solar cycles, respectively.

NASA database[1]. The data set we analyze here covers the solar cycles 12 to 23, i.e., from September in 1878 to January in 2008.

One important difference between the present work and others (e.g., Hathaway et al. 2003; Li et al. 2003) is that when the latitudinal distribution of sunspots is calculated we consider the area of sunspots in terms of center-of-latitude (COL). As pointed out above, this is important since the conventional butterfly diagram is dominated by small and short-lived sunspots, which randomly scatter over wide latitude ranges than larger ones, so that it looks very noisy. The center-of-latitude is given by

$$\mathrm{COL} = \sum_i (A_i \times |L_i|) / \sum_i A_i, \tag{1}$$

where A_i is the i-th sunspot's area and L_i is its latitude. We have carried out the summation over a period of one calendar month, instead of the Carrington month. We collectively compute COL for sunspots taking the absolute value of the latitude rather than the latitude itself to consider regardless of the Northern and Southern hemispheres. We have employed the double Gaussian function rather than a single function to describe the distribution (cf., Li et al. 2003). The double Gaussian function is given by $A_1 \exp(-[x-x_1]^2/2\sigma_1^2) + A_2 \exp(-[x-x_2]^2/2\sigma_2^2)$, where A_1 and A_2, x_1 and x_2, σ_1 and σ_2, represent the amplitude of each component, the central value, the width, respectively. By adopting the double Gaussian function, we explore in details the characteristics of the latitudinal distribution of COL.

In Figure 1 we show the 'center-of-latitude (COL)' as a function of time for the data set currently analyzed. Thick and thin curves represent the yearly averaged COL and the monthly averaged COL, respectively. The summation is carried out over a calendar year and a calendar month, instead of the Carrington month. In the upper panel, we separately calculate and plot the COL for sunspots appearing in the northern and southern hemispheres. It should be noted that COL is not *monotonically* decreasing as commonly assumed, which can be clearly seen with the yearly averaged COL. Humps appear in fact around every solar maxima over time. In the middle panel, for comparison, the averaged COL for sunspots appearing in both hemispheres is plotted taking into account the sign of the latitudes. We also note that the difference is large at around the solar minima and far from the random distribution showing that the wings of the butterfly diagram are not symmetrical. In the bottom panel, we plot the averaged COL for sunspots appearing in two hemispheres, but the latitude is considered as unsigned.

In Figure 2, we show the histogram of COL and their best fit using the double Gaussian function. The histogram is obtained from COL of sunspots appearing in both North and South hemispheres during the period from the solar cycles 12 to 23. All the histograms in the following result from monthly averaged COL. We

[1] `http://solarscience.msfc.nasa.gov/greenwch.shtml`

Table 1
Parameters from fits to the distributions of COL from the sunspot data set.

	A_1	$\overline{COL}_1$	σ_1	A_2	$\overline{COL}_2$	σ_2
All	123.6±0.49	11.7±0.03	5.3±0.04	48.4±0.61	21.1±0.07	4.2±0.08
Long	56.6±0.49	11.2±0.06	5.1±0.08	21.3±0.56	20.7±0.15	4.3±0.19
Short	62.7±0.47	11.9±0.05	5.7±0.08	27.0±0.57	22.1±0.11	4.4±0.14

Shown here are the 6 parameters obtained from fitting the double Gaussian function to the distributions of COL as a result from the data set of sunspots. In the first row, we show fitting results from the distributions of COL of sunspots appearing during the period from solar cycles 12 to 23. In the second and third rows, we show fitting results from the distributions of COL of sunspots appearing in the long and short solar cycles, respectively. A_k, $\overline{COL}_k$, and σ_k are the amplitude, the central position, and FWHM of the individual Gaussian profile, respectively, and subscripts 1 and 2 stand for the primary and the secondary components of the double Gaussian function.

count the number of COL into bins with a width of 1°. We then fit a double Gaussian function to the histogram by adjusting 6 parameters (two amplitudes, two central values, and two widths) simultaneously. Solid curve represents the best fit of the double Gaussian function. Dashed and dotted curves represent two individual Gaussian functions. As for a goodness of a fit, discussion on the quantity χ^2 can be found in Chang (2012).

As shown in Figure 2, the distribution of COL as a whole is bimodal, which is well represented by the double Gaussian function having maxima of $\sim 12°$ and $\sim 21°$ as listed in Table 1. Of interest is that the profiles are skewed such that the main component is closer to the equator with the secondary component towards the higher latitudes, which reflects the fact that the drift of sunspots slows down as a solar cycle proceeds (e.g., Li et al. 2001; Hathaway et al. 2003). What we would like to stress here is that the bimodality could imply that as the sunspot formation sites move from high latitudes to low latitudes, there are two preferential sunspot formation sites. This is somewhat surprising in the sense that the widely accepted paradigm has pretended that sunspots are scattered around a mean latitude, which steadily drifts equatorward.

3. LONG SOLAR CYCLES VS. SHORT SOLAR CYCLES

In Figure 3, we show histograms of COL resulting from two subsets. Here we divide the whole data set of the solar cycles from 12 to 23 into two subsets according to the duration of solar cycles[2], which is defined from Kane (2002). In the upper and lower panels, results of the histograms from the sunspots in the long and short cycles are shown using the absolute value of the latitude of sunspots, respectively. We have identified a cycle as long and short if its duration is longer and shorter than 11 years. This criterion is indeed arbitrary, but not totally absurd considering that the mean value of the duration of solar cycles is 11.1 years. The data set for the long solar cycles hence consists of the solar cycles 12 (11.3 yrs), 13 (11.9 yrs), 14 (11.5 yrs), 20 (11.7 yrs), and 23 (11.7 yrs). The short solar cycles include the solar cycles 15 (10.0 yrs), 16 (10.1 yrs), 17 (10.4 yrs), 18 (10.2 yrs), 19 (10.5 yrs), 21 (10.3 yrs), and 22 (9.7 yrs). Solid, dashed, and dotted curves represent the observed histogram, the best fit of the double Gaussian function, and the individual Gaussian function, respectively.

When comparing histograms resulting from long and short cycles, one may find key features, such as, bimodality. Several interesting points can be also noticed. As long as the main component of the double Gaussian function is concerned, the variation seems not critical in the central position and in the full-width-at-half-maximum (FWHM), except in the amplitude. They are all centered at $\sim 11°$ with FWHM of $\sim 5°$, with quite small errors. The change in the amplitude is due to the number of solar cycles added. That is, the number of short solar cycles is 7, and the number of long cycles is 5. On the other hand, the secondary component of the double Gaussian function at higher latitudes seems to be modified in the sense that even though their width remains fixed at $\sim 4°$, the central position varies systematically. In other words, the central position due to the short cycles peaks at $\sim 22.1°$ while that due to the long cycles peaks at somewhat a lower latitude, $\sim 20.7°$, as summarized in Table 1. Our finding agrees with the earlier suggestion by Hathaway et al. (2003) that the duration of solar cycles is anticorrelated with the drift rate at sunspot cycle maximum. Hence, we conclude that sunspots in the short cycles emerge at higher latitude and are likely to spread in a wider range of latitudes than over the long cycles.

In Figure 4, histograms of COL obtained from the solar cycles 23 and 22 are shown, respectively. They are one of the longest and the shortest solar cycles. As listed in Table 2, a general trend we observe in Figure 3 can be seen Figure 4. However, the errors are too large to make any conclusive statements (cf., Gopalswamy et al. 2012). In fact, we tried to calculate a correlation between parameters of the distributions of COL and the duration of an individual cycle, as attempted in Chang (2011). Unfortunately, we failed to obtain statistically significant results. To isolate sunspots for a given solar cycle, as before, we have followed a criterion that is commonly adopted in researches of similar purpose (e.g., Li et al. 2002; Solanki et al. 2008). The first sunspots of a new cycle appear at high latitudes during the decline of the old cycle, whereas the last sunspots of an old cycle appear near the equator during the rise of the new cycle. Thus, for each cycle, sunspots at low latitudes (lower than $\sim 15°$) in the beginning phase should not be counted since we consider them as belongings to

[2]Cf. `https://en.wikipedia.org/wiki/List_of_solar_cycles`

Table 2
Similar to Table 1, but for sunspots in solar cycles 23 and 22, one of the longest and the shortest solar cycles, respectively.

	A_1	$\overline{COL}_1$	σ_1	A_2	$\overline{COL}_2$	σ_2
Cycle 23	10.9±0.68	10.3±0.34	4.7±0.40	5.0±0.44	20.9±0.99	6.6±1.27
Cycle 22	7.8±0.60	11.3±0.50	4.7±0.58	4.9±0.48	21.3±0.91	5.8±1.16

the previous solar cycle. Similarly, sunspots at high latitudes (higher than $\sim 25°$) in the declining phase are not counted since we consider them as belongings to the following solar cycle. The boundary between cycles have been estimated by eye near the ends of solar cycles. A reason we think that the significance is low is not because a way of isolating a particular solar cycle is coarse.

4. Summary and Conclusions

The distribution of area-weighted latitude of sunspots appearing in the solar cycles 12 to 23 has been studied. We have calculated center-of-latitude (COL) by averaging the latitude with the weight function in area. Then the latitudinal distribution of COL was formed for sunspots appearing in the long and short cycles separately. We have further obtained the best fit of the observed distribution with the double Gaussian function and compare results, though the signed average of COL from two hemispheres results in a single Gaussian function. We have repeated the same analysis with sunspots appearing in an individual cycle. The summary of what we have found is as follows:

(1) As long as the main component of the double Gaussian function is concerned, the variation seems not critical in the central position and in the full-width-at-half-maximum (FWHM), except in the amplitude. They are all centered at $\sim 11°$ with FWHM of $\sim 5°$ with quite small errors.

(2) The secondary component of the double Gaussian function at higher latitudes seems to have a role in the sense that even though their width remains fixed at $\sim 4°$ their central position peaks at $\sim 22.1°$ for the short cycles and at $\sim 20.7°$ for the long cycles with quite small errors, which agrees with earlier claims (Hathaway et al. 2003). It may imply that sunspots in the short cycles emerge at higher latitude and spread in a wider range of latitudes than in the long cycles.

(3) We have tried to calculate a correlation between parameters and the duration of an individual cycle, and failed to have statistically significant outcomes due to large errors.

Nonetheless, we would like to make a comment on a possible implication of investigating a distribution of individual cycle. The solar cycle 4 (from 1784 to 1798) is particularly interesting for two reasons. First, just after the solar cycle 4 an epoch of low solar activity named the Dalton Minimum began. Second, it is the longest solar cycle since the numbering began, whose duration is 13.7 years. Due to its unusually long duration there came an interesting suggestion that the solar cycle 4 is a superposition of two cycles and the latter is missing: a normal 10-year long cycle between 1784 and 1793 followed by a short and weak cycle from 1793 to 1798 (Usoskin et al. 2001, 2002, 2003, 2009). Arguments against the lost cycle hypothesis appeared immediately. Krivova et al. (2002) have performed some statistical tests of sunspot records. They also considered ^{10}Be, ^{14}C, and auroral proxy data and argued that the presumably lost cycle appears even more peculiar than the single solar cycle 4. Zolotova & Ponyavin (2007) have suggested that the unusual duration of the solar cycle 4 could be explained by outstanding phase asynchrony between the northern and southern hemispheric activities reaching a delay up to 4.5 years. We are aware that several tests can be made to check this hypothesis, such as, the phase shift of the Gnevyshev-Ohl rule (Gnevyshev & Ohl 1948), the Waldmeier relation (Waldmeier 1935, 1939), and the parity of the solar magnetic field. In addition to these, since we have seen that long and short cycles have possibly different distribution of COL and the duration of 13.7 years might be long enough to make any statistical difference, we suggest therefore the arguments for the proposal that cycle 4 is a superposed sunspot cycle can be further reviewed if we could afford to reconstruct the butterfly diagram by direct sunspot records found in the past (e.g., Arlt 2008, 2009a, 2009b).

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