

## A MULTIVARIATE JUMP DIFFUSION PROCESS FOR COUNTERPARTY RISK IN CDS RATES

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**ABSTRACT.** We consider counterparty risk in CDS rates. To do so, we use a multivariate jump diffusion process for obligors' default intensity, where jumps (i.e. magnitude of contribution of primary events to default intensities) occur simultaneously and their sizes are dependent. For these simultaneous jumps and their sizes, a homogeneous Poisson process. We apply copula-dependent default intensities of multivariate Cox process to derive the joint Laplace transform that provides us with joint survival/default probability and other relevant joint probabilities. For that purpose, the piecewise deterministic Markov process (PDMP) theory developed in [7] and the martingale methodology in [6] are used. We compute survival/default probability using three copulas, which are Farlie-Gumbel-Morgenstern (FGM), Gaussian and Student-t copulas, with exponential marginal distributions. We then apply the results to calculate CDS rates assuming deterministic rate of interest and recovery rate. We also conduct sensitivity analysis for the CDS rates by changing the relevant parameters and provide their figures.

### 1. INTRODUCTION

In practice, the insolvency of one firm can cause an increase in other firms' default intensities due to business links or ties between firms. The mismanagement of subprime mortgages in the US in the year 2007 which had far reaching consequences provide a perfect illustration in this effect, and thereby emphasizing the importance for incorporating shocks and dependence structure in financial modeling.

The jump diffusion process that has been used to represent variables such as the default intensity, asset returns as well as interest rate (such as the work by [10], [26], [28], [30] and [5]) allows us to capture the effects of shocks. Shock elements can arrive due to primary events such

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as oil and commodity prices, governments fiscal and monetary policies, the release of corporate financial reports, political and social decisions, rumours of mergers and acquisitions among firms, the collapse and bankruptcy of firms, the September 11 World Trade Centre catastrophe and Hurricane Katrina. Each of these events cause jumps in the variable being modelled. Readers are referred to [36] and [26] for a further discussion of the various motivations for using a jump diffusion process.

This paper is based on the jump diffusion approach for the case when the firms in the complementary or substitute industry/sector are affected by a common external event. Numerous papers have examined the modelling for the dependence of default intensities via a point process for the purpose of pricing derivative instruments (such as [35], [24], [6], [37], [17] and [32]). The use of univariate jump diffusion model to represent the reference credit intensity in pricing the CDS instrument was also explored in [2]. The analytical expression for CDS and CDS swaptions prices offered in the literature was obtained using the Jamshidian option decomposition trick as in [20].

Besides the construction of a point process, considerable attention was given to the default dependence between the obligors. The work by [11] considered joint jumps in the default intensity for this effect. [25] and [23] developed it further considering the possibility of default-event triggers that cause joint default. Another approach to incorporate default dependence between obligors is through the use of copulas ([27], [35], [24], [16] and [28]). The use of FGM copula with multivariate shot noise process has been explored in [22] which was then extended in [28] by adding diffusion term to the intensity processes. Both papers adopted martingale methodology and PDMP technique to derive the survival probability. Using the same methodology and technique, we examine a multivariate default intensity process where the jumps occur simultaneously.

We structure the article in the following order: In section 2.2 we define the multivariate jump diffusion process for obligors' default intensity and derive the relevant joint Laplace transform using the PDMP theory and the martingale methodology. These joint Laplace transforms then lead us to the joint survival/default probability and other relevant joint probabilities. This is followed by a numerical example showing how the joint probabilities can be generated capturing the dependence structure between the vector of event jumps, using three copulas as examples which are the Farlie-Gumbel-Morgenstern (FGM) copula, Gaussian copula and Student-t copula. In section 3, we then illustrate how this jump diffusion process can be applied to calculate CDS rates considering counterparty risk. For that purpose, we assume that the jumps of default intensities of the CDS seller and reference credit (RC) occur simultaneously and that the dependence structure between their jump sizes are captured by the three copulas. We also assume deterministic short rate of interest and a deterministic recovery rate for simplicity. This is then followed by a sensitivity analysis of the CDS rates with respect to relevant parameters such as the diffusion rate, the constant reversion level, the decay rate at which the default intensity would retract back to the constant reversion level as well as the jump size of both obligors. Section 4 contains some concluding remarks.

## 2. MODEL SETUP AND THEORETICAL RESULTS

For  $i = 1, 2, \dots, n$  denoting obligor  $i$  involved in the financial transaction, the multivariate default intensity model we consider has the following structure:

$$d\lambda_t^{(i)} = c^{(i)} \left( b^{(i)} + a^{(i)} \lambda_t^{(i)} \right) dt + \sigma^{(i)} \sqrt{\lambda_t^{(i)}} dW_t^{(i)} + dL_t^{(i)}, \quad L_t^{(i)} = \sum_{j=1}^{M_t} X_j^{(i)} \quad (2.1)$$

where

- $\{X_j^{(1)}, X_j^{(2)}, \dots, X_j^{(n)}\}_{j=1,2,\dots}$  is a vector sequence of dependent but not identically distributed random variables with distribution function  $F^{(i)}(x)$  ( $x > 0$ ),
- $M_t$  is the total number of events up to time  $t$ ,
- $W_t^{(i)}$  is a standard Brownian motion governing obligor  $i$ ,
- $a < 0$ ,  $b \geq 0$  and  $c > 0$  with  $c^{(i)}a^{(i)}$  being the rate of exponential decay for obligor  $i = 1, 2, \dots, n$  and  $c^{(i)}b^{(i)}$  being the constant reversion level for default intensity of obligor  $i$ ; and
- $\sigma^{(i)} > 0$  is the diffusion coefficient for obligor  $i$ .

We also make the additional assumption that the point process  $M_t$  is independent of the vector sequence of jump sizes and that the vector sequence  $\{X_k^{(1)}, X_k^{(2)}, \dots, X_k^{(n)}\}_{k=1,2,\dots}$  is independent of another vector sequence for  $k \neq j$ .  $L_t^{(i)}$  is a compound process for the default intensity of obligor  $i$ .

In this model, the dependence between the intensities  $\lambda_t^{(i)}$  comes from the common event arrival process  $M_t$ , together with the dependence between the vector of jumps  $(X_j^{(1)}, X_j^{(2)}, \dots, X_j^{(n)})$ . We assume that event arrival process  $M_t$ , (i.e. the simultaneous jump process) follows a homogeneous Poisson process with frequency  $\rho$  and the vector of jumps is modelled using copulas ([31] and [29]) - that is, the joint distribution of the vector  $(X_j^{(1)}, X_j^{(2)}, \dots, X_j^{(n)})$  is assumed to be of the form  $C(F^{(1)}, F^{(2)}, \dots, F^{(n)})$  with  $C$  being a given copula.

As specific examples for  $C$  in this paper, we use the FGM, the Gaussian and the Student-t copulas which are given in consecutive manner by

$$C^{FGM}(u_1, \dots, u_n) = \prod_{i=1}^n \left( 1 + \sum_{1 \leq i < j} \theta_{ij} (1 - u_i) \right) \quad (2.2)$$

$$C^G(u_1, \dots, u_n) = \int_{-\infty}^{\Phi^{-1}(u_1)} \dots \int_{-\infty}^{\Phi^{-1}(u_n)} \frac{1}{2\pi\sqrt{|\Theta|}} \exp\left(-\frac{1}{2}\omega^T \Theta^{-1} \omega\right) dudv \quad (2.3)$$

$$C_v^t(u_1, \dots, u_n) = \int_{-\infty}^{t_v^{-1}(u_1)} \dots \int_{-\infty}^{t_v^{-1}(u_n)} \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{(\pi\nu)^2 |\Theta|}} \left( 1 + \frac{\eta^T \Theta^{-1} \eta}{\nu} \right) dudv \quad (2.4)$$

where  $u_i \in [0, 1]$  for  $i = 1, \dots, n$ . For the elliptical copulas, the correlation parameter

$\theta \in [-1, 1]$  is contained in the correlation matrix  $\Theta = \begin{bmatrix} 1 & \dots & \theta_{1j} & \dots & \theta_{1n} \\ \vdots & & \ddots & & \vdots \\ \theta_{n1} & \dots & \theta_{nj} & \dots & 1 \end{bmatrix}$ . We also

define  $\omega = [\omega_1 \dots \omega_n]^T$  and  $\eta = [\eta_1 \dots \eta_n]^T$  where  $\omega_i = \Phi^{-1}(u_i)$  and  $\eta_i = t_v^{-1}(u_i)$  are the inverse Gaussian and inverse Student-t distribution with degrees of freedom  $v$  respectively taken on the variables  $u_i$ . For the marginal distributions of  $X_j^{(i)}$  in the vector of jumps  $(X_j^{(1)}, X_j^{(2)}, \dots, X_j^{(n)})$ , any continuous distribution can be considered.

With  $F^{(i)}(x_j) = 1 - e^{-\mu^{(i)}x_j}$  ( $\mu^{(i)} > 0, x_j > 0$ ), for  $i = 1, 2, \dots, n$  to represent the marginal distribution, the FGM copula, which is illustrated in Figure 2, is used in this study for its simplicity and analytical tractability, where it is also used in [22] and [28]. Its simplicity allows for the closed-form expressions of final results to be easily derived. It is also used to compare our numerical results against their counterparts in [28]. The Gaussian copula, shown in Figure 3, is chosen so as to examine the effect of elliptical copula on simultaneous jumps in the intensity process as it has not been explored previously in the context of CDS pricing with counterparty risk. We also choose the Student-t copula to incorporate the possibility of having more frequency of higher and/or smaller as well as opposing joint jumps size impact in the obligors' intensity, as shown in Figure 5.

The simulated paths of the jump diffusion process under each copula considered in this study with exponential jump size distributions is also shown in Figures 2, 4 and 6, where  $\theta = -0.95, 0$  and  $0.95$ .

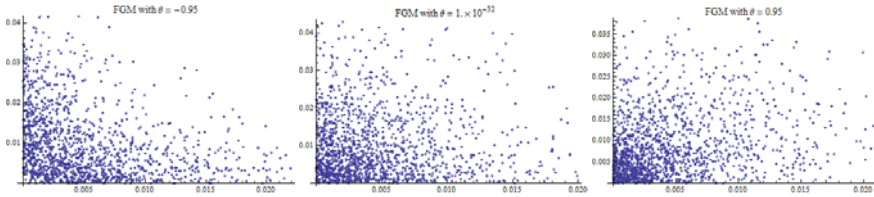


FIGURE 1. FGM copula with exponential margins and dependence parameter -0.95, 0, 0.95

**2.1. Survival and Default Probabilities.** Now, let us derive the joint survival probability and relevant joint probabilities. To do so, we use a multivariate Cox process  $(N_t^{(1)}, \dots, N_t^{(n)})$  with the integrated default intensities  $\Lambda_t^{(i)} = \int_0^t \lambda_s^{(i)} ds$  ( $i = 1, 2, \dots, n$ ) to model the joint default time. We define

$$\tau^{(i)} = \inf\{t : N_t^{(i)} = 1 \mid N_0^{(i)} = 0\}$$

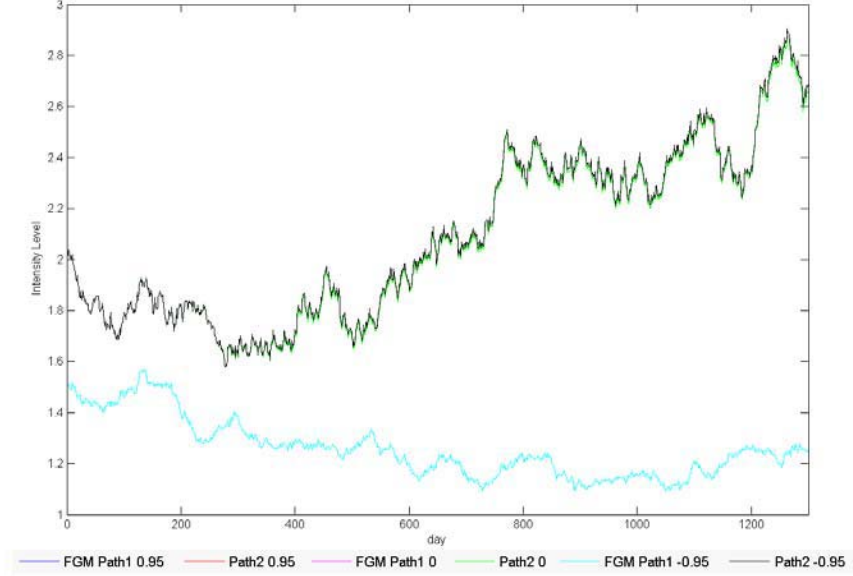


FIGURE 2. Simulated paths of jump diffusion process with dependence structure capture by FGM copula

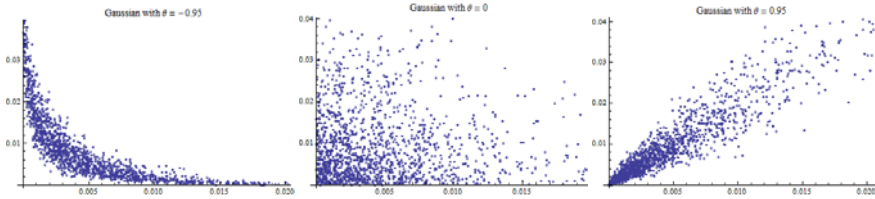


FIGURE 3. Gaussian copula with exponential margins and dependence parameter -0.95, 0, 0.95

as the default arrival time for the firm  $i = 1, \dots, n$ , that is equivalent to the first jump time of the Cox process  $N_t^{(i)}$  ( $i = 1, 2, \dots, n$ ) respectively.

We derive the joint Laplace transform of the vector  $(\Lambda_t^{(1)}, \dots, \Lambda_t^{(n)})$ , i.e.

$$\mathbb{E} \left( e^{-\sum_{i=1}^n \gamma^{(i)} \Lambda_t^{(i)}} \mid \lambda_0^{(1)}, \dots, \lambda_0^{(n)} \right) \quad (2.5)$$

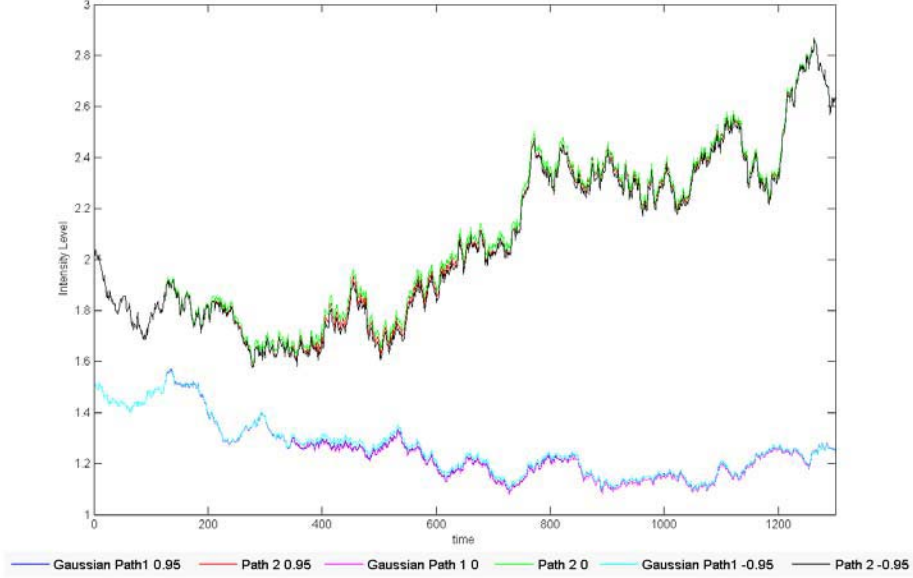


FIGURE 4. Simulated paths of jump diffusion process with dependence structure capture by Gaussian copula

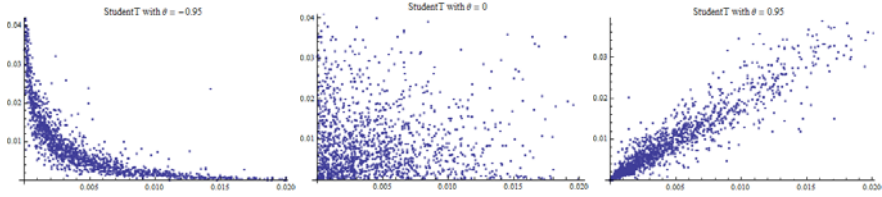


FIGURE 5. Student-T copula with exponential margins and dependence parameter  $-0.95, 0, 0.95$

where  $\gamma^{(i)} \geq 0$ , as it provides the joint survival/default probabilities by setting  $\gamma^{(i)} = 1$  in the equation (2.5) i.e.

$$\begin{aligned} & \Pr\left(\tau^{(1)} > t, \dots, \tau^{(n)} > t \mid \lambda_0^{(1)}, \dots, \lambda_0^{(n)}\right) \\ &= \mathbb{E}\left[e^{-\sum_{i=1}^n \Lambda_t^{(i)}} \mid \lambda_0^{(1)}, \dots, \lambda_0^{(n)}\right]. \end{aligned} \quad (2.6)$$

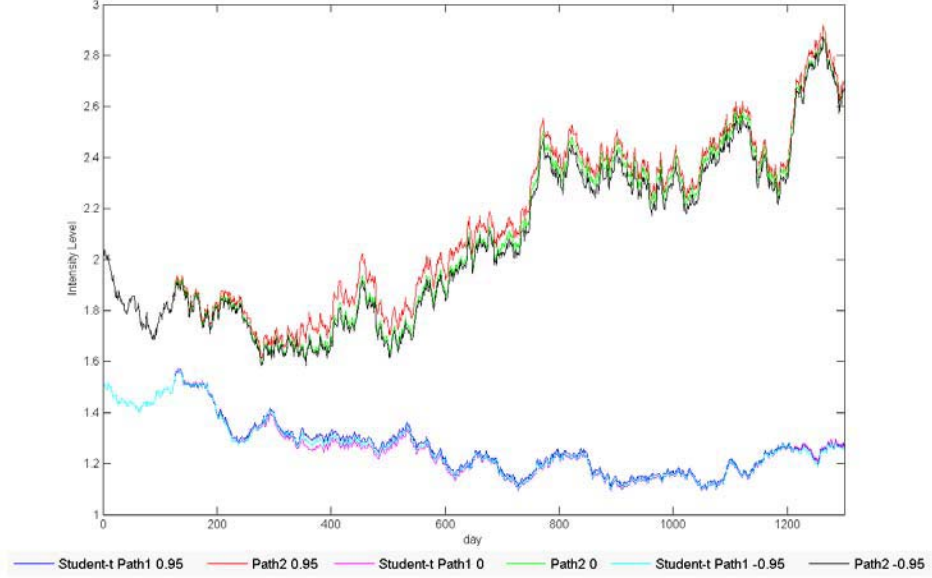


FIGURE 6. Simulated paths of jump diffusion process with dependence structure capture by student-T copula

Similarly, the expression for joint default probability represented by the following:

$$\begin{aligned} & \Pr\left(\tau^{(1)} \leq t, \dots, \tau^{(n)} \leq t \mid \lambda_0^{(1)}, \dots, \lambda_0^{(n)}\right) \\ &= \mathbb{E}\left[\left(1 - e^{-\Lambda_t^{(1)}}\right) \dots \left(1 - e^{-\Lambda_t^{(n)}}\right) \mid \lambda_0^{(1)}, \dots, \lambda_0^{(n)}\right]. \end{aligned} \quad (2.7)$$

can be obtained using equation (2.5). For that purpose, the PDMP theory developed by [7] and the martingale methodology by [6] are used.

Analogous to the univariate case in [21], the generator  $\mathcal{A}$  of the process  $(\Lambda_t^{(1)}, \dots, \Lambda_t^{(n)}, \lambda_t^{(1)}, \dots, \lambda_t^{(n)}, t)$  acting on a function  $f(\Lambda^{(1)}, \dots, \Lambda^{(n)}, \lambda^{(1)}, \dots, \lambda^{(n)}, t)$  belonging to its domain is given by

$$\begin{aligned} & \mathcal{A}f(\Lambda^{(1)}, \dots, \Lambda^{(n)}, \lambda^{(1)}, \dots, \lambda^{(n)}, t) \\ &= \frac{\partial f}{\partial t} + \sum_{i=1}^n \lambda^{(i)} \frac{\partial f}{\partial \Lambda^{(i)}} + \sum_{i=1}^n c^{(i)}(b^{(i)} + a^{(i)}\lambda^{(i)}) \frac{\partial f}{\partial \lambda^{(i)}} + \frac{1}{2} \sum_{i=1}^n \left( \sigma^{(i)} \sqrt{\lambda^{(i)}} \right)^2 \frac{\partial^2 f}{\partial \lambda^{(i)2}} \\ &+ \rho \left[ \int_0^\infty \dots \int_0^\infty f(\Lambda^{(1)}, \dots, \Lambda^{(n)}, \lambda^{(1)} + x_1, \dots, \lambda^{(n)} + x_n, t) \frac{\partial^n C(F_{X^{(1)}}(x_1), \dots, F_{X^{(n)}}(x_n))}{\partial x_1 \dots \partial x_n} \right. \\ &\quad \left. \times dx_1 \dots dx_n - f(\Lambda^{(1)}, \dots, \Lambda^{(n)}, \lambda^{(1)}, \dots, \lambda^{(n)}, t) \right] \end{aligned}$$

where  $\frac{\partial^n C(F_{X^{(1)}}(x_1), \dots, F_{X^{(n)}}(x_n))}{\partial x_1 \dots \partial x_n}$  is the joint density of event jump sizes.

For  $f(\Lambda^{(1)}, \dots, \Lambda^{(n)}, \lambda^{(1)}, \dots, \lambda^{(n)}, t)$  to belong to the domain of the generator  $\mathcal{A}$ , it is sufficient that the function  $(\Lambda^{(1)}, \dots, \Lambda^{(n)}, \lambda^{(1)}, \dots, \lambda^{(n)}, t)$  is differentiable w.r.t.  $\Lambda^{(i)}, \lambda^{(i)}, t$  for  $i = 1, \dots, n$  and that

$$\left\| \int_0^\infty \dots \int_0^\infty f(\cdot, \lambda^{(1)} + x_1, \dots, \lambda^{(n)} + x_n, \cdot) \frac{\partial^n C(F_{X^{(1)}}(x_1), \dots, F_{X^{(n)}}(x_n))}{\partial x_1 \dots \partial x_n} dx_1 \dots dx_n \right\| < \infty.$$

$$\left\| -f(\cdot, \lambda^{(1)}, \dots, \lambda^{(n)}, \cdot) \right\|$$

Now we find a suitable martingale to derive the joint Laplace transform of the vector  $(\Lambda^{(1)}, \dots, \Lambda^{(n)}, \lambda^{(1)}, \dots, \lambda^{(n)}, t)$  at time  $t$ .

**Theorem 2.1.** *Considering constant  $\gamma^{(i)} \geq 0$  and  $k^{(i)} \geq 0$ ,*

$$\begin{aligned} & \exp \left[ - \sum_{i=1}^n \left( \gamma^{(i)} \Lambda_t^{(i)} + A^{(i)}(t) \lambda_t^{(i)} + c^{(i)} b^{(i)} \int_0^t A^{(i)}(s) ds \right) \right] \\ & \times \exp \left[ \rho \int_0^t [1 - \hat{c}(A^{(1)}(s), \dots, A^{(n)}(s))] ds \right] \end{aligned}$$

is a martingale where

$$A^{(i)}(t) = \frac{[D^{(i)} + c^{(i)}a^{(i)}] + [D^{(i)} - c^{(i)}a^{(i)}] \exp\{D^{(i)}t - k^{(i)}\}}{(\sigma^{(i)})^2 (1 - \exp\{D^{(i)}t - k^{(i)}\})} \quad (2.8)$$

with

$$\begin{aligned} & \hat{c}(\zeta^{(1)}, \dots, \zeta^{(n)}) \\ &= \int_0^\infty \dots \int_0^\infty e^{-\sum_{i=1}^n \zeta^{(i)} x_i} \frac{\partial^2 C(F_{X^{(1)}}(x_1), \dots, F_{X^{(n)}}(x_n))}{\partial x_1 \dots \partial x_n} dx_1 \dots dx_n, \end{aligned}$$



and  $D^{(i)} = \sqrt{(c^{(i)}a^{(i)})^2 + 2(\sigma^{(i)})^2\gamma^{(i)}}$ .

*Proof.* The generator of the process has to satisfy  $\mathcal{A}f = 0$  for it to be a martingale. Setting  $f = e^{B(t) - \sum_{i=1}^n [\gamma^{(i)}\Lambda^{(i)} + A^{(i)}(t)\lambda^{(i)}]}$  obtains the equation

$$\begin{aligned} & - \sum_{i=1}^n \left[ \lambda^{(i)} A'^{(i)}(t) - c^{(i)} A^{(i)}(t) \left( b^{(i)} + a^{(i)} \lambda^{(i)} \right) - \lambda^{(i)} \gamma^{(i)} \right] \\ & - \frac{1}{2} \sum_{i=1}^n \left( \sigma^{(i)} \sqrt{\lambda^{(i)}} \right)^2 \frac{\partial^2 f}{\partial \lambda^{(i)2}} + B'(t) + \rho [\hat{c}(A^{(1)}(t), \dots, A^{(n)}(t)) - 1] = 0 \end{aligned}$$

and solving it results in

$$A^{(i)}(t) = \frac{(D^{(i)} + c^{(i)}a^{(i)}) + (D^{(i)} - c^{(i)}a^{(i)}) \exp(D^{(i)}t - k^{(i)})}{(\sigma^{(i)})^2 [1 - \exp(D^{(i)}t - k^{(i)})]}$$

$$\text{and } B(t) = \sum_{i=1}^n c^{(i)} b^{(i)} \int_0^t A^{(i)}(s) ds + \rho \int_0^t [1 - \hat{c}(A^{(1)}(s), \dots, A^{(n)}(s))] ds$$

$$\text{with } D^{(i)} = \sqrt{(c^{(i)}a^{(i)})^2 + 2(\sigma^{(i)})^2\gamma^{(i)}} \text{ for } i = 1, \dots, n.$$

Hence the result follows.  $\square$

Using the martingale in Theorem 2.1, we can easily obtain the joint Laplace transform of the vector  $(\Lambda^{(1)}, \dots, \Lambda^{(n)}, \lambda^{(1)}, \dots, \lambda^{(n)}, t)$  at time  $t$ .

**Corollary 2.2.** *Considering constants  $\alpha^{(i)} \geq 0$ , and  $\gamma^{(i)} \geq 0 \forall i = 1, \dots, n$  the joint Laplace transform of the vector  $(\Lambda^{(1)}, \dots, \Lambda^{(n)}, \lambda^{(1)}, \dots, \lambda^{(n)}, t)$  is given by*

$$\begin{aligned} & \mathbb{E} \left[ e^{-\sum_{i=1}^n \gamma^{(i)} \Lambda_t^{(i)} + \alpha^{(i)} \lambda_t^{(i)}} \mid \lambda_0^{(1)}, \dots, \lambda_0^{(n)} \right] \\ & = \prod_{i=1}^n \left[ H^{(i)}(t) \frac{2c^{(i)}b^{(i)}}{\sigma^{(i)2}} \right] e^{-\left( \sum_{i=1}^n G^{(i)}(t)\lambda_0^{(i)} + \rho \int_0^t [1 - \hat{c}\{G^{(1)}(s), \dots, G^{(n)}(s)\}] ds \right)} \end{aligned} \quad (2.9)$$

where  $t > 0$ , with

$$\begin{aligned} & G^{(i)}(t) \\ & = \frac{\alpha^{(i)} [(D^{(i)} + c^{(i)}a^{(i)}) + (D^{(i)} - c^{(i)}a^{(i)}) \exp(-D^{(i)}t)] + 2\gamma^{(i)}(1 - \exp\{-D^{(i)}t\})}{\sigma^{(i)2}\alpha^{(i)}[1 - \exp(-D^{(i)}t)] + (D^{(i)} - c^{(i)}a^{(i)}) + [D^{(i)} + c^{(i)}a^{(i)}] \exp(-D^{(i)}t)} \end{aligned}$$

and

$$\begin{aligned} & H^{(i)}(t) \\ & = \frac{2D^{(i)} \exp\left[-\frac{D^{(i)} + c^{(i)}a^{(i)}}{2}t\right]}{\sigma^{(i)2}\alpha^{(i)}[1 - \exp(-D^{(i)}t)] + (D^{(i)} - c^{(i)}a^{(i)}) + [D^{(i)} + c^{(i)}a^{(i)}] \exp(-D^{(i)}t)} \end{aligned}$$

*Proof.* Set  $A^{(i)}(T) = \alpha^{(i)}$  for  $i = 1, 2, \dots, n$  using (2.8) where  $t < T$ , then we have

$$k^{(i)} = D^{(i)}T - \ln \left[ \frac{c^{(i)}a^{(i)} + D^{(i)} - \alpha^{(i)}\sigma^{(i)2}}{c^{(i)}a^{(i)} - D^{(i)} - \alpha^{(i)}\sigma^{(i)2}} \right]. \quad (2.10)$$

Substitute (2.10) into (2.8) and the martingale in Theorem 2.1, the result follows immediately.  $\square$

**Corollary 2.3.** *The joint Laplace transform of the vector  $(\Lambda^{(1)}, \dots, \Lambda^{(n)}, t)$  is given by*

$$\begin{aligned} & \mathbb{E} \left[ e^{-\sum_{i=1}^n \gamma^{(i)} \Lambda_t^{(i)}} \mid \lambda_0^{(1)}, \dots, \lambda_0^{(n)} \right] \\ &= \exp \left[ -\sum_{i=1}^n G^{(i)}(t) \lambda_0^{(i)} \right] \times \prod_{i=1}^n \left[ H^{(i)}(t) \right]^{\frac{2c^{(i)}b^{(i)}}{\sigma^{(i)2}}} \\ & \quad \times \exp \left[ -\rho \int_0^t \left[ 1 - \hat{c} \left\{ G^{(1)}(s), \dots, G^{(n)}(s) \right\} \right] ds \right] \end{aligned} \quad (2.11)$$

*Proof.* Equation (2.11) follows immediately if we set  $\alpha^{(i)} = 0 \forall i = 1, \dots, n$  in equation (2.9).  $\square$

Using Corollary 2.3, we can easily derive the joint survival/default probability and other relevant joint probabilities. While FGM copula admits a simple analytical expression, the same can not be said for Gaussian and Student-t copulas. Hence, we evaluate the probabilities numerically by replacing the suitable copula formulae in the third component of (2.11). Due to the dependence of simultaneous event jumps of  $X^{(i)}$ 's with sharing event jump frequency rate  $\rho$ , we have that

$$\mathbb{E} \left[ e^{-\sum_{i=1}^n \Lambda_t^{(i)}} \right] \neq \mathbb{E} \left[ e^{-\Lambda_t^{(1)}} \right] \mathbb{E} \left[ e^{-\Lambda_t^{(2)}} \right] \dots \mathbb{E} \left[ e^{-\Lambda_t^{(n)}} \right].$$

If the event jump  $X^{(i)}$  for  $i = 1, 2, \dots, n$  occurs by a Poisson process  $M_t^{(i)}$  with its frequency rate  $\rho^{(i)}$  respectively and everything else is independent of each other, we have the joint survival probability of firm  $i = 1, 2, \dots, n$  at time  $t$ , which is the product of each marginal survival probability.

**2.2. Numerical Examples.** In this section, we use the results obtained in the previous section to calculate survival/default probabilities and relevant joint probabilities. We assume bivariate dependence structure and a 1-year period ( $t_1 = 0, t_2 = 1$ ) for the simplicity of computation. We also assume constant risk free rate,  $r = 0.023$  and average annual event occurrence  $\rho = 4$  per year. The following table summarizes the parameter values chosen:

In this example, Firm 1 is relatively more robust in terms of shock absorption than its counterpart, Firm 2. The strength of Firm 1 is characterized by a higher decay rate, a lower diffusion parameter, lower initial default intensity as well as higher jump size parameter (hence lower average jump size) as opposed to Firm 2.

TABLE 1. Parameter values for the intensity process in the hypothetical example

Firms	$c^{(i)}$	$a^{(i)}$	$b^{(i)}$	$\sigma^{(i)}$	$\mu^{(i)}$	$\rho^{(i)}$	$\lambda_0^{(i)}$
Firm 1	0.5	-1	0	0.025	20	4	0.04
Firm 2	0.05	-1	0	0.25	2	4	0.4

From the equations (2.6), (2.7) and relevant probabilities that accounts for the survival of each Firm 1 and Firm 2, given by

$$\begin{aligned} & \Pr(\tau^{(1)} > t, \tau^{(2)} < t \mid \lambda_0^{(1)}, \lambda_0^{(2)}) \\ &= \mathbb{E} \left[ (1 - e^{-\Lambda_t^{(2)}}) e^{-\Lambda_t^{(1)}} \mid \lambda_0^{(1)}, \dots, \lambda_0^{(n)} \right], \end{aligned} \quad (2.12)$$

and

$$\begin{aligned} & \Pr(\tau^{(1)} \leq t, \tau^{(2)} \geq t \mid \lambda_0^{(1)}, \lambda_0^{(2)}) \\ &= \mathbb{E} \left[ (1 - e^{-\Lambda_t^{(1)}}) e^{-\Lambda_t^{(2)}} \mid \lambda_0^{(1)}, \dots, \lambda_0^{(n)} \right], \end{aligned} \quad (2.13)$$

the calculations of the joint survival/default probabilities and relevant joint probabilities are shown in Table 3 and 4. The individual survival and default probabilities calculated for Firm 1 and Firm 2 are shown in Table 2.

TABLE 2. Individual survival and default probabilities.

	FGM	Gaussian	Student-t
$\Pr(\tau^{(1)} > 1)$	0.891870	0.891870	0.849264
$\Pr(\tau^{(1)} \leq 1)$	0.108130	0.108130	0.150736
$\Pr(\tau^{(2)} > 1)$	0.322700	0.322700	0.294917
$\Pr(\tau^{(2)} \leq 1)$	0.677300	0.677300	0.705083

While the individual survival and default probability under the Gaussian and FGM copulas are equal, those probabilities in Table 2 under the Student-t copula are different as dependent parameter value  $\theta = 0$  does not imply the case of independence, in line with [33]. We also found that the Student-t copula returns lower survival probability values and higher default probability values as opposed to its FGM and Gaussian counterparts by 5%. In comparison with the other 2 copulas, the default probability for Firm 2 (the weaker firm) is also greater under Student-t copula, suggesting that dependence structure under a Student-t copula could be a good candidate to depict a riskier environment.

Since Firm 1 is relatively stronger than Firm 2, the individual survival probability of Firm 1 is higher than its counterpart under all copula considered (see Table 2) with Student-t copula

TABLE 3. Joint survival and default probabilities.

$\Pr(\tau^{(1)} > 1, \tau^{(2)} > 1)$			$\Pr(\tau^{(1)} \leq 1, \tau^{(2)} \leq 1)$				
$\theta$	FGM	Gaussian	Student-t	$\theta$	FGM	Gaussian	Student-t
-0.95	0.292334	0.290216	0.260738	-0.95	0.077763	0.075646	0.116557
-0.9	0.292393	0.290359	0.260791	-0.9	0.077823	0.075788	0.116611
-0.5	0.292872	0.291711	0.261970	-0.5	0.078032	0.077140	0.117790
0	0.293472	0.293472	0.264586	0	0.078901	0.078901	0.120405
0.5	0.294072	0.295459	0.268433	0.5	0.079502	0.080889	0.124252
0.9	0.294554	0.297207	0.272726	0.9	0.079983	0.082636	0.128546
0.95	0.294614	0.297311	0.273423	0.95	0.080044	0.082740	0.129243

TABLE 4. Other relevant joint probabilities.

$\Pr(\tau^{(1)} > 1, \tau^{(2)} < 1)$			$\Pr(\tau^{(1)} < 1, \tau^{(2)} > 1)$				
$\theta$	FGM	Gaussian	Student-t	$\theta$	FGM	Gaussian	Student-t
-0.95	0.599536	0.601654	0.588526	-0.95	0.030367	0.032484	0.034179
-0.9	0.599477	0.601511	0.588472	-0.9	0.030307	0.032342	0.034125
-0.5	0.598998	0.600159	0.587293	-0.5	0.029828	0.030999	0.032946
0	0.598398	0.598398	0.584678	0	0.029229	0.029229	0.030331
0.5	0.597798	0.596411	0.580831	0.5	0.028628	0.027241	0.026484
0.9	0.597316	0.594663	0.576537	0.9	0.028147	0.025494	0.022190
0.95	0.597256	0.594560	0.575841	0.95	0.028086	0.025390	0.021494

giving the lowest value, (approximately 0.85) whereas the FGM and Gaussian copula return almost 0.90 probability of Firm 1 surviving after 1 year. Hence the joint probabilities given in the FGM and Gaussian columns of Table 3 and 4 where the survivorship of Firm 2 is concerned, approach the individual survival / default probabilities of Firm 2, which are approximately 0.3 and 0.7 as given in Table 2, respectively.

With a low individual default probability within 1 year of Firm 1 under each copula, the joint defaultability of both firms also approaches Firm 1's individual default probability. Combined with the low individual survival probability of Firm 2 within 1 year, the probability that Firm 2 would survive after 1 year with Firm 1 defaulting within the same period, is very low (between 0.02 and 0.03) under each copula.

The results in Table 3 and 4 also demonstrate that the FGM, Gaussian and Student-t copulas show the same pattern, i.e. either increasing or decreasing as the dependence structure represented by parameter  $\theta$  progress from negative to positive. We also note that the spread (i.e. the difference between probabilities corresponding to -0.95 and 0.95) is the widest under the

Student-t copula (126.8511 bps), followed by Gaussian copula (70.9428 bps) and FGM copula (22.8044 bps).

Table 3 shows that joint survival and default probability decrease as the value of copula parameter  $\theta$  moves from -0.95 to 0.95 as time to default for each firm moves in the same direction. Thus, when  $\theta = -0.95$ , we can consider applying the results to calculate joint survival and default probability for the firms in the substitute industry/sector. For example when  $\theta = -0.95$ , consider that Firm 1 produces cars run by petrol and Firm 2 produces cars run by battery. If the oil price surges due to an external event affecting the car manufacturing industry, consumers are likely to begin changing their petrol-run cars to battery-run cars.

In contrast, Table 4 show that joint probabilities increase as the value of copula parameter  $\theta$  becomes  $-0.95$  (or nearly -1) as time to default for each firm moves in the opposite direction. Hence when  $\theta = 0.95$  (or nearly 1) we can consider applying the results to calculate joint survival and default probability for the firms in the complementary industry/sector - for instance, Firm 1 being an air-liner and Firm 2 being a chain hotel. An occurrence of a catastrophic event such as the September 11 World Trade Centre attacks or the disappearance of Malaysia Airlines flight MH370 may cause consumers to travel less via air and subsequently causing hotel booking rates to fall.

When comparing joint default probability between complementary industries and substitute industries, it was found that the joint default probability of firms in complementary industries was higher than its counterpart in substitute industries, which is economically intuitive (see  $\Pr(\tau^{(1)} \leq 1, \tau^{(2)} \leq 1)$  in Table 3). When comparing the joint survival probability between complementary industries and substitute industries, we also found that the joint survival probability of firms in complementary industries was higher than its counterpart in substitute industries, which is also economically intuitive (see  $\Pr(\tau^{(1)} > 1, \tau^{(2)} > 1)$  in Table 3). The relevant joint probabilities of the firms in substitute industries are higher than their counterparts in complementary industries because it is more likely that one firm will fail (or survive) when the other firm survives (or fails) if they are in substitute industries (see Table 4).

### 3. APPLICATIONS

**3.1. CDS Pricing Under Counterparty Risk.** This section applies the results in Section 2 to the pricing of a financial product. For this purpose, the instrument credit default swaps (CDS) is chosen as there are three parties involved in this contract - a reference credit, a CDS buyer and a CDS seller. Note that the dependence is assumed only between the seller and the reference credit and that the buyer is assumed to be default free.

In calculating the CDS rate, we assume that the deterministic instantaneous rate of interest  $r = 0.0023$  for a zero-coupon default-free bond. Then its price at time 0, paying 1 at time  $t$  is given by  $B(0, t) = e^{-rt}$ .

We denote the default intensity process of the CDS buyer, seller and reference credit by  $\lambda_t^{(b)}$ ,  $\lambda_t^{(s)}$  and  $\lambda_t^{(RC)}$  respectively. The CDS rate formula, denoted by  $\bar{s}$ , as adopted from [34] is given

by

$$\bar{s} = (1 - \pi) \frac{\sum_{k=1}^{k_N} e^{rc,s}(0, t_{k-1}, t_k)}{\sum_{n=1}^N (t_{k_n} - t_{k_{n-1}}) \bar{B}^b(0, t_{k_n})} \quad (3.1)$$

where

$$\begin{aligned} & e^{rc,s}(0, t_{k-1}, t_k) \\ = & \mathbb{E} \left[ \exp \left( - \int_0^{t_k} r_s ds \right) \left[ \exp \left( - \int_0^{t_{k-1}} \lambda_s^{(RC)} ds \right) - \exp \left( - \int_0^{t_k} \lambda_s^{(RC)} ds \right) \right] \right. \\ & \left. \times \left[ \exp \left( - \int_0^{t_k} \lambda_s^{(s)} ds \right) \right] \middle| r_0, \lambda_0^{(RC)}, \lambda_0^{(s)} \right], \\ \bar{B}^b(0, t_{k_n}) = & \mathbb{E} \left[ \exp \left\{ - \int_0^{t_{k_n}} (r_s + \lambda_s^{(b)}) ds \right\} \middle| r_0, \lambda_0^{(b)} \right] \end{aligned} \quad (3.2)$$

and  $t_{k_1} < t_{k_2} < \dots < t_{k_n}$ .

We assume that  $r_t$  and  $\lambda_t^{(i)}$  are independent of each other and the recovery rate is deterministic. To keep the calculation simple, we use the case of 1-year CDS contract with premium paid by the buyer every 6 months, i.e.  $N = 2$ ,  $t_0 = 0$ ,  $t_{k_1} = 0.5$ , and  $t_{k_2} = 1$ , as well as recovery rate  $\pi$ . We may also use equation (3.2) to price defaultable bonds as well as credit spread between default-free bond and defaultable bond.

Assuming recovery rate  $\pi = 0.5$  with the parameter values used in section 2.2, the parameter values for the intensity process of the CDS counterparties are shown in Table 5 and the CDS rate values are shown in Table 6.

TABLE 5. Parameter values for the intensity process of the CDS counterparties

Counterparty	$c^{(i)}$	$a^{(i)}$	$b^{(i)}$	$\sigma^{(i)}$	$\mu^{(i)}$	Jump frequency
CDS Seller	0.5	-1	0	0.025	10	4
Reference Credit	0.05	-1	0	0.25	2	4
CDS Buyer	0.2	-1	0	0.1	7	3

As opposed to the elliptical copulas, the CDS rates under FGM copula do not show much difference as the dependence parameter varies from negative to positive dependence. This is shown by the value of spread of only 13.2196 bps (given by  $0.347543 - 0.346221$ ) as compared to the Gaussian copula (41.0831 bps) and Student-t copula (76.3648 bps). We also note that the CDS rates show a decreasing pattern under all copulas considered as  $\theta$  varies from negative correlation to positive correlation, which is a similar pattern to that seen in the survival probabilities (see Figure 7).

TABLE 6. CDS rates computed under various copulas dependence structure.

$\theta$	FGM	Gaussian	Student-t
-0.95	0.347543	0.348826	0.354295
-0.9	0.347509	0.348744	0.354262
-0.5	0.347231	0.347905	0.353553
0	0.346884	0.346884	0.351978
0.5	0.346535	0.345732	0.349662
0.9	0.346256	0.344721	0.347077
0.95	0.346221	0.344718	0.346658
Spread (bps)	13.2196	41.0831	76.3648

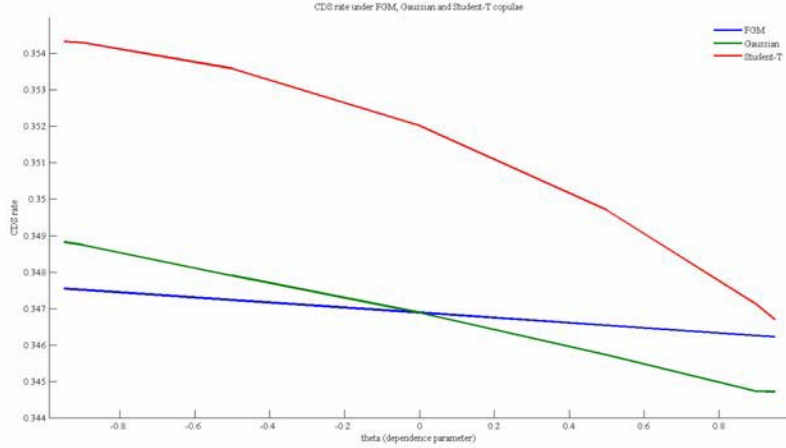


FIGURE 7. CDS rates under FGM, Gaussian and Student-t copulas.

**3.2. CDS rates calculation: Sensitivity analysis.** In this section, we conduct sensitivity analysis of CDS rates with respect to the seller's and reference credit's jump size rate, frequency rate, diffusion rate, decay rate and the reversion level. Since the patterns of CDS rates sensitivity analysis are the same under all copulas, only the findings under Student-t copula are presented here and we refer the readers to Appendix A for the rest of findings under the Gaussian and FGM copulas.

As shown in Figure 8 and Figure 11, the CDS rate is converging to 0 as the values of  $\mu^{(RC)}$  and  $c^{(RC)}$  are increased. In contrast, CDS rate also converge to 0 as the value of  $\mu^{(s)}$  and  $c^{(s)}$  are decreased for the CDS seller. These findings are similar to that of [28] in which

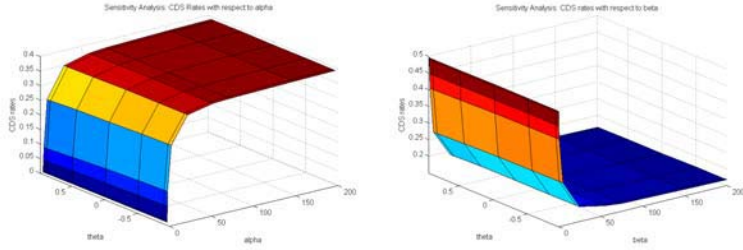


FIGURE 8. Sensitivity of CDS rates under Student-t copula with respect to seller's (left) and RC's (right) jump sizes,  $\mu^{(s)}$  and  $\mu^{(RC)}$  respectively.

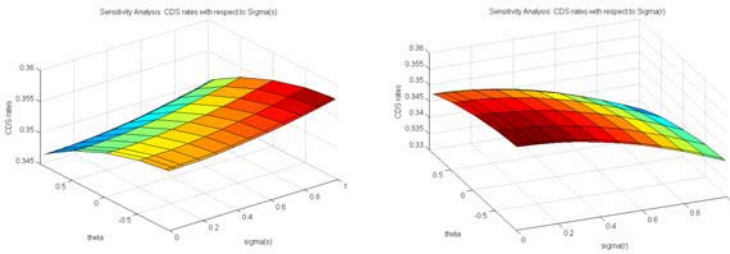


FIGURE 9. Sensitivity of CDS rates under Student-t copula with respect to seller's and RC's diffusion rates, i.e.  $\sigma^{(s)}$  and  $\sigma^{(r)}$  respectively.

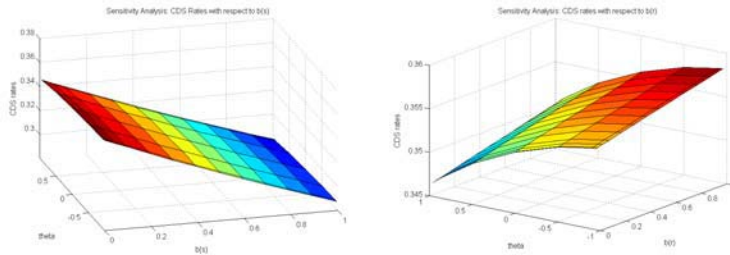


FIGURE 10. Sensitivity of CDS rates under Student-t copula with respect to the constant reversion level of seller,  $b^{(s)}$ , and RC  $b^{(RC)}$ , with  $c^{(s)} = c^{(RC)} = 1$  and  $a^{(s)} = a^{(RC)} = -1$ .

increasing the value of the jump size and decay rate parameter,  $c^{(i)}$  for  $i = s, rc$  will result in a monotonically increasing value of CDS rates (for changes in  $\mu^{(s)}$  and  $c^{(s)}$ ) and decreasing (for changes in  $\mu^{(RC)}$  and  $c^{(RC)}$ ). Intuitively, from the CDS buyers' point of view, a CDS contract



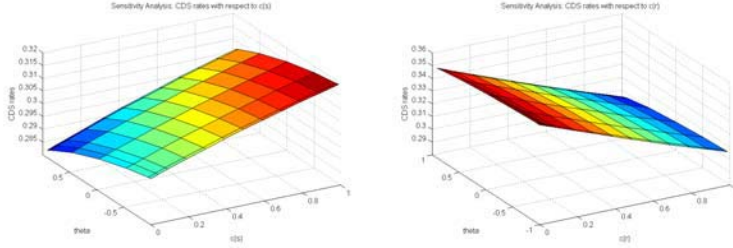


FIGURE 11. Sensitivity of CDS rates under Student-t copula with respect to seller’s and RC’s decay rate,  $c^{(s)}$  and  $c^{(r)}$  respectively, where  $b^{(s)} = b^{(RC)} = 1$  and  $a^{(s)} = a^{(RC)} = -1$ .

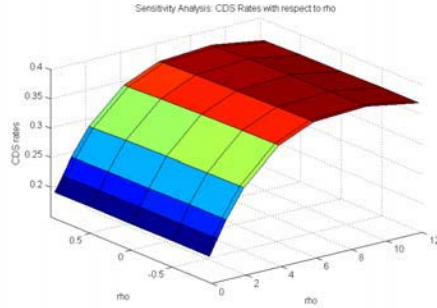


FIGURE 12. Sensitivity of CDS rates under Student-t copula with respect to frequency of yearly jump events,  $\rho$ .

is more attractive when the CDS seller is less likely to default. As long as the CDS seller’s credit is strong enough, they can hedge against the default risk of the reference credit using a CDS contract. Hence the lower the CDS rate, the more likely the CDS seller defaults. The worst case scenario for the CDS buyer is when both the reference credit and the CDS seller default.

Figure 9 shows a decreasing CDS rates as we increase the value of  $\sigma^{(RC)}$ , as well as an increasing CDS rates as we increase the value of  $\sigma^{(s)}$ . Intuitively, an increasing values of reference credit’s diffusion rate  $\sigma^{(RC)}$  will reduce the CDS rates because the CDS contract is deemed as less safe since the defaultability of the reference credit becomes more certain, thereby reducing the survival probability of the reference credit, as can be seen in equation (3.2). In contrast, while it is slightly difficult to see the intuition behind the increasing CDS rates as we increase the seller’s diffusion rate  $\sigma^{(s)}$ , closely examining the numerator of

CDS rate (equation (3.2)) easily verifies that the changes in numerator moves in upward direction as we increase seller's diffusion rate  $\sigma^{(s)}$ , bearing in mind that

$$\mathbb{E} \left[ \exp \left( - \int_0^{t_k} \lambda_s^{(s)} ds \right) \middle| r_0, \lambda_0^{(RC)}, \lambda_0^{(s)} \right]$$

have the same form as the default free bond price, as presented in Table 4.3 of [21], which increased as  $\sigma$  increased.

We also found that the CDS rates show a monotonically increasing and decreasing behaviours with respect to changes in  $\sigma^{(s)}$  and  $\sigma^{(RC)}$  respectively. These are different from the findings shown in Section 4 of [28] which presented a graph showing instability in the values of the CDS rates resulting from the changes in the two parameters.

Comparing to other parameters of each counterparty, the constant reversion level parameters  $b^{(s)}$  and  $b^{(RC)}$  give an opposite direction of changes in the CDS rates, as in Figure 10. Even though the default threshold level will be discernible only after default occurs, higher  $b^{(s)}$  and  $b^{(RC)}$  implies that the default is more likely to happen. Therefore, assuming that the seller has strong credibility, higher  $b^{(RC)}$  allows the seller to demand the buyer to pay higher premium for the CDS contract as the default event is likely to happen. This is parallel to the justification of insurers demanding higher premium from smokers for a life insurance contract as opposed to a non-smoker. On the other hand, higher  $b^{(s)}$  implies that the seller is likely to default. Hence, the CDS rates decrease since reduced credibility of the seller will make the CDS contract less attractive and induce the CDS buyer to obtain the protection from another seller.

By changing the values of the event jump frequency,  $\rho$ , we notice that the value of the CDS rates will increase up to a certain threshold level under all copula, and decrease thereafter, as seen in Figure 12. This implies that while initially the seller was able to withstand the default risk of the reference credit, its ability to absorb that risk declines as the event jumps occur more frequently. This is not examined extensively in the section 4 of [28] where they presented a table showing an increasing values of the CDS rates up to  $\rho = 3.9$  only. When the jump occurrence is too frequent to the extent that it affects both the CDS seller and reference credit, there is an increasing chance of both counterparties going bust. As aforementioned, this would be the worst scenario for the CDS buyers and would subsequently make the CDS rates less valuable from the buyers' perspective.

#### 4. CONCLUSION

For default intensities, we used the multivariate jump diffusion process in which jumps (i.e. the magnitude of contribution of primary events to default intensities) occur simultaneously and their sizes are dependent. To count simultaneous event jumps in default intensities, a homogeneous Poisson process was used, while to model the dependence structure between event jump sizes, the FGM copula, Gaussian copula and Student-t copula were used, together with exponential margins. Simulated paths of the jump diffusion intensity processes under the three copulas were also presented with various dependence parameter values,  $\theta$ .

By applying copula-dependent default intensity to the multivariate Cox process, joint survival/default probability and other relevant joint probabilities were derived via the joint Laplace transforms for which the PDMP theory and standard martingale methodology were used. We then showed an example to calculate joint survival/default probability, with an application to CDS rate considering counterparty risk. We also conduct sensitivity analyses with respect to the parameter values involved.

In this study, the multivariate jump diffusion process examined was used to model counterparty risk in CDS rates. This process also has the potential to be applicable to a variety of problems where multiple transition rates are involved in the realms of economics, finance and insurance, which could be the object of further research.

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## APPENDIX A. CDS RATES SENSITIVITY ANALYSIS

### A.1. FGM Copula.

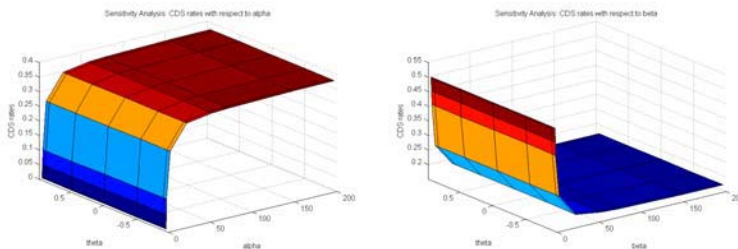


FIGURE 13. Sensitivity of CDS rates under FGM copula with respect to seller's and RC's jump size ( $\alpha$  and  $\beta$  respectively)

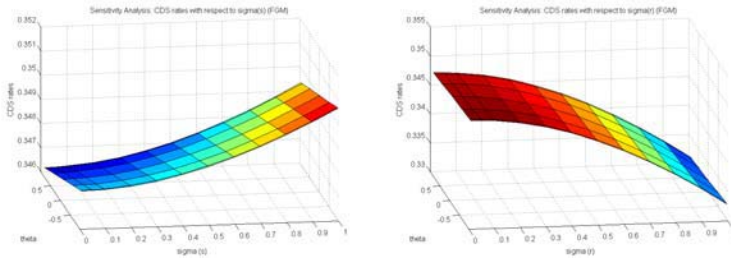


FIGURE 14. Sensitivity of CDS rates under FGM copula with respect to seller's and RC's diffusion rates ( $\sigma^{(s)}$  and  $\sigma^{(r)}$  respectively)

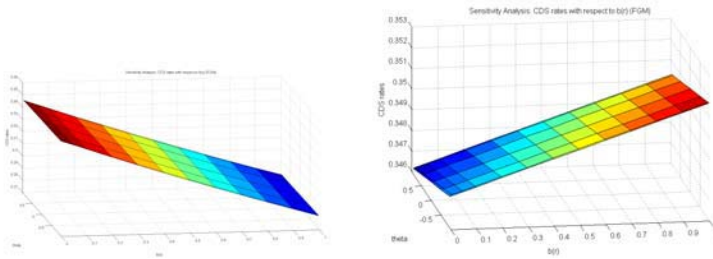


FIGURE 15. Sensitivity of CDS rates under FGM copula with respect to seller's and RC's long term mean ( $b^{(s)}$  and  $b^{(r)}$  respectively)

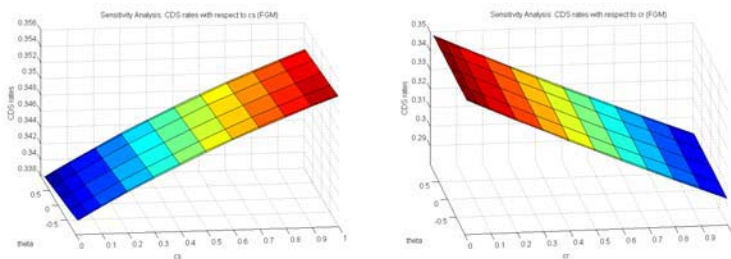


FIGURE 16. Sensitivity of CDS rates under FGM copula with respect to seller's and RC's decay rate ( $c^{(s)}$  and  $c^{(r)}$  respectively)

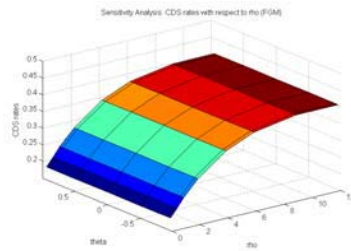


FIGURE 17. Sensitivity of CDS rates under FGM copula with respect to frequency of yearly jump events,  $\rho$

### A.2. Gaussian Copula.

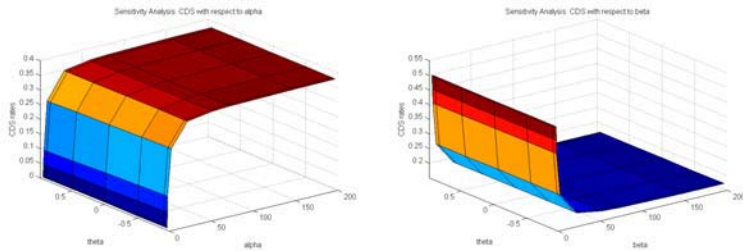


FIGURE 18. Sensitivity of CDS rates under Gaussian copula with respect to seller's and RC's jump size ( $\alpha$  and  $\beta$  respectively)

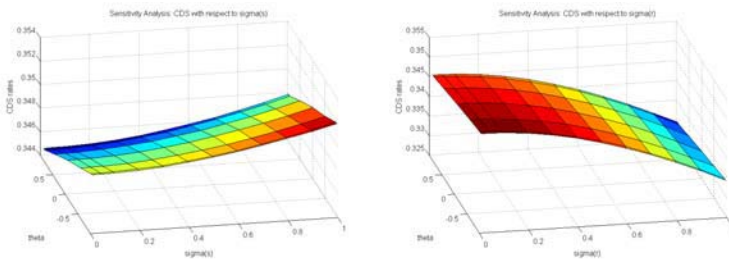


FIGURE 19. Sensitivity of CDS rates under Gaussian copula with respect to seller's and RC's diffusion rates ( $\sigma^{(s)}$  and  $\sigma^{(r)}$  respectively)

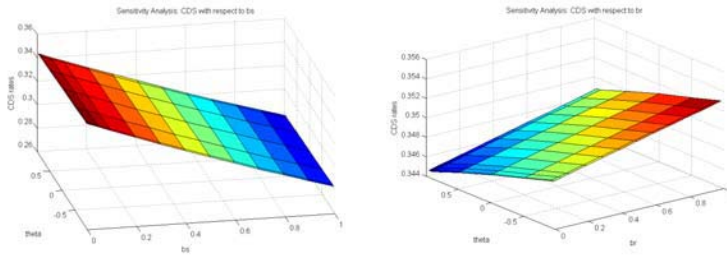


FIGURE 20. Sensitivity of CDS rates under Gaussian copula with respect to seller's and RC's long term mean ( $b^{(s)}$  and  $b^{(r)}$  respectively)

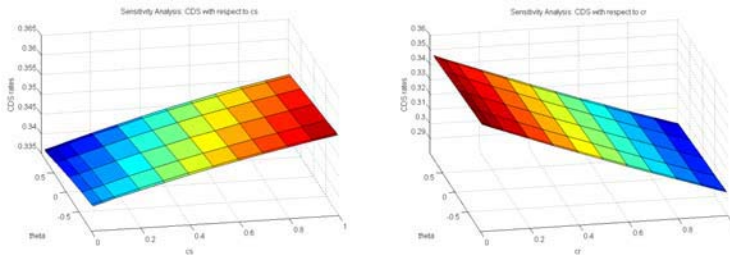


FIGURE 21. Sensitivity of CDS rates under Gaussian copula with respect to seller's and RC's decay rate ( $c^{(s)}$  and  $c^{(r)}$  respectively)

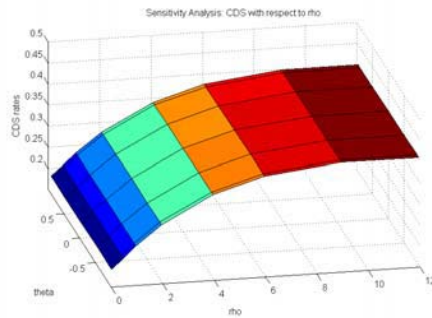


FIGURE 22. Sensitivity of CDS rates under Gaussian copula with respect to frequency of yearly jump events, ( $\rho$ )